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A SURVEY OF POLARIZATION ASYMMETRICS PREDICTED BY QCD

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E R R A T A

Page 12 - The relationship in equation 3.4 between $G_1 + G_2$ and $\delta_2 D_N^{\,i}$, which follows from the naive parton model with $m_q = M_p$ Ref. 3, does not reproduce the OPE result. However the latter approach also appears to be in a completely unsatisfactory state (see references 37,38,39), since the result depends heavily on the renormalization scheme. For example it is claimed in Ref. 38 that in the Calvo on mass shell renormalization scheme [40], an additional twist 3 operator, which is proportional to the quark mass, appears and mixes with the main twist 2 operators. Our impression is that the issue needs further study. In any case, the measurement of transverse polarization will be a very valuable means of giving us some insight into this question. Naively it may tell us something about the role of quark masses in QCD perturbation theory.

Page 24 - Equation 3.29 should read

$$A_{LL}^{ii}\,(q\;g) = \frac{\hat{s}^2 - \hat{t}^2}{\hat{s}^2 + \hat{t}^2}$$

and in Eq. 3.30 the factors $\hat{u}/\hat{s}$ and $\hat{t}/\hat{s}$ in the numerator should be replaced by $(\hat{s}^2 - \hat{t}^2)/-\hat{t}\,\hat{s}$ and $(\hat{s}^2 - \hat{u}^2)/-\hat{u}\,\hat{s}$ respectively.

Page 43 - The second paragraph should read:

The asymmetry in prompt photon production in proton-proton collisions is determined by the subprocess $g\,q \longrightarrow g\,q$, which has an asymmetry $A_{LL}^{ii} \sim .6$ or $\theta_{CMS} = 90^0$. However we have no model of gluon polarization, which is much open to question. We can write it as a rough guide at $\theta_{CM} = 90^0$

$$A_{LL}^{ii}\,(pp \longrightarrow \gamma X) = .6\ \frac{\Delta_2 D_p^{\,q}}{D_p^{\,q}}\ \frac{\Delta_2 D_p^{\,g}}{D_p^{\,g}} \sim .24\ \frac{\Delta_2 D_p^{\,g}}{D_p^{\,g}}$$

Page 46 - On table 4.2 $\Delta g(x) = 0$ and not that given in Ref. 2.

Fig. 14 - should be replaced by the following curve.

In Table 3.1 - $f(h_1, h_2) = 1 - \delta_{h_1, h_2}$ $\qquad g(h_1, h_2) = \frac{1}{2}(h_1 - h_2)$

$h(h_1, h_2) = \delta_{h_1, h_2}$ $\qquad k(h_1, h_2) = \frac{1}{2}[h_1 + h_2]$

Additional References (37,38,39):

37. M.A. Ahmad and G.G. Ross, Phys. Letters B56, 385 (1975); Nucl. Phys. B111, 1141 (1976).
38. J. Kodaira, S. Matsuda, K. Sasaki and T. Uematsu, Nucl. Phys. B159, 99 (1979).
39. Antoniadas and Kounnas, preprint Ecole Polytechnique 1980.
40. M. Calvo, Phys. Rev. D15, 730 (1977).

Note Added:

We wish to thank K. Hidaka for pointing out some of these erratum and we would like to refer to his study of prompt photons with polarized beams and targets. Ref., Westfield College London, preprint 1980.

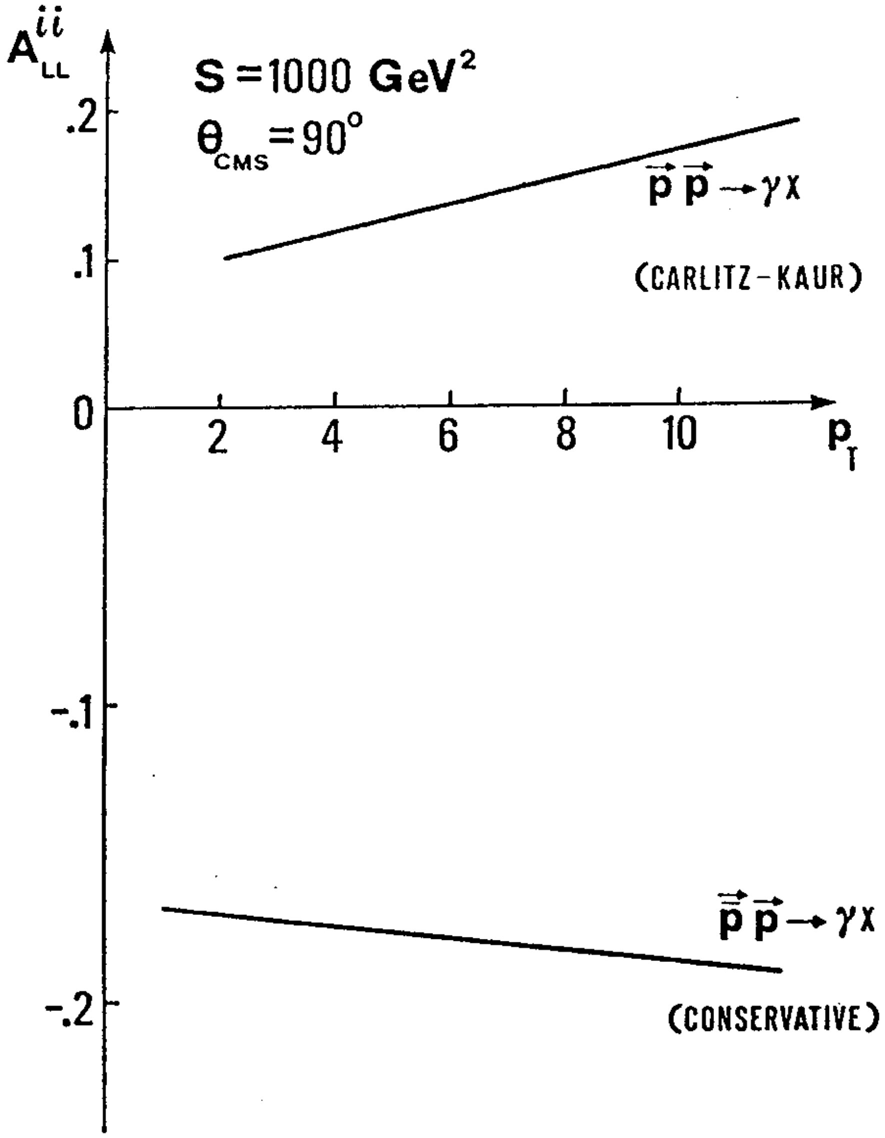
A_{LL}^{ii}
S = 1000 GeV²
θ_CMS = 90°
.2
.1
0
-.1
-.2
2
4
6
8
10
p_T
$\vec{p}\,\vec{p} \to \gamma X$
(CARLITZ-KAUR)
$\vec{\bar{p}}\,\vec{p} \to \gamma X$
(CONSERVATIVE)

FIG 14

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International Atomic Energy Agency
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United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A SURVEY OF POLARIZATION ASYMMETRIES PREDICTED BY QCD *

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ABSTRACT

We give a comprehensive review of the polarization asymmetries predicted by QCD and the parton model for hard processes. The article discusses the interrelation between these processes which include deep inelastic inclusive, massive lepton pairs, reflected and transmitted asymmetries in hadron production at large p_T, prompt photon production, etc. We also discuss the validity of the formulae derived and the potential effects of non-leading logarithmic and power orders.

I. INTRODUCTION

Recently there has been a considerable interest in high energy and deep inelastic experiments with polarized beams and targets, since this provides a valuable means of testing the parton model and QCD. In particular, these experiments will provide a very valuable way of studying factorization beyond the leading logarithmic and power orders. In this article we survey the large number of interrelatedprocesses, which admit a parton model interpretation and for which cross-sections and polarization asymmetries are calculable in the leading orders of QCD. We also provide estimates of the expected sizes of the various asymmetries in the leading order in QCD, however one must stress that much of the basic input needed in such estimates is not yet available, because this class of polarization experiments has barely begun [1]. This article complements and extends the work of Sivers and collaborators [2], who discussed mainly what we shall call reflected (beam-target) asymmetries in large p_T inclusive experiments. We shall discuss how transmitted asymmetries may also be measured with a polarized beam or target facility. The other processes analysed in this article include deep inelastic electron or muon and neutrino scattering with polarized targets and the various asymmetries in massive muon pair production and prompt photons at large p_T.

The article is divided up as follows. Sec.II is devoted to the notion of factorization and how the basic asymmetries are derived in any short distance (i.e. hard) process, which admits a parton model interpretation and by the same token, perturbative QCD thought to be applicable. In Sec.III we give the basic formulae for the asymmetries for the various processes listed in the following list of contents:

Sec.IV will be devoted to estimates of the asymmetries given in Sec.III, while finally in Sec.V we shall discuss the rôle of the next to leading orders in QCD (both logarithmic and powers).

Here we try to emphasise the importance of polarization experiments in giving us an insight into many aspects of QCD not yet understood properly. The totality of the experiments discussed here will be a powerful means of testing the basic premises of the parton model and the application of perturbative QCD. We however cannot go into the feasibility of each of the experiments discussed, except we might add as a theoretician one never ceases to be amazed at the ability of our experimental colleagues to measure even the most unlikely of processes, thus often providing a crucial new valuable insight.

II. GENERAL PARTON SPIN FORMALISM AND FACTORIZATION IN THE LEADING ORDER IN QCD

In this section we shall set up our procedure with which spin asymmetries associated with processes admitting a parton model (PM) interpretation are defined in the framework of quantum chromodynamics (QCD) in the leading logarithmic regime (LLR). Before doing that, however, we shall discuss briefly the important concept of factorization which is at the base of all our next developments.

In the naive PM [3] when considering processes involving hadrons A,B,... and leptons ℓ, ℓ' in the kinematical region in which all the relevant observable variables (energies, momentum transfers) are large enough, one writes the physical cross-section as a convolution of distribution and fragmentation functions (which measure probabilities of finding partons in hadrons and hadrons as products of parton decays with appropriate fractions of longitudinal momenta, respectively) with cross-sections involving partons and the gauge particles associated with the electroweak interactions. That is, one obtains a complete factorized probabilistic picture.

As is well known the previous scheme still goes through in the LLR of QCD and this is shown for example in Refs.4 and 5, where non-covariant gauge techniques are used. We are led to a picture in which physical cross-sections are calculable again as convolutions, but now in terms of parton densities which depend on the frequency scale at which the hard process occurs and with the parton cross-sections evaluated to lowest order of QCD perturbation theory. Indeed, as a brief sketch of the arguments involved in the factorization proof, consider a process involving at least the two quark-hadron channels (a,A) and (b,B) in which $Q^2 \equiv p_A \cdot p_B$ defines the large mass scale (Fig.1(a)). Diagrams involving gluons which are exchanged between different parton-hadron channels (see Fig.1(b)) will in general not lead to factorization of parton

probabilities and fragmentation functions because such interference terms correspond to the kind of coherent effects one normally excludes in the naive parton model description. In a covariant gauge one would expect such interference terms to be overwhelmingly important, however factorization still emerges because one can choose a non-covariant gauge ($n \cdot A = 0$, where the vector $n = p_A + p_B$), for which such cross diagrams give zero contribution in the leading order in QCD. To see how this works, one makes a Sudakov decomposition of the momentum in each channel according to $k_a = x_a p_A + y_a n + k_{aT}$ and $k_b = x_b p_B + y_b n + k_{bT}$ and similarly for the primed quantities. In the region of integration responsible for the leading logarithmic behaviour (LLR) ($x_a, x_b, x_a', x_b' \sim 1$, $y_a, y_b, y_a', y_b' << 1$ and $k_T^2 << 2\, p_A \cdot p_B$), the vertex $\not{k}_a \gamma^\mu \not{k}_a'$ can be replaced by $2p_A^\mu \not{k}_a$ and the gluon insertion leads to the factor $[p_A^\mu p_B^\nu D_{\mu\nu}(k,n) + O(y, k_T^2/Q^2)]$, where

$$D_{\mu\nu}(k,\eta) = \left[-g_{\mu\nu} + (k_\mu \eta_\nu + k_\nu \eta_\mu)/k\cdot\eta - \eta^2 k_\mu k_\nu/(k\cdot\eta)^2\right]/k^2 \tag{2.1}$$

is the gluon propagator in the n gauge. This type of contribution vanishes in the leading order (LLR) since the second term in the gluon propagator cancels the contribution from the $(-g^{\mu\nu})$ term and the third term is of $O(y^2)$ and contributes only in non-leading orders. Thus in this gauge the leading order in QCD corresponds to the factorized insertions shown in Fig.1(c), plus the corresponding virtual insertions on, respectively, the p_A and p_B lines. The infra-red divergence corresponding to soft gluons cancel between real and virtual gluons in each channel (Fig.1(d)) and the coherence effects in leading order lay hidden in the anomalous dimensions, which govern the scaling violations of the parton densities. In the next to leading order important coherent effects will be associated with the gauge fixing vector n.

Apart from these distinctions, we have block-wise factorization of the various Green functions describing the process in the LLR of QCD as in the covariant parton model description, leading to a formula of the type

$$\sigma^{AB\ldots}(p_A, p_B, \ldots) = \sum_{a,b} \int dx_a\, D_A^a(x_a, Q^2) \int dx_b\, D_B^b(x_b, Q^2) \quad \sigma^{ab\ldots}(x_a p_A, x_b p_B, \ldots) \quad , \tag{2.2}$$

where

$$D_A^a(x_a,Q^2) = \frac{4Q^2}{x_a}\int\frac{d^4k_a}{(2\pi)^3}\,\delta^{(4)}((k_a+Q)^2)\,A_A^a(k_a)\,Z_{3a}^{-1}(k_a^2) \qquad (2.3)$$

In deriving (2.2) one makes use of the fact that in the LLR the relevant part of the parton-hadron Bethe-Salpeter (B-S) Green's functions is given by $T_A^a = P_{ij}(k_a)\,A_A^a(k_a)$, where P_{ij}^a is the spin projection operator for parton a in the limit $k_a^2 = 0$. $Z_{3a}^{1/2}$ is the wave function renormalization factor associated with the a^{th} parton and $\sigma^{ab\cdots}$ is the parton cross-section in the lowest order in QCD. *) In (2.2) we do not distinguish initial and final state hadrons (i.e. parton distributions or fragmentation functions) because they are treated in precisely the same way in the leading order in QCD [5] (except the variable $x_a \to x_a^{-1}$).

There exists a general proof of factorization, which holds to all orders in perturbation theory in the renormalized coupling constant $\alpha_s = g_s^2/4\pi$ and in all logarithms of Q^2, based on a treatment of the mass singularities [6]. However, the proof is formal in the sense that there is an ambiguity in the definition of the parton probabilities and basic cross-section beyond the leading order. It also does not obviously correspond to block-wise factorization of the various Green's functions, which means that the transmission of polarization information may not be in general as simple as in the leading order. We shall return to a discussion of the latter in Sec.V. If we write the cross-section $\sigma^{AB\cdots}(s_A, s_B)$ for polarized hadrons and allow for polarization effects in the parton densities and cross-section, then we can express it in the form

$$\sigma^{AB\cdots}(s_A,s_B,\ldots) = \sum_{s_a,s_b\ldots}\int D_A^a(s_A,s_a)\,D_B^b(s_B,s_b)\cdots\,\sigma^{ab\cdots}(s_a,s_b) \qquad (2.4)$$

where the spin dependent parton distribution functions are given by the same integral as in (2.3) but now over the polarized parton-hadron B-S amplitudes.

*) The logarithms associated with the renormalization group behaviour of the basic parton Green's funtions are cancelled by the square root of the logarithmic factors in the parton legs, so all the logarithmic behaviour is incorporated into the $D_A^a(x,Q^2)$.

If we define the spin differential $\delta_a F(s_a) = F(\uparrow) - F(\downarrow)$ the final state spins and $\delta_a F(s_a) = \frac{1}{2}[F(\uparrow) - F(\downarrow)]$ for initial spins, following the usual convention of spin average for initial spins, then by repeated use of the trivial identity $2A_1B_1 + 2A_2B_2 = [A_1 + A_2][B_1 + B_2] + [A_1 - A_2][B_1 - B_2]$, Eq.(2.4) can be written in the form

$$\begin{aligned}\sigma^{AB\cdots}(s_A,s_B,\ldots) = {} & \int D_A^a(s_A)\,D_B^b(s_B)\cdots\,\sigma^{ab\cdots} + \int \delta_a D_A^a(s_A,s_a)\,D_B^b(s_B)\cdots\,\delta_a\sigma^{ab\cdots}(s_a) \\ & + \int D_A^a(s_A)\,\delta_b D_B^b(s_B,s_b)\cdots\,\delta_b\sigma^{ab\cdots}(s_b) \\ & + \int \delta_a D_A^a(s_A,s_a)\,\delta_b D_B^b(s_B,s_b)\cdots\,\delta_a\delta_b\,\sigma^{ab\cdots}(s_a,s_b) \quad ,\end{aligned} \qquad (2.5)$$

where we have used the notation of not exhibiting the spin argument when it is averaged over, e.g. $D_A^a(s_A) = \frac{1}{2}[D_A^a(s_A,\uparrow) + D_A^a(s_A,\downarrow)]$. Only the first term in (2.5) is present in the LLR, however formally any block-wise factorized cross-section can be expressed in this form if all partons have only two spin states (i.e. taking only transverse gluons).

Although in general one would write the cross-section involving the parton-hadron channel (a,A) in the form

$$\sigma^A = \int Tr\,\{D^a_{A\,ss'}\,\Sigma^a_{s's}\} \qquad (2.6)$$

one can always choose a spin base in which $\Sigma_{ss'}$ is diagonal, i.e. one can always find a unitary matrix U for which $U\Sigma U^{-1} = \sigma_s^a\,\delta_{ss'}$, so that (2.6) can be expressed as

$$\sigma^A = \sum_s\int\,[U D_A^a U^{-1}]_{ss}\;\sigma_s^a \qquad (2.7)$$

We list here for completeness useful changes of basis of this kind. For spin $\frac{1}{2}$ states polarized along the y axis, we can write $|\uparrow\rangle = \frac{1}{\sqrt{2}}[-i|+\rangle + |-\rangle]$ and $|\downarrow\rangle = \frac{1}{\sqrt{2}}[-|+\rangle + i|-\rangle]$, where $|\pm\rangle$ refers to helicity states. Similarly, we can write linearly polarized gluon states in terms of helicity states, by expressing the polarization vectors $\epsilon_\mu^{(1)} = (0,1,0,0)$ and

$\epsilon_\mu^{(2)} = (0,0,1,0)$ in the form $\epsilon_\mu^{(1)} = \frac{1}{2}[\epsilon_\mu^{(+)} - \epsilon_\mu^{(-)}]$ and $\epsilon_\mu^{(2)} = \frac{1}{i\sqrt{2}}[\epsilon_\mu^{(+)} + \epsilon_\mu^{(-)}]$.
From the former linear relations we can derive the familiar identities:

$$|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| = |+\rangle\langle+| + |-\rangle\langle-|$$

$$|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| = i\{-|+\rangle\langle-| + |-\rangle\langle+|\} \qquad . \tag{2.8}$$

Eq.(2.8) can be used to derive (2.5) with the help of the parity relations $D(+,+) = D(-,-)$ and $D(+,-) = D(-,+)$ which lead to identities like those expressed in Fig.2.

Returning to (2.5) only the first term is present in leading order in QCD, the other terms appearing only in second order (i.e. α_s^2 corrections to the leading order). However if we form the single spin asymmetry $\delta_A \sigma^{AB}(s_A)$, then three of the four terms potentially contribute in order α_s *) and we can write this differential cross-section in the form

$$\delta_A \sigma^{AB\ldots}(s_A) = \int \delta_A D_A^a(s_a) D_B^b \ldots \sigma^{ab\ldots} + \int \delta_A \delta_a D_A^a(s_A, s_a) D_B^b \delta_a \sigma^{ab\ldots}(s_a)$$
$$+ \int \delta_A D_A^a(s_A) \delta_b D_B^b(s_b) \delta_b \sigma^{ab\ldots}(s_b) + \int \delta_A \delta_a D_A^a(s_A, s_a) \delta_b D_B^b(s_b) \delta_a \delta_b \sigma^{ab\ldots}(s_a, s_b) \quad , \tag{2.9}$$

where the double spin differentials (i.e. asymmetries) $\delta_2 D_A^a \equiv \delta_A \delta_a D_A^a(s_A, s_a)$ are non-zero in leading order of QCD. However because of terms like $\delta_A D_A^a(s_A)$ and $\delta_a \sigma^{ab\ldots}(s_a)$, (2.9) still vanishes in leading order. On the other hand, if we form the double spin asymmetry $\delta_A \delta_B \sigma^{AB\ldots}(s_A, s_B)$, then the last term in (2.5) and (2.9) becomes a leading order term in QCD and forms the basis of a test of QCD proposed by Sivers and collaborators [2]. In the latter case we write

$$\delta_A \delta_B \sigma^{AB\ldots}(s_A, s_B) = \int \delta_A \delta_a D_A^a(s_A, s_a) \quad \delta_B \delta_b D_B^b(s_B, s_b) \ldots \delta_a \delta_b \sigma^{ab\ldots}(s_a, s_b) \tag{2.10a}$$

*) In fact, since zero mass quarks have no spin asymmetry, it turns out for the qq term, $\delta_A \sigma^{A\ldots}$ has only contributions in order α_s^2 beyond LLR.

or in more familiar notation

$$A_{\mu\nu}^{AB} \sigma^{AB\ldots} = \int \delta_2 D_A^a \quad \delta_2 D_B^b \ldots a_{\mu\nu}^{ab} \sigma^{ab\ldots} \qquad . \tag{2.10b}$$

The double spin differential of parton density is given explicitly by

$$\delta_2 D_A^a = \delta_A \delta_a D_A^a(s_A, s_a)$$
$$= \frac{1}{2}\left[D_A^a(\uparrow,\uparrow) - D_A^a(\uparrow,\downarrow) - D_A^a(\downarrow,\uparrow) + D_A^a(\downarrow,\downarrow)\right] \tag{2.11}$$

and it satisfies the following Bethe-Salpeter equation in the leading order in QCD:

$$\delta_A \delta_a D_A^a(x, Q^2, s_A, s_a) = \delta_A \delta_a D_A^a(x, Q_0^2, s_A, s_a)$$
$$+ \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} \delta_a \delta_b P(z, s_a, s_b) \delta_A \delta_b D_A^b\left(\frac{x}{z}, k^2, s_A, s_b\right) \tag{2.12}$$

or since the non-singlet quark distribution functions do not mix with gluon distributions, we have in that case the simpler form

$$\delta_2 D(x, Q^2) = \delta_2 D(x, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} \delta_2 P(z) \delta_2 D\left(\frac{x}{z}, k^2\right) \quad , \tag{2.13}$$

where

$$\delta_2 P(z) = \lim_{\delta \to 0} C_F \left[\frac{2z}{1-z+\delta} - \delta(z-1) \int_0^{1-\delta} dz' \frac{(1+z'^2)}{1-z'} \right]$$

and we denote the corresponding anomalous dimensions

$$\gamma_n^{\delta FF} = \int_0^1 dz\, z^{n-1}\, \delta_z P(z)$$

$$= -\frac{C_F}{2}\left[1 + 4\sum_{j=2}^{n}\frac{1}{j}\right] \qquad (2.14)$$

We shall return to a more detailed discussion of these evolution equations in Sec.IV, where all the relevant cases will be discussed.

Finally we note that if we form longitudinal polarization or helicity asymmetries, the same considerations go through and one obtains corresponding formulae with the spin differential replaced by helicity differences, which we denote by $\Delta_a F(\lambda_a) = \frac{1}{2}\{F(+) - F(-)\}$ in order to distinguish the latter from the former. If we include effects of longitudinal gluons in next to leading order, then the above formulae must be modified to include the additional terms.

III. SURVEY OF THE MAIN ASYMMETRIES PREDICTED IN THE LEADING ORDER IN QCD

In this section we shall apply the techniques developed in Sec.II to a number of hard processes, which admit a parton model description and derive formulae describing in the leading order of QCD spin and/or helicity asymmetries associated with them. In Sec.IV we shall provide detailed estimates for a number of the cases below, while for the others, rough estimates and a review of present-day literature will be presented.

3.1 Total inclusive deep inelastic electroproduction

Consider the process $e(\lambda) + N(\Lambda) \to \langle e' \rangle + X$, where λ refers to the initial lepton helicity, Λ is the longitudinal polarization of the nucleon (along the beam direction) and $\langle e' \rangle$ denotes the unpolarized final lepton. The double differential cross-section is given by

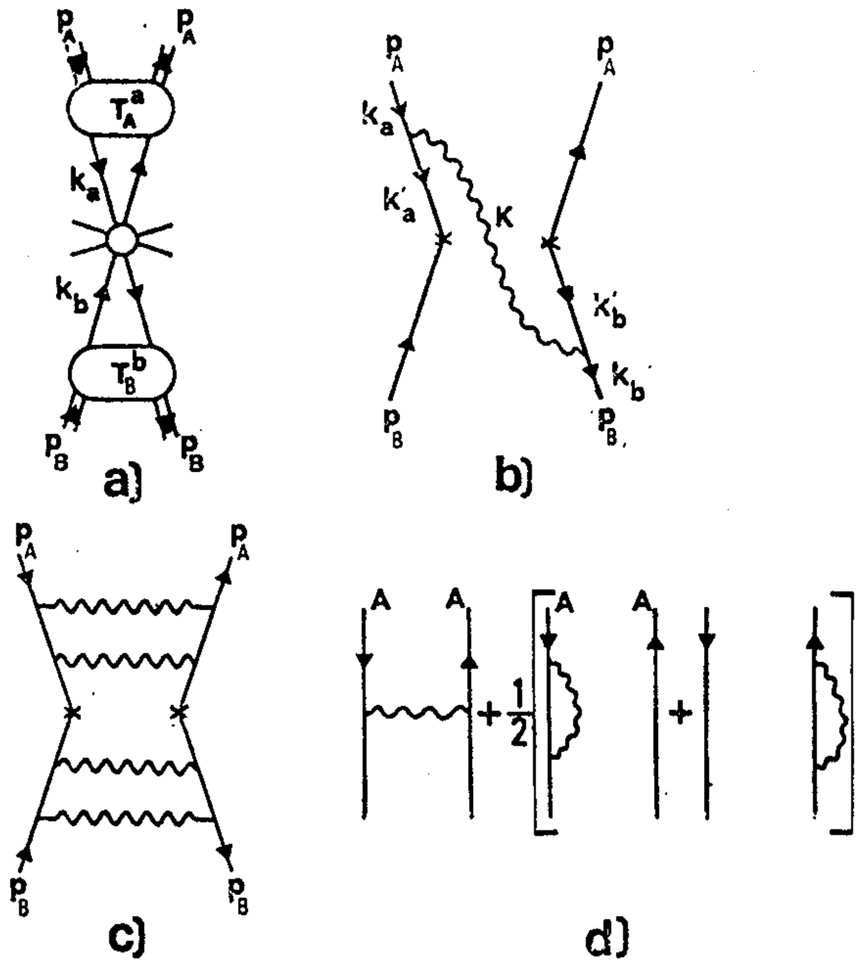


Fig.1 (a) Block-wise factorization of Bethe-Salpeter amplitudes in the parton hadron channels(a,A) and (b,B), respectively.
(b) Crossed graph which could mix (a,A) and (b,B) channels and destroy factorization.
(c) Gluon exchange in LLR in the η gauge.
(d) Real and virtual gluons in a given channel.

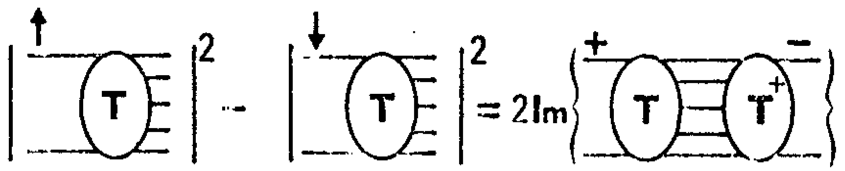


Fig.2 Relationship between spin asymmetry $\sigma(\uparrow) - \sigma(\downarrow)$ and discontinuity in the helicity basis.

- - - - -

$$\left(\frac{d\sigma}{dt\,d\nu}\right)_{e(\lambda)N(\Lambda)\to(e')X} \equiv \sigma(\lambda,\Lambda)$$

$$= \frac{2\pi\alpha^2}{s^2t^2}\,\frac{1}{s+\nu}\sum_i x e_i^2 \sum_{\lambda'}\left[(s^2+\nu^2)+\lambda\lambda'(s^2-\nu^2)\right] D_N^i(x,Q^2;\Lambda,\lambda') \tag{3.1}$$

where $x = -t/(s+u) = Q^2/2M_N\nu$, $s = 2M_N E$, $u = -2M_N E'$, $t = -Q^2 = -4EE'\sin^2\theta/2$, $\nu = E-E'$, E and E' are the beam and final lepton energies, respectively, and θ the scattering angle in the laboratory frame, $\sum_{\lambda'}$ refers to the sum over the helicities of the strucked parton i (quarks and antiquarks) and e_i^2 is its squared charge. $D_N^i(\Lambda,\lambda')$ measures the probability of finding a quark (antiquark) i with helicity λ' in a nucleon with longitudinal polarization Λ. From Eq.(3.1) it is easy to obtain for the longitudinal asymmetry *)

*) Since in the CM frame the nucleon is in a definite helicity state we may also refer to A_{LL} as a helicity asymmetry.

$A_{LL} \equiv \frac{\sigma(+,+)-\sigma(+,-)}{\sigma(+,+)+\sigma(+,-)}$ the expression

$$A_{LL}(x,y,Q^2) = \frac{1-(1-y)^2}{1+(1-y)^2}\;\frac{\sum_i e_i^2\,\Delta_2 D_N^i(x,Q^2)}{\sum_i e_i^2\,D_N^i(x,Q^2)} \tag{3.2}$$

with $y = \nu/E$ and $\sum_i = \sum_{q,\bar q}$. In deriving Eq.(3.2) we have used the relations $D_N^i(+,+) = D_N^i(-,-)$ and $D_N^i(+,-) = D_N^i(-,+)$ which are a consequence of parity invariance of the strong interactions. The spin structure of the functions $\Delta_2 D_N^i$ is defined in Eq.(2.11). We note in Eq.(3.2) the presence of a y-dependent kinematical factor in front of what may be called "parton asymmetry". A rough estimate of the latter can be given in the framework of the SU(6) model, which is reliable near $x = 1$ and which gives $5/9$ and zero for proton and neutron targets, respectively.

One can also analyse the spin dependence of deep inelastic scattering in terms of the four structure functions W_1, W_2, G_1 and G_2 defined by the decomposition [3)]

$$W^{\mu\nu} = \sum_X \langle p,s|J^\mu|X\rangle\langle X|J^\nu|p,s\rangle\;\delta(M_X^2-(p+q)^2)$$
$$= (-g^{\mu\nu}+\cdots)W_1 + (p^\mu p^\nu/M+\cdots)W_2$$
$$+\frac{4i}{\nu}\epsilon^{\mu\nu\lambda\sigma}q_\lambda\left[s_\sigma G_1 + \left\{s_\sigma - \frac{(s\,q)}{p\,q}q_\sigma\right\}G_2\right] \tag{3.3}$$

The structure functions G_1 and G_2 are simply related to the spin and helicity differentials $\delta_2 D_N^i$ and $\Delta_2 D_N^i$ by

$$2G_1(x,Q^2) = \sum_i e_i^2\,\Delta_2 D_N^i(x,Q^2)$$

$$2\left[G_1+G_2\right] = \sum_i e_i^2\,\delta_2 D_N^i(x,Q^2) \tag{3.4}$$

The G structure functions satisfy the Bjorken sum rules [7)] (see Ref.3 for details) in the form

$$2\int_0^1 dx \left[G_1^p(x) - G_1^n(x) \right] = \frac{1}{3} \frac{G_A}{G_V} \quad , \tag{3.5}$$

where $G_A/G_V \simeq 1.23$ is the ratio of axial vector and vector coupling constants.

$$\int_0^1 dx \; G_2^p(x) = \int_0^1 dx \; G_2^n(x) = 0 \quad . \tag{3.6}$$

The latter corresponds to angular momentum conservation. This implies, respectively, for the spin and helicity parton densities the identical sum rules

$$\int_0^1 dx \left[\delta_2 D_p^u - \delta_2 D_p^d + \delta_2 D_p^{\bar{u}} - \delta_2 D_p^{\bar{d}} \right] = \frac{G_A}{G_V} \quad , \tag{3.7}$$

$$\int_0^1 dx \left[\Delta_2 D_p^u - \Delta_2 D_p^d + \Delta_2 D_p^{\bar{u}} - \Delta_2 D_p^{\bar{d}} \right] = \frac{G_A}{G_V} \quad . \tag{3.8}$$

The sum rules can be used to make models of the parton polarization distributions in the absence of data.

3.2 Deep inelastic neutrino (antineutron) nucleon scattering

The reactions $\nu\vec{N} \to \mu + X$ and $\bar{\nu}\vec{N} \to \mu + X$ provide a novel polarization asymmetry experiment [8] because the neutrino (or antineutrino) is 100% polarized. The class of experiments in this area deserves a careful study and we shall only touch on it here. We shall consider here only the total inclusive deep inelastic processes initiated with $\nu_\mu(\bar{\nu}_\mu)$ beams involving the charged weak currents $\nu\mu^-(\bar{\nu}\mu^+)$ and with target nucleons polarized longitudinally (i.e. along the beam direction). Again, as in case 3.1 in the CM system the nucleon is in a definite helicity state and therefore we shall discuss the following single helicity asymmetries. The spin dependent double differential cross-sections for the processes $\nu N(\Lambda) \to \langle \mu^- \rangle + X$

and $\bar{\nu}N(\Lambda) \to \langle \mu^+ \rangle + X$ ($\langle \mu \rangle$: outgoing unpolarized muon) in the leading order of QCD are

$$\frac{d\sigma}{dt\,du}\left(\nu N(\Lambda) \to \langle\mu^-\rangle X\right) \equiv \sigma^{\nu}(\Lambda) \sim \sum_q D_N^q(x,Q^2;\Lambda,-) + (1-y)^2 \sum_{\bar{q}} D_N^{\bar{q}}(x,Q^2;\Lambda,+) \quad ,$$

$$\frac{d\sigma}{dt\,du}\left(\bar{\nu} N(\Lambda) \to \langle\mu^+\rangle X\right) \equiv \sigma^{\bar{\nu}}(\Lambda) \sim (1-y)^2 \sum_q D_N^q(x,Q^2;\Lambda,-) + \sum_{\bar{q}} D_N^{\bar{q}}(x,Q^2;\Lambda,+) \quad , \tag{3.9}$$

respectively, with the variables x,y,t and u denoting the same physical quantities as in the case 3.1 above. The D's functions are the same as in electroproduction and the sums extend in principle over all quark and anti-quark flavours. From Eqs.(3.9) one obtains for the asymmetries $A_{LL}^{\nu(\bar{\nu})} \equiv \dfrac{\sigma_{(+)}^{\nu(\bar{\nu})} - \sigma_{(-)}^{\nu(\bar{\nu})}}{\sigma_{(+)}^{\nu(\bar{\nu})} + \sigma_{(-)}^{\nu(\bar{\nu})}}$ the expressions

$$A_{LL}^{\nu}(x,y,Q^2) = \frac{-\sum_q \Delta_2 D_N^q(x,Q^2) + (1-y)^2 \sum_{\bar{q}} \Delta_2 D_N^{\bar{q}}(x,Q^2)}{\sum_q D_N^q(x,Q^2) + (1-y)^2 \sum_{\bar{q}} D_N^{\bar{q}}(x,Q^2)} \tag{3.10a}$$

and

$$A_{LL}^{\bar{\nu}}(x,y,Q^2) = \frac{-(1-y)^2 \sum_q \Delta_2 D_N^q(x,Q^2) + \sum_{\bar{q}} \Delta_2 D_N^{\bar{q}}(x,Q^2)}{(1-y)^2 \sum_q D_N^q(x,Q^2) + \sum_{\bar{q}} D_N^{\bar{q}}(x,Q^2)} \quad , \tag{3.10b}$$

respectively. In the framework of the Cabibbo model for charged weak interactions with $\cos^2\theta_c \sim 1$ and $\sin^2\theta_c \sim 0$ and in a kinematical region below charm production, Eqs.(3.10) reduce to

$$A_{LL}^{\nu} = \frac{-\Delta_2 D_N^d(x,Q^2) + (1-y)^2 \Delta_2 D_N^{\bar{u}}(x,Q^2)}{D_N^d(x,Q^2) + (1-y)^2 D_N^{\bar{u}}(x,Q^2)} \quad , \tag{3.11a}$$

$$A_{LL}^{\bar{\nu}} = \frac{-(1-y)^2 \Delta_2 D_N^{\nu}(x,Q^2) + \Delta_2 D_N^{\bar{d}}(x,Q^2)}{(1-y)^2 D_N^{\nu}(x,Q^2) + D_N^{\bar{d}}(x,Q^2)} \qquad (3.11b)$$

The interesting feature of these asymmetries is that through their measurement in different y regions one would be able to separate in a very clean way the spin asymmetries for the distribution functions of valence quarks from those associated with the distributions of antiquarks. Detailed predictions for A_{LL}^{ν} and $A_{LL}^{\bar{\nu}}$ will be gone into elsewhere. A rough estimate of their order of magnitude can be done near x = 1, where the sea contribution can be ignored and as a consequence they become y independent. In the SU(6) model one easily obtains $(A_{LL}^{\nu}, A_{LL}^{\bar{\nu}}) = (1/3, -2/3)$ and $(A_{LL}^{\nu}, A_{LL}^{\bar{\nu}}) = (-2/3, 1/3)$ for proton and neutron targets, respectively.

3.3 Semi-inclusive deep inelastic electroproduction

The interest in this process lies in the fact that it involves in a rather simple fashion, besides asymmetries for distribution functions, those associated with the fragmentation functions $\bar{D}_i^h$. We shall consider here the reaction $\langle e \rangle\ H(S) \to \langle e' \rangle\ h(S') + X$ (Fig.3) in which the initial and final electrons are unpolarized and S, S' denote the spin components of the target and the produced hadron, respectively, along a direction normal to the production plane. If we denote with z the fraction of longitudinal momentum of the fragmenting quark carried by the outgoing hadron h and with s,t and u the same quantities as in the case 3.1, the cross-section is

$$\frac{d\sigma^h}{dz}(s,x,Q^2;S,S') \equiv \sigma^h(S,S')$$

$$= \frac{\pi\alpha^2}{s^2t^2}\,\frac{s^2+u^2}{s+u}\sum_i e_i^2 x \left\{ \mathcal{F}_{1\,H,i}^{\,i,h} + \left[\frac{(s+u)^2}{s^2+u^2} - 1\right] \mathcal{F}_{2\,H,i}^{\,i,h} \right\} \qquad (3.12)$$

with

$$\mathcal{F}_{1\,H,i}^{\,i,h}(x,z,Q^2;S,S') = \sum_{\sigma\sigma'} D_H^i(x,Q^2;S,\sigma)\ \bar{D}_i^h(z,Q^2;\sigma',S') \qquad (3.13a)$$

and

$$\mathcal{F}_{2\,H,i}^{\,i,h}(x,z,Q^2;S,S') = \sum_{\sigma\sigma'} D_H^i(x,Q^2;S,\sigma)\ (\sigma\cdot\sigma')\ \bar{D}_i^h(z,Q^2;\sigma',S') \qquad (3.13b)$$

$\sum_{\sigma\sigma'}$ denotes the sum over the polarizations of the strucked and fragmenting partons, $D_H^i(S,\sigma)$ measures the probability of finding parton i in hadron H with spin components S and σ, respectively, and $\bar{D}_i^h(\sigma',S')$ that of finding hadron h as a fragment of parton i with spin components S' and σ', respectively. If we use the fact that for both D's and $\bar{D}$'s functions $D(\uparrow,\uparrow) - D(\downarrow,\downarrow) \approx D(\uparrow,\downarrow) - D(\downarrow,\uparrow) \sim O(k_\perp)$, where $k_\perp$ is the relative transverse momentum for the parton-hadron system and that in the present approximation we neglect terms of $O(k_T^2/Q^2)$ we obtain for the asymmetry $A_{NN} \equiv \frac{\sigma^h(\uparrow,\uparrow) - \sigma^h(\uparrow,\downarrow)}{\sigma^h(\uparrow,\uparrow) + \sigma^h(\uparrow,\downarrow)}$ the expression

$$A_{NN}(x,y,z,Q^2) = \frac{2(1-y)}{1+(1-y)^2}\ \frac{\sum_i e_i^2\ \delta_2 D_H^i(x,Q^2)\ \delta_2 \bar{D}_i^h(z,Q^2)}{\sum_i e_i^2\ D_H^i(x,Q^2)\ \bar{D}_i^h(z,Q^2)} . \qquad (3,14)$$

Again as in the previous cases a y-dependent kinematical factor appears in front of the "parton asymmetry". The spin structure of $\delta_2 D_H^i$ and $\delta_2 \bar{D}_i^h$ is given in Eq.(2.11). We note that in this case the asymmetry is a function of the four independent variables Q^2 (or s) x,y and z. The main problem of this kind of experiment will be in determining the helicity of the final state hadron. In the case of vector mesons ρ, ω, K^*, ϕ, etc. a density matrix analysis through angular distributions in final stable meson system could be tried. In the case of stable strange baryons, the asymmetries in the decay distribution provides a natural asymmetry monitor. In the case of $ep^{\uparrow} \to \ell'\Lambda^{\uparrow} + X$ the asymmetry is approximately given by

$$A_{NN}(x,y,z,Q^2) \simeq \frac{2(1-y)}{1+(1-y)^2} \; \frac{\delta_2 D_p^s(x,Q^2)}{D_p^s(x,Q^2)} \; \frac{\delta_1 \bar{D}_s^{\Lambda}(z,Q^2)}{\bar{D}_s^{\Lambda}(z,Q^2)} , \tag{3.15}$$

where the last factor is likely to be large because of leading quark effects on the helicity. The important question as to what degree the polarization of the nucleon is reflected in the sea distribution, can be answered in the latter experiments.

3.4 Asymmetries in massive lepton pair production

It is popularly believed that the Drell-Yan parton model cross-section describes massive lepton pair production in high-energy hadron-hadron collisions, particularly if one includes both leading and next to leading corrections in QCD perturbation theory [9]. For this reason the following simple and easily measurable asymmetries of the Drell-Yan mechanism have become important items to study. We shall consider two types of asymmetry: the initial-initial or reflected asymmetry involving both colliding hadrons in definite states of polarization and unpolarized muons and the initial-final transmitted asymmetry, which involves the measurement of the polarizations of one initial hadron and one final lepton, the other observable particles being unpolarized. The latter was studied in Ref.10, however we differ with the formulae given, so we give the basic helicity dependent cross-sections in full so the formulae we derived can be verified. The basic cross-section for $q(\lambda_a)\,\bar{q}(\lambda_b) \to \mu^+(\lambda_+)\;\mu^-(\lambda_-)$, $\lambda_a, \lambda_b, \lambda_+$ and λ_- being the respective helicities is given by

$$\left(\frac{d\sigma}{d\hat{t}}\right)^{\lambda_a\lambda_b}_{\lambda_+\lambda_-}$$

$$= \frac{2\pi\alpha^2}{\hat{s}^2}\, e_i^2 \left[\frac{1}{4}(1-\delta_{\lambda_a\lambda_b})(1-\delta_{\lambda_+\lambda_-})\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - \frac{1}{4}(\lambda_a-\lambda_b)(\lambda_+-\lambda_-)\,\frac{\hat{t}^2-\hat{u}^2}{\hat{s}^2}\right] \tag{3.16}$$

where

$$\hat{t} = x_a\, t \quad \text{and} \quad t = (p_A - p_{\mu^+})^2$$
$$\hat{u} = x_b\, u \qquad\qquad u = (p_B - p_{\mu^+})^2 .$$
$$\hat{s} = x_a\, x_b\, s .$$

The cross-section for $\bar{q}q \to \mu^+\mu^-$ is obtained by the substitution $\lambda_a \to -\lambda_a$.

The basic initial-initial asymmetries are given by

$$a_{LL}^{ii} = \frac{(d\sigma)^{++} - (d\sigma)^{+-}}{(d\sigma)^{++} + (d\sigma)^{+-}} = -1 \tag{3.17}$$

since $(d\sigma/dt)^{++} = 0$ by angular momentum conservation. Hence for the whole process $\vec{A}\vec{B} \to \mu^+\mu^- + X$ we have

$$A_{LL}^{ii} = -\frac{\sum_i e_i^2\left[\Delta_2 D_A^i(x_a)\;\Delta_2 D_B^{\bar{i}}(x_b) + \Delta_2 D_A^{\bar{i}}(x_a)\;\Delta_2 D_B^i(x_b)\right]}{\sum_i e_i^2\left[D_A^i(x_a)\;D_B^{\bar{i}}(x_b) + D_A^{\bar{i}}(x_a)\;D_B^i(x_b)\right]} \tag{3.18}$$

with

$$x_a = \frac{1}{2}\left[\left(x_F^2 + 4Q^2/s\right)^{1/2} + x_F\right] \quad ; \quad x_b = \frac{1}{2}\left[\left(x_F^2 + 4Q^2/s\right)^{1/2} - x_F\right] ,$$

where the asymmetries have been written for the cross-section $d\sigma/dQ^2\,dX_F$. For the integrated cross-section $d\sigma/dQ^2$ one replaces Eq.(3.18) by one with the appropriate integrals in the numerator and the denominator.

For the transmitted asymmetries $\vec{A}B \to \vec{\mu}^+\mu^- + X$ introduced in Ref.10, we note that the basic cross-section $\vec{q}q \to \vec{\mu}^+\mu^-$ is given by

$$\left(\frac{d\sigma}{d\hat{t}}\right)^{\lambda_a}_{\lambda_+} = \frac{2\pi\alpha^2}{\hat{s}^2}\, e_i^2\, \frac{1}{2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - \lambda_a\lambda_+\,\frac{\hat{t}^2-\hat{u}^2}{\hat{s}^2}\right] . \tag{3.19}$$

Hence the basic trasmitted asymmetry is given by

$$a_{LL}^{if}\,(q\bar{q}\to\mu^+\mu^-) = \frac{(d\sigma)_+^+ - (d\sigma)_-^+}{(d\sigma)_+^+ + (d\sigma)_-^+}$$

$$= -\frac{\hat{t}^2 - \hat{u}^2}{\hat{t}^2 + \hat{u}^2} \quad . \tag{3.20}$$

The same formula holds with the opposite sign for the case $\vec{\bar{q}}q \to \vec{\mu}^+ \bar{\mu} + X$ because of the substitution for $(q_a\,\bar{q}_b) \to (\bar{q}_a\,q_b)$ $\lambda_a \to -\lambda_a$. From this basic asymmetry, we can derive the transmitted asymmetry for the whole process $\vec{A}B \to \vec{\mu}^+\mu^- + X$, with the result, again written for the Drell-Yan cross-section $d\sigma/dQ^2\,dX_F$, integrating out the muon angular distribution

$$A_{LL}^{if} = \mp\left\{\frac{x_a^2 - x_b^2}{x_a^2 + x_b^2}\right\}\frac{\sum_i e_i^2\left[\Delta_2 D_A^i(x_a)\,D_B^{\bar{i}}(x_b) - \Delta_2 D_A^{\bar{i}}(x_a)\,D_B^i(x_b)\right]}{\sum_i e_i^2\left[D_A^i(x_a)\,D_B^{\bar{i}}(x_b) + D_A^{\bar{i}}(x_a)\,D_B^i(x_b)\right]} \tag{3.21}$$

The initial (±) sign referring to μ^+ and μ^-, respectively. If we do not integrate out the muon angle then the expression in the curly bracket is replaced by

$$\{\;\} = \frac{x_a^2\,(1-\cos\theta_\mu)^2 - x_b^2\,(1+\cos\theta_\mu)^2}{x_a^2\,(1-\cos\theta_\mu)^2 + x_b^2\,(1+\cos\theta_\mu)^2} \tag{3.22}$$

3.5 Hadron production at large p_T

Consider in general the large p_T single hadron production reaction $A(s_A)\,B(s_B) \to C(s_C,p_T) + X$ where A and B are hadrons and s_A, s_B and s_C refer to particle helicities or spin components along a direction transverse to the production plane. The differential cross-section is given by

$$E_c\left(\frac{d\sigma}{d^3 p_c}\right)_{A(s_A)B(s_B)\to C(s_c,p_T)+X} \equiv \sigma\,(s_A, s_B, s_C)$$

$$= \sum_{ab,cd}\int_{x_{a\,min}}^1 dx_a \int_{x_{b\,min}}^1 dx_b \int_0^1 dx_c^{-1}\,\frac{\hat{s}}{\pi}\,\delta(\hat{s}+\hat{t}+\hat{u}) \sum_{s_a,s_b,s_c,s_d} D_A^a(x_a,Q^2;s_A,s_a)\,D_B^b(x_b,Q^2;s_B,s_b)\;\cdot$$

$$\bar{D}_c^C(x_c,Q^2;s_c,s_C)\left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{a(s_a)\,b(s_b)\to c(s_c)\,d(s_d)} \tag{3.23}$$

where $x_{a\,min} = \frac{x_\perp \operatorname{cotg}\theta_{CM}/2}{2-x_\perp \operatorname{tg}\theta_{CM}/2}$, $x_{b\,min} = \frac{x_a x_T \operatorname{tg}\theta_{CM}/2}{2x_a - x_T \operatorname{cotg}\theta_{CM}/2}$, $x_T = 2p_T/\sqrt{s}$ and θ_{CM} is the scattering angle in the CM . $\sum_{ab,cd}$ extends over all possible types of partons (quarks, antiquarks and gluons) and $\sum_{s_a,s_b,s_c,s_d}$ sums over their spin degrees of freedom. $\hat{s} = x_a x_b s$, $\hat{t} = x_a/x_c\,t$ and $\hat{u} = x_b/x_c\,u$ are the Mandelstam variables associated with the elementary sub-process $ab \to cd$ with cross-section $d\hat{\sigma}/d\hat{t}$.

Unlike the previous situations, the identification of a scale variable Q^2 with some combination of the invariants is problematic because there are many choices. In the leading logarithmic order, these differences are unimportant, however they will have a decisive bearing on the role of next to leading logarithmic corrections. We shall return to this question in Sec.V.

In what follows we shall consider both double and single asymmetries. The former include the so-called reflected asymmetry, which we briefly review for completeness in this article (for a detailed treatment see the work of Babcock, Monsay and Sivers [2]) and Ranft and Ranft [11]) and also transmitted asymmetries (details of which are given by us in Ref.12). As far as single asymmetries are concerned we shall briefly discuss the non-vanishing (though small) asymmetry associated with the process

$$A(s_A) + B \to \pi(p_T) + \text{jet}\ (p_{out}) + X,$$

in which the spin asymmetry out of the production plane - p_{out} correlation is measured.

3.5 (a) Reflected asymmetry for $\vec{A}\vec{B} \to C + X$

In this case $\vec{A}$ represents either a polarized proton or antiproton and $\vec{B}$ a polarized nucleon or nuclear target. [We shall restrict our attention to single nucleons.] The trigger or detected particle can be many things (i.e. $\pi^\pm$, π^0, K, $\bar{K}$, p, $\bar{p}$,...) and the dependence on particle type has proved important in singling out the nature of the production mechanism. It is also worthwhile to consider a jet or calorimeter trigger, in which C is a jet of particles. In the latter case the cross-section is some hundred times that of the single particle trigger [13]. The general asymmetry formula for the case of a single particle trigger can be obtained from (3.23) and is given by

$$A_{LL}^{ii} = \frac{\sum_{a,b,c} \int_{\text{parton ph.sp.}} \Delta_2 D_A^a(x_a,Q^2)\, \Delta_2 D_B^b(x_b,Q^2)\, \bar{D}_c^C(x_c,Q^2)\, a_{LL}^{ii} \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)}{\sum_{a,b,c} \int_{\text{parton ph.sp.}} D_A^a(x_a,Q^2)\, D_B^b(x_b,Q^2)\, \bar{D}_c^C(x_c,Q^2) \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)}, \tag{3.24}$$

where $\frac{d\hat{\sigma}}{d\hat{t}}$ and a_{LL}^{ii} refer to the basic lowest order QCD cross-section and asymmetry for the parton sub-process $ab \to cd$. The parton phase space integral is given in (3.23). Here it turns out that helicity asymmetries are far more important than spin polarizations perpendicular to the production plane [14].

Defining $\sigma_{s_b s_d}^{s_a s_c} = \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{s_a s_b \to s_c s_d}$, then the basic parton reflected asymmetry a_{LL}^{ii} is defined by

$$a_{LL}^{ii} = \left(\sigma_{++}^{++} + \sigma_{+-}^{+-} - \sigma_{--}^{++} - \sigma_{-+}^{+-}\right) / 2\langle\sigma\rangle \tag{3.25}$$

where $\langle\sigma\rangle$ is the initial spin average final spin sum. For completeness we give the basic differential cross-sections (lowest order in QCD) for $q_\alpha(s_a)\, q_\beta(s_b) \to q_\alpha(s_c)\, q_\beta(s_d)$ and $\bar{q}_\alpha(s_a)\, q_\beta(s_b) \to \bar{q}_\gamma(s_a)\, q_\delta(s_d)$ ...

Tables 3.1 and 3.2 for the cases, in which s_i refers to helicity and spin transverse to interaction plane. In Table 3.3 we list the initial-initial helicity asymmetries and differential unpolarized cross-sections for the subprocesses shown in Figs.4a-f, namely: a) $q_\alpha q_\beta \to q_\alpha q_\beta$, b) $\bar{q}_\alpha q_\alpha \to \bar{q}_\beta q_\beta$, c) $q\bar{q} \to gg$, d) $gq \to gq$, e) $gg \to q\bar{q}$, f) $gg \to gg$ (α,β: flavour indices).

3.5 b) Transmitted asymmetry $\vec{A}B \to \vec{C} + X$

If one considers the production of a spin $\frac{1}{2}$ baryon C at large p_T in a definite helicity state, from hard collision of polarized baryon A on an unpolarized hadron B (e.g. $\vec{p}p \to \vec{\Lambda} + X$), then one has the analogue of the case 3.5(a), in which one studies how helicity is transmitted to the final state. In Ref.12 we give details of such an experiment in the case of Λ or $\bar{\Lambda}$ production. The potential interest arises because Λ polarizations have been detected to quite large transverse momenta and the Λ trigger selects only one or two basic parton sub-processes so is in general clearer than the reflected case, in which C is a meson. The basic asymmetry formula for this kind of experiment is given by

$$A_{LL}^{if} = \frac{\sum_{a,b,c} \int_{\text{parton ph.sp.}} \Delta_2 D_A^a(x_a,Q^2)\, D_B^b(x_b,Q^2)\, \Delta_2 \bar{D}_c^C(x_c,Q^2)\, a_{LL}^{if} \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)}{\sum_{a,b,c} \int_{\text{parton ph.sp.}} D_A^a(x_a,Q^2)\, D_B^b(x_b,Q^2)\, \bar{D}_c^C(x_c,Q^2) \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)} \tag{3.26}$$

with

$$a_{LL}^{if} = \left(\sigma_{++}^{++} - \sigma_{+-}^{+-} - \sigma_{-+}^{+-} + \sigma_{--}^{++}\right) / 2\langle\sigma\rangle, \tag{3.27}$$

again using the notation $\sigma_{s_b s_d}^{s_a s_c} = \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{s_a s_b \to s_c s_d}$ and $\langle\sigma\rangle$ is the summed-averaged cross-section. For the case $\vec{p}p \to \vec{\Lambda} + X$, the relevant dominant mechanism $q\bar{q} \to s\bar{s}$ is shown in Fig.4b and the corresponding basic asymmetries and cross-section are given by

$$a_{LL}^{if} = \frac{\hat{t}^2 - \hat{u}^2}{\hat{t}^2 + \hat{u}^2}$$

and

$$\left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{q\bar{q}\to gg} = \frac{4\pi}{9}\,\frac{1}{\hat{s}^2}\,\alpha_s^2(Q^2)\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right) \qquad (3.28)$$

The two gluon annihilation in Fig.4(e) will be an important background at small $x_T = 2\,p_T/\sqrt{s}$. This is analysed in Ref.12.

3.6 Prompt photon production at large p_T

Another hard scattering process which has received much interest recently [15),16)] as a relatively clean way of testing QCD perturbation theory is the study of prompt photon production at large transverse momentum in the reactions $PP \to \gamma + X$ and $P\bar{P} \to \gamma + X$. Although there are a number of possible mechanisms in each case, the dominant ones are just those shown in Figs.5(a) and (b) involving the subprocesses $qg \to \gamma q$ and $q\bar{q} \to \gamma g$, respectively. A straightforward trace calculation based on the amplitudes

$$M_{h_1h_2}(q_i g_a \to \gamma q_j)$$
$$= -e_f\, g\, \bar{u}_j(p_4,h_4)\left[T^a_{ji}\, \not\epsilon_2(h_2)[\not p_1 - \not k_3]\not\epsilon_3 + \not\epsilon_3[\not p_1 + \not k_2]\not\epsilon_2(h_2)\right] u_i(p_1,h_1)$$

for $qg \to \gamma q$ *) and

$$M_{h_1h_2}(q_i \bar{q}_j \to \gamma g_a)$$
$$= -e_f\, g\, \bar{v}_j(p_2,h_2)\left[T^a_{ji}\, \not\epsilon_4[\not p_1 - \not k_3]\not\epsilon_3 + \not\epsilon_3[\not p_1 - \not k_4]\not\epsilon_4\right] u_i(p_1,h_1)$$

*) A usual identity in the case of gluons of fixed helicity is
$$\epsilon^{\mu}(q,\lambda)\,\epsilon^{\nu}(q,\lambda) = \frac{1}{2}\left[-g^{\mu\nu} - i\lambda\,\epsilon^{\mu\nu\alpha\beta}\,q_\alpha\, p_\beta / q\cdot p\right] \quad \text{with} \quad p = p_1 .$$

for $q\bar{q} \to \gamma g$, respectively, yields $|M_{++}|^2 = 0$ for the latter, while for the former $qg \to \gamma q$ we have

$$a_{LL}^{ii}(qg) = \frac{|M_{++}|^2 - |M_{+-}|^2}{|M_{++}|^2 + |M_{+-}|^2} = \frac{\hat{t}\,\hat{u}}{\hat{s}^2 + \hat{t}^2} . \qquad (3.29)$$

The case $gq \to \gamma q$ is obtained through the substitution $\hat{t} \leftrightarrow \hat{u}$. This leads to, respectively, the asymmetry formulae for (a) $\vec{P}\vec{P} \to \gamma + X$

$$A_{LL}^{ii} = -\frac{\sum_{f=u,d} e_f^2 \int_{\text{parton ph.sp.}} \hat{s}^{-2}\left[\Delta_2 D_p^{q^f}(x_1)\,\Delta_2 D_p^{g}(x_2)(\hat{u}/\hat{s}) + \Delta_2 D_p^{g}(x_1)\,\Delta_2 D_p^{q^f}(x_2)\left(\frac{\hat{t}}{\hat{s}}\right)\right]}{\sum_{f=u,d} e_f^2 \int_{\text{parton ph.sp.}} \hat{s}^{-2}\left[D_p^{q^f}(x_1)\,D_p^{g}(x_2)\left(\frac{\hat{s}^2+\hat{t}^2}{-\hat{t}\hat{s}}\right) + D_p^{g}(x_1)\,D_p^{q^f}(x_2)\left(\frac{\hat{s}^2+\hat{u}^2}{-\hat{u}\hat{s}}\right)\right]} , \qquad (3.30)$$

where

$$\left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{qg\to\gamma q} = e_f^2\,\pi\,\frac{\alpha\alpha_s}{\hat{s}^2}\,\frac{1}{3}\left(\frac{\hat{s}^2+\hat{t}^2}{-\hat{t}\hat{s}}\right) \qquad (3.31)$$

and for (b) $\vec{P}\vec{\bar{P}} \to \gamma + X$

$$A_{LL}^{ii} = -\frac{\sum_{f=u,d}\int_{\text{parton ph.sp.}} \Delta_2 D_p^{q^f}(x_1)\,\Delta_2 D_{\bar{p}}^{\bar{q}^f}(x_2)\,(d\hat{\sigma}/d\hat{t})}{\sum_{f=u,d}\int_{\text{parton ph.sp.}} D_p^{q^f}(x_1)\,D_{\bar{p}}^{\bar{q}^f}(x_2)\,(d\hat{\sigma}/d\hat{t})} , \qquad (3.32)$$

where

$$\frac{d\hat{\sigma}}{d\hat{t}} = -e_f^2\,\pi\,\frac{\alpha\alpha_s}{\hat{s}^2}\,\frac{8}{9}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\right) . \qquad (3.33)$$

The parton phase space integral is given by

$$\int_{\substack{\text{parton}\\ \text{ph. sp.}}} = \int_0^1 dx_1 dx_2 \, \frac{x_1 x_2 S}{\pi} \, \delta(x_1 x_2 S + x_1 t + x_2 u) \tag{3.34}$$

where $s = (p_A + p_B)^2$, $t = (p_A - p_\gamma)^2$ and $u = (p_B - p_\gamma)^2$. These formulae show that one may expect the asymmetry to be of order 10-20% and in the case of $\overleftrightarrow{P}\bar{P} \to \gamma + X$ to be negative, reflecting the underlying asymmetry $q\bar{q} \to \gamma g$, hence providing a relative clean test of QCD.

3.7 Single asymmetry. p_{out} correlation

As we mentioned in the introduction to this section we shall study here the process (see Fig.6(a)) $A(s_A)\langle B\rangle \to \pi(p_T) + \text{Jet}(p_{out}) + X$ where the initial hadron A is prepared in a state with a definite spin component (up or down) along a direction normal to the production plane, i.e. the plane defined by the beam and the outgoing pion (see Fig.6(b)). We recall the reader's attention to the discussion we made in Subsec.3.3 in connection with asymmetries involving spin components along a definite direction in space. From it, it turns out immediately that single asymmetries vanish as $k_\perp \to 0$ i.e. when the relevant large mass scale in the problem (Q^2) is much larger than the transverse momenta in the block-wise factorized baryon-hadron systems. A non-negligible $k_\perp$ however can be introduced into the fragmenting parton c - observable hadron C (π in this case) system through the measurement of a large p_T pion (trigger) simultaneously with the detection of the away side jet with a large momentum ($\equiv p_{out} < p_T$) transverse to the production plane. (For studies of such towards side-away side correlations see for example Ref.17. (If z-x is the production plane, then $p_{out} \equiv |k_{dy}|$ (see Fig.6(a)).)By transverse momentum conservation $k_\perp^c$ can then be large enough to allow for a non-vanishing single asymmetry. Of course one expects the cross-sections for such events to be small, which would make their measurements rather difficult. It turns out from a detailed theoretical and numerical analysis, which we shall briefly discuss in Sec.IV, that this leads to a small value of 3%. The basic formula in this case is complex and involves resummation effects of soft gluon radiation, which gives an eikonal type smearing of the basic parton processes [5]. In the case of the unpolarized p_{out} distribution of large p_T events, the basic formula is given by (see Ref.5 for details)

$$E_\pi \frac{d\sigma}{d^3 \vec{p}_\pi dp_o}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} db \, e^{ip_o b} \int_{\substack{\text{parton}\\ \text{ph. sp.}}} e^{\chi_A(s,b)} D_A^a(x_a, 1/b^2) \, e^{\chi_B(s,b)} D_B^b(x_b, 1/b^2)$$

$$e^{\chi_c(s,b)} \bar{D}_c^\pi(x_c, 1/b^2) \left(\frac{d\hat{\sigma}}{d\hat{t}}\right)_{ab \to cd} , \tag{3.35}$$

where we have assumed $s \sim p_T^2 \gg p_{out}^2 = p_0^2 \gg 1$ and the parton phase space is the same as earlier. The eikonal phase factors χ_A, χ_B and χ_C contain the effect of soft gluon radiation on the process and are given by the expression (for details as to the conditions under which such formulae are valid see Refs.5 and 18)

$$\chi(s,b) = \frac{C_F}{2\pi} \int_0^s \frac{dk_T^2}{k_T^2} \alpha_s(k_T^2) \left[J_0(bk_T) - 1 \right] \log s/k_T^2 \, . \tag{3.36}$$

If one neglects the effect of soft gluons, for example in calculating asymmetries, so the effect on the overall normalization is presumably not too important, then (3.35) can be written in the form

$$E_\pi \frac{d\sigma}{d^3 \vec{p}_\pi dp_o}$$

$$= \frac{1}{p_o} \int_{\substack{\text{parton}\\ \text{ph. sp.}}} p_o^2 \frac{\partial}{\partial p_o^2} \left[D_A^a(x_a, p_o^2) D_B^b(x_b, p_o^2) \bar{D}_c^\pi(x_c, p_o^2) \right] \left\langle \frac{d\hat{\sigma}}{d\hat{t}} \right\rangle . \tag{3.37}$$

The single asymmetry $A_{out} \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$ where $\sigma(s_A) = E \frac{d\sigma}{d^3\vec{p}_\pi dp_0}$ $(A(s_A)B \to \pi + \text{jet} + X)$ turns out in the same approximation to be given by

$$A_{out}(s, p_T, p_o)$$

$$= \frac{p_o}{p_T} \frac{\int_{\substack{\text{parton}\\ \text{ph. sp.}}} x_c \, \partial_z D_A^a \, D_B^b \, p_o^2 \frac{\partial}{\partial p_o^2} \, \delta \bar{D}_c^\pi \, a_{NN}^{if} \left\langle \frac{d\hat{\sigma}}{d\hat{t}} \right\rangle}{\int_{\substack{\text{parton}\\ \text{ph. sp.}}} p_o^2 \frac{\partial}{\partial p_o^2} \left(D_A^a D_B^b \bar{D}_c^\pi \right) \left\langle \frac{d\hat{\sigma}}{d\hat{t}} \right\rangle} , \tag{3.38}$$

where $p_0^2 \frac{\partial}{\partial p_0^2} D_a(x,p_0^2) = \frac{\alpha_s(p_0^2)}{2\pi} \int_0^1 \frac{dx'}{x'} \; P_{ab}\left(\frac{x}{x'}\right) D_b(x',p_0^2)$ which is the appropriate Altarelli-Parisi evolution equation [19] and $\delta\bar{D}_c^{\pi}(x_c,Q^2)$ is a single asymmetry associated with jet emerging from a polarized quark. Naively one would assume the latter to be zero in the parton model because one assumes the process to be incoherent. However in QCD the gluon radiation makes the process intrinsically a coherent effect even in the special non-covariant gauges. In the latter the virtual contribution appears as interference terms which cancel the infra-red singularities coming from the coherent real processes (i.e. the QCD analogue of the Bloch-Nordsieck cancellation for QED). This lack of incoherence is reflected in a dependence on the gauge fixing vector n introduced in Sec.II. The second ingredient responsible for a non-vanishing single asymmetry is the specific mechanism in Fig.6(c) which naturally emerges from the dynamics of confined quarks and jet production suggested sometime ago by Craigie and Preparata [20]. Here $\pi\rho - \omega$ interference naturally couples quark states of different helicity. Further, a detailed computation based on Ref.20 which agrees with the same calculation within the quark pion sigma model, shows that the helicity flip analogue of fragmentation function, i.e. $\bar{D}_{+-}$ is of comparable size to $\bar{D}$ due to the large strength of the $\rho\omega\pi$ coupling. However the combination of factors, all less than unity, makes this asymmetry small. Numerical results will be given in Sec.IV.

	$\frac{d\sigma}{dt}$
$q_\alpha(1)q_\beta(2) + q_\alpha(3)q_\beta(4)$	$\pi\left(\frac{2}{3s}\right)^2 \alpha_s^2 \Big\{ h(h_1,h_4)\,h(h_2,h_3)\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + k(h_1,h_4)\,k(h_2,h_3)\,\frac{\hat{s}^2-\hat{u}^2}{\hat{t}^2}$ $+ \delta_{\alpha\beta}\left[h(h_1,h_3)\,h(h_2,h_4)\,\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} + k(h_1,h_3)\,k(h_2,h_4)\,\frac{\hat{s}^2-\hat{t}^2}{\hat{u}^2}\right]$ $- \frac{1}{6}\,(1 + h_1h_2 + h_1h_3 + h_1h_4 + h_2h_3 + h_2h_4 + h_3h_4 + h_1h_2h_3h_4)\,\frac{\hat{s}^2}{\hat{t}\hat{u}}\Big\}$
$q_\alpha(1)\bar{q}_\beta(2) + q_\delta(4)\bar{q}_\gamma(3)$	$\pi\left(\frac{2}{3s}\right)^2 \alpha_s^2 \Big\{\delta_{\alpha\beta}\,\delta_{\delta\gamma}\left[f(h_1,h_2)\,f(h_3,h_4)\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - g(h_1,h_2)\,g(h_3,h_4)\,\frac{\hat{t}^2-\hat{u}^2}{\hat{s}^2}\right]$ $+ \delta_{\alpha\delta}\,\delta_{\beta\gamma}\left[h(h_1,h_4)\,h(h_2,h_3)\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + k(h_1,h_4)\,k(h_2,h_3)\,\frac{\hat{s}^2-\hat{u}^2}{\hat{t}^2}\right]$ $- \frac{1}{3}\,\delta_{\alpha\beta}\,\delta_{\delta\gamma}\,\delta_{\alpha\delta}\left[f(h_1,h_2)\,f(h_3,h_4) - g(h_1,h_2)\,g(h_3,h_4) + h(h_1,h_4)\,h(h_2,h_3) - k(h_1,h_4)\,k(h_2,h_3)\right]\frac{\hat{u}^2}{\hat{s}\hat{t}}\Big\}$

Table 3.1

Table 3.2

	$\frac{d\sigma}{dt}$
$q_\alpha(1)q_\beta(2) \to q_\beta(3)q_\alpha(4)$	$\frac{1}{\pi}\left(\frac{\pi}{3\hat s}\right)^2 \alpha_s^2\Big\{(1-(s_1s_4))\,(1-(s_2s_3))\,\frac{\hat s^2+\hat u^2}{\hat t^2} + (1-(s_1s_4))\,(s_2s_3) + (1-(s_2s_3))\,(s_1s_4) + (s_1s_2)\,(s_3s_4) + (s_1s_3)\,(s_2s_4)$ $+\,\delta_{\alpha\beta}\Big\{(1-(s_1s_3))\,(1-(s_2s_4))\,\frac{\hat s^2+\hat t^2}{\hat u^2} + (1-(s_1s_3))\,(s_2s_4) + (1-(s_2s_4))\,(s_1s_3) + (s_1s_2)\,(s_3s_4) + (s_1s_4)\,(s_2s_3)$ $+\,\frac{1}{3}\frac{1}{\hat t\hat u}\Big\{\hat s^2\Big[(s_1s_2)\,(1-(s_3s_4)) + (s_3s_4)\,(1-(s_1s_2)) - (1-(s_2s_3)) - (1-(s_2s_4)) - (1-(s_1s_3)) - (1-(s_1s_4)) + 2\Big]$ $+\,\hat t^2\Big[-(s_2s_3)\,(1-(s_1s_4)) - (s_1s_4)\,(1-(s_2s_3)) + (s_2s_4)\,(1-(s_1s_3)) + (s_1s_3)\,(1-(s_2s_4)) - (s_1s_2) + (s_3s_4))\Big]$ $+\,\hat u^2\Big[(s_2s_3)\,(1-(s_1s_4)) + (s_1s_4)\,(1-(s_2s_3)) - (s_2s_4)\,(1-(s_1s_3)) - (s_1s_3)\,(1-(s_2s_4)) - ((s_1s_2) + (s_3s_4))\Big]\Big\}\Big\}\Big\}$
$q_\alpha(1)\bar q_\beta(2) \to q_\delta(4)\bar q_\gamma(3)$	$\frac{1}{\pi}\left(\frac{\pi}{3\hat s}\right)^2 \alpha_s^2\Big\{\delta_{\alpha\beta}\,\delta_{\delta\gamma}\Big[(1-(s_1s_2))\,(1-(s_3s_4))\,\frac{\hat t^2+\hat u^2}{\hat s^2} + (s_1s_2)\,(1-(s_3s_4)) + (s_3s_4)\,(1-(s_1s_2)) + (s_1s_3)\,(s_2s_4) + (s_1s_4)\,(s_2s_3)\Big]$ $+\,\delta_{\alpha\delta}\,\delta_{\beta\gamma}\Big[(1-(s_1s_4))\,(1-(s_2s_3))\,\frac{\hat s^2+\hat u^2}{\hat t^2} + (s_1s_4)\,(1-(s_2s_3)) + (s_2s_3)\,(1-(s_1s_4)) + (s_1s_3)\,(s_2s_4) + (s_1s_2)\,(s_3s_4)\Big]$ $+\,\frac{1}{3}\,\delta_{\alpha\beta}\,\delta_{\delta\gamma}\,\delta_{\alpha\delta}\,\frac{1}{\hat s\hat t}\Big\{\hat s^2\Big[-(s_1s_2)\,(1-(s_3s_4)) - (s_3s_4)\,(1-(s_1s_2)) + (s_1s_4)\,(1-(s_2s_3)) + (s_2s_3)\,(1-(s_1s_4)) - ((s_1s_3) + (s_2s_4)$ $+\,\hat t^2\Big[(s_1s_2)\,(1-(s_3s_4)) + (s_3s_4)\,(1-(s_1s_2)) - (s_1s_4)\,(1-(s_2s_3)) - (s_2s_3)\,(1-(s_1s_4)) - ((s_1s_3) + (s_2s_4))\Big]$ $+\,\hat u^2\Big[(s_1s_3)\,(1-(s_2s_4)) + (s_2s_4)\,(1-(s_1s_3)) - (1-(s_1s_2)) - (1-(s_1s_4)) - (1-(s_2s_3)) - (1-(s_3s_4)) + 2\Big]\Big\}\Big\}$

Table 3.3

Process ab → cd	a_{LL}^{ii}	$\frac{d\hat\sigma}{dt}$
$qq \to qq$	$\dfrac{\frac{\hat s^2-\hat u^2}{\hat t^2} + \frac{\hat s^2-\hat t^2}{\hat u^2} - \frac{2}{3}\frac{\hat s^2}{\hat t\hat u}}{\frac{\hat s^2+\hat u^2}{\hat t^2} + \frac{\hat s^2+\hat t^2}{\hat u^2} - \frac{2}{3}\frac{\hat s^2}{\hat t\hat u}}$	$\frac{4}{9}\left(\frac{\hat s^2+\hat u^2}{\hat t^2} + \frac{\hat s^2+\hat t^2}{\hat u^2} - \frac{2}{3}\frac{\hat s^2}{\hat t\hat u}\right)$
$qq' \to qq'$	$\frac{\hat s^2-\hat u^2}{\hat s^2+\hat u^2}$	$\frac{4}{9}\,\frac{\hat s^2+\hat u^2}{\hat t^2}$
$q\bar q \to q'\bar q'$	-1	$\frac{4}{9}\,\frac{\hat t^2+\hat u^2}{\hat s^2}$
$q\bar q \to q\bar q$	$\dfrac{\frac{\hat s^2-\hat u^2}{\hat t^2} - \frac{\hat t^2+\hat u^2}{\hat s^2} + \frac{2}{3}\frac{\hat u^2}{\hat s\hat t}}{\frac{\hat s^2+\hat u^2}{\hat t^2} + \frac{\hat t^2+\hat u^2}{\hat s^2} - \frac{2}{3}\frac{\hat u^2}{\hat s\hat t}}$	$\frac{1}{2}\left(\frac{\hat s^2}{\hat t^2} + \frac{\hat u^2}{\hat t^2} + \frac{\hat t^2+\hat u^2}{\hat s^2} - \frac{2}{3}\,\frac{\hat u^2}{\hat s\hat t}\right)$
$q\bar q \to gg$ (*)	-1	$\frac{32}{27}\,\frac{\hat t^2+\hat u^2}{\hat u\hat t} - \frac{8}{3}\,\frac{\hat t^2+\hat u^2}{\hat s^2}$
$qg \to qg$ (*)	$\dfrac{\frac{\hat s^2-\hat u^2}{\hat t^2} - \frac{4}{9}\,\frac{\hat s^2-\hat u^2}{\hat u\hat s}}{\frac{\hat s^2+\hat u^2}{\hat t^2} - \frac{4}{9}\,\frac{\hat s^2+\hat u^2}{\hat u\hat s}}$	$\frac{\hat s^2+\hat u^2}{\hat t^2} - \frac{4}{9}\,\frac{\hat s^2+\hat u^2}{\hat u\hat s}$
$gg \to q\bar q$ (*)	-1	$\frac{1}{2}\left(\frac{1}{3}\,\frac{\hat u^2+\hat t^2}{\hat u\hat t} - \frac{3}{4}\,\frac{\hat t^2+\hat u^2}{\hat s^2}\right)$
[illegible] (*)	$-3 + \frac{2\hat s^2}{\hat u\hat t} + \frac{\hat u\hat t}{\hat s^2}$ / [illegible]	[illegible]

TABLE CAPTIONS

<u>Table 3.1</u> Differential cross-sections for the sub-processes $qq \to qq$ and $q\bar{q} \to q\bar{q}$. $\alpha, \beta, \gamma, \delta$ flavour indices, $\hat{s} = (p_1+p_2)^2$, $\hat{t} = (p_1-p_4)^2$, $\hat{u} = (p_2-p_4)^2$. $\{h_i\} = \pm$: helicities $i = 1,2,3,4$. $f(h_i,h_j) = 1 - \delta_{h_i h_k}$, $g(h_i h_j) = \frac{1}{2}(h_i - h_j)$, $h(h_i,h_j) = \delta_{h_i h_j}$, $k(h_i h_j) = \frac{1}{2}(h_i + h_j)$.

<u>Table 3.2</u> Differential cross-sections for the sub-processes $qq \to qq$ and $q\bar{q} \to q\bar{q}$. $\alpha, \beta, \gamma, \delta$ flavour indices, $\hat{s}, \hat{t}$ and $\hat{u}$ as defined in Table 3.1. $\{s_i\} = \uparrow, \downarrow$: spin component normal to the scattering plane, $(\uparrow \cdot \uparrow) = (\downarrow \cdot \downarrow) = -1$, $(\uparrow \cdot \downarrow) = (\downarrow \cdot \uparrow) = +1$.

<u>Table 3.3</u> Basic asymmetries and unpolarized cross-sections for the parton-parton processes $ab \to cd$ discussed in the text. ((*): These have been taken from Ref.24.) All cross-sections contain a common factor $\pi(\alpha_s/\hat{s})^2$. $\hat{s} + \hat{t} + \hat{u} = 0$.

IV. INPUTS AND ESTIMATES

We discuss here the parton distributions and fragmentation functions and their asymmetries or spin differentials involved in the calculation of the processes described in Sec.III. Where possible we will give estimates of the expected asymmetry based on our present knowledge of the inputs. However we must stress that the spin differentials of the parton densities are essentially unknown, since all one can predict in QCD perturbation theory is their evolution away from some point $Q^2 = Q_0^2$, i.e. the primordial distributions must be determined directly from experiment. There are on the other hand some constraints (for example the Bjorken sum rules) on these functions which can be used to make models. Estimates based on various models will be given below. Inspection of the cross-sections listed in Sec.III, shows that the whole system is over determined, so a complete set of experiments on spin and helicity asymmetries could in principle test QCD very precisely. We anticipate that valuable information on the non-leading orders in QCD will be obtained by such studies. Formulae of the type wehave been considering have the general structure

$$A = \frac{\int \delta_z D_A^a \; \delta_z D_B^b \; \dots \; (a \; d\sigma)_{ab\to\dots}}{\int D_A^a \; D_B^b \; \dots \; (d\sigma)_{ab\to\dots}} , \tag{4.1}$$

where a is the corresponding asymmetry at the parton level. One can crudely estimate the asymmetry given by (4.1) by approximating it by

$$A = \frac{\langle \delta_z D_A^a \rangle}{\langle D_A^a \rangle} \; \frac{\langle \delta_z D_B^b \rangle}{\langle D_B^b \rangle} \; \dots \; \langle a \rangle , \tag{4.2}$$

where $\langle \;\; \rangle$ denotes an average value over the parton phase space $dx_a \, dx_b \dots$. Clearly this can at best be a very crude guide, however it is nevertheless useful and implies for example, if $A \sim 1$ that each factor in (4.2) must be individually of order unity, since

$$\left. \begin{array}{l} -1 \leqslant \delta_z D_A^a / D_A^a \leqslant 1 \\ -1 \leqslant a \leqslant 1 \end{array} \right\} . \tag{4.3}$$

This means that a large measured asymmetry implies large spin (or helicity) differentials for all factorized parts of the process in the parton model.

Parton distributions

The parton distributions satisfy the Bethe-Salpeter equation

$$D_A^a(x,Q^2) = D_A^a(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{ab}(z) D_A^b\left(\frac{x}{z},k^2\right) \quad . \tag{4.4}$$

Similarly the spin differential satisfies the equation

$$\delta_2 D_A^a(x,Q^2) = \delta_2 D_A^a(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} \delta_2 P_{ab}(z)\, \delta_2 D_A^b\left(\frac{x}{z},k^2\right) \quad . \tag{4.5}$$

The calculation of the asymmetry kernels is relatively straightforward in the leading order in QCD and the technique we use is described in the appendix. We also note that the same equations hold for helicity differentials with the replacement of $\delta_2 D$ by $\Delta_2 D$.

Given an input or primordial parton distribution $\delta_2 D_A^a(x,Q_0^2)$ for some chosen initial Q_0^2, these equations can be used to calculate the evolution with Q^2. This can be done in the form of a Gaemers and Buras [21] analysis or for the purpose of making estimates, by iterating once; which is the procedure we have adopted by writing

$$D_A^a(x,Q^2) = D_A^a(x,Q_0^2) + \frac{\alpha_s(Q_0^2)}{2\pi} \log Q_0^2/\Lambda^2 \; \eta(Q^2) \int_x^1 \frac{dz}{z} P_{ab}(z)\, D_A^b\left(\frac{x}{z},Q_0^2\right) \tag{4.6}$$

and

$$\delta_2 D_A^a(x,Q^2) = \delta_2 D_A^a(x,Q_0^2) + \frac{\alpha_s(Q_0^2)}{2\pi} \log \frac{Q_0^2}{\Lambda^2} \; \eta(Q^2) \int_x^1 \frac{dz}{z} \delta_2 P_{ab}(z)\, \delta_2 D_A^b\left(\frac{x}{z},Q_0^2\right) \quad , \tag{4.7}$$

where $\eta(Q^2) = \log\{\log(Q^2/\Lambda^2)/\log(Q_0^2/\Lambda^2)\}$.

We shall principally be interested in the case $A = p$ or $\vec{p}$, so we shall list all the relevant primordial distributions for only these processes for which $a = u,d,s$ and g. For most cases we shall neglect the strange quark component at least in estimating the asymmetries. We adopt the following parametrization of the primordial distributions:

$$D_p^f(x) = \sum_{i=1}^{8} c_i(f)\, x^{q_i} (1-x)^{p_f} \quad , \quad (f:\ \text{flavor}) \quad . \tag{4.8}$$

We have used the Feynman and Field fit [22] and fitted the constants $(c_i(f), \ldots)$. For $a = u,d,\bar{u},\bar{d}$ and s we list (c_i, p_f) in Table 4.1, while $q_i = (-1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2})$ for $i = 1,\ldots,8$ respectively, independent of flavour. For the gluon distribution we have used the so-called "naive" parametrization of Ref. ..., i.e. $D_p^g(x) = \frac{3(1-x)^5}{x}$. For the spin differentials or asymmetries of the parton distributions we have the following two models, which have been presented in detail in Refs.24, 25 and 11. They are constrained by the Bjorken sum rule [7]

$$2\int_0^1 dx \left[G_1^p(x,Q^2) - G_1^n(x,Q^2) \right] = \frac{1}{3}\frac{G_A}{G_V} \tag{4.9}$$

which is equivalent to

$$\int_0^1 dx \left[\Delta_2 D_p^u - \Delta_2 D_p^d + \Delta_2 D_p^{\bar{u}} - \Delta_2 D_p^{\bar{d}} \right] = \frac{G_A}{G_V} \tag{4.10}$$

where $G_A/G_V \approx 1.23$. Also we have

$$\int_0^1 dx \quad G_2(x, Q^2) = 0 \tag{4.11}$$

which follows from angular momentum conservation so that we also have

$$\int_0^1 dx \left[\delta_2 D_p^u - \delta_2 D_p^d + \delta_2 D_p^{\bar{u}} - \delta_2 D_p^{\bar{d}} \right] = \frac{G_A}{G_V} \tag{4.12}$$

The first model which we call the conservative model following Ref.24, assumes the spin of the proton is carried by the valence quarks. From the SU(6) wave function this would imply

$$\int_0^1 dx \quad \Delta_2 D_p^u = \int_0^1 dx \, \delta_2 D_p^u = \frac{4}{3}$$

$$\int_0^1 dx \quad \Delta_2 D_p^d = \int_0^1 dx \, \delta_2 D_p^d = -\frac{1}{3} \tag{4.13}$$

which does not satisfy the Bjorken sum rule. This is corrected in the model by allowing the sea quarks to carry a compensating part of the spin asymmetry. The resulting parametrization is given in Table 4.2. In our calculations we have neglected the gluon contribution to the spin asymmetry, however in general it will also be correlated with the proton helicity.

The second model is due to Carlitz and Kaur [25]. This would assume that the valence quarks loose their memory of the spin of the proton as we go to small x and the sea component becomes more important. However they always assume that the sea quarks are unpolarized. The particular form used in Ref.11 is given by the parametrization in Table 4.3 and will form the basis of our estimates in terms of this second model. We use the same parametrization for spin and helicity, since they satisfy the same Bjorken sum rules. However, we suspect this will turn out to be an over simplification.

In Table 4.4 we list the complete set of kernels and corresponding anomalous dimensions. Here we only consider the effect of transverse gluons, longitudinal gluons will enter if we go beyond the leading logarithmic order in QCD.

Because the soft gluon regularization of the branching kernels cannot be handled numerically, we modify the equations (4.7) by integrating by parts with the result

$$D^q(x,Q^2) = D^q(x,Q_0^2) + y\, c_F \sum_i f^q(x,p_i,q_i) \; c_i\, x^{p_i} (1-x)^{q_i} ,$$

$$y = \frac{\alpha_s(Q_0^2)}{2\pi} \log\frac{Q_0^2}{\Lambda^2} \log\left\{ \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right\}$$

where

$$f^q(x,p,q) = x + \frac{x^2}{2} + 2\log(1-x) + \int_0^1 dy \; \phi(y,x,p,q)$$

with

$$\phi(y,x,p,q) = \begin{cases} (1+z^2)/y \left[\dfrac{(1-y)^p}{z^{p+q+1}} - 1 \right] , & 10^{-4} \le y \le 1 \\ 2\left[(p+q+1)(1-x) - p \right] + O(y) , & y \le 10^{-4} \end{cases} ,$$

$$z = 1 - y(1-x) . \tag{4.14}$$

For the case $\delta_2 D^q$ we have

$$\delta_2 D^q(x,Q^2) = \delta_2 D^q(x,Q_0^2) + y\, c_F \sum_i f^{asym.}(x,p_i,q_i) \; c_i\, x^{p_i} (1-x)^{q_i} ,$$

where

$$f^{asym.}(x,p,q) = 2x + 2\log(1-x) + \int_0^1 dy \; \phi^{asym.}(y,x,p,q)$$

with

$$\phi^{asym}(y,x,p,q) = \begin{cases} 2z\left[\dfrac{(1-y)^p}{z^{p+q+1}} - 1\right]/y & , \quad 10^{-4} \lesssim y \leq 1 \\ 2\left[(p+q+1)(1-x) - p\right] & , \quad y \leq 10^{-4} \end{cases} \qquad (4.15)$$

<u>Fragmentation functions</u>

The same evolution equations govern the fragmentation functions, with a slightly different kernel $\bar{P}_{ab}(z)$ namely

$$\bar{D}_a^A(x,Q^2) = \bar{D}_a^A(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} \bar{P}_{ab}(z)\, \bar{D}_b^A\left(\frac{x}{z}, k^2\right) , \qquad (4.16)$$

where

$$\bar{P}_{ab}(z) = \frac{1}{z} P_{ba}\left(\frac{1}{z}\right) \qquad (4.17)$$

For $q \to q$ $P_{qq}(z)$ satisfies

$$\int_0^1 dz\, P_{qq}(z) = 0 \qquad (4.18)$$

which is the Bloch-Nordsieck cancellation of real and virtual gluon contributions. This ensures the quark charge sum rule

$$\int_0^1 dx\, D_A^q(x,Q^2)\Big|_{val.} = \int_0^1 dx\, D_A^q(x,Q_0^2)\Big|_{val.} = n_A^q \qquad (4.19)$$

(i.e. number of valence quarks is independent of Q^2).

The corresponding statement for fragmentation functions is the momentum sum rule

$$\sum_A \int_0^1 dx\, x\, \bar{D}_q^A(x,Q^2) = \sum_A \int_0^1 dx\, x\, \bar{D}_q^A(x,Q_0^2) = 1 \qquad (4.\ )$$

which follows from

$$\int_0^1 dz\, z\, \bar{P}_{qq}(z) = 0 . \qquad (4.\ )$$

Hence Bloch-Nordsieck cancellations between real and virtual gluons is intimately tied up with the momentum sum rule in the case of fragmentation functions.

We shall be concerned with the following set of fragmentation functions:

$$u \to \pi^+, \pi^0, \pi^-$$
$$d \to \pi^+, \pi^0, \pi^-$$
$$s \to \Lambda \quad .$$

For our purpose it will be sufficient to take only those fragmentation functions arising from valence quarks, assuming the others are negligible. In this way one arrives at the simple parametrization

$$\bar{D}_u^{\pi^+} = \bar{D}_d^{\pi^-} = \bar{D}_{d,u}^{\pi^0} = \bar{D}(z) \qquad (4.\ )$$

with

$$\bar{D}(z) = 1.05\,(1-x)^3 + .05 \quad . \qquad (4.\ )$$

The latter is the parametrization used by Sehgal in Ref.26. A non-valence quark fragmentation can be parametrized by a function $\omega(z) = D_u^{\pi^-}(z)/D_u^{\pi^+}(z)$ as in the Feynman and Field parameterization [22], however we have not done so in what follows.

For the $s \to \Lambda$ fragmentation function, we use the simple plausible ansatz

$$z\, \bar{D}_s^{\Lambda}(z) = 2(1-z)^3 \quad . \tag{4.24}$$

The form is dictated by the quark counting rules [27] as $z \to 1$, and the momentum sum rule normalization condition

$$\int_0^1 dz\, z \left[\bar{D}_s^{\Lambda} + \bar{D}_s^{\Sigma} \right] = 1 \quad , \tag{4.25}$$

where we neglect the Ξ and spin $\frac{3}{2}$ contributions. However it should be stressed that the results are quite sensitive to the $(1-z)^3$ power and new data on $\Lambda\bar{\Lambda}$ production in e^+e^- collisions would be very useful in providing a better phenomenological understanding of the $\bar{D}_f^{\Lambda}$ decay functions.

For the spin asymmetry in the case of $s^{\uparrow} \to \Lambda^{\uparrow}$ we use the simple leading quark ansatz (i.e. all the spin is carried by the leading quark)

$$\delta_2 \bar{D}_s^{\Lambda}(z) = \lambda\, \bar{D}_s^{\Lambda}(z) \quad . \tag{4.26}$$

Estimates of the asymmetries

With the above input, we are in a position to make some rough estimates of the asymmetries discussed in Sec.III, beginning with the simple process, namely deep inelastic scattering with polarized electrons off polarized protons. Using the two models of the spin differentials $\delta_2 D_p^q$ (i.e. the conservative and Carlitz-Kaur) we obtain the curves in Figs.7(a) and (b) plotted for the combination of helicity dependent structure functions defined in Subsec.3.1

$$G_1(x,Q^2) + G_2(x,Q^2) = \sum_{f=u,d} e_f^2\, \delta_2 D^f(x,Q^2) \quad . \tag{4.27}$$

One sees the same sort of scaling violation as for the unpolarized structure functions, the effect being more marked however for the Carlitz-Kaur model.

In Figs.8(a) and (b) we plot the ratio $\delta_2 D_p^u(x,Q^2)/D_p^u(x,Q^2)$ for the two models. We notice that the perturbative effects of QCD, i.e. gluon radiation and pair creation, only influence the small x region, where the effect is to modestly depolarize the quarks.

In the case of helicity asymmetries, the ratio $\Delta_2 D_p^u/D_p^u$ has no scaling violations as $x \to 1$ because $\Delta_2 D_p^u$ satisfies the same Altarelli-Parisi evolution equation as D_p^u. However as $x \to 0$ there will be a Q^2 dependence due to the mixing with gluon operators. If we neglect the latter the $Q^2 = 10\ \mathrm{GeV}^2$ curves in Figs.8(a) and (b) can be taken as an indication also of $\Delta_2 D_p^u/D_p^u$. The latter ratio approximately determines the neutrino asymmetry discussed in Subsec.3.2, in particular for large y, where

$$A_{LL}^{\nu}(x,y,Q^2) \underset{y\to 1}{\simeq} - \frac{\Delta_2 D_p^d(x,Q^2)}{D_p^d(x,Q^2)} \tag{4.28}$$

which depending on the model we take has values ranging between 30-50% for large x. Hence neutrino interactions with polarized nucleons are potentially a valuable source of information about quark helicity densities inside the protons. For anti-neutrinos in the large y limit we have

$$A_{LL}^{\bar{\nu}}(x,y,Q^2) \underset{y\to 1}{\simeq} - \frac{\Delta_2 D_p^{\bar{d}}(x,Q^2)}{D_p^{\bar{d}}(x,Q^2)} \quad , \tag{4.29}$$

which in the Carlitz-Kaur model is zero, being different from zero in the conservative model. Hence the latter measurement would be a useful means of testing these models and how the helicity of the nucleon is reflected in the sea distribution.

Another region of interest is when $x \to 1$ and valence quarks dominate. In this case (because $u^- \to d$ and $d^+ \to u$) we have approximately

$$A_{LL}^{\nu} \underset{x\to 1}{\simeq} - \frac{\Delta_2 D_p^d}{D_p^d} \simeq .3$$

while

$$A_{LL}^{\bar{\nu}} \underset{x\to 1}{\simeq} - \frac{\Delta_2 D_p^u}{D_p^u} \simeq -.4$$

where these numbers correspond to the conservative model discussed above.

Before leaving deep inelastic processes there is another potentially interesting area which can be explored, namely semi-inclusive processes like $e\vec{p} \to e'\vec{\Lambda} + X$ (see Subsec.3.3). Here the polarization of the outgoing Λ is studied through its decay distribution. This latter process is interesting because it picks out the strange quark distribution in the proton. The order of magnitude of the asymmetry given in Subsec.3.3 (Eq.(3.14))can be estimated from the approximate form

$$A^{if}_{NN} = \left\langle \frac{2(1-y)}{1+(1-y)^2} \right\rangle \frac{\langle \delta_2 D^s_p \rangle}{\langle D^s_p \rangle} \frac{\langle \delta_2 \bar{D}^{\Lambda}_s \rangle}{\langle \bar{D}^{\Lambda}_s \rangle} \tag{4.30}$$

where $\langle \quad \rangle$ denotes the average values over the kinematical region we are interested in.

We can expect the last factor to be of order unity if the leading s quark transmits all its helicity to the Λ. On the other hand, the factor $\delta_2 D^s_p/D^s_p = 0$ in the Carlitz-Kaur model, while in the conservative model [24] $\delta_2 D^s_p \sim 0.2 \times (2-x) D^s_p$, which gives an average value $\langle \delta_2 D^s_p \rangle / \langle D^s_p \rangle \sim 10\%$. Hence this process provides another sensitive measurement of the detailed nature of parton polarizations. A point analysis of all the deep inelastic processes, of which we have only considered a sample, will provide a complete picture of the nature of parton spin and helicity distributions inside polarized nucleons.

The Drell-Yan mechanism and massive lepton

Pair production in hadron-hadron collisions provides another area in which QCD has been intensively studied recently [9]. One of the main points of discussion is the role of next to leading logarithms, which are thought to be of major importance even at the largest Q^2 measured. For this reason the measurement of asymmetries will be very valuable. We have made estimates of both the reflected A^{ii}_{LL} and the transmitted asymmetries discussed in Subsec.3.4. In the latter the polarization of one of the outgoing muons is monitored as suggested by Soffer and Taxil [10]. As we pointed out in Subsec.3.4 we disagree with the explicit formulae given in Ref.10.and hence the curves we give below differ from those in the latter reference. The beam-target reflected asymmetry for $\vec{P}\vec{P}$ scattering is plotted as a function of Q^2 for $x_F = 0$ in Fig.9(a). For this case the valence quarks dominate and because of $q \to \bar{q}$ symmetry we have approximately

$$A^{ii}_{LL}(p\bar{p}) = -\left[\frac{\Delta_2 D^v_p(x)}{D^v_p(x)}\right]^2 \tag{4.31}$$

where $x = \sqrt{Q^2}/\sqrt{s}$.

In the case of proton-proton, the asymmetry depends crucially on the polarization of sea quarks and will vanish in the case of the Carlitz-Kaur model. We have plotted the distribution given by the conservative model in Fig.9(b) for fixed $x_F = 0$, for which the following formula is a good approx ion:

$$A^{ii}_{LL}(pp) = -\frac{\Delta_2 V(x)}{V(x)} \frac{\Delta_2 S(x)}{S(x)} \tag{4.3}$$

where $V(x) = \frac{4}{9} D^u_p + \frac{1}{9} D^d_p$ is the valence quark distribution and $S(x)$ is the sea quark density.

In our calculations no consideration was given to next to leading logarithms in QCD and the possible breakdown of factorization discussed in Sec.II. We shall return to this question in Sec.V.

In the case of the transmitted asymmetry A^{if}_{LL} we have the curve in Fig.9(c). Here we follow Ref.10 and plot the curve as a function of x_F of the virtual photon for fixed Q^2 ($Q^2 = 50\ \mathrm{GeV}^2$). Fig.9(c) corresponds to $\vec{P}P \to \vec{\mu}\mu^- + X$. We see in agreement with Ref.10 that there is a sizeable asymmetry. However in contrast we find the asymmetry changes sign at $x_F =$ which is a consequence of the difference in the asymmetry formula.

Hadron production at large transverse momentum

This is another interested area to measure polarization asymmetries. The beam target polarization (reflected) asymmetry has been studied extensiv in Refs.2 and 11 to which we refer for a detailed numerical analysis. For completeness we give here the driving asymmetries of the parton sub-process a) $\vec{u}\vec{u} \to uu$, b) $\vec{u}\vec{d} \to ud$, c) $qg \to qg$, d) $gg \to gg$ and e) $q\bar{q} \to q\bar{q}$. These are plotted as a function of the variable $z = -\hat{u}/\hat{s}$ in Figs.10(a)-(e). As a gui to the order of magnitude of the expected asymmetry in leading order in QCD, we plotted in Fig.11 the expected asymmetry for (a) $\vec{P}\vec{P} \to \pi^0 X$; (b) $\vec{P}\vec{P} \to$ JET and $\vec{P}\vec{P} \to \pi^{\pm} X$ in (c) the conservative and (d) the Carlitz-Kaur models. [The curves have been taken from Ref.24.]

The transmitted asymmetry can be measured in the processes $\vec{P}\vec{P} \to \vec{\Lambda} + X$ and $\vec{P}P \to \vec{\Lambda} + X$. These possibilities have been given in detail by us in Ref.12, so we give here only a summary. The driving asymmetry $\vec{u}\vec{u} \to \vec{s}\vec{s}$ is shown in Fig.12. The resulting $\vec{P}\vec{P} \to \vec{\Lambda} + X$ asymmetries are given in Figs.13(a) and (b) for trigger angles $\theta_{tr} = 60^\circ$ and 30° ($s = 1000$ GeV^2). In Figs.13(a) and (b) we have neglected the effect of the sub-process $gg \to s\bar{s}$. The latter is shown in Fig.13(c) using only the conservative distributions.

Prompt photons at large p_T

The asymmetry in prompt photon production in proton-proton collisions is determined by the dominant sub-process $g\vec{q} \to gq$, which has a maximal asymmetry $a_{LL}^{if} = -1$, since $|M_{++}|^2 = 0$. However we have made no model of gluon polarization which is very much an open question. We can write as a rough guide

$$A_{LL}^{ii}(pp \to \gamma X) = -\frac{\langle \Delta_2 D_p^g \rangle}{\langle D_p^g \rangle} \frac{\langle \Delta_2 D_p^q \rangle}{\langle D_p^q \rangle} \simeq -.4 \frac{\langle \Delta_2 D_p^q \rangle}{\langle D_p^q \rangle} , \tag{4.33}$$

Hence a sizeable asymmetry for this process would imply that the gluons inside a proton of definite helicity are appreciably polarized. By the same token this process is a good way of analysing the latter question. The conservative model gives zero for this asymmetry, while the Carlitz-Kaur model predicts a small but non-vanishing effect (see Fig.14).

A relative clean test of QCD comes from the reaction $\bar{P}P \to \gamma + X$, since we can expect approximately

$$A_{LL}^{ii} = -\frac{\langle \Delta_2 D_p^g \rangle^2}{\langle D_p^g \rangle^2} \simeq 10 - 30\ \% .$$

(The dominant sub-process $q\bar{q} \to \gamma g$ has an asymmetry $a_{LL}^{ii} = -1$.) We have made an estimate of the expected asymmetry based on the conservative model and plotted this in Fig.14 for $s = 1000$ GeV^2 and $p_T = 3$-10 GeV/c with a CMS trigger angle of 90°. In Fig.14 we have also plotted the estimated asymmetry for $PP \to \gamma X$ based on the Carlitz-Kaur model.

Finally we give an estimate of the single spin asymmetry - p_{out} correlation in the process $\vec{P}P \to \pi^+(p_T) + \text{Jet}(p_{out}) + X$. We have used the following additional inputs:

$$t\frac{d}{dt}\, \delta \bar{D}_q^\pi(z,t) = \frac{\alpha_s(t)}{2\pi}\int_z^1 \frac{dz'}{z'}\, \delta_2 P_{qq}\left(\frac{z}{z'}\right) \delta\, \bar{D}_q^\pi(z',t) \tag{4.34}$$

with the single spin differential

$$\delta\, \bar{D}_q^\pi(z) = \bar{D}_q^\pi(\uparrow) - \bar{D}_q^\pi(\downarrow) = \frac{1}{2}\, \bar{D}_q^\pi(z) . \tag{4.35}$$

This large asymmetry is suggested by the $q \to \pi,\rho,\omega$ cascade model discussed in Subsec.3.7 and emerges naturally in the dynamics of confined quarks in Ref.20. The estimated A^{out} asymmetry according to Eq.(3.38) is plotted in Fig.15 as a function of $x_{out} = p_{out}/p_T$ for π^+, π^0 and π^-, respectively. The effect is small but in principle measurable and shows an interesting dependence on trigger charge.

All the estimates given in this section are based on factorization in the parton model, with its simple parton probabilistic interpretation, in which the partons are treated like particles with definite spin. This picture is known to hold in leading order in QCD [4),5)]. However it is not clear if this kind of factorization holds beyond the leading order, although something like it has been suggested to hold [28)]. Clearly, any departure from it will show up in all the curves we have shown. Further, since non-leading terms, for example in Drell-Yan have a 50% effect, we have potentially important deviations. in asymmetry measurements which by the same token will play an important role in analysing this question. We end this article by discussing non-leading effects.

Table 4.1

	f				
	u	d	s,$\bar{s}$	$\bar{u}$	$\bar{d}$
c_1	0.17	0.17	0.10	0.17	0.17
c_2	1.545	0.932	0	0	0
c_3	7.12	-2.398	0	0	0
c_4	-66.282	19.126	0	0	0
c_5	240.104	-23.696	0	0	0
c_6	-353.608	-6.448	0	0	0
c_7	216.992	21.568	0	0	0
c_8	-43.872	-7.904	0	0	0
p_f	3	4	8	10	7

Table 4.2

$$\Delta u(x) = 0.44\, u_{val.}(x)$$

$$\Delta d(x) = -0.35\, d_{val.}(x)$$

$$\Delta\bar{q}(x) = 0.13\, (1-x)^{10}\, (2-x)$$

$$\Delta g(x) = 0.65\, \frac{(1-x)^6}{x}\, (1-(1-x)^2)$$

Table 4.3

$$\Delta u(x) = u_{val.}(x) - \frac{2}{3}\, d_{val.}(x)$$

$$\Delta d(x) = -\frac{1}{3}\, d_{val.}(x)$$

$$\Delta s(x) = \Delta\bar{q}(x) = 0$$

$$\Delta g(x) = 0.43\, (2-x)\, (1-x)^6$$

TABLE CAPTIONS

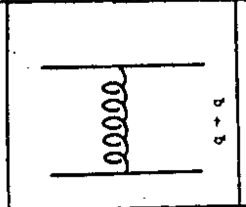
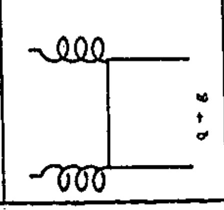
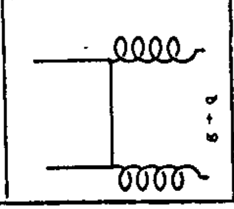
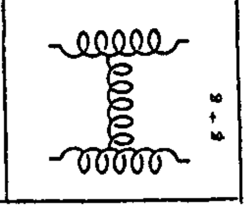
Table 4.1 Feynman and Field parametrization of unpolarized distribution functions (see Eq.(4.8)).

Table 4.2 Conservative SU(6) model for asymmetries of parton distributions.

Table 4.3 Carlitz-Kaur model for asymmetries of parton distributions as given in Ref.11.

Table 4.4 Unpolarized kernels (P), spin and helicity differentials ($\delta_2 P$ and $\Delta_2 P$ respectively) and corresponding anomalous dimensions, γ_n, γ_n^δ and γ_n^Δ. $\left\{N_f \equiv \text{number of flavours}; \int_0^1 dz\, \frac{F(z)}{(1-z)_+} \equiv \int_0^1 dz\, \ln(1-z)\, \frac{dF(z)}{dz}.\right\}$

Table 4.4

	P	$\delta_2 P$	$\Delta_2 P$	γ_n	γ_n^δ	γ_n^Δ
 q → q	$\lim_{\delta \to 0} \langle\frac{4}{3}\rangle \left[\frac{1+z^2}{1-z+\delta} - \delta(1-z)\left\{\int_0^{1-\delta} \frac{1+z'^2}{1-z'}\, dz'\right\}\right]$	$\lim_{\delta \to 0} \langle\frac{4}{3}\rangle \left[\frac{2z}{1-z+\delta} - \delta(1-z)\left\{\int_0^{\delta} \frac{1+z'^2}{1-z'}\, dz'\right\}\right]$	$\lim_{\delta \to 0} \langle\frac{4}{3}\rangle \left[\frac{1+z^2}{1-z+\delta} - \delta(1-z)\left\{\int_0^{\delta} \frac{1+z'^2}{1-z'}\, dz'\right\}\right]$	$\langle\frac{4}{3}\rangle \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n} \frac{1}{j}\right]$	$-\langle\frac{4}{3}\rangle \left[\frac{1}{2} + 2\sum_{j=2}^{n} \frac{1}{j}\right]$	$\langle\frac{4}{3}\rangle \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n} \frac{1}{j}\right]$
 q → g	$\langle\frac{4}{3}\rangle \frac{1+(1-z)^2}{z}$	0	$\langle\frac{4}{3}\rangle \frac{1-(1-z)^2}{z}$	$\langle\frac{4}{3}\rangle \frac{2+n+n^2}{n(n^2-1)}$	0	$\langle\frac{4}{3}\rangle \frac{n+2}{n(n+1)}$
 g → q	$\langle\frac{1}{2}\rangle \left[z^2 + (1-z)^2\right]$	0	$\langle\frac{1}{2}\rangle \left[z^2 - (1-z)^2\right]$	$\langle\frac{1}{2}\rangle \frac{2+n+n^2}{n(n+1)(n+2)}$	0	$\langle\frac{1}{2}\rangle \frac{n-1}{n(n+1)}$
 g → g	$2\langle 3\rangle \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \left(\frac{11}{2} - \frac{N_f}{3}\right)\delta(z-1)$	$\langle 3\rangle \left[\frac{2z}{(1-z)_+}\right] + \left(\frac{11}{6} - \frac{N_f}{9}\right)\delta(z-1)$	$\langle 3\rangle \left[\frac{1+z^4}{z} + \frac{1+z^4}{(1-z)_+} - \frac{(1-z)^3}{z}\right] + \left(\frac{11}{6} - \frac{N_f}{9}\right)\delta(1-z)$	$\langle 3\rangle \left[-\frac{1}{6} - \frac{N_f}{9} + \frac{2}{n(n-1)} + \frac{2}{(n+1)(n+2)} - 2\sum_{j=2}^{n} \frac{1}{j}\right]$	$\langle -3\rangle \left\{\frac{1}{6} + \frac{N_f}{9} + 2\sum_{j=2}^{n} \frac{1}{j}\right\}$	$\langle 3\rangle \left[\frac{11}{6} - \frac{N_f}{9} + \frac{2}{n} - \frac{4}{n+1} - 2\sum_{j=1}^{n-1} \frac{1}{j}\right]$

V. THE EFFECT OF NON-LEADING ORDERS IN QCD

Here we briefly touch on the crucial area of non-leading orders in QCD and the influence this will have on the asymmetry formulae discussed in Sec.III for the leading order in QCD. These formulae were derived on the basis of block-wise factorization of parton Green's functions in special gauges, so each parton line can be thought to carry definite momentum and helicity. However it is far from obvious that this should hold beyond the leading order. In the case of gluons one has the additional problem that longitudinal gluons contribute at sub-leading logarithmic orders, which is analogous to deviations from the Callan-Gross relation in next to leading order [29].

In deep inelastic scattering in a suitable non-covariant gauge only factorized diagrams like Fig.16(a) contribute in leading order. However if we are interested in next to leading contributions also Fig.16(b) contributes. We can take into account such contributions by writing $F_2(x,Q^2)$ in the form

$$F_2(x,Q^2) = \sum_i \int \frac{dx'}{x'} D_p^i(x',Q^2)\, \sigma^i\left(\frac{x}{x'},Q^2\right) , \tag{5.1}$$

where the cross-section $\sigma^i(z,Q^2)$ can be thought to be that of the sub-process $q_i\gamma_V \to X$ and is given to order $\alpha_s(Q^2)$ as:[9]

$$\sigma^i(z,Q^2) = e_i^2\,\delta(z-1) + e_i^2 \frac{\alpha_s(Q^2)}{2\pi} c_F\, 2z + \left(\sum_i e_i^2\right) \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{2}\, 4z(1-z) + \cdots \tag{5.2}$$

The first term corresponds to the Born cross-section without radiative corrections, while the second includes the effect of radiative corrections. However here we have an ambiguity because many of these corrections can be incorporated into the D's, for example, by making them satisfy a modified evolution equation of the form [30]

$$D^i(x,Q^2) = D^i(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \int_x^1 \frac{dz}{z} P_{ij}(k^2,z)\, D^j\left(\frac{x}{z},k^2\right) ,$$

where

$$P_{ij}(k^2,z) = \frac{\alpha_s(k^2)}{2\pi} c_F \left(\frac{1+z^2}{1-z}\right)_+ \quad \frac{\alpha_s^2(k^2)}{2\pi} c_F f(z) + \cdots \tag{5.3}$$

In fact one might resolve this ambiguity by simply defining the D's by requiring

$$F_2(x,Q^2) \equiv \sum_i e_i\, D_p^i(x,Q^2) \tag{5.4}$$

to all orders in perturbation theory. In the latter case one has to identify this quantity in all other hard processes. Before discussing how this has been done in the case of the Drell-Yan mechanism, we might ask what is the corresponding statement for the asymmetry. Here one would expect

$$2G_1(x,Q^2) = \sum_i \int_x^1 \frac{dx'}{x'}\, \delta_z D_p^i(x',Q^2)\; \delta_z \sigma^i\left(\frac{x}{x'},Q^2\right) \tag{5.5}$$

or by a similar redefinition to that given above ,

$$2G_1(x,Q^2) = \sum_i e_i^2\, \delta_z D_p^i(x,Q^2) . \tag{5.6}$$

However this procedure is likely to have a big influence on the size of the predicted asymmetries as we go to more complex processes. The reason is that the asymmetry formulae in Sec.III were derived on the basis that all the spin or helicity is transmitted from one "part" of the process to another on a single parton line, which takes one of two values (i.e. $\pm\frac{1}{2}$ for quarks and ± 1 for gluons). Clearly this is unlikely <u>a priori</u> to be true in general, since the above corrections involve radiative corrections to the basic sub-processes, in which a quark line is replaced by a gluon plus a quark. Furthermore, longitudinal gluon polarizations are likely to play a role.

Another important point concerns the spinology of parton dynamics. One generally assumes always spin averaged quantities enter. However just as the Callan-Gross relation only holds to leading order, single asymmetries start to contribute in lower QCD orders even in the case of unpolarized cross-sections. In Sec.II we saw that a given hard scattering process involving hadrons A,B,... and partons a,b,... respectively

$$\sigma_{AB\ldots} = \int D_A^a D_B^b \ldots \sigma^{ab\ldots} + \int \delta_a D_A^a D_B^b \ldots \delta_a \sigma^{ab\ldots} + \int D_A^a\, \delta_b D_B^b \ldots \delta_b \sigma^{ab\ldots} + \cdots \tag{5.7}$$

In general for all orders in QCD the notion that unpolarized parton densities can be used to describe $\sigma_{AB\ldots}$ is itself open to question. One might however expect that one can write the single asymmetries as a convolution of the type

$$\frac{d}{dt}\left(\delta D_A^a\right) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\,\delta P_{ab}(z)\; D_A^b\left(\frac{x}{z},t\right) \quad , \quad t = \log Q^2/\Lambda^2 \quad . \tag{5.8}$$

However juggling these factors between different processes would have to be carefully thought out.

For the above reasons we believe that the study of asymmetries in hard processes will do much to enhance our understanding of what kind of factorization comes out of QCD perturbation theory and further, how one can interpret what is happening in terms of the naive parton notions.

In the case of the Drell-Yan mechanism one can attempt to say something about the effect of next to leading order in the asymmetry formulae discussed in Subsec.3.4, following the formidable work of Altarelli, Ellis and Martinelli [9]. Let us consider the contribution of the sub-process $q\bar{q} \to \gamma^* g$ to this process (Fig.17(a)). The first point to note is that in leading logarithmic order, the whole contribution is contained in the definition of $D_p^u(x,Q^2)$ and in the case of the asymmetry $\delta_2 D_p^u(x,Q^2)$ to that order. However using the procedure in Eq.(5.4), when all the effects of $\sigma_{q_1}\ \gamma^* + X$ in (5.2) are incorporated into the definition of D_p^i, one finds, after identifying the same D_p^i in the Drell-Yan mechanism, a new residual effective cross-section $\sigma_{q\bar{q}} \to \gamma^*$ in the following convolution:

$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9s}\sum_i \int_{x_1}^1 \frac{dx_1'}{x_1'}\, D_p^i(x_1',Q^2)\int_{x_2}^1 \frac{dx_2'}{x_2'}\, D_p^{\bar{i}}(x_2',Q^2)\; \sigma_i\left(\frac{x_1}{x_1'}, \frac{x_2}{x_2'}, Q^2\right) \tag{5.9}$$

where

$$\sigma_i(z,z',Q^2) = e_i^2\Big\{\Big[1 + \frac{\alpha_s}{2\pi}c_F\left(\frac{4}{3}\pi^2+1\right)\Big]\delta(z-1)\,\delta(z'-1) + \delta(z-1)\frac{\alpha_s}{2\pi}c_F\, g(z') + \delta(z'-1)\frac{\alpha_s}{2\pi}c_F\, g(z) + \frac{\alpha_s}{2\pi}c_F\, h(z,z',\ldots)\Big\} \quad ,$$

$$g(z) = \left[-2 - 3z + (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{3}{2}\frac{1}{(1-z)_+}\right] \tag{5.10}$$

where $h(z,z',\ldots)$ is a somewhat more complicated mixed term [see Ref.9 for details].

The first term in (5.10) corresponds to the usual Drell-Yan piece (except for a constant which is due to the effect of radiative correction of the type in Fig.17(b) and the fact that Q^2 is positive so logarithms of the type $\mathrm{Re}(\log^2(-Q^2)) = (\log|Q^2|)^2 + \pi^2$ enter). One might now ask how the corresponding asymmetry formula looks like. Here there is a simplification, since for $q\bar{q} \to \gamma^* g$, $|M_{++}|^2 = 0$, the basic asymmetry of the sub-process is -1 like $q\bar{q} \to \gamma^*$ (i.e. $\delta_2\sigma_i = -\sigma_i$). This would naively lead to expect the appropriate formula to be

$$A_{LL}^{ii} = -\frac{\sum_i \int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}\;\delta_2 D_p^i(x_1',Q^2)\;\delta_2 D_p^{\bar{i}}(x_2',Q^2)\;\sigma^i\left(x_1/x_1', x_2/x_2', Q^2\right)}{\sum_i \int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}\; D_p^i(x_1',Q^2)\; D_p^{\bar{i}}(x_2',Q^2)\;\sigma^i\left(x_1/x_1', x_2/x_2', Q^2\right)} \quad . \tag{5.11}$$

However identifying $\delta_2 D$ in both deep inelastic and Drell-Yan is a very strong assumption about how helicity is transported into the process. Clearly the exercise in Ref.9 has to be repeated. If next to leading logarithms contribute 50% of the unpolarized cross-section as reported on by Altarelli in Ref.31, then the asymmetry A_{LL}^{ii} will be very dependent on how we treat the next to leading logarithms. Our purpose here is to raise the basic issues involved in asymmetry measurement at short distances.

Before leaving the Drell-Yan mechanism, one might ask if one can expect non-vanishing single asymmetries in next to leading order in QCD. There are two types of terms that contribute to the latter, namely

$$A_L(d\sigma) = \iint \delta_A D_p^i (s_A) D_p^{\bar{i}} \sigma^i + \iint \delta_2 D_p^i D_p^{\bar{i}} \delta\sigma^i \qquad (5.12)$$

The single asymmetry of the basic Born amplitude $q\bar{q} \to g\gamma^*$ vanishes identically, so the second term does not contribute even at next to leading order. The single spin asymmetry of the parton density $\delta_A D_A^i$ can be obtained from the equation

$$\delta_A D_A^i = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \, \delta P_{ij}(z) \, D_A^j \qquad (5.13)$$

where δD is the asymmetry kernel $|q_+ \to qg|^2 - |q_- \to qg|^2$, which vanishes identically for single gluon exchange. Hence the single asymmetry in Drell-Yan can only be non-zero at order α_s^2 beyond the leading order.

In the case of large p_T, Ellis et al. [32] carried out a similar analysis as that described above for Drell-Yan. However in this case there was the additional problem of the value of Q^2 in $\alpha_s(Q^2)$, which is not determined by the parton model calculation, except when everything scales like $s \sim p_T^2 \sim Q^2 \sim t$ etc., the choice does not effect the leading order. By trying different choices, for example

$$(1) \quad Q^2 = 2\hat{s}\hat{t}\hat{u}/[\hat{s}^2+\hat{t}^2+\hat{u}^2] \; ,$$

$$(2) \quad Q^2 = \hat{t}\hat{u}/\hat{s} \qquad (5.14)$$

(one could also choose different scales for different parts of the process) Ref.32 claims large effects depending on the choice and hence strong next to leading effects. One could try to minimize the effects of next to leading order, except that which is incorporated into the parton density $D_A^i(x,Q^2)$ by the appropriate choice of Q^2. However a priori this is not likely to have the same effect in the numerator of the asymmetry formula. This implies that the basic asymmetries A_{LL}^{ii}, A_{LL}^{if} in hadron production at large p_T are likely to be sensitive quantities to test this issue.

Finally we come to the ~~important area of~~ higher power (called higher twist in deep inelastic scattering) corrections. Here one also faces a purely kinematic effect associated with an improved scaling variable $x \to \xi = x + M^2 f(x)/Q^2 + O(Q^{-4})$. In deep inelastic scattering the variable turns out to be [33]

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}} \qquad (5.15)$$

However in other processes such as massive lepton pair production and hadron production at large p_T a substitution of the form $\xi_i = x_i + f_i(x_1,x_2,\ldots)M_i^2/Q^2$ is likely to be needed. This is a neglected and difficult kinematic problem, which can have an important bearing on the next to leading powers. We can perhaps call the latter the spill over higher power (or twist) contributions. The usual higher power terms correspond to diagrams in which more than one parton is exchanged between different parts of the process. In deep inelastic scattering for example one would consider the diagram Fig.18(a), which corresponds to the operators $\bar{q}\,0\,q\,\bar{q}\,0\,q$ in the O.P.E. In the case of massive pair production one has analogous diagrams like Fig.18(b) and in large p_T contributions like that in Fig.18(c). For a general process involving hadrons A,B,... Politzer has recently pointed out [34] that one can always write a general parton formula

$$\sigma^{AB\ldots}(p_A,p_B,\ldots) = \int dx_a \int dx_{a'} \ldots D_A^{aa'}(x_a,x_{a'},\ldots) \int dx_b \int dx_{b'} \, D_B^{bb'}(x_b,x_{b'},\ldots)$$

$$\ldots \; \sigma^{\{a\}\{b\}\ldots}(\{x_a p_A\},\{x_b p_B\},\ldots) \; . \qquad (5.16)$$

The corresponding asymmetry will be given by the formula

$$\Delta_2 \sigma^{AB} = \int \Delta_{A\{a\}} D_A^{\{a\}} \int \Delta_{B\{b\}} D_B^{\{b\}} \ldots \; \Delta_{\{a\}\{b\}} \, \sigma^{\{a\}\{b\}} \qquad (5.17)$$

where

$$\Delta_{A\{a\}} = \Delta_A \left[\Delta_a + \Delta_{a'} + \Delta_a \Delta_{a'} + \ldots \right]$$

$$\Delta_{\{a\}\{b\}} = \left(\Delta_a + \Delta_{a'} + \Delta_a \Delta_{a'} \right) \left(\Delta_b + \Delta_{b'} + \Delta_b \Delta_{b'} \right) \; .$$

[Recalling $\Delta_a D(s_a) = \frac{1}{2}[D(+) - D(-)]$.] The function $D_A^{aa'}(x_a, x_a')$ for example describes the probability of finding a parton of type a with fractional momentum x_a and parton a' with fractional momentum x_a' inside hadron A. The spin differentials are defined in a straightforward way from our notation and would typically describe the way helicity is divided between more than one parton, summing over all orbital angular momentum in each two parton channels. In each case one can in the parton model framework derive an asymmetry formula and we end this article by briefly illustrating this with an example which shows the potential of asymmetry measurements in resolving the higher power controversy in large p_T reactions. Let us consider the constituent interchange strategy of Blankenbecler, Brodsky and Gunion [35) for large p_T reactions, in which the cross-section for $AB \to CX$ is written as a series

$$E_c \frac{d\sigma}{d^3 p_c} = \frac{C_2}{(p_T^2 + m_2^2)^2} \epsilon^{N_2} + \frac{C_4}{(p_T^2 + m_4^2)^4} \epsilon^{N_4} + \dots$$

where $\epsilon = (1 - x_T)$ and each term corresponds to a power counting argument based on the number of quarks involved in the hard scattering process. [For example we have p_T^{-4} for four quarks, p_T^{-8} for 6 quarks as in $qM \to qM$, where M denotes a meson.] Let us consider $P\bar{P} \to \pi^+ + X$ which at the level of p_T^{-8} terms will involve the quark fusion mechanism $q\bar{q} \to MM$ in Fig.19. The basic formula for the unpolarized cross-section is given by

$$\left(E\frac{d\sigma}{d^3p}\right)_{pp\to\pi X} = \int dx_a\, dx_b\, D_p^q(x_a)\, D_p^{\bar{q}}(x_b) \left(E\frac{d\sigma}{d^3p}\right)_{q\bar{q}\to\pi X} = \frac{C_8}{p_T^8}(1-x_T)^{n_8},$$

$(n_8 \simeq 9)$.

The corresponding asymmetry is given by

$$A_{LL}^{ii}\left(E\frac{d\sigma}{d^3p}\right) = \int dx_a\, dx_b\, \Delta_2 D_p^q\, \Delta_2 D_p^{\bar{q}}\; a_{LL}^{ii}\left(E\frac{d\sigma}{d^3p}\right)_{q\bar{q}\to\pi X}.$$

The basic asymmetry can be worked out on the assumption $\bar{q}q \to \pi\pi$ with a simple γ_5 structure at the meson quark vertex *). A trivial calculation shows

*) What is relevant here is the asymptotic behaviour of meson wave function. Some progress has been made recently in determining the latter in QCD [36).

$|M_{++}(q\bar{q} \to \pi\pi)|^2 = 0$, so that $a_{LL}^{ii} = -1$. Assuming (as in the conservative model) $\Delta_2 D_p^q = \lambda D_p^q$ one obtains

$$A_{LL}^{ii}\left(E\frac{d\sigma}{d^3p}\right) = -\lambda^2 \frac{C_8}{p_T^8}(1-x_T)^{n_8}.$$

Hence the total asymmetry including the p_T^{-4} term is given by

$$A_{LL}^{ii} = -\frac{C_A(1-x_T)^{n_A} - \lambda^2 C_8\, p_T^{-4}(1-x_T)^{n_8}}{C_U(1-x_T)^{n_U} + C_8\, p_T^{-4}(1-x_T)^{n_8}}.$$

The scaling violations in the leading term give a behaviour of $p_T^{-n_{eff}(p_T)}$ (with $n_{eff}(p_T) < 4$ for smaller p_T) instead of p_T^{-4} in this formula. However we still expect a rapid transition in the asymmetry as we pass through the region in which the power behaved term dies out.

In conclusion we hope by this article to have demonstrated the potential physics interest in hard scattering short distance processes with polarized beams and target, in particular, if one assumes that the QCD perturbative framework can be used to describe these processes.

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APPENDIX

In this appendix we present the calculation of the asymmetry kernels in Eq.(2.13) in a transversity basis, in the leading order of QCD. Finally, we show how to obtain from the latter kernels in the helicity basis.

As far as the kernels involving quarks and gluons are concerned (but not gluons alone) the basic diagram we have to consider is that of Fig.20. There

$$p_{1\mu} \simeq (\beta_1 P;\, \vec p_{1\perp},\, \beta_1 P) \quad , \quad \vec p_{1\perp} = p_{1\perp}(\cos\varphi_1,\, \sin\varphi_1)$$

$$p_{2\mu} \simeq (\beta_2 P;\, \vec p_{2\perp},\, \beta_2 P) \quad , \quad \vec p_{2\perp} = p_{2\perp}(\cos\varphi_2,\, \sin\varphi_2)$$

$$q_\mu \simeq \left((\beta_1-\beta_2)P;\, \vec q_\perp = \vec p_{1\perp} - \vec p_{2\perp},\, (\beta_1-\beta_2)P\right) \quad , \quad \theta \simeq \frac{q_\perp}{(\beta_1-\beta_2)P} \ll 1 \,. \tag{A.1}$$

β_i denote the fractions of a large momentum P along the z direction and $\vec p_\perp$ are transverse momenta, here assumed small with respect to P. The amplitude corresponding to this diagram is proportional to

$$T_{s_1 s_2 s_3} = T^{(c)}\, \bar u(2)\, \not\epsilon(3)\, u(1) \tag{A.2}$$

where $T^{(c)}$ is the group colour factor, u's are the relativistic spinors corresponding to massless quarks

$$u(p,s) = \sqrt{E}\begin{pmatrix} 1 \\ \vec\sigma\cdot\vec p/E \end{pmatrix}\chi_s \quad , \quad \chi_{\uparrow,\downarrow} = \frac{1}{\sqrt2}\begin{pmatrix} \pm 1 \\ 1 \end{pmatrix} \tag{A.3}$$

($\vec\sigma$ are the Pauli matrices) and ϵ_μ are the gluon polarization vectors

$$\epsilon_\mu(\uparrow) \simeq (0;\, 1,\, 0,\, -\theta\cos\phi)\,,$$

$$\epsilon_\mu(\downarrow) \simeq (0;\, 0,\, 1,\, -\theta\sin\phi)\,. \tag{A.4}$$

A straightforward calculation gives for the expressions in Eq.(A.2)

$$T_{s_1 s_2 \uparrow} = -T^{(c)} P\sqrt{\beta_1\beta_2}\left[\left\{-2\theta\cos\phi + \frac{p_{1\perp}}{\beta_1 P}\cos\varphi_1 + \frac{p_{2\perp}}{\beta_2 P}\cos\varphi_2\right\}\delta_{s_1 s_2} - i\left\{\frac{p_{1\perp}}{\beta_1 P}\sin\varphi_1 - \frac{p_{2\perp}}{\beta_2 P}\sin\varphi_2\right\}\delta_{s_1,-s_2}\right] \,,$$

$$T_{s_1 s_2 \downarrow} = -T^{(c)} P\sqrt{\beta_1\beta_2}\left[\left\{-2\theta\sin\phi + \frac{p_{1\perp}}{\beta_1 P}\sin\varphi_1 + \frac{p_{2\perp}}{\beta_2 P}\sin\varphi_2\right\}\delta_{s_1 s_2} + i\left\{\frac{p_{1\perp}}{\beta_1 P}\cos\varphi_1 - \frac{p_{2\perp}}{\beta_2 P}\cos\varphi_2\right\}\delta_{s_1,-s_2}\right] \,. \tag{A.5}$$

The cases we are interested in are shown in Figs 21 (a), (b) and (c) for which the four vectors in Eq.(A.1) are identified, respectively, as follows:

(a)

$$p_{1\mu} \equiv k'_\mu = (\alpha' P;\, \vec 0,\, \alpha' P) \quad , \quad \vec p_{1\perp} = \vec 0 \,,$$

$$p_{2\mu} \equiv k_\mu = (\alpha P;\, \vec k_\perp,\, \alpha P) \quad , \quad \varphi_2 = \varphi \,,$$

$$q_\mu \equiv K_\mu \quad , \quad \theta \simeq \frac{k_\perp}{P(\alpha'-\alpha)} \quad , \quad \phi = \varphi + \pi \;; \tag{A.6}$$

(b)

$$p_{1\mu} \equiv k'_\mu = (\alpha' P;\, \vec 0,\, \alpha' P) \quad , \quad \vec p_{1\perp} = \vec 0 \quad , \quad \beta_1 = \alpha' \,,$$

$$q_\mu \equiv k_\mu = (\alpha P;\, \vec k_\perp,\, \alpha P) \quad , \quad \theta \simeq \frac{k_\perp}{\alpha P} \quad , \quad \phi = \varphi \,,$$

$$p_{2\mu} \equiv K_\mu = \left((\alpha'-\alpha)P;\, -\vec k_\perp,\, (\alpha'-\alpha)P\right) \quad , \quad \varphi_2 = \varphi + \pi \quad , \quad \beta_2 = \alpha'-\alpha \quad , \quad p_{2\perp} = k_\perp \;; \tag{A.7}$$

(c)

$$p_{1\mu} \equiv K_\mu = \left((\alpha-\alpha')P;\, \vec k_\perp,\, (\alpha-\alpha')P\right) \quad , \quad \beta_1 = \alpha-\alpha' \quad , \quad p_{1\perp} = k_\perp \quad , \quad \varphi_1 = \varphi_2 = \varphi \,,$$

$$p_{2\mu} \equiv k_\mu = (\alpha P;\, \vec k_\perp,\, \alpha P) \quad , \quad \beta_2 = \alpha \quad , \quad p_{2\perp} = k_\perp \,,$$

$$-q_\mu \equiv k'_\mu = (\alpha' P;\, \vec 0,\, \alpha' P) \quad , \quad \theta = 0 \,. \tag{A.8}$$

Defining the variable $z \equiv \alpha/\alpha'$ the corresponding spin dependent kernels are given by

(a)

$$P_{qq}(s_1, s_1'; z) = \frac{1}{2}\,\frac{z(1-z)}{k_\perp^2}\ \underset{color}{av.\sum}\ \sum_{s_3} T_{s_1 s_2 s_3} T^*_{s_1' s_2 s_3} \ :$$

$$P_{qq}(\uparrow,\uparrow; z) = P_{qq}(\downarrow,\downarrow; z) = \frac{1}{2}\langle 4/3\rangle \frac{(1+z)^2}{1-z} \ ,$$

$$P_{qq}(\downarrow,\uparrow; z) = P_{qq}(\uparrow,\downarrow; z) = \frac{1}{2}\langle 4/3\rangle (1-z) \ ; \qquad \text{(A.9)}$$

(b)

$$P_{Gq}(s_3, s_4; z) = \frac{1}{2}\,\frac{z(1-z)}{k_\perp^2}\ \underset{color}{av.\sum}\ \sum_{s_2} T_{s_1 s_2 s_3} T^*_{s_4 s_2 s_3} \ :$$

$$P_{Gq}(\uparrow,\uparrow; z) = P_{Gq}(\uparrow,\downarrow; z) = \frac{1}{2}\langle \tfrac{4}{3}\rangle \left[\frac{(2-z)^2}{z}\cos^2\varphi + z\sin^2\varphi\right] ,$$

$$P_{Gq}(\downarrow,\downarrow; z) = P_{Gq}(\downarrow,\uparrow; z) = \frac{1}{2}\langle \tfrac{4}{3}\rangle \left[z\cos^2\varphi + \frac{(2-z)^2}{z}\sin^2\varphi\right] , \qquad \text{(A.10)}$$

(c)

$$P_{qG}(s_2, s_3; z) = \frac{1}{2}\,\frac{z(1-z)}{k_\perp^2}\ \underset{color}{av.\sum}\ \sum_{s_1} T_{s_1 s_2 s_3} T^*_{s_1 s_2 s_3} \ :$$

$$P_{qG}(\uparrow,\uparrow; z) = P_{qG}(\downarrow,\uparrow; z) = -\frac{1}{2}\langle \tfrac{1}{2}\rangle \left[(2z-1)^2\cos^2\varphi + \sin^2\varphi\right] ,$$

$$P_{qG}(\downarrow,\downarrow; z) = P_{qG}(\uparrow,\downarrow; z) = -\frac{1}{2}\langle \tfrac{1}{2}\rangle \left[\cos^2\varphi + (2z-1)^2\sin^2\varphi\right] . \qquad \text{(A.11)}$$

For the calculation of the spin dependent kernels involving only gluons we have to consider the diagram in Fig.22(a). The corresponding amplitude is proportional to

$$T_{s_1 s_2 s_3} = T^{(c)}\, V_{\mu\lambda\nu}\, \epsilon^{\mu}(k', s_1)\, \epsilon^{\lambda}(K, s_3)\, \epsilon^{\nu}(k, s_2) \qquad \text{(A.12)}$$

with the basic three-gluon vertex in Fig.22(b) given by

$$V_{\mu\lambda\nu}(q_1, q_2, q_3) = (q_1 - q_2)_\nu\, g_{\mu\lambda} + (q_2 - q_3)_\mu\, g_{\lambda\nu} + (q_3 - q_1)_\lambda\, g_{\nu\mu} \ . \qquad \text{(A.13)}$$

Then the kernels are given by

$$P_{GG}(s_2, s_4; z) = \frac{1}{2}\,\frac{z(1-z)}{k_\perp^2}\ \underset{color}{av.\sum}\ \sum_{s_3} T_{s_1 s_2 s_3} T^*_{s_1 s_4 s_3} \ :$$

$$P_{GG}(\uparrow,\uparrow; z) = 2\langle 3\rangle \frac{1}{1-z}\left[\frac{(z-z^2-1)^2}{z}\cos^2\phi + z\sin^2\phi\right] ,$$

$$P_{GG}(\downarrow,\downarrow; z) = 2\langle 3\rangle \frac{1}{1-z}\left[z\cos^2\phi + \frac{(z-z^2-1)^2}{z}\sin^2\phi\right] ,$$

$$P_{GG}(\uparrow,\downarrow; z) = 2\langle 3\rangle z(1-z)\left[\cos^2\phi + \frac{1}{z^2}\sin^2\phi\right] ,$$

$$P_{GG}(\downarrow,\uparrow; z) = 2\langle 3\rangle z(1-z)\left[\frac{1}{z^2}\cos^2\phi + \sin^2\phi\right] . \qquad \text{(A.14)}$$

The asymmetry kernels in Eq.(2.13) are defined by

$$\delta_2 P_{ab}(z) \equiv \frac{1}{2}\left[P_{ab}(\uparrow,\uparrow; z) + P_{ab}(\downarrow,\downarrow; z) - P_{ab}(\uparrow,\downarrow; z) - P_{ab}(\downarrow,\uparrow; z)\right] . \qquad \text{(A.15)}$$

From Eqs.(A.9), (A.10), (A.11) and (A.14) it is straightforward to obtain

$$\delta_2 P_{qq}(z) = \langle \tfrac{4}{3}\rangle \frac{2z}{1-z} ,$$

$$\delta_2 P_{Gq}(z) = \delta_2 P_{qG}(z) = 0$$

$$\delta_2 P_{GG}(z) = \langle 3\rangle \frac{2z}{1-z} . \qquad \text{(A.16)}$$

We see that in a transversity basis and in the leading order in QCD there is no quark-gluon mixing in the asymmetry kernels.

As a final comment we want to remark here that an alternative and easy way to derive Eqs.(A.15) is to start from the asymmetry kernels in a helicity basis as given by Altarelli and Parisi in Ref. 19 and to use the transformation matrix relating this basis to the transversity basis:

$$\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 \\ -1 & i \end{pmatrix} \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} \qquad (A.17)$$

for quarks and

$$\begin{pmatrix} \epsilon_\mu(\uparrow) \\ \epsilon_\mu(\downarrow) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -i & -i \end{pmatrix} \begin{pmatrix} \epsilon_\mu(+) \\ \epsilon_\mu(-) \end{pmatrix} \qquad (A.18)$$

for gluons.

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FIGURE CAPTIONS (Sec.III.)

Fig.3 Parton process $e\vec{H} \to e' + \vec{h} + X$.

Fig.4 Basic lowest order QCD parton processes in large p_T.
(a) $q_\alpha q_\beta \to q_\alpha q_\beta$, (b) $\bar{q}_\alpha q_\alpha \to \bar{q}_\beta q_\beta$, (c) $q\bar{q} \to gg$,
(d) $gq \to gq$, (e) $gg \to q\bar{q}$, (f) $gg \to gg$, (g) dominant mechanism for the large - p_T reaction $\bar{P}P \to \Lambda + X$.

Fig.5 Basic parton process for prompt photon production at large p_T.

Fig.6 (a) Hard large p_T process $AB \to C + \text{jet} + X$.
(b) Kinematics of large p_T process with a definite P_0.
(c) Interference mechanism responsible for non-vanishing $\delta\bar{D}^{\pi}$, the (+,-) denote the flow of helicity.

FIGURE CAPTIONS (Sec.IV.)

Fig.7 Sum of the helicity dependent structure functions $G_1(x,Q^2)$ and $G_2(x,Q^2)$ as a function of x at $Q^2 = 10$, 50 and 100 GeV^2 in the (a) conservative and (b) Carlitz-Kaur models.

Fig.8 Ratio of the spin dependent $(\delta_2 D_p^u(x,Q^2))$ and unpolarized $(D_p^u(x,Q^2))$ distributions as a function of x at $Q^2 = 10$, 50 and 100 GeV^2 in the (a) conservative and (b) Carlitz-Kaur models.

Fig.9 Drell-Yan processes. (a) Beam-target reflected asymmetry for $\vec{P}\vec{P}$ collisions as a function of Q^2 at $x_F = 0$ in the conservative and Carlitz-Kaur models; (b) Beam-target reflected asymmetry for $\vec{P}\vec{\bar{P}}$ collisions as a function of Q^2 at $x_F = 0$ in the conservative model; (c) Transmitted asymmetry for $\vec{P}P$ collisions as a function of x_F at $Q^2 = 20$, 50 and 100 GeV^2 and $s = 1000$ GeV^2 in the Carlitz-Kaur model.

Fig.10 The basic parton asymmetries in lowest order in QCD. The processes $gg \to q\bar{q}$, $q\bar{q} \to gg$ and $u\bar{u} \to s\bar{s}$ all have $a_{LL}^{ii} = -1$.

Fig.11 Reflected asymmetry A_{LL}^{ii} for the reactions (a) $\vec{P}\vec{P} \to \pi^0 X$, (b) $\vec{P}\vec{P} \to \text{Jet} + X$ and $\vec{P}\vec{P} \to \pi^{\pm} X$ in (c) the conservative and (d) the Carlitz and Kaur models.

Fig.12 Transmitted asymmetry $a_{LL}^{if}(x)$ for $q\bar{q} \to s\bar{s}$ $(z = -\hat{t}/\hat{s})$.

Fig.13 Plots of transmitted asymmetry for $\vec{P}\vec{P} \to \Lambda + X$: (a) $\theta_{trigg} = 60^\circ$, (b) $\theta_{trigg} = 30^\circ$ and (c) $\theta_{trigg} = 60^\circ$ with (solid) and without (dashed) gluon contribution.

Fig.14 Reflected double asymmetry for PP and $P\bar{P}$ collisions. In the case of $PP \to \gamma X$ the Carlitz-Kaur model was used because the conservative model assumes $\Delta_2 G(x) = 0$.

Fig.15 Single spin asymmetry - p_{out} correlation for $\vec{P}P \to \pi^+(p_T) + \text{Jet}(p_{out}) + X$ as a function of $X_{out} = p_{out}/p_T$ for the $\pi = \pi^+, \pi^0$ and π^- cases at $\theta_{trigg} = 60^\circ$, $s = 10{,}000$ GeV^2 and $p_T = 20$ GeV/c in the conservative and Carlitz-Kaur models.

FIGURE CAPTIONS (Sec.V.)

Fig.16 Parton diagrams in deep inelastic scattering:(a) factorized leading contribution in non-covariant gauge; (b) next to leading contributions.

Fig.17 (a) $q\bar{q} \to \gamma^* g$ contribution to massive pair production; (b) radiative correction to the Drell-Yan mechanism.

Fig.18 Higher twist contributions in (a) deep inelastic scattering, (b) massive pair production and (c) large p_T.

Fig.19 Quark fusion mechanism in $P\bar{P} \to \pi^+ + X$.

(Appendix)

Fig.20 Basic diagram for the calculation of the asymmetry kernels involving quarks and gluons.

Fig.21 (a) $P_{qq}(s_2,s_1)$, (b) $P_{Gq}(s_3,s_1)$ and (c) $P_{qG}(s_2,s_3)$.

Fig.22 (a) Basic diagram for the calculation of the asymmetry kernels involving gluons alone, (b) three-gluon vertex with all the momenta flowing in.

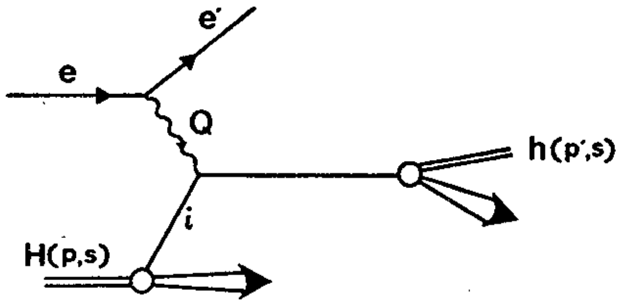

e'
e
Q
h(p',s)
i
H(p,s)

FIG 3

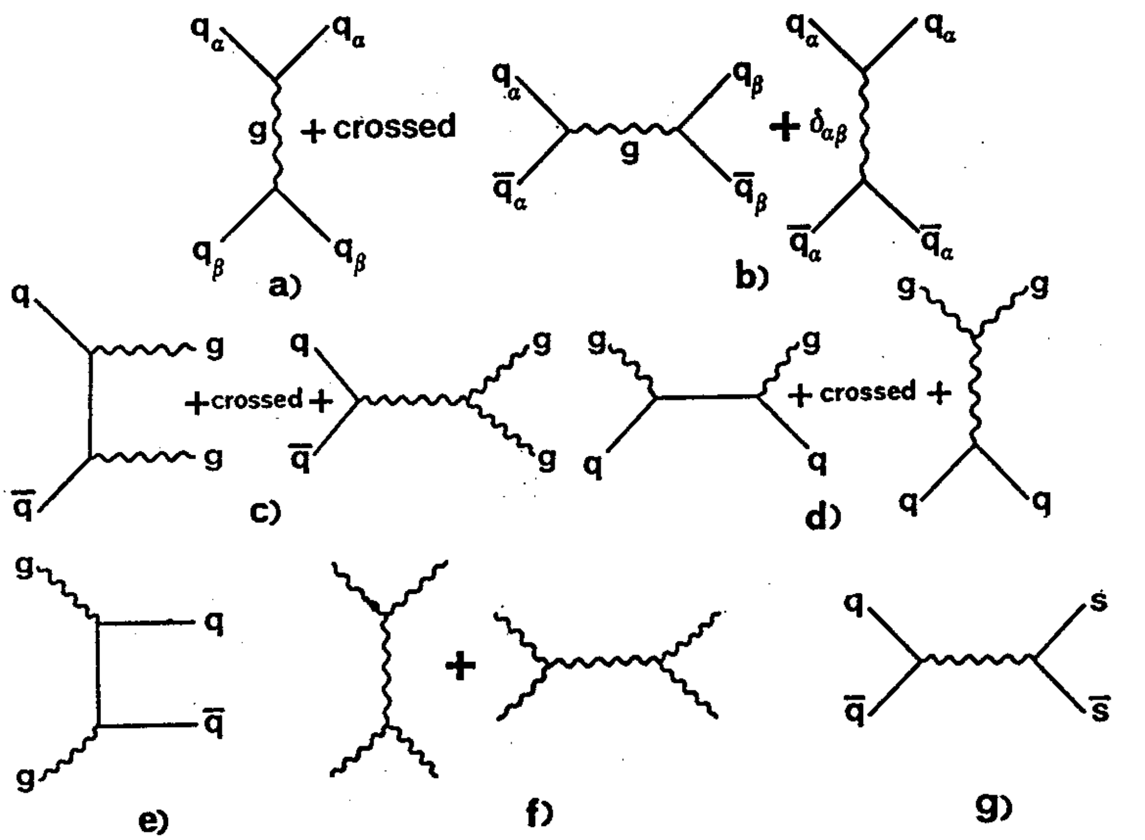

q_α
q_α
g
+ crossed
q_β
q_β
a)
q_α
$\bar{q}_\alpha$
g
q_β
$\bar{q}_\beta$
+ $\delta_{\alpha\beta}$
q_α
q_α
$\bar{q}_\alpha$
$\bar{q}_\alpha$
b)
q
$\bar{q}$
g
g
+crossed+
q
$\bar{q}$
g
g
c)
g
q
g
q
+ crossed +
g
g
q
q
d)
g
g
q
$\bar{q}$
e)
+
f)
q
$\bar{q}$
s
$\bar{s}$
g)

FIG 4

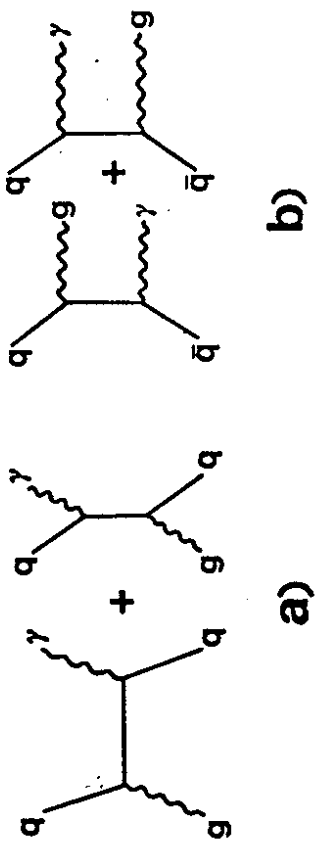

a)
b)
q
g
γ
q̄
+

FIG 5

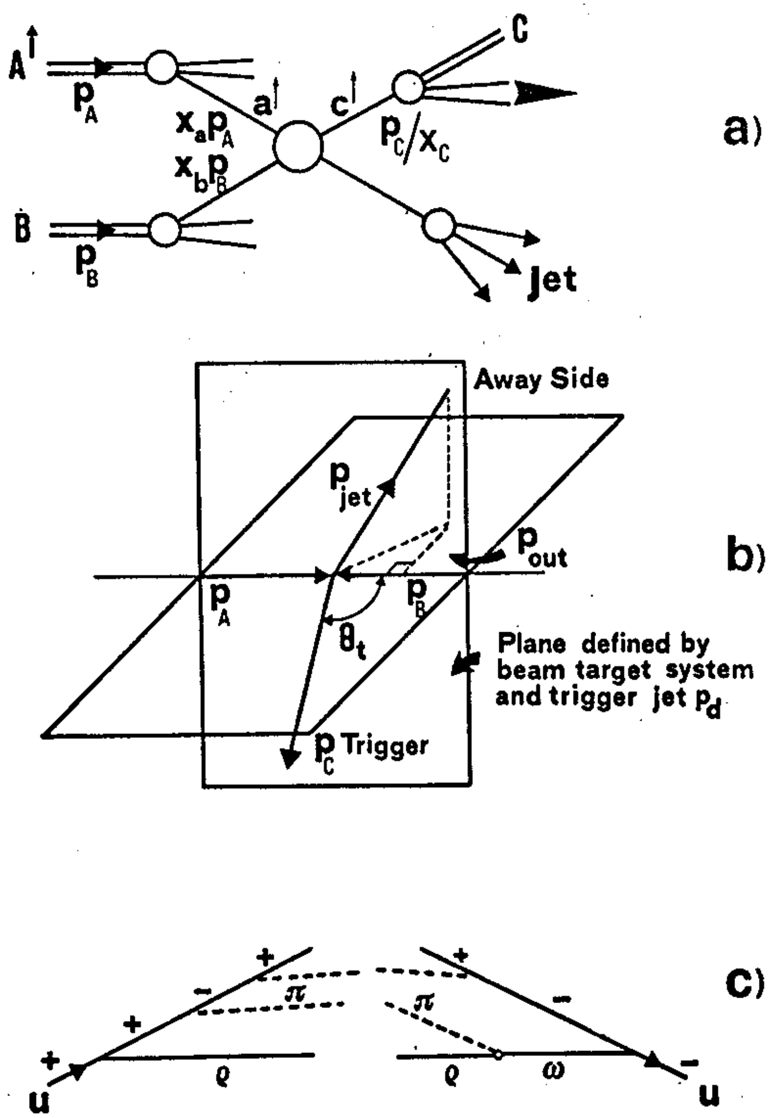

a)
A↑
B
p_A
p_B
$x_a p_A$
$x_b p_B$
a↑
c↑
C
p_C/x_C
jet
b)
Away Side
p_{jet}
p_{out}
p_A
p_B
θ_t
Plane defined by
beam target system
and trigger jet p_d
p_C Trigger
c)
u
π
ρ
ω

FIG 6

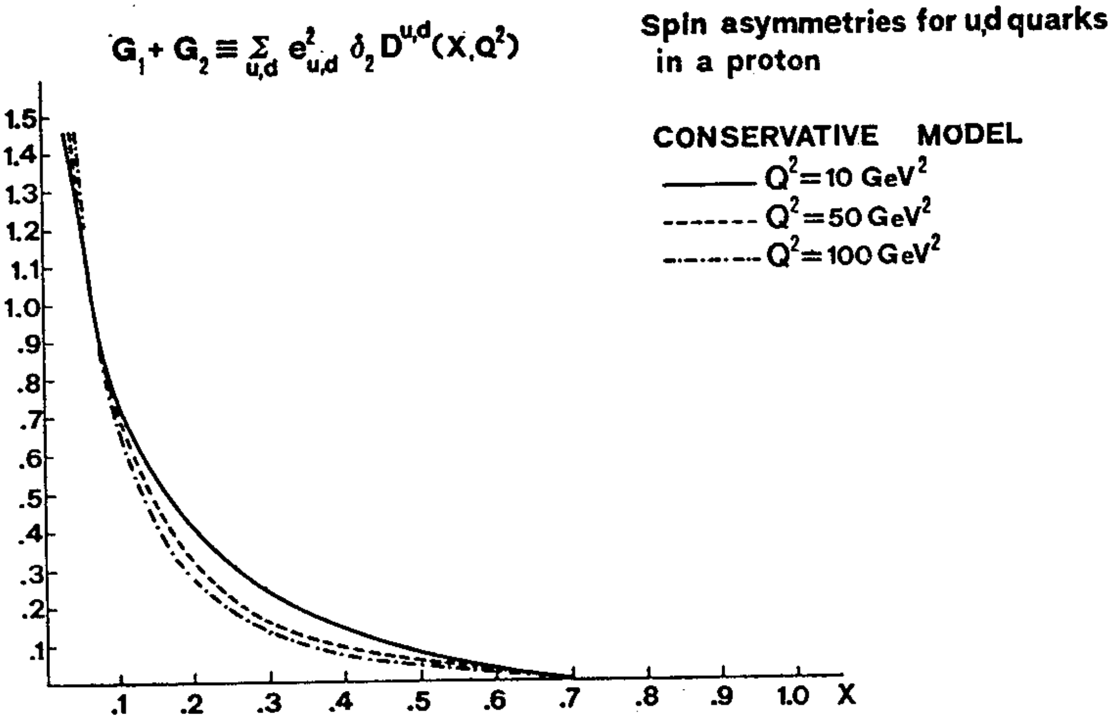

$G_1 + G_2 \equiv \sum_{u,d} e^2_{u,d}\, \delta_2 D^{u,d}(X,Q^2)$
Spin asymmetries for u,d quarks in a proton
CONSERVATIVE MODEL
$Q^2 = 10$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 100$ GeV^2
X

FIG 7a

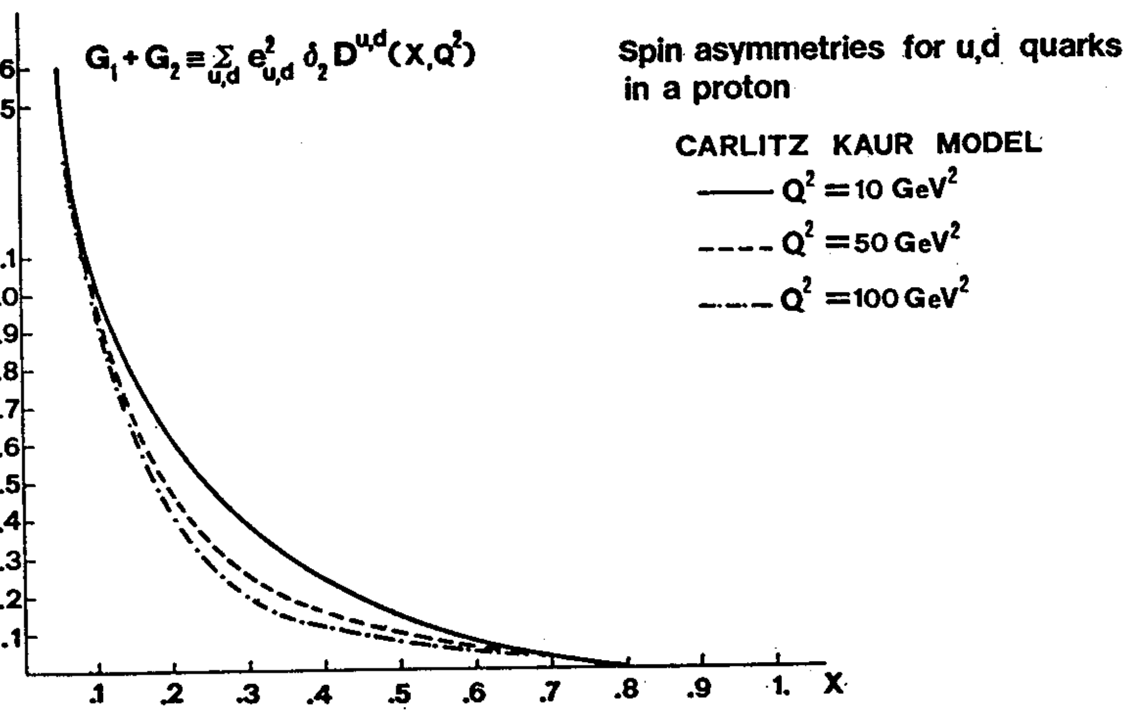

$G_1 + G_2 \equiv \sum_{u,d} e^2_{u,d}\, \delta_2 D^{u,d}(X,Q^2)$
Spin asymmetries for u,d quarks in a proton
CARLITZ KAUR MODEL
$Q^2 = 10$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 100$ GeV^2
X

FIG 7b

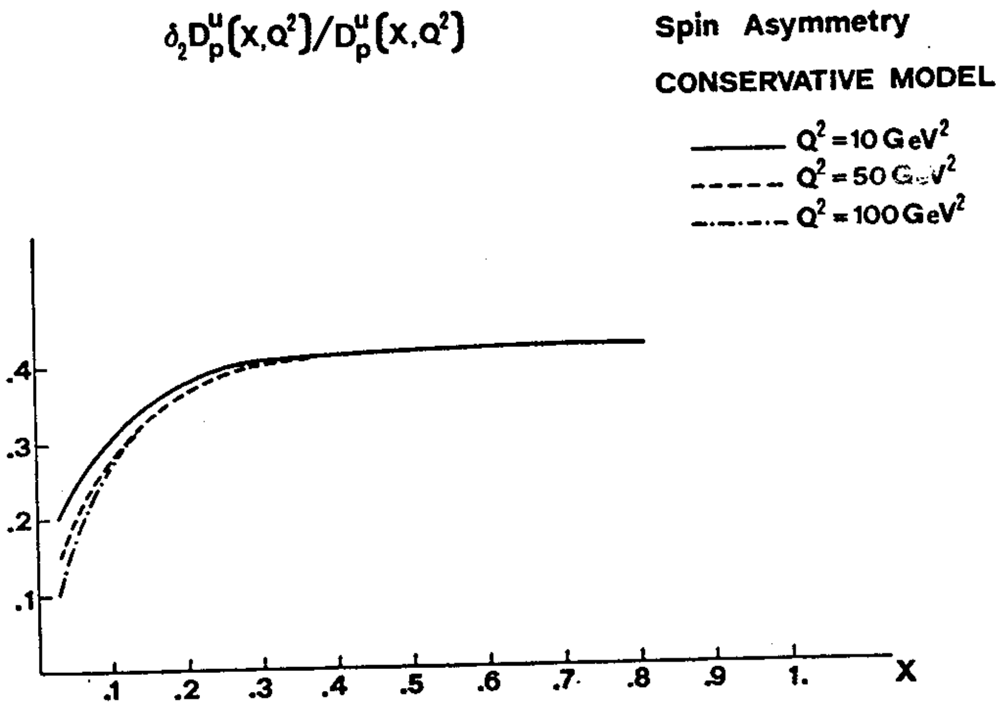

$\delta_2 D_p^u(x,Q^2)/D_p^u(x,Q^2)$
Spin Asymmetry
CONSERVATIVE MODEL
$Q^2=10$ GeV^2
$Q^2=50$ GeV^2
$Q^2=100$ GeV^2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.
X

FIG 8a

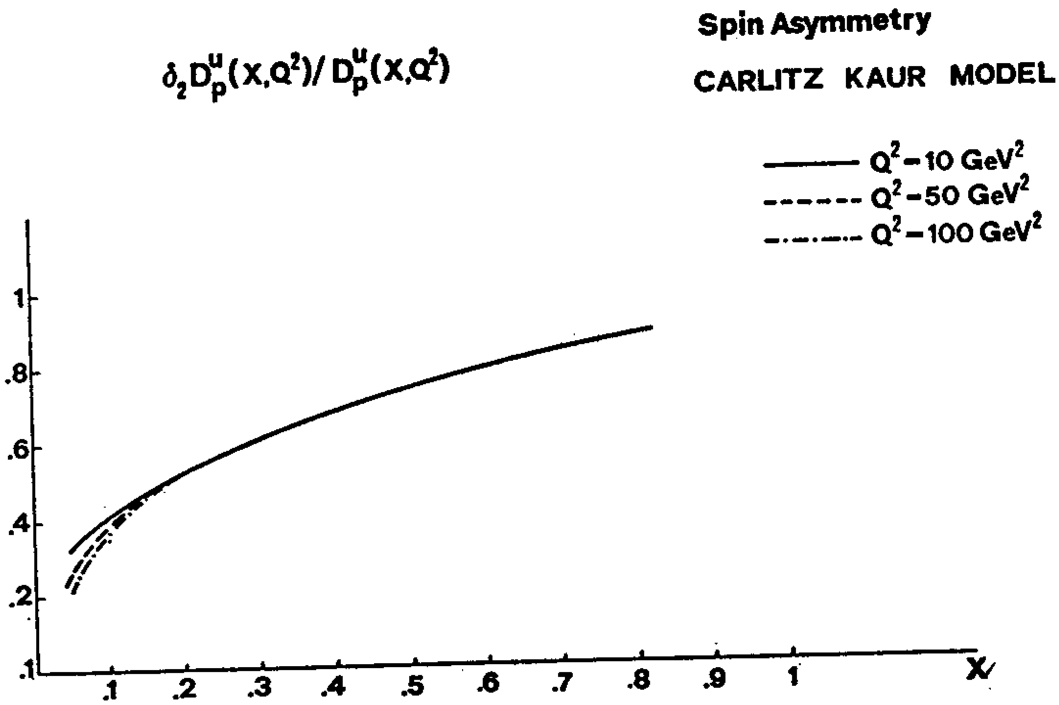
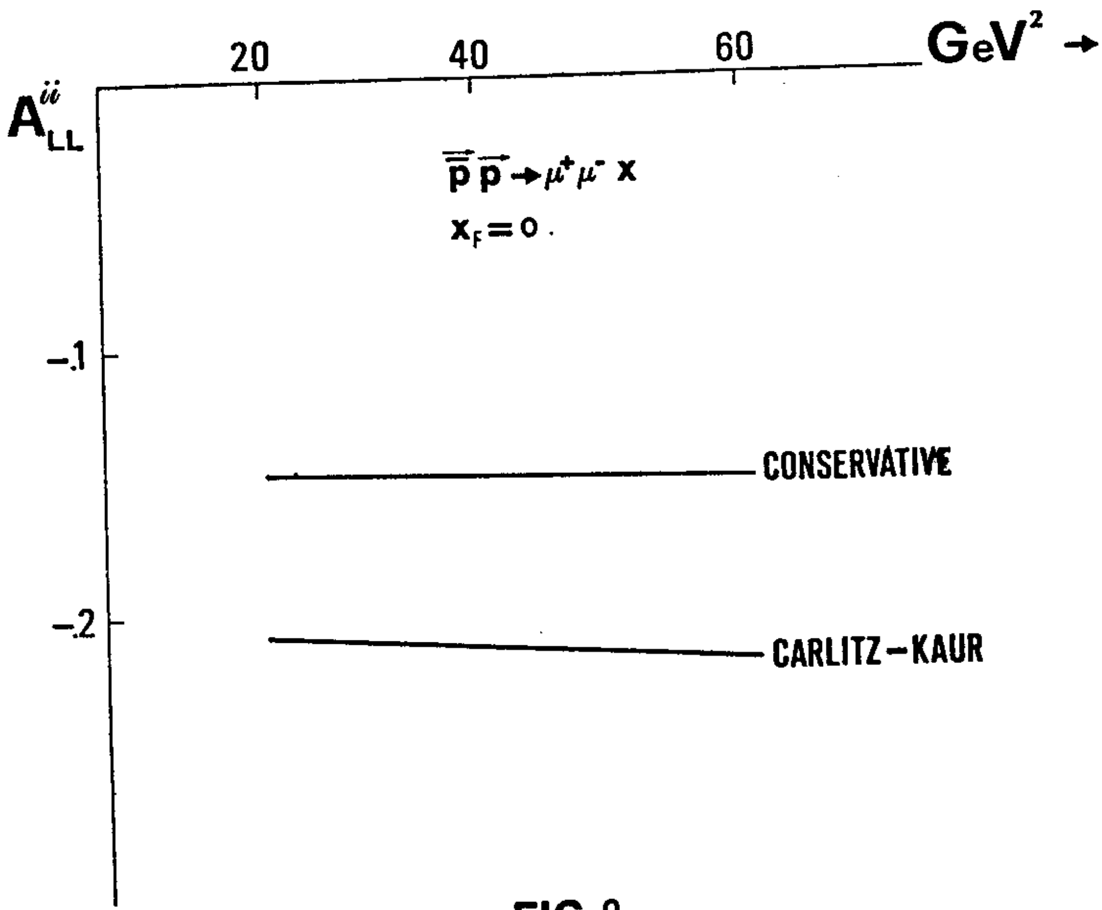
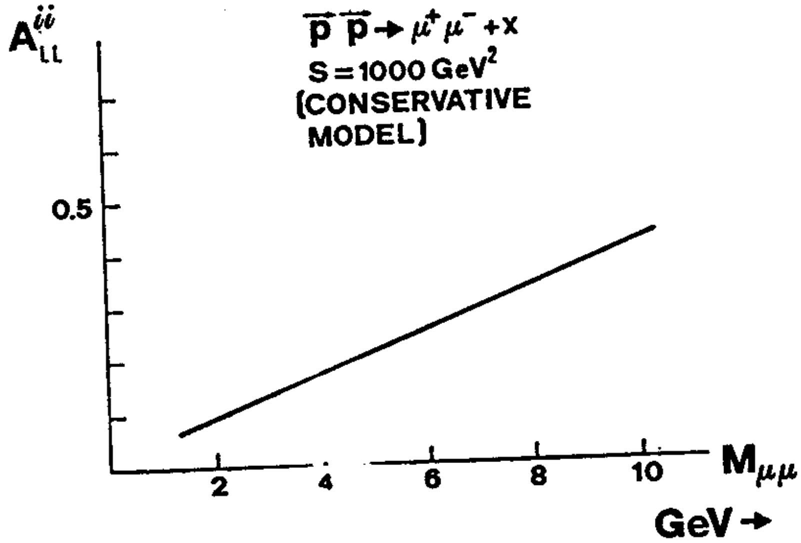

Spin Asymmetry
$\delta_2 D_p^u(x,Q^2)/D_p^u(x,Q^2)$
CARLITZ KAUR MODEL
Q^2-10 GeV^2
Q^2-50 GeV^2
Q^2-100 GeV^2
1
.8
.6
.4
.2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
X

FIG 8b



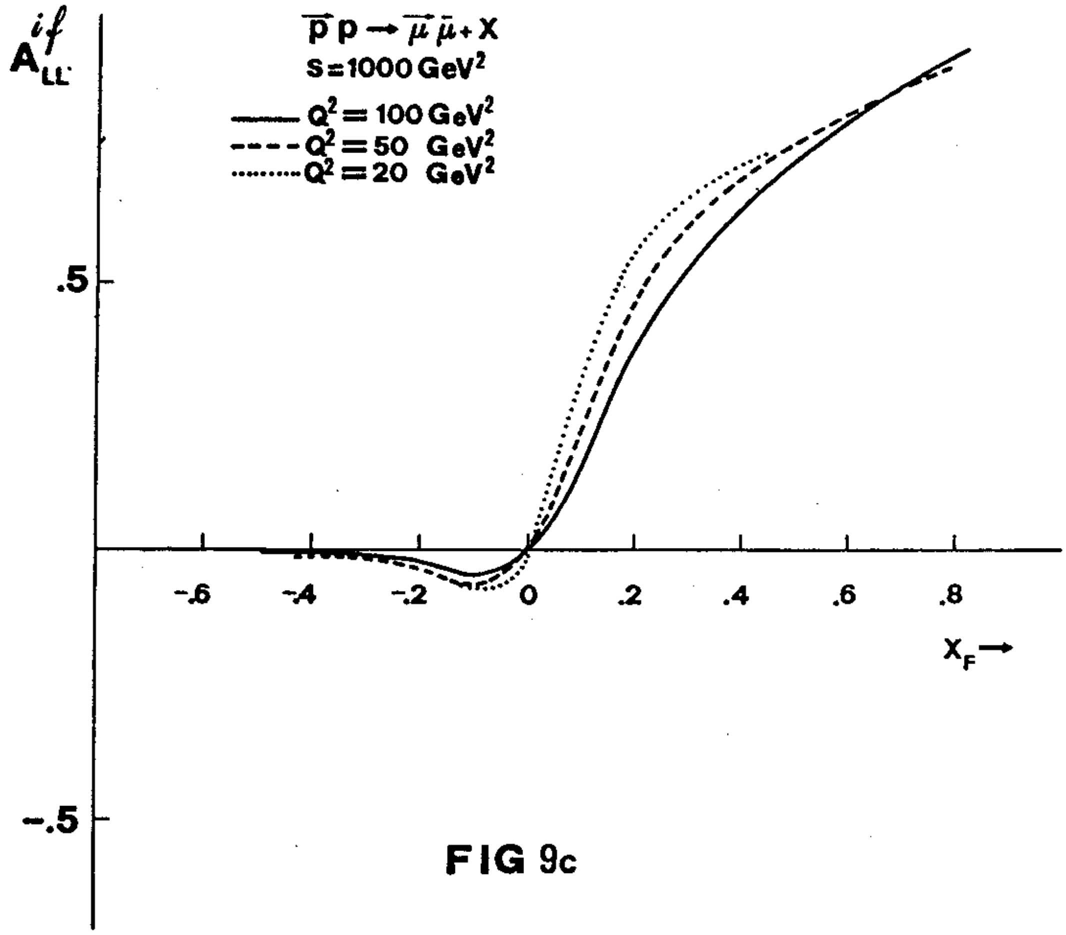
20
40
60
GeV²
A_{LL}^{ii}
$\vec{\vec{p}}\,\vec{p} \to \mu^+\mu^- X$
$x_F = 0$
-.1
-.2
CONSERVATIVE
CARLITZ-KAUR

FIG 9a



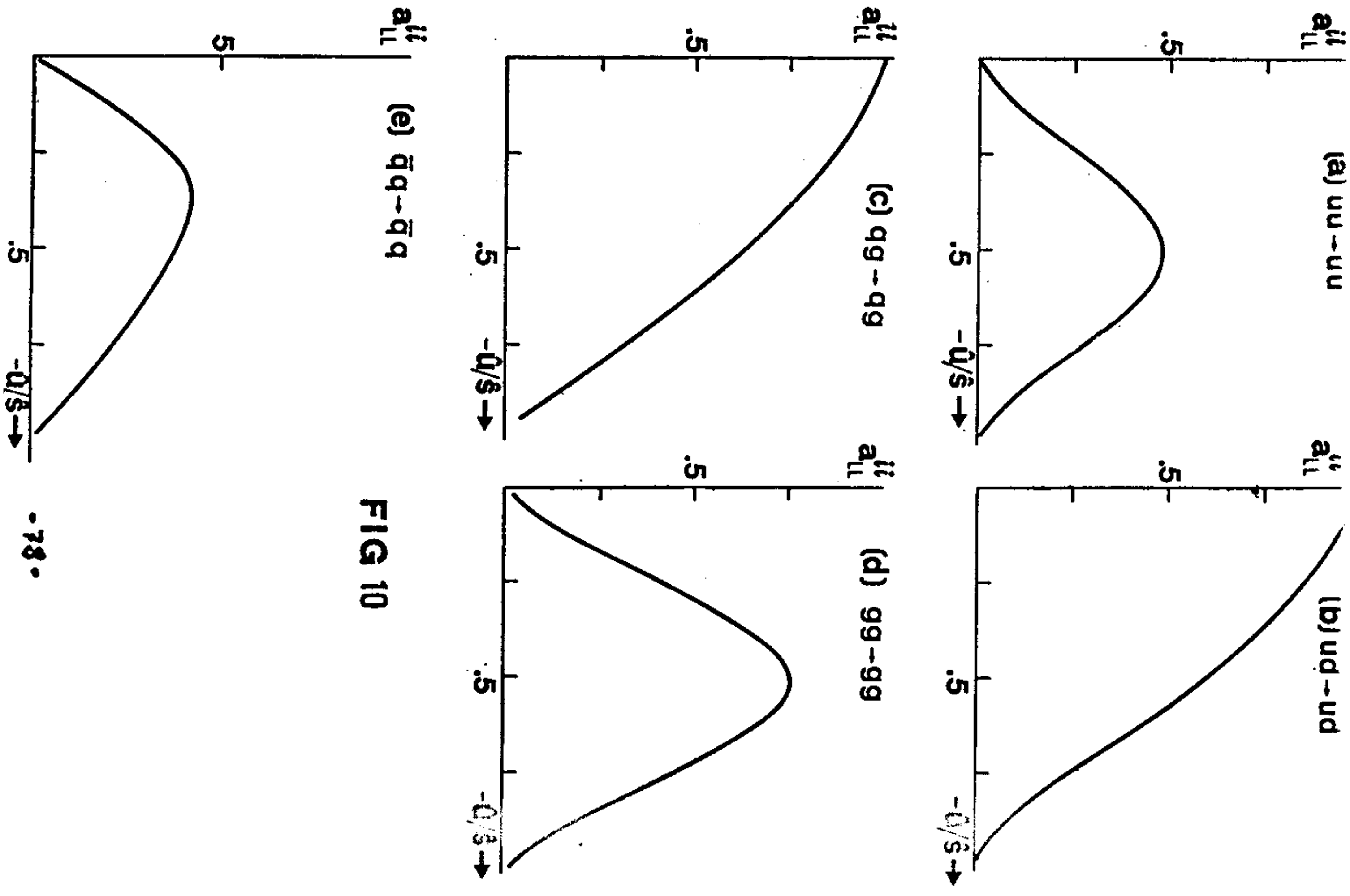
A_{LL}^{ii}
$\vec{p}\,\vec{p} \to \mu^+\mu^- + X$
S = 1000 GeV²
(CONSERVATIVE MODEL)
0.5
2
4
6
8
10
$M_{\mu\mu}$
GeV

FIG 9b



$\vec{p}\,p \rightarrow \vec{\mu}\,\bar{\mu} + X$
$s = 1000\ GeV^2$
$Q^2 = 100\ GeV^2$
$Q^2 = 50\ GeV^2$
$Q^2 = 20\ GeV^2$
A_{LL}^{if}
.5
-.5
-.6
-.4
-.2
0
.2
.4
.6
.8
$X_F \rightarrow$

FIG 9c



(a) uu→uu
(b) ud→ud
(c) qg→qg
(d) gg→gg
(e) $\bar{q}q \rightarrow \bar{q}q$
a_{LL}^{ij}
.5
$-\hat{u}/\hat{s} \rightarrow$

FIG 10

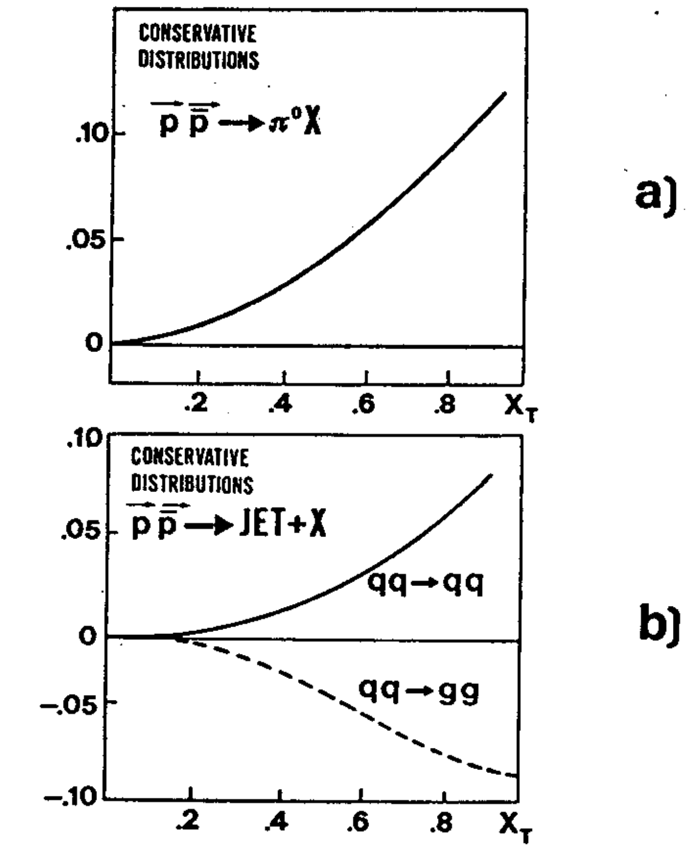
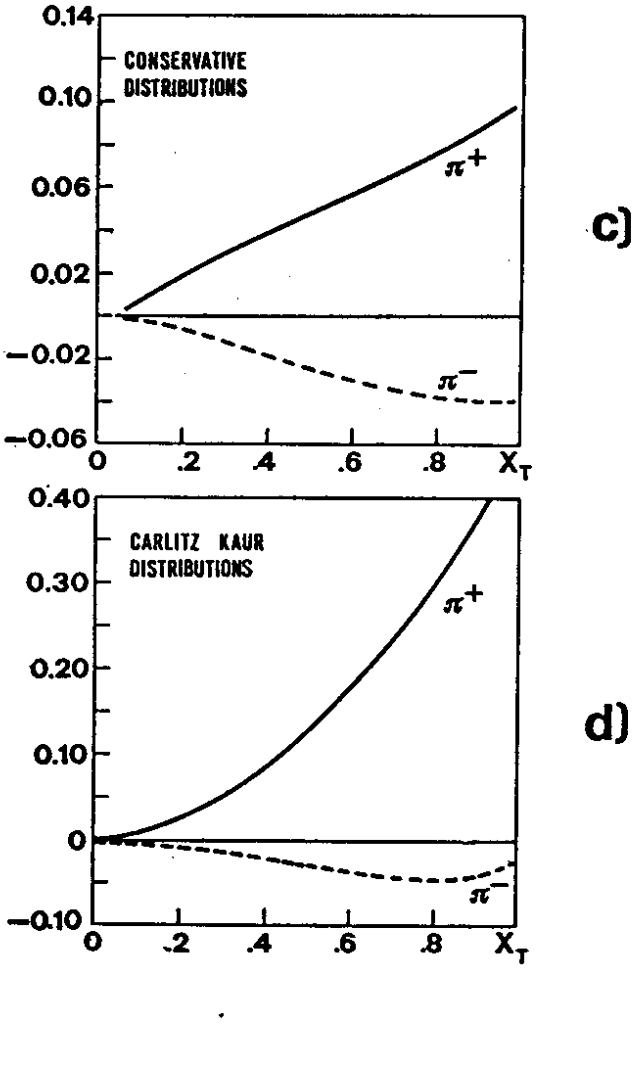

CONSERVATIVE
DISTRIBUTIONS
$\vec{p}\,\vec{\bar{p}} \to \pi^0 X$
.10
.05
0
.2
.4
.6
.8
X_T
a)
.10
CONSERVATIVE
DISTRIBUTIONS
$\vec{p}\,\vec{\bar{p}} \to$ JET+X
.05
qq→qq
0
qq→gg
–.05
–.10
.2
.4
.6
.8
X_T
b)

FIG 11


0.14
CONSERVATIVE
DISTRIBUTIONS
0.10
0.06
π^+
0.02
–0.02
π^-
–0.06
0
.2
.4
.6
.8
X_T
c)
0.40
CARLITZ KAUR
DISTRIBUTIONS
0.30
π^+
0.20
0.10
0
π^-
–0.10
0
.2
.4
.6
.8
X_T
d)

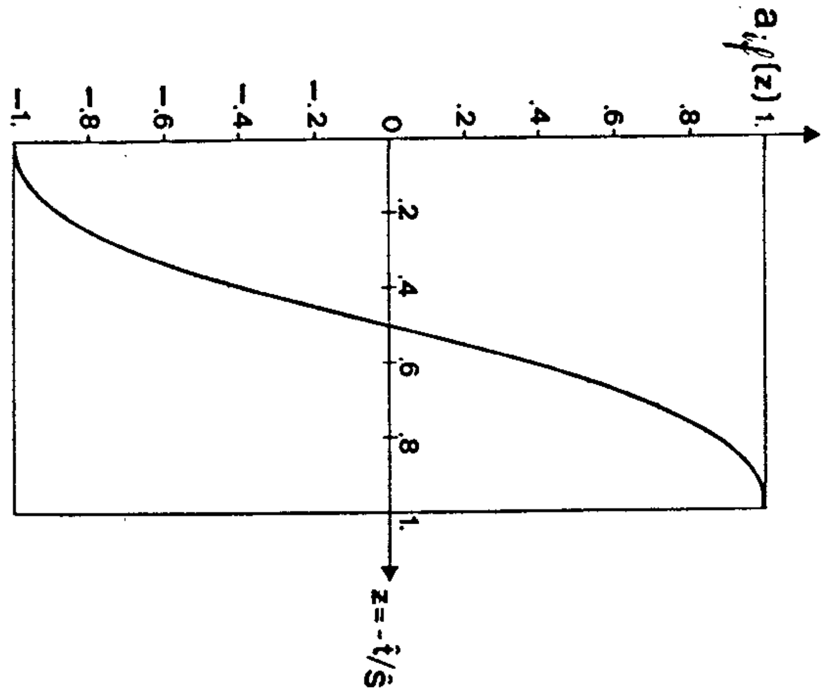

$a_{if}(z)$
1.
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.
.2
.4
.6
.8
1.
$z = -\hat{t}/\hat{s}$

FIG 12

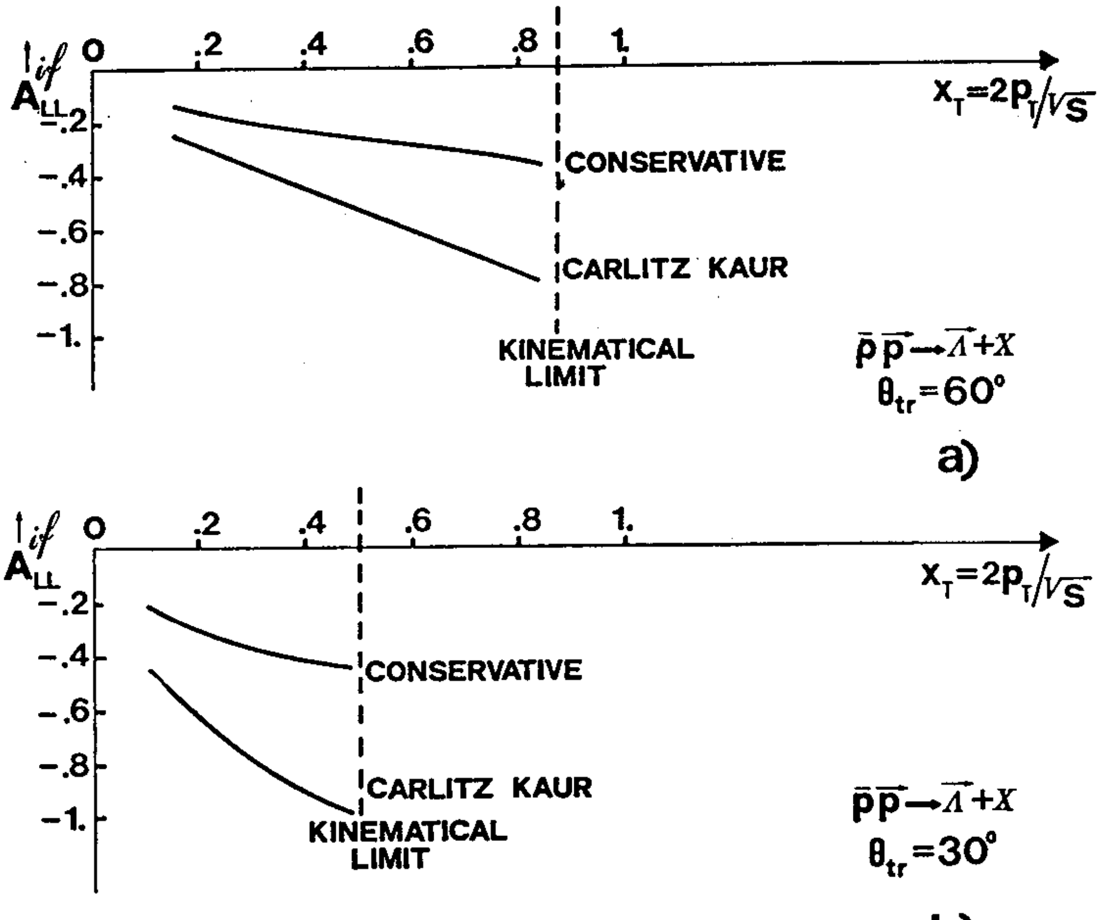
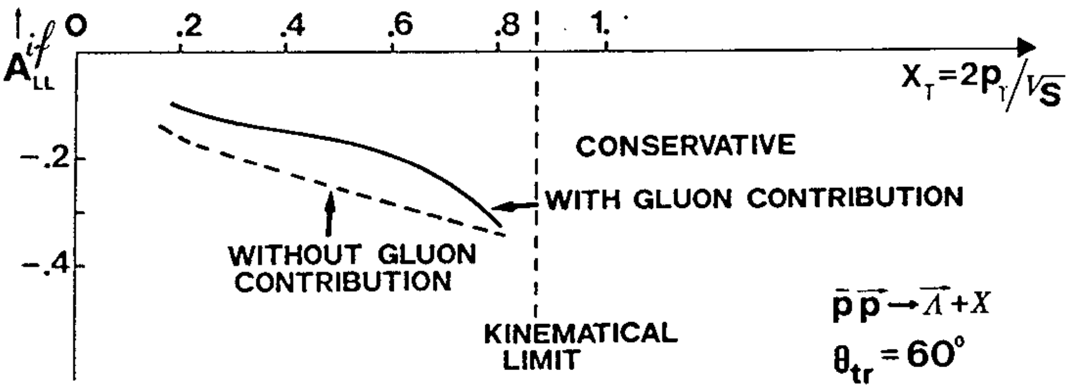

A_{LL}^{if}
0
.2
.4
.6
.8
1.
$x_T = 2p_T/\sqrt{s}$
-.2
-.4
-.6
-.8
-1.
CONSERVATIVE
CARLITZ KAUR
KINEMATICAL LIMIT
$\bar{p}\vec{p} \to \vec{\Lambda} + X$
$\theta_{tr} = 60°$
a)
A_{LL}^{if}
0
.2
.4
.6
.8
1.
$x_T = 2p_T/\sqrt{s}$
-.2
-.4
-.6
-.8
-1.
CONSERVATIVE
CARLITZ KAUR
KINEMATICAL LIMIT
$\bar{p}\vec{p} \to \vec{\Lambda} + X$
$\theta_{tr} = 30°$
b)

FIG 13



c)

FIG 13

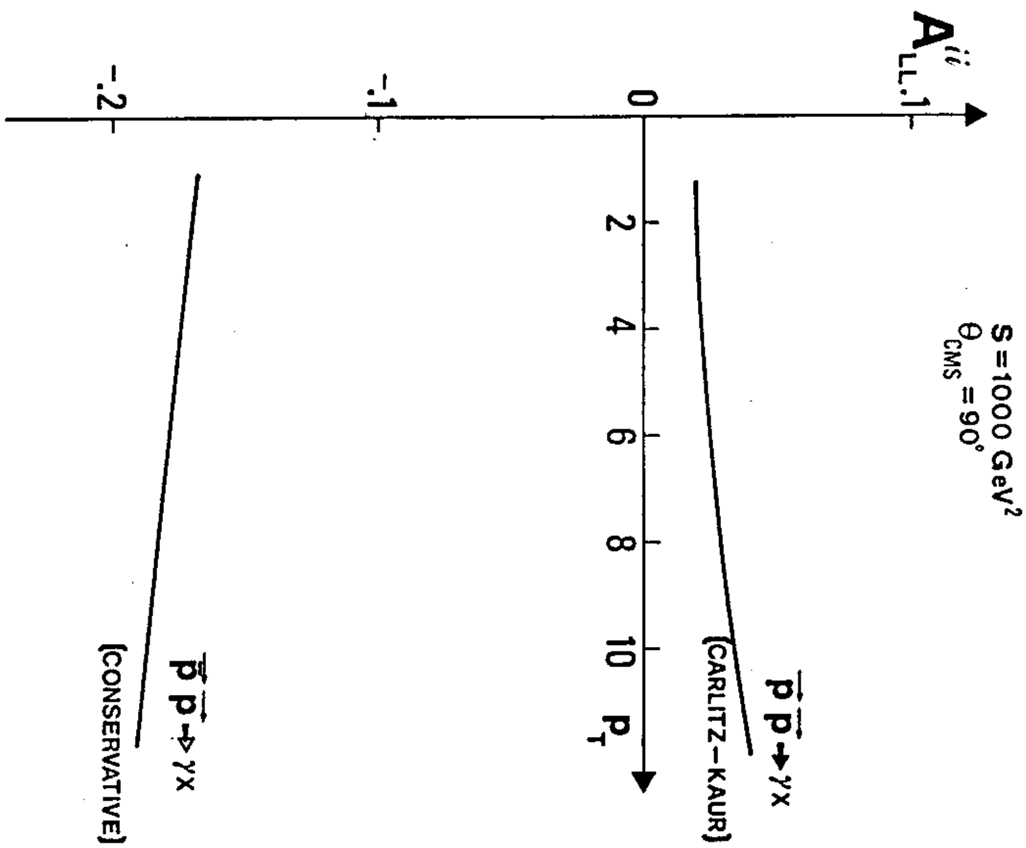


FIG 14

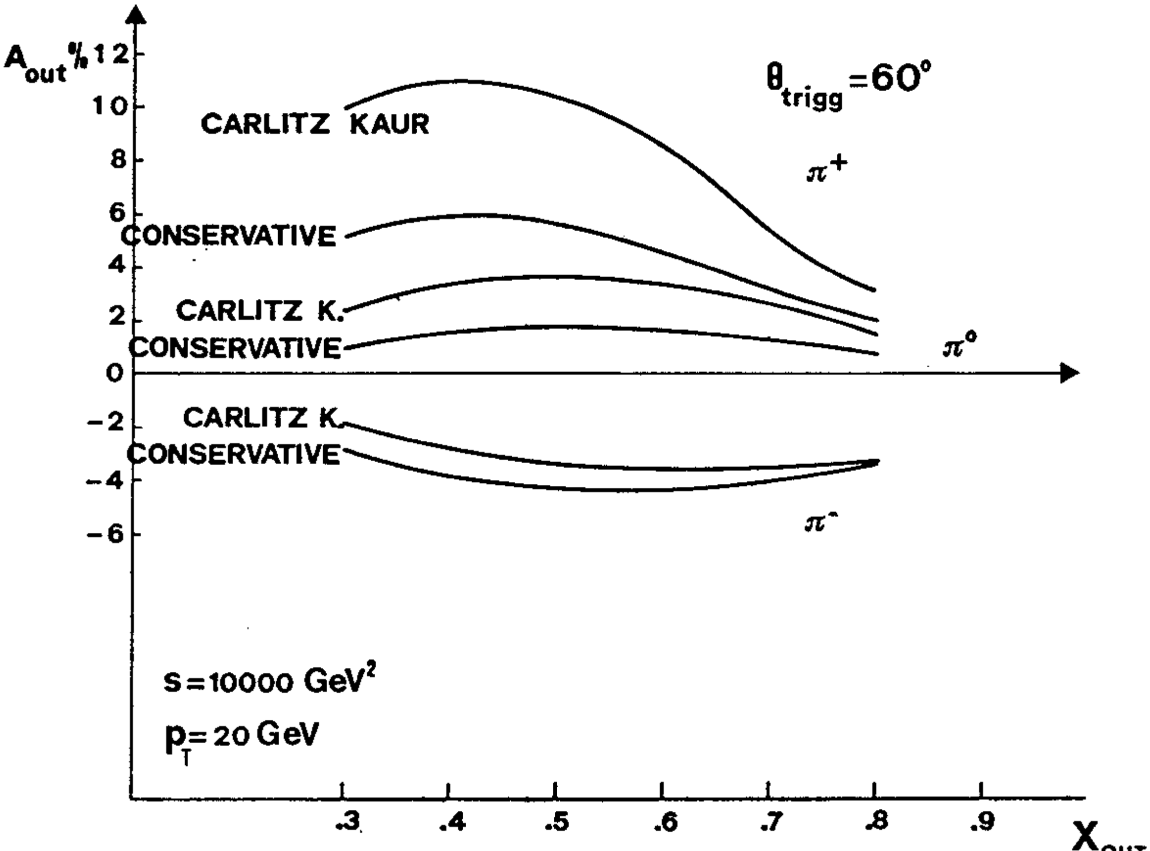

A_{out} %
12
10
8
6
4
2
0
-2
-4
-6
CARLITZ KAUR
CONSERVATIVE
CARLITZ K.
CONSERVATIVE
CARLITZ K.
CONSERVATIVE
$\theta_{trigg} = 60°$
π^+
π^0
π^-
$s = 10000$ GeV^2
$p_T = 20$ GeV
.3
.4
.5
.6
.7
.8
.9
X_{OUT}

FIG 15

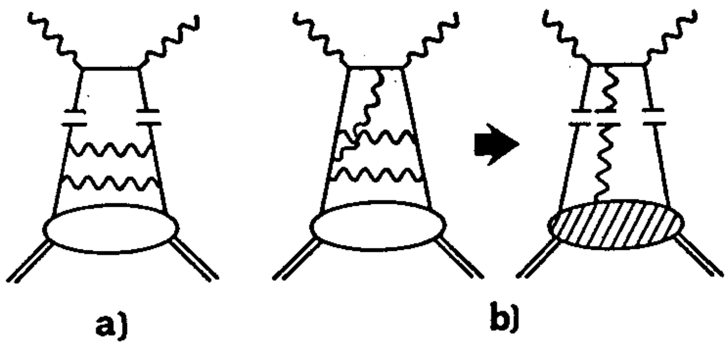

a)
b)

FIG 16

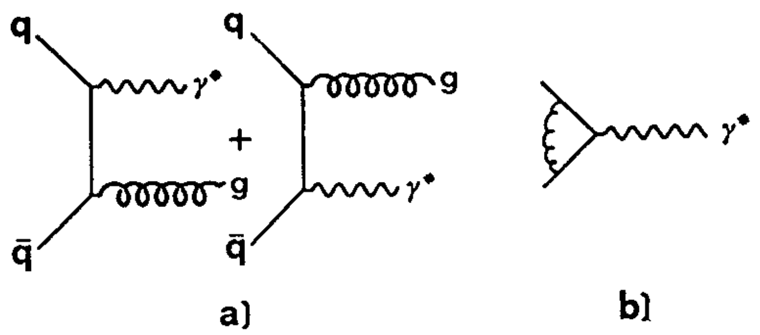


FIG 17

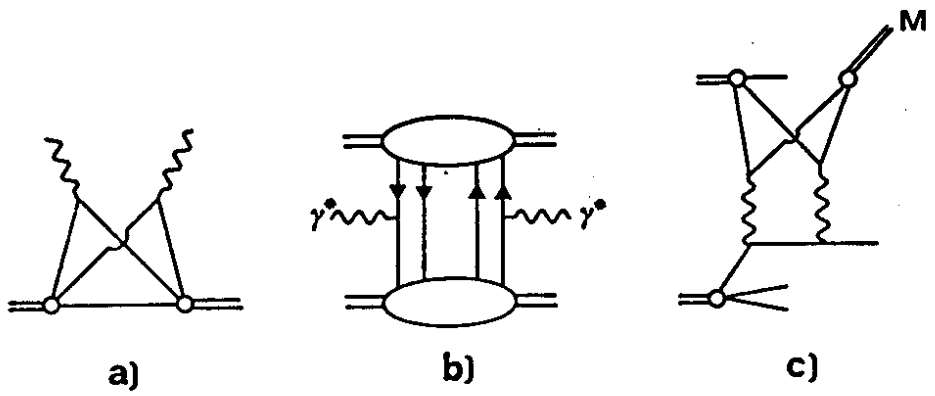


FIG 18

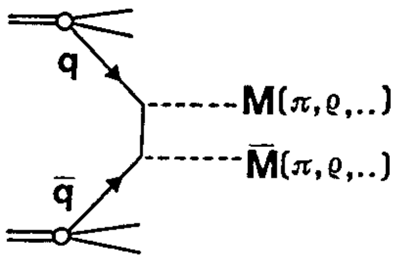


FIG 19

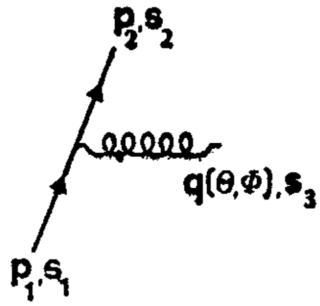


FIG 20

/5 -

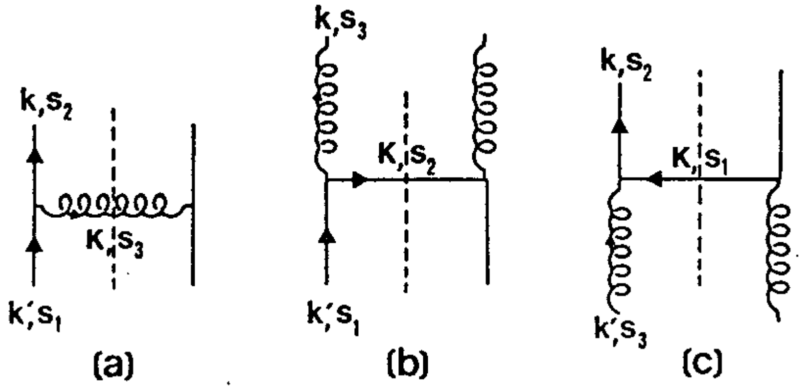


FIG 21

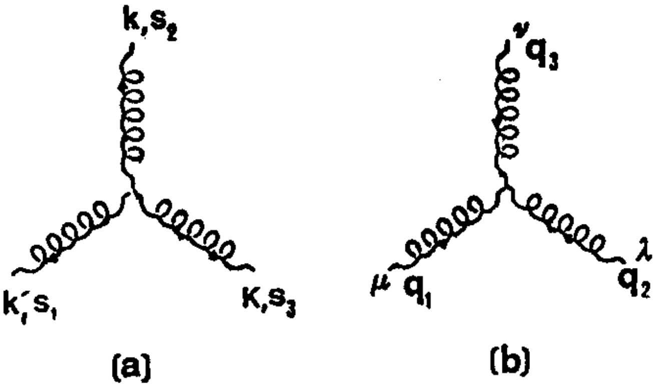


FIG 22