A Survey of Relaxations and Approximations of the Power Flow Equations

Other titles in Foundations and Trends® in Electric Energy Systems

Sustainable Transportation with Electric Vehicles
Fanxin Kong and Xue Liu
ISBN: 978-1-68083-388-1

Unit Commitment in Electric Energy Systems
Miguel F. Anjos and Antonio J. Conejo
ISBN: 978-1-68083-370-6

Reliability Standards for the Operation and Planning of Future Electricity Networks
Goran Strbac, Daniel Kirschen and Rodrigo Moreno
ISBN: 978-1-68083-182-5

Toward a Unified Modeling and Control for Sustainable and Resilient Electric Energy Systems
Marija D. Ilic
ISBN: 978-1-68083-226-6

A Survey of Relaxations and Approximations of the Power Flow Equations

Daniel K. Molzahn
Georgia Institute of Technology
molzahn@gatech.edu

Ian A. Hiskens
University of Michigan
hiskens@umich.edu

Foundations and Trends® in Electric Energy Systems

Foundations and Trends® in Electric Energy Systems
Volume 4, Issue 1-2, 2019
Editorial Board

Editorial Scope

Topics

Foundations and Trends® in Electric Energy Systems publishes survey and tutorial articles in the following topics:

- Advances in power dispatch
- Demand-side and grid scale data analytics
- Design and optimization of electric services
- Distributed control and optimization of distribution networks
- Distributed sensing for the grid
- Distribution systems
- Fault location and service restoration
- Integration of physics-based and data-driven modeling of future electric energy systems
- Integration of Power electronics, Networked FACTS
- Integration of renewable energy sources
- Interdependence of power system operations and planning and the electricity markets
- Microgrids

- Modern grid architecture
- Power system analysis and computing
- Power system dynamics
- Power system operation
- Power system planning
- Power system reliability
- Power system transients
- Security and privacy
- Stability and control for the whole multi-layer (granulated) network with new load models (to include storage, DR, EVs) and new generation
- System protection and control
- The new stability guidelines and control structures for supporting high penetration of renewables (>50%)
- Uncertainty quantification for the grid
- System impacts of HVDC

Information for Librarians

Contents

A Survey of Relaxations and Approximations of the Power Flow Equations

Daniel K. Molzahn[1] and Ian A. Hiskens[2]

[1] *Georgia Institute of Technology; molzahn@gatech.edu*
[2] *University of Michigan; hiskens@umich.edu*

ABSTRACT

The power flow equations relate the power injections and voltages in an electric power system and are therefore key to many power system optimization and control problems. Research efforts have developed a wide variety of relaxations and approximations of the power flow equations with a range of capabilities and characteristics. This monograph surveys relaxations and approximations of the power flow equations, with a particular emphasis on recently proposed formulations.

1

Introduction

The power flow equations model the relationship between voltage phasors and power injections at nodes (buses) in an electric power system. These equations are fundamental in the analysis and operation of power systems. Accordingly, they form the key constraints in many optimization and control problems relevant to electric power systems, including optimal power flow (OPF), unit commitment, state estimation, contingency evaluation, voltage stability assessment, and dynamic stability analysis. The power flow equations are nonlinear and result in non-convex optimization problems. Moreover, at least some optimization problems containing the power flow equations (e.g., OPF problems) are generally NP-Hard [1], even for systems with radial network topologies [2], and may have multiple local solutions [3]. This inherent complexity is immediately apparent in the simple examples presented at the end of Chapter 2.

There exists a voluminous literature regarding the power flow equations. The intent of this monograph is to review various representations of the power flow equations, with a particular focus on those proposed in the last decade.

The power flow representations in this monograph are primarily presented in the context of optimization problems. However, note that while optimization plays an important role in many problems relevant to the design and operation of power systems (e.g., OPF, state estimation, unit commitment, transmission switching, expansion planning, etc. [4, 5]), various power flow representations are relevant to other important problems (stability analyses, dynamic simulations, analysis of control strategies such as volt/var control and automatic generation control, etc. [6, 7]). Moreover, while much of the literature develops power flow representations in the context of certain applications, this monograph focuses on the power flow representations themselves rather than specific problems. The reader interested in a specific problem or solution algorithm is referred to the surveys and tutorials that exist for power flow [8, 9], different formulations of optimal power flow [10–21] (and various extensions to consider, e.g., security constraints [22–25] and transient-stability constraints [26, 27]), unit commitment [28–31], state estimation [32–35], transmission switching [36], infrastructure planning [19], voltage stability analysis [37–40], cascading failure [41], distributed optimization and control methods [42–45], complex network theory [46], and more general power system stability concepts [6]. Several recent references of particular relevance are the surveys in [47] and [48] as well as the video lectures in [49], all of which review some of the topics covered in this monograph. Also note that reference implementations for several of the power flow representations presented in this monograph are provided in the software packages MATPOWER [50] and PowerModels.jl [51].

The power flow representations surveyed in this monograph are categorized as either *relaxations* or *approximations*. Figure 1.1 shows conceptual examples of a relaxation and an approximation of a non-convex feasible space. Relaxations enclose the non-convex feasible spaces associated with the power flow equations in a larger space. The larger space is typically chosen to be convex to enable the application of theory and algorithms developed for convex optimization problems.

Approximations use assumptions regarding certain quantities to simplify the power flow equations. Power flow approximations are capable of closely representing system behavior when the associated assumptions

(a) Convex Relaxation   **(b)** Approximation
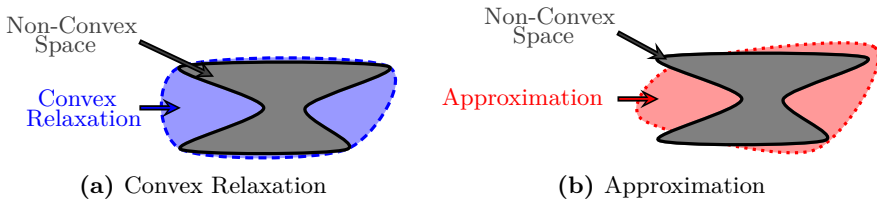
Figure 1.1: Conceptual illustrations showing a convex relaxation (blue region on the left) and an approximation (red region on the right) for the gray non-convex space.

are valid. Many power flow approximations are reasonably accurate for "typical" operating conditions.

In general, solutions to optimization problems that use power flow relaxations and approximations do not exactly satisfy the actual power flow equations. Rather, relaxations and approximations are typically employed in attempts to obtain tractable formulations which adequately represent the actual power flow physics. Optimization problems that use convex relaxations additionally provide bounds on the optimal objective value for the original non-convex problem as well as sufficient conditions for certifying problem infeasibility. Some convex relaxations also have associated sufficient conditions which guarantee their ability to provide global optima for certain limited classes of power system optimization problems. Some of these sufficient conditions can be evaluated prior to solving the relaxation based solely on the problem parameters and network topology, while other conditions are checked after solving a relaxation. In contrast, note that approximations do not provide any of the aforementioned theoretical guarantees provided by relaxations.

Solutions to relaxations and approximations may not exactly satisfy the power flow equations. This may be unacceptable for some applications, necessitating the deployment of algorithms that return a feasible power flow solution, possibly at the cost of increased computational difficulty or the lack of theoretical guarantees. A wide variety of nonlinear programming techniques have been applied to power system optimization problems. Starting from specified initializations, these techniques typically seek *local optima* for power system optimization problems, which are feasible points with objective values that are superior to all
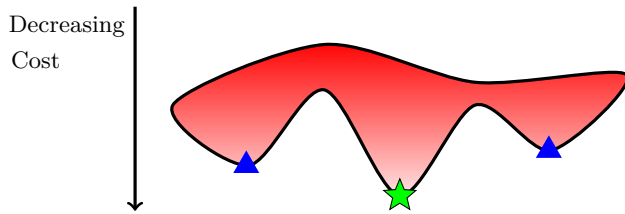
Figure 1.2: Conceptual illustration showing local optima (blue triangles) and the global optimum (green star).

nearby points but potentially inferior to the global optimum. Figure 1.2 provides a conceptual example showing the distinction between local and global optima. While surveying the power system optimization literature regarding local solution techniques is largely beyond the scope of this monograph, a brief summary of traditional nonlinear programming techniques is presented in §6. The interested reader is directed to other reviews of traditional local solution techniques, such as [13–17] for further details. Additionally, some of the power flow representations considered in this monograph form the basis of recently developed algorithms for computing local optima or "nearly globally optimal" feasible points. This monograph also reviews several such algorithms in §6.

The capabilities of various power flow relaxations and approximations are, in many ways, complementary rather than competitive with the capabilities of local solution algorithms. Local solution algorithms can benefit from the outputs resulting from power flow relaxations and approximations (e.g., using the decision variable values and the set of binding constraints to initialize certain local solution algorithms). Moreover, optimization problems may combine various power flow representations in order to balance accuracy and computational tractability. For instance, an optimization problem may have a "base case" that uses a detailed model of the power flow physics and multiple "scenarios" that use simplified power flow representations for the sake of computational tractability. As another example, an algorithm could decompose the solution of a complicated mixed-integer nonlinear program into two steps: first solve a mixed-integer problem with a simplified power flow model to select values for the discrete variables, and then apply a local

solution algorithm to the continuous optimization problem that results
from fixing the discrete variables and employing a higher-fidelity power
flow model.

The theoretical guarantees provided by relaxations also complement
the capabilities of local solution algorithms. Infeasibility of a relaxation
certifies that the original optimization problem is infeasible, but fea-
sibility of a relaxation is not sufficient to guarantee feasibility of the
original problem. Conversely, a local solution algorithm can show that
a problem is feasible, but failure of a local algorithm to converge to a
feasible point does not guarantee that the original problem is infeasible.
Thus, relaxations and local solution algorithms have complementary
capabilities with respect to the question of problem feasibility. Fur-
thermore, many global solution algorithms compute an *optimality gap*
by comparing the objective value bound from a relaxation with the
achievable objective value from a feasible point obtained via a local
solution algorithm. In order to provably obtain a global optimum, these
algorithms then use a variety of techniques to shrink the optimality gap.
Also note that the objective value bounds can be directly useful, for
instance, in algorithms that aim to achieve robustness with respect to
a set of possible uncertainty realizations, compute bounds on voltage
stability margins, etc. The references at the end of §7.2 provide examples
of these and other synergistic uses of various power flow representations.

The remainder of this monograph is organized as follows. Chapter 2
describes the power flow equations. Chapter 3 overviews the optimization
tools which form the basis for the power flow representations. Chapters 4
and 5 review the literature of power flow relaxations and approximations,
respectively. Chapter 6 overviews various techniques for obtaining a
feasible point, focusing on recent developments. Chapter 7 concludes
the monograph and discusses open research topics.

References

[1] D. Bienstock and A. Verma, "Strong NP-hardness of AC Power Flows Feasibility," *arXiv:1512.07315*, December 2015.

[2] K. Lehmann, A. Grastien, and P. Van Hentenryck, "AC-Feasibility on Tree Networks is NP-Hard," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 798–801, January 2016.

[3] W. A. Bukhsh, A. Grothey, K. I. M. McKinnon, and P. A. Trodden, "Local Solutions of the Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780–4788, 2013.

[4] A. M. Sasson and H. M. Merrill, "Some Applications of Optimization Techniques to Power Systems Problems," *Proceedings of the IEEE*, vol. 62, no. 7, pp. 959–972, July 1974.

[5] J. A. Momoh, *Electric Power System Applications of Optimization*. CRC Press, 2008.

[6] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Prentice Hall, 1998.

[7] A. J. Wood, B. F. Wollenberg, and G. B. Sheble, *Power Generation, Operation and Control*. John Wiley and Sons, Inc., third ed., 2013.

[8] B. Stott, "Review of Load-Flow Calculation Methods," *Proceedings of the IEEE*, vol. 62, no. 7, pp. 916–929, July 1974.

[9] D. Mehta, D. K. Molzahn, and K. Turitsyn, "Recent Advances in Computational Methods for the Power Flow Equations," in *American Control Conference (ACC)*, (Boston, MA, USA), pp. 1753–1765, July 2016.

[10] H. H. Happ, "Optimal Power Dispatch – A Comprehensive Survey," *IEEE Transactions on Power Apparatus and Systems*, vol. 96, no. 3, pp. 841–854, 1977.

[11] S. N. Talukdar and F. F. Wu, "Computer-Aided Dispatch for Electric Power Systems," *Proceedings of the IEEE*, vol. 69, no. 10, pp. 1212–1231, October 1981.

[12] J. L. Carpentier, "Optimal Power Flows: Uses, Methods and Developments," in *IFAC Symposium on Planning and Operation of Electric Energy Systems*, vol. 18, (Rio de Janeiro, Brazil), pp. 11–21, July 1985.

[13] M. Huneault and F. D. Galiana, "A Survey of the Optimal Power Flow Literature," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 762–770, May 1991.

[14] J. A. Momoh, R. Adapa, and M. E. El-Hawary, "A Review of Selected Optimal Power Flow Literature to 1993. Parts I and II," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 96–111, February 1999.

[15] Z. Qiu, G. Deconinck, and R. Belmans, "A Literature Survey of Optimal Power Flow Problems in the Electricity Market Context," in *IEEE Power Systems Conference and Exposition (PSCE)*, (Seattle, WA, USA), pp. 1–6, March 2009.

[16] S. Frank, I. Steponavice, and S. Rebennack, "Optimal Power Flow: A Bibliographic Survey, Parts I and II," *Energy Systems*, vol. 3, no. 3, pp. 221–289, 2012.

[17] A. Castillo and R. P. O'Neill, "Survey of Approaches to Solving the ACOPF (OPF Paper 4)," tech. rep., US Federal Energy Regulatory Commission, March 2013.

[18] P. Panciatici, M. C. Campi, S. Garatti, S. H. Low, D. K. Molzahn, X. A. Sun, and L. Wehenkel, "Advanced Optimization Methods for Power Systems," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–18, August 2014.

[19] J. A. Taylor, *Convex Optimization of Power Systems*. Cambridge University Press, April 2015.

[20] S. Frank and S. Rebennack, "An Introduction to Optimal Power Flow: Theory, Formulation, and Examples," *IIE Transactions*, vol. 48, no. 12, pp. 1172–1197, 2016.

[21] H. Abdi, S. D. Beigvand, and M. L. Scala, "A Review of Optimal Power Flow Studies applied to Smart Grids and Microgrids," *Renewable and Sustainable Energy Reviews*, vol. 71, pp. 742–766, May 2017.

[22] B. Stott, O. Alsaç, and A. J. Monticelli, "Security Analysis and Optimization," *Proceedings of the IEEE*, vol. 75, no. 12, pp. 1623–1644, December 1987.

[23] B. Stott and O. Alsaç, "Optimal Power Flow–Basic Requirements for Real-Life Problems and their Solutions," in *12th Symposium of Specialists in Electric Operational and Expansion Planning (SEPOPE)*, (Rio de Janeiro, Brazil), May 2012.

[24] F. Capitanescu, J. L. M. Ramos, P. Panciatici, D. Kirschen, A. M. Marcolini, L. Platbrood, and L. Wehenkel, "State-of-the-Art, Challenges, and Future Trends in Security Constrained Optimal Power Flow," *Electric Power Systems Research*, vol. 81, no. 8, pp. 1731–1741, 2011.

[25] F. Capitanescu, "Critical Review of Recent Advances and Further Developments Needed in AC Optimal Power Flow," *Electric Power Systems Research*, vol. 136, pp. 57–68, 2016.

[26] S. Abhyankar, G. Geng, M. Anitescu, X. Wang, and V. Dinavahi, "Solution Techniques for Transient Stability-Constrained Optimal Power Flow – Part I," *IET Generation, Transmission & Distribution*, vol. 11, pp. 3177–3185, August 2017.

[27] G. Geng, S. Abhyankar, X. Wang, and V. Dinavahi, "Solution Techniques for Transient Stability-Constrained Optimal Power Flow – Part II," *IET Generation, Transmission & Distribution*, vol. 11, pp. 3186–3193, August 2017.

[28] S. Sen and D. P. Kothari, "Optimal Thermal Generating Unit Commitment: A Review," *International Journal of Electrical Power & Energy Systems*, vol. 20, no. 7, pp. 443–451, 1998.

[29] N. P. Padhy, "Unit Commitment–A Bibliographical Survey," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1196–1205, May 2004.

[30] M. Tahanan, W. Van Ackooij, A. Frangioni, and F. Lacalandra, "Large-Scale Unit Commitment Under Uncertainty," *4OR*, vol. 13, no. 2, pp. 115–171, 2015.

[31] W. van Ackooij, I. Danti Lopez, A. Frangioni, F. Lacalandra, and M. Tahanan, "Large-Scale Unit Commitment Under Uncertainty: An Updated Literature Survey," *Annals of Operations Research*, vol. 271, no. 1, pp. 11–85, December 2018.

[32] F. F. Wu, "Power System State Estimation: A Survey," *International Journal of Electrical Power & Energy Systems*, vol. 12, no. 2, pp. 80–87, 1990.

[33] A. Monticelli, *State Estimation in Electric Power Systems.* Kluwer Academic Publishers, 1999.

[34] A. Abur and A. Gómez Expósito, *Power System State Estimation: Theory and Implementation.* Marcel Dekker, 2004.

[35] V. Kekatos, G. Wang, H. Zhu, and G. B. Giannakis, "PSSE Redux: Convex Relaxation, Decentralized, Robust, and Dynamic Approaches," *arXiv:1708.03981*, August 2017.

[36] K. W. Hedman, S. S. Oren, and R. P. O'Neill, "A Review of Transmission Switching and Network Topology Optimization," in *IEEE Power & Energy Society General Meeting*, pp. 1–7, July 2011.

[37] C. W. Taylor, "Modeling of Voltage Collapse Including Dynamic Phenomena." CIGRE Task Force 38-02-10 Report, 1993.

[38] N. D. Hatziargyriou, J. van Hecke, and T. van Cutsem, "Indices Predicting Voltage Collapse Including Dynamic Phenomena." CIGRE Task Force 38.02.11 Report, 1994.

[39] I. Dobson, T. Van Cutsem, C. Vournas, C. L. DeMarco, M. Venkatasubramanian, T. Overbye, and C. A. Canizares, "Voltage Stability Assessment: Concepts, Practices and Tools." IEEE Power Engineering Society, Power System Stability Subcommittee Special Publication SP101PSS, August 2002.

[40] C. W. Taylor, *Power System Voltage Stability.* McGraw-Hill, 1994.

[41] M. Vaiman, K. Bell, Y. Chen, B. Chowdhury, I. A. Dobson, P. Hines, M. Papic, S. Miller, and P. Zhang, "Risk Assessment of Cascading Outages: Methodologies and Challenges," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 631–641, May 2012.

[42] M. Yazdanian and A. Mehrizi-Sani, "Distributed Control Techniques in Microgrids," *IEEE Transactions on Smart Grid*, vol. 5, no. 6, pp. 2901–2909, November 2014.

[43] H. Han, X. Hou, J. Yang, J. Wu, M. Su, and J. M. Guerrero, "Review of Power Sharing Control Strategies for Islanding Operation of AC Microgrids," *IEEE Transactions on Smart Grid*, vol. 7, no. 1, pp. 200–215, January 2016.

[44] A. Kargarian, J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, and R. Baldick, "Toward Distributed/Decentralized DC Optimal Power Flow Implementation in Future Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2574–2594, July 2018.

[45] D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, and J. Lavaei, "A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2939–2940, November 2017.

[46] G. A. Pagani and M. Aiello, "The Power Grid as a Complex Network: A Survey," *Physica A: Statistical Mechanics and its Applications*, vol. 392, no. 11, pp. 2688–2700, 2013.

[47] F. Geth, R. D'Hulst, and D. Van Hertem, "Convex Power Flow Models for Scalable Electricity Market Modelling," in *24th International Conference & Exhibition on Electricity Distribution (CIRED)*, no. 1, (Glasgow, Scotland, UK), pp. 989–993, June 2017.

[48] G. Wang and H. Hijazi, "Mathematical Programming Methods for Microgrid Design and Operations: A Survey on Deterministic and Stochastic Approaches," *Computational Optimization and Applications*, June 2018.

[49] C. Coffrin and L. A. Roald, "Convex Relaxations in Power System Optimization: A Brief Introduction," *arXiv:1807.07227*, July 2018. videos available at https://www.youtube.com/watch?v=gB43TmcoUpA&list=PLeuOzWTGxj2ZZ_XUutDwNFvNfSWwWCgR5.

[50] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.

[51] C. Coffrin, R. Bent, K. Sundar, Y. Ng, and M. Lubin, "PowerModels.jl: An Open-Source Framework for Exploring Power Flow Formulations," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[52] B. C. Lesieutre and D. Wu, "An Efficient Method to Locate All the Load Flow Solutions – Revisited," in *53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 381–388, September 2015.

[53] D. Wu, D. K. Molzahn, B. C. Lesieutre, and K. Dvijotham, "A Deterministic Method to Identify Multiple Local Extrema for the AC Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 654–668, January 2018.

[54] Y. C. Chen and S. V. Dhople, "Power Divider," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 5135–5143, 2016.

[55] Y. C. Chen, A. Al-Digs, and S. V. Dhople, "Mapping Nodal Power Injections to Branch Flows in Connected LTI Electrical Networks," in *IEEE International Symposium on Circuits and Systems (ISCAS)*, pp. 2146–2149, May 2016.

[56] D. Chéverez-González and C. L. DeMarco, "Characterization of Feasible LMPs: Inclusion of Losses and Reactive Power," in *39th North American Power Symposium*, (Las Cruces, NM, USA), pp. 440–447, September 2007.

[57] J. Lin and D. Chéverez-González, "Network-Driven Dynamic Congestion based on Mutually Orthogonal LMP Decomposition," in *8th IREP Symposium on Bulk Power System Dynamics and Control*, (Rio de Janeiro, Brazil), pp. 1–8, August 2010.

[58] B. Park, J. Netha, M. C. Ferris, and C. L. DeMarco, "Sparse Tableau Formulation for Optimal Power Flow Applications," *arXiv:1706.01372*, June 2017.

[59] R. P. O'Neill, A. Castillo, and M. B. Cain, "The IV Formulation and Linear Approximations of the AC Optimal Power Flow Problem (OPF Paper 2)," tech. rep., US Federal Energy Regulatory Commission, December 2012.

[60] D. M. Bromberg, M. Jereminov, X. Li, G. Hug, and L. Pileggi, "An Equivalent Circuit Formulation of the Power Flow Problem with Current and Voltage State Variables," in *IEEE Eindhoven PowerTech*, (Eindhoven, Netherlands), pp. 1–6, June 2015.

[61] M. Jereminov, D. M. Bromberg, X. Li, G. Hug, and L. Pileggi, "Improving Robustness and Modeling Generality for Power Flow Analysis," in *IEEE PES Transmission and Distribution Conference and Exposition (PES T&D)*, (Dallas, TX, USA), pp. 1–5, May 2016.

[62] M. Jereminov, D. M. Bromberg, A. Pandey, X. Li, G. Hug, and L. Pileggi, "An Equivalent Circuit Formulation for Three-Phase Power Flow Analysis of Distribution Systems," in *IEEE Power & Energy Society Transmission and Distribution Conference and Exposition (PES T&D)*, (Dallas, TX, USA), pp. 1–5, May 2016.

[63] A. Pandey, M. Jereminov, X. Li, G. Hug, and L. Pileggi, "Unified Power System Analyses and Models using Equivalent Circuit Formulation," in *IEEE PES Conference on Innovative Smart Grid Technologies (ISGT)*, (Minneapolis, MN, USA), pp. 1–5, September 2016.

[64] A. Castillo, P. Lipka, J. Watson, S. S. Oren, and R. P. O'Neill, "A Successive Linear Programming Approach to Solving the IV-ACOPF," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 2752–2763, July 2016.

[65] S. Misra, L. A. Roald, M. Vuffray, and M. Chertkov, "Fast and Robust Determination of Power System Emergency Control Actions," in *10th IREP Symposium on Bulk Power System Dynamics and Control*, (Espinho, Portugal), August 2017.

[66] A. Trias, "HELM: The Holomorphic Embedding Load-Flow Method. Foundations and Implementations," *Foundations and Trends in Electric Energy Systems*, vol. 3, no. 3–4, pp. 140–370, December 2018.

[67] T. Chen and D. Mehta, "On the Network Topology Dependent Solution Count of the Algebraic Load Flow Equations," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1451–1460, March 2018.

[68] Y. V. Makarov, D. J. Hill, and I. A. Hiskens, "Properties of Quadratic Equations and their Application to Power System Analysis," *International Journal of Electrical Power & Energy Systems*, vol. 22, pp. 313–323, 2000.

[69] M. E. Baran and F. F. Wu, "Optimal Capacitor Placement on Radial Distribution Systems," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 725–734, January 1989.

[70] M. Baran and F. F. Wu, "Optimal Sizing of Capacitors Placed on a Radial Distribution System," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 735–743, January 1989.

[71] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "DistFlow Extensions for AC Transmission Systems," *arXiv:1506.04773*, June 2014.

[72] R. A. Jabr, "A Conic Quadratic Format for the Load Flow Equations of Meshed Networks," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 2285–2286, November 2007.

[73] S. Bose, S. H. Low, T. Teeraratkul, and B. Hassibi, "Equivalent Relaxations of Optimal Power Flow," *IEEE Transactions on Automatic Control*, vol. 60, no. 3, pp. 729–742, 2015.

[74] S. H. Low, "Convex Relaxation of Optimal Power Flow–Part I: Formulations and Equivalence," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 1, pp. 15–27, March 2014.

[75] S. H. Low, "Convex Relaxation of Optimal Power Flow–Part II: Exactness," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 2, pp. 177–189, June 2014.

[76] N. Biggs, *Algebraic Graph Theory*. Cambridge University Press, 1993.

[77] T. Kavitha, C. Liebchen, K. Mehlhorn, D. Michail, R. Rizzi, T. Ueckerdt, and K. A. Zweig, "Cycle Bases in Graphs Characterization, Algorithms, Complexity, and Applications," *Computer Science Review*, vol. 3, no. 4, pp. 199–243, 2009.

[78] J. Baillieul and C. Byrnes, "Geometric Critical Point Analysis of Lossless Power System Models," *IEEE Transactions on Circuits and Systems*, vol. 29, no. 11, pp. 724–737, 1982.

[79] D. K. Molzahn, D. Mehta, and M. Niemerg, "Toward Topologically Based Upper Bounds on the Number of Power Flow Solutions," in *American Control Conference (ACC)*, (Boston, MA, USA), pp. 5927–5932, July 2016.

[80] O. Coss, J. D. Hauenstein, H. Hoon, and D. K. Molzahn, "Locating and Counting Equilibria of the Kuramoto Model with Rank One Coupling," *SIAM Journal on Applied Algebra and Geometry*, vol. 2, no. 1, pp. 45–71, 2018.

[81] W. Ma and S. Thorp, "An Efficient Algorithm to Locate All the Load Flow Solutions," *IEEE Transactions on Power Systems*, vol. 8, no. 3, pp. 1077–1083, 1993.

[82] D. Mehta, H. D. Nguyen, and K. Turitsyn, "Numerical Polynomial Homotopy Continuation Method to Locate all the Power Flow Solutions," *IET Generation, Transmission & Distribution*, vol. 10, no. 12, pp. 2972–2980, 2016.

[83] A. Zacariah, Z. Charles, N. Boston, and B. C. Lesieutre, "Distributions of the Number of Solutions to the Network Power Flow Equations," in *International Symposium on Circuits and Systems (ISCAS)*, (Florence, Italy), May 2018.

[84] J. Y. Jackson, "Interpretation and Use of Generator Reactive Capability Diagrams," *IEEE Transactions on Industry and General Applications*, no. 6, pp. 729–732, 1971.

[85] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*. McGraw-hill New York, 1994.

[86] D. Kosterev, A. Meklin, J. Undrill, B. Lesieutre, W. Price, D. Chassin, R. Bravo, and S. Yang, "Load Modeling in Power System Studies: WECC Progress Update," in *IEEE Power & Energy Society General Meeting*, (Pittsburgh, PA, USA), pp. 1–8, 2008.

[87] E. C. M. Stahl, "Economic Loading of Generating Stations," *Electrical Engineering*, vol. 50, no. 9, pp. 722–727, September 1931.

[88] M. J. Steinberg and T. H. Smith, *Economy Loading of Power Plants and Electric Systems*. Wiley, 1943.

[89] L. K. Kirchmeyer, *Economic Operation of Power Systems*. Wiley New York, 1958.

[90] J. B. Ward and H. W. Hale, "Digital Computer Solution of Power-Flow Problems," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 75, no. 3, pp. 398–404, January 1956.

[91] J. E. Van Ness, "Iteration Methods for Digital Load Flow Studies," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 78, no. 3, pp. 583–586, April 1959.

[92] N. Sato and W. F. Tinney, "Techniques for Exploiting the Sparsity or the Network Admittance Matrix," *IEEE Transactions on Power Apparatus and Systems*, vol. 82, no. 69, pp. 944–950, December 1963.

[93] W. F. Tinney and C. E. Hart, "Power Flow Solution by Newton's Method," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-86, no. 11, pp. 1449–1460, November 1967.

[94] H. H. Spencer and H. L. Hazen, "Artificial Representation of Power Systems," *Transactions of the American Institute of Electrical Engineers*, vol. XLIV, pp. 72–79, January 1925.

[95] R. B. Squires, "Economic Dispatch of Generation Directly From Power System Voltages and Admittances," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 79, no. 3, pp. 1235–1244, April 1960.

[96] J. L. Carpentier, "Contribution a l'Etude du Dispatching Economique," *Bulletin de la Societe Francoise des Electriciens*, vol. 8, no. 3, pp. 431–447, 1962.

[97] R. D. Zimmerman, "AC Power Flows, Generalized OPF Costs and their Derivatives using Complex Matrix Notation," tech. rep., MATPOWER Technical Note 2, February 2010.

[98] R. Madani, S. Sojoudi, and J. Lavaei, "Convex Relaxation for Optimal Power Flow Problem: Mesh Networks," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 199–211, January 2015.

[99] B. Park, L. Tang, M. C. Ferris, and C. L. DeMarco, "Examination of Three Different ACOPF Formulations With Generator Capability Curves," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 2913–2923, July 2017.

[100] I. A. Hiskens and R. J. Davy, "Exploring the Power Flow Solution Space Boundary," *IEEE Transactions on Power Systems*, vol. 16, no. 3, pp. 389–395, August 2001.

[101] B. C. Lesieutre and I. A. Hiskens, "Convexity of the Set of Feasible Injections and Revenue Adequacy in FTR Markets," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1790–1798, November 2005.

[102] Y. V. Makarov, Z. Y. Dong, and D. J. Hill, "On Convexity of Power Flow Feasibility Boundary," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 811–813, May 2008.

[103] D. K. Molzahn, B. C. Lesieutre, and C. L. DeMarco, "Investigation of Non-Zero Duality Gap Solutions to a Semidefinite Relaxation of the Power Flow Equations," in *47th Hawaii International Conference on Systems Sciences (HICSS)*, (Waikoloa, HI, USA), pp. 2325–2334, January 2014.

[104] J. Lavaei, D. Tse, and B. Zhang, "Geometry of Power Flows and Optimization in Distribution Networks," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 572–583, March 2014.

[105] S. Chandra, D. Mehta, and A. Chakrabortty, "Equilibria Analysis of Power Systems Using a Numerical Homotopy Method," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[106] B. Polyak and E. Gryazina, "Convexity/Nonconvexity Certificates for Power Flow Analysis," in *Proceedings of the First International Symposium on Energy System Optimization (ISESO)*, (Heidelberg, Germany), pp. 221–230, November 2015.

[107] D. K. Molzahn, "Computing the Feasible Spaces of Optimal Power Flow Problems," *IEEE Transactionson Power Systems*, vol. 32, no. 6, pp. 4752–4763, November 2017.

[108] H. D. Chiang and C. Y. Jiang, "Feasible Region of Optimal Power Flow: Characterization and Applications," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 236–244, January 2018.

[109] M. R. Narimani, D. K. Molzahn, D. Wu, and M. L. Crow, "Empirical Investigation of Non-Convexities in Optimal Power Flow Problems," in *American Control Conference (ACC)*, (Milwaukee, WI, USA), June 2018.

[110] D. K. Molzahn, "Identifying and Characterizing Non-Convexities in Feasible Spaces of Optimal Power Flow Problems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 65, no. 5, pp. 672–676, May 2018.

[111] K. Bestuzheva and H. Hijazi, "Invex Optimization Revisited," *Journal of Global Optimization*, April 2018.

[112] F. Thams, A. Venzke, R. Eriksson, and S. Chatzivasileiadis, "Efficient Database Generation for Data-driven Security Assessment of Power Systems," *arXiv:1806.01074*, June 2018.

[113] A. Dymarsky and K. Turitsyn, "Convexity of Solvability Set of Power Distribution Networks," *arXiv:1803.11197*, March 2018.

[114] A. Dymarsky, E. Gryazina, S. Volodin, and B. Polyak, "Geometry of Quadratic Maps via Convex Relaxation," *arXiv:1810.00896*, October 2018.

[115] M. Slater, "Lagrange Multipliers Revisited," *Cowles Commission Discussion Paper No. 403*, November 1950.

[116] X. Cao, J. Wang, and B. Zeng, "A Study on the Strong Duality of Conic Relaxation of AC Optimal Power Flow in Radial Networks," *arXiv:1807.08785*, July 2018.

[117] A. Hauswirth, S. Bolognani, G. Hug, and F. Dörfler, "Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow," *arXiv:1806.06615*, June 2018.

[118] D. Peterson, "A Review of Constraint Qualifications in Finite-Dimensional Spaces," *SIAM Review*, vol. 15, no. 3, pp. 639–654, 1973.

[119] K. C. Almeida and F. D. Galiana, "Critical Cases in the Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 11, no. 3, pp. 1509–1518, August 1996.

[120] C. Coffrin, "Visualizations of AC Power Flow over a Line." https://github.com/ccoffrin/ac-powerflow-vis, February 2017.

[121] D. K. Molzahn, S. S. Baghsorkhi, and I. A. Hiskens, "Semidefinite Relaxations of Equivalent Optimal Power Flow Problems: An Illustrative Example," in *IEEE International Symposium on Circuits and Systems (ISCAS)*, (Lisbon, Portugal), pp. 1887–1890, May 2015.

[122] D. K. Molzahn and I. A. Hiskens, "Convex Relaxations of Optimal Power Flow Problems: An Illustrative Example," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 5, pp. 650–660, May 2016.

[123] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2009.

[124] J. Nocedal and S. Wright, *Numerical Optimization*. Springer-Verlag New York, 2nd ed., 2006.

[125] G. P. McCormick, "Computability of Global Solutions to Factorable Nonconvex Programs: Part I–Convex Underestimating Problems," *Mathematical Programming*, vol. 10, no. 1, pp. 147–175, December 1976.

[126] M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of Second-Order Cone Programming," *Linear Algebra and its Applications*, vol. 284, no. 1-3, pp. 193–228, 1998.

[127] MOSEK ApS, "Modeling Cookbook." http://docs.mosek.com/MOSEKModelingCookbook-letter.pdf.

[128] G. Strang, *Linear Algebra and its Applications*. Orlando, FL: Harcourt Brace Jovanovich, 3rd ed., 1988.

[129] C. Josz and D. K. Molzahn, "Lasserre Hierarchy for Large Scale Polynomial Optimization in Real and Complex Variables," *SIAM Journal on Optimization*, vol. 28, no. 2, pp. 1017–1048, 2018.

[130] J. C. Gilbert and C. Josz, "Plea for a Semidefinite Optimization Solver in Complex Numbers," tech. rep., LAAS–Laboratoire d'analyse et d'architecture des systèmes (Toulouse, France), Mach 2017.

[131] J. Löfberg, "YALMIP: A Toolbox for Modeling and Optimization in MATLAB," in *IEEE International Symposium on Computer Aided Control Systems Design (CACSD)*, (New Orleans, LA, USA), pp. 284–289, September 2004.

[132] M. Grant and S. Boyd, "CVX: Matlab Software for Disciplined Convex Programming, version 2.1." http://cvxr.com/cvx, March 2017.

[133] MOSEK ApS, "The MOSEK Optimization Toolbox." https://www.mosek.com/documentation/.

[134] M. Kočvara and M. Stingl, "PENSDP User's Guide (Version 2.2)." http://www.penopt.com/pensdp.html.

[135] J. F. Sturm, "Using SeDuMi 1.02, A MATLAB Toolbox for Optimization Over Symmetric Cones," *Optimization Methods and Software*, vol. 11, no. 1, pp. 625–653, 1999.

[136] R. H. Tütüncü, K. C. Toh, and M. J. Todd, "Solving Semidefinite-Quadratic-Linear Programs using SDPT3," *Mathematical Programming*, vol. 95, no. 2, pp. 189–217, 2003.

[137] M. Yamashita, K. Fujisawa, M. Fukuda, K. Kobayashi, K. Nakata, and M. Nakata, "Latest Developments in the SDPA Family for Solving Large-Scale SDPs," in *Handbook on Semidefinite, Conic and Polynomial Optimization* (M. F. Anjos and J. B. Lasserre, eds.), (Boston, MA), pp. 687–713. Boston, MA: Springer US, 2012.

[138] B. Borchers, "CSDP, A C Library for Semidefinite Programming," *Optimization Methods and Software*, vol. 11, no. 1-4, pp. 613–623, 1999.

[139] T. Gally, M. E. Pfetsch, and S. Ulbrich, "A Framework for Solving Mixed-Integer Semidefinite Programs," *Optimization Methods and Software*, vol. 33, no. 3, pp. 594–632, 2018.

[140] M. Lubin, E. Yamangil, R. Bent, and J. P. Vielma, "Extended Formulations in Mixed-Integer Convex Programming," in *18th International Conference on Integer Programming and Combinatorial Optimization (IPCO)*, (Liège, Belgium), pp. 102–113, June 2016.

[141] D. K. Molzahn, J. T. Holzer, B. C. Lesieutre, and C. L. DeMarco, "Implementation of a Large-Scale Optimal Power Flow Solver Based on Semidefinite Programming," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3987–3998, November 2013.

[142] M. Andersen, A. Hansson, and L. Vandenberghe, "Reduced-Complexity Semidefinite Relaxations of Optimal Power Flow Problems," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1855–1863, July 2014.

[143] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "The QC Relaxation: A Theoretical and Computational Study on Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 3008–3018, July 2016.

[144] D. K. Molzahn, "Incorporating Squirrel-Cage Induction Machine Models in Convex Relaxations of Optimal Power Flow Problems," *IEEE Transactions on Power Systems*, vol. 32, no. 6, pp. 4972–4974, November 2017.

[145] D. K. Molzahn, B. C. Lesieutre, and C. L. DeMarco, "Approximate Representation of ZIP Loads in a Semidefinite Relaxation of the OPF Problem," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1864–1865, July 2014.

[146] Z. Wang, D. S. Kirschen, and B. Zhang, "Accurate Semidefinite Programming Models for Optimal Power Flow in Distribution Systems," *arXiv:1711.07853*, December 2017.

[147] R. A. Jabr, "Optimal Power Flow Using an Extended Conic Quadratic Formulation," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1000–1008, August 2008.

[148] M. Baradar, M. R. Hesamzadeh, and M. Ghandhari, "Second-Order Cone Programming for Optimal Power Flow in VSC-Type AC-DC Grids," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4282–4291, November 2013.

[149] S. Bahrami, F. Therrien, V. W. S. Wong, and J. Jatskevich, "Semidefinite Relaxation of Optimal Power Flow for AC–DC Grids," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 289–304, January 2017.

[150] L. Cupelli, J. T. Ruiz, and A. Monti, "Optimal Voltage Control in Power Systems: Inclusion of Discrete Decision Variables," in *19th IEEE Mediterranean Electrotechnical Conference (MELECON)*, (Marrakech, Morocco), pp. 138–143, May 2018.

[151] A. Venzke and S. Chatzivasileiadis, "Convex Relaxations of Probabilistic AC Optimal Power Flow for Interconnected AC and HVDC Grids," *arXiv:1804.00035*, April 2018.

[152] E. Briglia, S. Alaggia, and F. Paganini, "Distribution Network Management based on Optimal Power Flow: Integration of Discrete Decision Variables," in *51st Annual Conference on Information Sciences and Systems (CISS)*, (Baltimore, MD, USA), pp. 1–6, March 2017.

[153] C.-Y. Chang, S. Martinez, and J. Cortes, "Virtual-Voltage Partition-Based Approach to Mixed-Integer Optimal Power Flow Problems," in *55th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 307–314, September 2017.

[154] B. Kocuk, S. S. Dey, and X. A. Sun, "New Formulation and Strong MISOCP Relaxations for AC Optimal Transmission Switching Problem," *IEEE Transactions on Power Systems*, vol. 32, no. 6, pp. 4161–4170, November 2017.

[155] K. Bestuzheva, H. L. Hijazi, and C. Coffrin, "Convex Relaxations for Quadratic On/Off Constraints and Applications to Optimal Transmission Switching," Preprint: *http://www.optimization-online.org/DB_FILE/2016/07/5565.pdf*, 2016.

[156] Y. Liu, J. Li, and L. Wu, "Coordinated Optimal Network Reconfiguration and Voltage Regulator/DER Control for Unbalanced Distribution Systems," to appear in *IEEE Transactions on Smart Grid*, 2019.

[157] B. A. Robbins, H. Zhu, and A. Domínguez-Garciá, "Optimal Tap Setting of Voltage Regulation Transformers in Unbalanced Distribution Systems," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 256–267, January 2016.

[158] W. Wu, Z. Tian, and B. Zhang, "An Exact Linearization Method for OLTC of Transformer in Branch Flow Model," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 2475–2476, May 2017.

[159] Y. Liu, J. Li, L. Wu, and T. Ortmeyer, "Chordal Relaxation Based ACOPF for Unbalanced Distribution Systems with DERs and Voltage Regulation Devices," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 970–984, 2018.

[160] M. Bazrafshan, N. Gatsis, and H. Zhu, "Optimal Tap Selection of Step-Voltage Regulators in Multi-Phase Distribution Networks," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[161] M. Bazrafshan, N. Gatsis, and H. Zhu, "Optimal Power Flow with Step-Voltage Regulators in Multi-Phase Distribution Networks," *arXiv:1901.04566*, January 2019.

[162] N. Z. Shor, "Quadratic Optimization Problems," *Soviet Journal of Computer and Systems Sciences*, vol. 25, pp. 1–11, 1987.

[163] X. Bai, H. Wei, K. Fujisawa, and Y. Wang, "Semidefinite Programming for Optimal Power Flow Problems," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 6-7, pp. 383–392, 2008.

[164] J. Lavaei and S. H. Low, "Zero Duality Gap in Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 92–107, February 2012.

[165] J. Lavaei and S. H. Low, "Convexification of Optimal Power Flow Problem," in *48th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 223–232, September 2010.

[166] University of Washington Department of Electrical Engineering, "Power Systems Test Case Archive." http://www.ee.washington.edu/research/pstca/.

[167] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed Optimal Power Flow for Smart Microgrids," *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1464–1475, September 2013.

[168] L. Gan and S. H. Low, "Convex Relaxations and Linear Approximation for Optimal Power Flow in Multiphase Radial Networks," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–9, August 2014.

[169] C. Zhao, E. Dall'Anese, and S. H. Low, "Convex Relaxation of OPF in Multiphase Radial Networks with Delta Connections," in *10th IREP Symposium on Bulk Power System Dynamics and Control*, (Espinho, Portugal), August 2017.

[170] B. C. Lesieutre, D. K. Molzahn, A. R. Borden, and C. L. DeMarco, "Examining the Limits of the Application of Semidefinite Programming to Power Flow Problems," in *49th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), September 2011.

[171] B. Kocuk, S. S. Dey, and X. A. Sun, "Inexactness of SDP Relaxation and Valid Inequalities for Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 642–651, January 2016.

[172] I. Zorin, S. Vasilyev, and E. Gryazina, "Fragility of the Semidefinite Relaxation for the Optimal Power Flow Problem," in *IEEE International Conference on the Science of Electrical Engineering (ICSEE)*, (Eilat, Israel), pp. 1–5, November 2016.

[173] W. Wang and N. Yu, "Chordal Conversion based Convex Iteration Algorithm for Three-phase Optimal Power Flow Problems," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1603–1613, March 2018.

[174] T. W. K. Mak, L. Shi, and P. Van Hentenryck, "Phase Transitions for Optimality Gaps in Optimal Power Flows: A Study on the French Transmission Network," *arXiv:1807.05460*, July 2018.

[175] B. Park, "Examining the rank of Semi-definite Programming for Power System State Estimation," *arXiv:1802.06094*, February 2018.

[176] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Strengthening the SDP Relaxation of AC Power Flows with Convex Envelopes, Bound Tightening, and Valid Inequalities," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3549–3558, September 2017.

[177] A. Eltved, J. Dahl, and M. S. Andersen, "On the Robustness and Scalability of Semidefinite Relaxation for Optimal Power Flow Problems," *arXiv:1806.08620*, June 2018.

[178] C. Coffrin, D. Gordon, and P. Scott, "NESTA, The NICTA Energy System Test Case Archive," *arXiv:1411.0359*, August 2016.

[179] IEEE PES Task Force on Benchmarks for Validation of Emerging Power System Algorithms, "Power Grid Lib – Optimal Power Flow." https://github.com/power-grid-lib/pglib-opf.

[180] ARPA-E, "Generating Realistic Information for Development of Distribution and Transmission Algorithms (GRID DATA)." https://arpa-e.energy.gov/?q=pdfs/grid-data-de-foa-0001357, June 2015. Funding Opportunity Announcement Number DE-FOA-0001357.

[181] S. Y. Abdelouadoud, R. Girard, F. P. Neirac, and T. Guiot, "Optimal Power Flow of a Distribution System based on Increasingly Tight Cutting Planes Added to a Second Order Cone Relaxation," *International Journal of Electrical Power & Energy Systems*, vol. 69, pp. 9–17, July 2015.

[182] H. Gao, J. Liu, L. Wang, and Y. Liu, "Cutting Planes based Relaxed Optimal Power Flow in Active Distribution Systems," *Electric Power Systems Research*, vol. 143, pp. 272–280, 2017.

[183] C. Coffrin, R. Bent, B. Tasseff, K. Sundar, and S. Backhaus, "Relaxations of AC Maximal Load Delivery for Severe Contingency Analysis," to appear in *IEEE Transactions on Power Systems*, 2019.

[184] B. Zhang and D. Tse, "Geometry of Injection Regions of Power Networks," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 788–797, May 2013.

[185] H. Mahboubi and J. Lavaei, "Analysis of Semidefinite Programming Relaxation of Optimal Power Flow for Cyclic Networks," in *IEEE 57th Annual Conference on Decision and Control (CDC)*, (Miami Beach, FL, USA), December 2018.

[186] R. Madani, S. Sojoudi, G. Fazelnia, and J. Lavaei, "Finding Low-Rank Solutions of Sparse Linear Matrix Inequalities using Convex Optimization," *SIAM Journal on Optimization*, vol. 27, no. 2, pp. 725–758, 2017.

[187] R. Madani, M. Ashraphijuo, and J. Lavaei, "Promises of Conic Relaxation for Contingency-Constrained Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1297–1307, March 2016.

[188] D. K. Molzahn, C. Josz, I. A. Hiskens, and P. Panciatici, "A Laplacian-Based Approach for Finding Near Globally Optimal Solutions to OPF Problems," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 305–315, 2017.

[189] R. Louca, P. Seiler, and E. Bitar, "A Rank Minimization Algorithm to Enhance Semidefinite Relaxations of Optimal Power Flow," in *51st Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 1010–1020, October 2013.

[190] R. Gron, C. R. Johnson, E. M. Sá, and H. Wolkowicz, "Positive Definite Completions of Partial Hermitian Matrices," *Linear Algebra and its Applications*, vol. 58, pp. 109–124, 1984.

[191] L. Vandenberghe and M. S. Andersen, "Chordal Graphs and Semidefinite Optimization," *Foundations and Trends in Optimization*, vol. 1, no. 4, pp. 241–433, 2015.

[192] T. A. Davis, J. R. Gilbert, S. Larimore, and E. Ng, "Algorithm 836: COLAMD, a Column Approximate Minimum Degree Ordering Algorithm," *ACM Transactions on Mathematical Software*, vol. 30, no. 3, pp. 377–380, September 2004.

[193] R. E. Tarjan and M. Yannakakis, "Simple Linear-Time Algorithms to Test Chordality of Graphs, Test Acyclicity of Hypergraphs, and Selectively Reduce Acyclic Hypergraphs," *SIAM Journal on Computing*, vol. 13, pp. 566–579, 1984.

[194] R. A. Jabr, "Radial Distribution Load Flow using Conic Programming," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1458–1459, August 2006.

[195] J. Dancis, "Positive Semidefinite Completions of Partial Hermitian Matrices," *Linear Algebra and its Applications*, vol. 175, pp. 97–114, 1992.

[196] M. Fukuda, M. Kojima, K. Murota, and K. Nakata, "Exploiting Sparsity in Semidefinite Programming via Matrix Completion I: General Framework," *SIAM Journal on Optimization*, vol. 11, no. 3, pp. 647–674, 2001.

[197] R. A. Jabr, "Exploiting Sparsity in SDP Relaxations of the OPF Problem," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 1138–1139, May 2012.

[198] J. A. Kersulis, I. A. Hiskens, C. Coffrin, and D. K. Molzahn, "Topological Graph Metrics for Detecting Grid Anomalies and Improving Algorithms," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[199] R. Y. Zhang and J. Lavaei, "Sparse Semidefinite Programs with Guaranteed Near-Linear Time Complexity via Dualized Clique Tree Conversion," *arXiv:1710.03475*, August 2018.

[200] J. Löfberg, "Dualize It: Software for Automatic Primal and Dual Conversions of Conic Programs," *Optimization Methods and Software*, vol. 24, no. 3, pp. 313–325, 2009.

[201] S. Sojoudi and J. Lavaei, "Physics of Power Networks Makes Hard Optimization Problems Easy to Solve," in *IEEE Power & Energy Society General Meeting*, (San Diego, CA, USA), pp. 1–8, July 2012.

[202] C. Bingane, M. F. Anjos, and S. Le Digabel, "Tight-and-Cheap Conic Relaxation for the AC Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 7181–7188, 2018.

[203] H. L. Hijazi, C. Coffrin, and P. Van Hentenryck, "Polynomial SDP Cuts for Optimal Power Flow," in *19th Power Systems Computation Conference (PSCC)*, (Genoa, Italy), pp. 1–7, June 2016.

[204] J. B. Lasserre, "On Representations of the Feasible Set in Convex Optimization," *Optimization Letters*, vol. 4, no. 1, pp. 1–5, 2010.

[205] J. B. Lasserre, "On Convex Optimization without Convex Representation," *Optimization Letters*, vol. 5, no. 4, pp. 549–556, 2011.

[206] A. Barzegar, D. K. Molzahn, and R. Su, "A Method for Quickly Bounding the Optimal Objective Value of an OPF Problem Using a Semidefinite Relaxation and a Local Solution," *arXiv:1808.04557*, August 2018.

[207] M. Yamashita, K. Fujisawa, M. Fukuda, K. Nakata, and M. Nakata, "Algorithm 925: Parallel Solver for Semidefinite Programming Problem Having Sparse Schur Complement Matrix," *ACM Transactions on Mathematical Software*, vol. 39, no. 1, November 2012.

[208] J. Mareček and M. Takáč, "A Low-Rank Coordinate-Descent Algorithm for Semidefinite Programming Relaxations of Optimal Power Flow," *Optimization Methods and Software*, vol. 32, no. 4, pp. 849–871, 2017.

[209] J. Liu, A. C. Liddell, J. Mareček, and M. Takáč, "Hybrid Methods in Solving Alternating-Current Optimal Power Flows," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2988–2998, November 2017.

[210] A. Lam, B. Zhang, and D. N. Tse, "Distributed Algorithms for Optimal Power Flow Problem," in *IEEE 51st Annual Conference on Decision and Control (CDC)*, (Maui, HI, USA), pp. 430–437, December 2012.

[211] A. Y.S. Lam, B. Zhang, A. Domínguez-Garciá, and D. Tse, "An Optimal and Distributed Method for Voltage Regulation in Power Distribution Systems," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1714–17726, July 2015.

[212] R. Madani, A. Kalbat, and J. Lavaei, "ADMM for Sparse Semidefinite Programming with Applications to Optimal Power Flow," in *IEEE 54th Annual Conference on Decision and Control (CDC)*, (Osaka, Japan), pp. 5932–5939, December 2015.

[213] R. Madani, A. Kalbat, and J. Lavaei, "A Low-Complexity Parallelizable Numerical Algorithm for Sparse Semidefinite Programming," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 4, pp. 1898–1909, December 2018.

[214] Q. Peng and S. H. Low, "Distributed Algorithm for Optimal Power Flow on an Unbalanced Radial Network," in *IEEE 54th Annual Conference on Decision and Control (CDC)*, (Osaka, Japan), pp. 6915–6920, December 2015.

[215] Q. Peng and S. H. Low, "Distributed Optimal Power Flow Algorithm for Radial Networks, I: Balanced Single Phase Case," *IEEE Transactions on Smart Grid*, vol. 9, no. 1, pp. 111–121, 2018.

[216] C. Josz, S. Fliscounakis, J. Maeght, and P. Panciatici, "AC Power Flow Data in MATPOWER and QCQP Format: iTesla, RTE Snapshots, and PEGASE," *arXiv:1603:01533*, March 2016.

[217] A. Raghunathan and A. Knyazev, "Degeneracy in Maximal Clique Decomposition for Semidefinite Programs," in *American Control Conference (ACC)*, (Boston, MA, USA), pp. 5605–5611, July 2016.

[218] V. Kungurtsev and J. Marecek, "A Two-Step Pre-Processing for Semidefinite Programming," *arXiv:1806.10868*, June 2018.

[219] D. K. Molzahn, C. Josz, I. A. Hiskens, and P. Panciatici, "Solution of Optimal Power Flow Problems using Moment Relaxations Augmented with Objective Function Penalization," in *IEEE 54th Annual Conference on Decision and Control (CDC)*, (Osaka, Japan), pp. 31–38, December 2015.

[220] D. K. Molzahn, B. C. Lesieutre, and C. L. DeMarco, "A Sufficient Condition for Global Optimality of Solutions to the Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 978–979, March 2014.

[221] J. B. Lasserre, "Global Optimization with Polynomials and the Problem of Moments," *SIAM Journal on Optimization*, vol. 11, no. 3, pp. 796–817, 2001.

[222] J. B. Lasserre, *Moments, Positive Polynomials and Their Applications*, vol. 1. Imperial College Press, 2010.

[223] C. Josz and D. Henrion, "Strong Duality in Lasserre's Hierarchy for Polynomial Optimization," *Optimization Letters*, vol. 10, no. 1, pp. 3–10, January 2016.

[224] J. Nie, "Optimality Conditions and Finite Convergence of Lasserre's Hierarchy," *Mathematical Programming*, vol. 146, no. 1-2, pp. 97–121, August 2014.

[225] P. A. Parrilo, "Semidefinite Programming Relaxations for Semialgebraic Problems," *Mathematical Programming*, vol. 96, pp. 293–320, 2003.

[226] R. Y. Zhang, C. Josz, and S. Sojoudi, "Conic Optimization Theory: Convexification Techniques and Numerical Algorithms," in *American Control Conference (ACC)*, (Milwaukee, WI, USA), June 2018.

[227] D. K. Molzahn and I. A. Hiskens, "Sparsity-Exploiting Moment-Based Relaxations of the Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3168–3180, November 2015.

[228] D. Henrion and J. B. Lasserre, "Detecting Global Optimality and Extracting Solutions in GloptiPoly," in *Positive Polynomials in Control* (D. Henrion and A. Garulli, eds.), pp. 293–310, Springer Berlin Heidelberg, 2005.

[229] D. K. Molzahn, C. Josz, I. A. Hiskens, and P. Panciatici, "Computational Analysis of Sparsity-Exploiting Moment Relaxations of the OPF Problem," in *19th Power Systems Computation Conference (PSCC)*, (Genoa, Italy), pp. 1–7, June 2016.

[230] C. Josz and D. K. Molzahn, "Moment/Sums-of-Squares Hierarchy for Complex Polynomial Optimization," *arXiv:1508.02068*, August 2015.

[231] J. Harmouch, H. Khalil, and B. Mourrain, "Structured Low Rank Decomposition of Multivariate Hankel Matrices," *Linear Algebra and its Applications*, vol. 542, pp. 162–185, 2018.

[232] C. Josz, J. B. Lasserre, and B. Mourrain, "Sparse Polynomial Interpolation: Compressed Sensing, Super Resolution, or Prony?," *arXiv:1708.06187*, August 2017.

[233] D. K. Molzahn and I. A. Hiskens, "Mixed SDP/SOCP Moment Relaxations of the Optimal Power Flow Problem," in *IEEE Eindhoven PowerTech*, (Eindhoven, Netherlands), pp. 1–6, June 2015.

[234] X. Kuang, L. Zuluaga, B. Ghaddar, and J. Naoum-Sawaya, "Approximating the ACOPF Problem with a Hierarchy of SOCP Problems," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[235] X. Kuang, B. Ghaddar, J. Naoum-Sawaya, and L. F. Zuluaga, "Alternative LP and SOCP Hierarchies for ACOPF Problems," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 2828–2836, July 2017.

[236] A. A. Ahmadi and A. Majumdar, "DSOS and SDSOS Optimization: LP and SOCP-based Alternatives to Sum of Squares Optimization," in *48th Annual Conference on Information Sciences and Systems (CISS)*, (Princeton, NJ, USA), pp. 1–5, 2014.

[237] C. Josz, "Counterexample to Global Convergence of DSOS and SDSOS Hierarchies," *arXiv:1707.02964*, July 2017.

[238] X. Kuang, B. Ghaddar, J. Naoum-Sawaya, and L. F. Zuluaga, "Alternative SDP and SOCP Approximations for Polynomial Optimization," *EURO Journal on Computational Optimization*, pp. 1–23, August 2018.

[239] A. A. Ahmadi, S. Dash, and G. Hall, "Optimization over Structured Subsets of Positive Semidefinite Matrices via Column Generation," *Discrete Optimization*, vol. 24, pp. 129–151, 2017.

[240] G. Hall, *Optimization over Nonnegative and Convex Polynomials With and Without Semidefinite Programming.* dissertation, Princeton University, June 2018.

[241] D. K. Molzahn and I. A. Hiskens, "Moment-Based Relaxation of the Optimal Power Flow Problem," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–7, August 2014.

[242] C. Josz, J. Maeght, P. Panciatici, and J. C. Gilbert, "Application of the Moment-SOS Approach to Global Optimization of the OPF Problem," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 463–470, January 2015.

[243] B. Ghaddar, J. Marecek, and M. Mevissen, "Optimal Power Flow as a Polynomial Optimization Problem," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 539–546, January 2016.

[244] J. Tian and H. Wei, "Global Optimization for the OPF Problem via Two-Degree SDP Method," *IEEJ Transactions on Electrical and Electronic Engneering*, vol. 10, no. 1, pp. 109–111, January 2015.

[245] D. K. Molzahn and I. A. Hiskens, "Moment Relaxations of Optimal Power Flow Problems: Beyond the Convex Hull," in *IEEE Global Conference on Signal and Informtion Processing (GlobalSIP)*, (Washington, DC, USA), pp. 856–860, December 2016.

[246] T. Weisser, L. A. Roald, and S. Misra, "Chance-Constrained Optimization for Non-Linear Network Flow Problems," *arXiv:1803.02696*, March 2018.

[247] E. Dumon, M. Ruiz, H. Godard, and J. Maeght, "SDP Resolution Techniques for the Optimal Power Flow with Unit Commitment," in *IEEE Manchester PowerTech*, (Manchester, England, UK), pp. 1–6, June 2017.

[248] R. Pedersen, C. Sloth, and R. Wisniewski, "Verification of Power Grid Voltage Constraint Satisfaction – A Barrier Certificate Approach," in *European Control Conference (ECC)*, (Aalborg, Denmark), pp. 447–452, June 2016.

[249] M. Anghel, F. Milano, and A. Papachristodoulou, "Algorithmic Construction of Lyapunov Functions for Power Grid Stability Analysis," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, pp. 2533–2546, 2013.

[250] C. Josz, D. K. Molzahn, M. Tacchi, and S. Sojoudi, "Transient Stability Analysis of Power Systems via Occupation Measures," *arXiv:1811.01372*, November 2018.

[251] M. Tacchi, B. Marinescu, M. Anghel, S. Kundu, S. Benahmed, and C. Cardozo, "Power System Transient Stability Analysis Using Sum Of Squares Programming," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[252] A. A. Ahmadi and A. Majumdar, "Response to "Counterexample to Global Convergence of DSOS and SDSOS Hierarchies"," *arXiv:1710.02901*, October 2017.

[253] H. Waki, S. Kim, M. Kojima, and M. Muramatsu, "Sums of Squares and Semidefinite Program Relaxations for Polynomial Optimization Problems with Structured Sparsity," *SIAM Journal on Optimization*, vol. 17, no. 1, pp. 218–242, 2006.

[254] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Strengthening the SDP Relaxation of AC Power Flows with Convex Envelopes, Bound Tightening, and Lifted Nonlinear Cuts," *arXiv:1512.04644*, January 2016.

[255] B. Kocuk, S. S. Dey, and X. A. Sun, "Strong SOCP Relaxations of the Optimal Power Flow Problem," *Operations Research*, vol. 64, no. 6, pp. 1177–1196, 2016.

[256] B. Kocuk, S. S. Dey, and X. A. Sun, "Matrix Minor Reformulation and SOCP-based Spatial Branch-and-Cut Method for the AC Optimal Power Flow Problem," *Mathematical Programming C*, vol. 10, no. 4, pp. 557–596, December 2018.

[257] A. Gómez Expósito and E. Romero Ramos, "Reliable Load Flow Technique for Radial Distribution Networks," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1063–1069, August 1999.

[258] H. Hijazi, C. Coffrin, and P. Van Hentenryck, "Convex Quadratic Relaxations for Mixed-Integer Nonlinear Programs in Power Systems," Preprint: *http://www.optimization-online.org/DB_HTML/2013/09/4057.html*, September 2013.

[259] H. Hijazi, C. Coffrin, and P. Van Hentenryck, "Convex Quadratic Relaxations for Mixed-Integer Nonlinear Programs in Power Systems," *Mathematical Programming Computation*, vol. 9, no. 3, pp. 321–367, September 2017.

[260] C. A. Meyer and C. A. Floudas, *Trilinear Monomials with Positive or Negative Domains: Facets of the Convex and Concave Envelopes*, pp. 327–352. Boston, MA: Springer US, 2004.

[261] C. A. Meyer and C. A. Floudas, "Trilinear Monomials with Mixed Sign Domains: Facets of the Convex and Concave Envelopes," *Journal of Global Optimization*, vol. 29, no. 2, pp. 125–155, 2004.

[262] A. D. Rikun, "A Convex Envelope Formula for Multilinear Functions," *Journal of Global Optimization*, vol. 10, no. 4, pp. 425–437, 1997.

[263] M. R. Narimani, D. K. Molzahn, and M. L. Crow, "Improving QC Relaxations of OPF Problems via Voltage Magnitude Difference Constraints and Envelopes for Trilinear Monomials," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[264] M. Lu, H. Nagarajan, R. Bent, S. D. Eksioglu, and S. J. Mason, "Tight Piecewise Convex Relaxations for Global Optimization of Optimal Power Flow," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[265] M. R. Narimani, D. K. Molzahn, H. Nagarajan, and M. L. Crow, "Comparison of Various Trilinear Monomial Envelopes for Convex Relaxations of Optimal Power Flow Problems," in *IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, (Anaheim, CA, USA), November 2018.

[266] K. Sundar, H. Nagarajan, S. Misra, M. Lu, C. Coffrin, and R. Bent, "Optimization-Based Bound Tightening using a Strengthened QC-Relaxation of the Optimal Power Flow Problem," *arXiv:1809.04565*, September 2018.

[267] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Strengthening Convex Relaxations with Bound Tightening for Power Network Optimization," in *21st International Conference on Principles and Practice of Constraint Programming (CP)* (G. Pesant, ed.), pp. 39–57, Cork, Ireland: Springer International Publishing, August 2015. Lecture Notes in Computer Science 9255.

[268] S. Mhanna, G. Verbič, and A. C. Chapman, "Tight LP Approximations for the Optimal Power Flow Problem," in *19th Power Systems Computation Conference (PSCC)*, (Genoa, Italy), pp. 1–7, June 2016.

[269] D. Bienstock and G. Muñoz, "On Linear Relaxations of OPF Problems," *arXiv:1411.1120*, November 2014.

[270] D. Bienstock and G. Muñoz, "Approximate Method for AC Transmission Switching Based on a Simple Relaxation for ACOPF Problems," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[271] B. Subhonmesh, S. H. Low, and K. M. Chandy, "Equivalence of Branch Flow and Bus Injection Models," in *50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 1893–1899, October 2012.

[272] A. R. Bergen and V. Vittal, *Power Systems Analysis.* Prentice-Hall, 1999.

[273] D. S. Callaway and I. A. Hiskens, "Achieving Controllability of Electric Loads," *Proceedings of the IEEE*, vol. 99, no. 1, pp. 184–199, November 2011.

[274] K. Christakou, D.-C. Tomozei, J.-Y. Le Boudec, and M. Paolone, "AC OPF in Radial Distribution Networks – Part I: On the Limits of the Branch Flow Convexification and the Alternating Direction Method of Multipliers," *Electric Power Systems Research*, vol. 143, pp. 438–450, February 2017.

[275] M. Nick, R. Cherkaoui, J. Y. LeBoudec, and M. Paolone, "An Exact Convex Formulation of the Optimal Power Flow in Radial Distribution Networks Including Transverse Components," *IEEE Transactions on Automatic Control*, vol. 63, no. 3, pp. 682–697, March 2018.

[276] S. Sojoudi and J. Lavaei, "Convexification of Optimal Power Flow Problem by Means of Phase Shifters," in *IEEE International Conference on Smart Grid Communications (SmartGridComm)*, (Vancouver, BC, Canada), pp. 756–761, October 2013.

[277] S. Sojoudi, S. Fattahi, and J. Lavaei, "Convexification of Generalized Network Flow Problem," to appear in *Mathematical Programming Series A*, 2019.

[278] S. Huang, Q. Wu, J. Wang, and H. Zhao, "A Sufficient Condition on Convex Relaxation of AC Optimal Power Flow in Distribution Networks," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1359–1368, 2016.

[279] L. Gan and S. H. Low, "Optimal Power Flow in Direct Current Networks," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 2892–2904, November 2014.

[280] C. W. Tan, D. W. H. Cai, and X. Lou, "Resistive Network Optimal Power Flow: Uniqueness and Algorithms," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 263–273, January 2015.

[281] P. Scott and S. Thiébaux, "Dynamic Optimal Power Flow in Microgrids using the Alternating Direction Method of Multipliers," *arXiv:1410.7868*, October 2014.

[282] P. Scott and S. Thiébaux, "Distributed Multi-Period Optimal Power Flow for Demand Response in Microgrids," in *ACM Sixth International Conference on Future Energy Systems (e-Energy)*, pp. 17–26, July 2015.

[283] Z. Yuan and M. R. Hesamzadeh, "A Modified Benders Decomposition Algorithm to Solve Second-Order Cone AC Optimal Power Flow," to appear in *IEEE Transactions on Smart Grid*, 2019.

[284] S. Mhanna, A. Chapman, and G. Verbic, "Accelerated Methods for the SOCP-relaxed Component-based Distributed Optimal Power Flow," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[285] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Network Flow and Copper Plate Relaxations for AC Transmission Systems," in *19th Power Systems Computation Conference (PSCC)*, (Genoa, Italy), pp. 1–8, June 2016.

[286] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Network Flow and Copper Plate Relaxations for AC Transmission Systems," *arXiv:1506.05202*, November 2015.

[287] S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott, and O. Alsaç, "Studies on Power System Load Flow Equivalencing," *IEEE Transactions on Power Apparatus and Systems*, no. 6, pp. 2301–2310, 1980.

[288] J. A. Taylor and F. S. Hover, "Linear Relaxations for Transmission System Planning," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2533–2538, November 2011.

[289] Z. Qin, Y. Hou, and Y. Chen, "Convex Envelopes of Optimal Power Flow with Branch Flow Model in Rectangular Form," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[290] Q. Gemine, D. Ernst, Q. Louveaux, and B. Cornélusse, "Relaxations for Multi-Period Optimal Power Flow Problems with Discrete Decision Variables," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–7, August 2014.

[291] M. Bynum, A. Castillo, J.-P. Watson, and C. D. Laird, "Strengthened SOCP Relaxations for ACOPF with McCormick Envelopes and Bounds Tightening," in *13th International Symposium on Process Systems Engineering (PSE 2018)*, vol. 44, (San Diego. CA, USA), pp. 1555–1560, July 2018.

[292] D. Bienstock and G. Muñoz, "LP Formulations for Polynomial Optimization Problems," *SIAM Journal on Optimization*, vol. 28, no. 2, pp. 1121–1150, 2018.

[293] F. Glover, "Improved Linear Integer Programming Formulations of Nonlinear Integer Problems," *Management Science*, vol. 22, no. 4, pp. 455–460, December 1975.

[294] K. Dvijotham, M. Chertkov, P. Van Hentenryck, M. Vuffray, and S. Misra, "Graphical Models for Optimal Power Flow," *Constraints*, vol. 22, no. 1, pp. 24–49, 2017.

[295] J. D. Foster, *Mixed-Integer Quadratically-Constrained Programming, Piecewise-Linear Approximation and Error Analysis with Applications in Power Flow.* dissertation, The University of Newcastle, Australia, School of Mathematical and Physical Sciences, November 2013.

[296] C. Chen, A. Atamtürk, and S. S. Oren, "Bound Tightening for the Alternating Current Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3729–3736, September 2016.

[297] M. Bynum, A. Castillo, J.-P. Watson, and C. Laird, "Tightening McCormick Relaxations Toward Global Solution of the ACOPF Problem," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 814–817, January 2019.

[298] D. Shchetinin, "Efficient Bound Tightening Techniques for Convex Relaxations of AC Optimal Power Flow," submitted to *IEEE Transactions on Power Systems*, 2018.

[299] Z. Miao, L. Fan, H. G. Aghamolki, and B. Zeng, "Least Square Estimation-Based SDP Cuts for SOCP Relaxation of AC OPF," *IEEE Transactions on Automatic Control*, vol. 63, no. 1, pp. 241–248, January 2018.

[300] C. Chen, A. Atamtürk, and S. S. Oren, "A Spatial Branch-and-Cut Method for Nonconvex QCQP with Bounded Complex Variables," *Mathematical Programming, Series A*, vol. 165, no. 2, pp. 549–577, October 2017.

[301] J. Liu, M. Bynum, A. Castillo, J.-P. Watson, and C. D. Laird, "A Multitree Approach for Global Solution of ACOPF Problems using Piecewise Outer Approximations," *Computers & Chemical Engineering*, vol. 114, pp. 145–157, 2018.

[302] C. Coffrin, "Lifted Nonlinear Cuts Animation." source code available at https://github.com/ccoffrin/lnc-animation, animation available at https://imgur.com/gallery/7JjPfy6, April 2016.

[303] A. Gopalakrishnan, A. Raghunathan, D. Nikovski, and L. T. Biegler, "Global Optimization of Optimal Power Flow using a Branch & Bound Algorithm," in *50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 609–616, October 2012.

[304] D. Phan, "Lagrangian Duality and Branch-and-Bound Algorithms for Optimal Power Flow," *Operations Research*, vol. 60, no. 2, pp. 275–285, 2012.

[305] D. Phan and J. Kalagnanam, "Some Efficient Optimization Methods for Solving the Security-Constrained Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 863–872, March 2014.

[306] M. Tawarmalani and N. V. Sahinidis, *Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming: Theory, Algorithms, Software, and Applications*, vol. 65. Springer Science & Business Media, 2002.

[307] H. Nagarajan, M. Lu, E. Yamangil, and R. Bent, "Tightening McCormick Relaxations for Nonlinear Programs via Dynamic Multivariate Partitioning," in *22nd International Conference on Principles and Practice of Constraint Programming (CP)* (M. Rueher, ed.), pp. 369–387, Toulouse, France: Springer International Publishing, September 2016. Lecture Notes in Computer Science 9892.

[308] H. Nagarajan, M. Lu, S. Wang, R. Bent, and K. Sundar, "An Adaptive, Multivariate Partitioning Algorithm for Global Optimization of Nonconvex Programs," *arXiv:1707.02514*, July 2017.

[309] J. Liu, A. Castillo, J.-P. Watson, and C. D. Laird, "Global Solution Strategies for the Network-Constrained Unit Commitment Problem with AC Transmission Constraints," to appear in *IEEE Transactions on Power Systems*, 2019.

[310] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Alternating Current (AC) Power Flow Analysis in an Electrical Power Network," 2012-05-23. Patent US20150088439A1.

[311] M. Baradar and M. R. Hesamzadeh, "AC Power Flow Representation in Conic Format," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 546–547, January 2015.

[312] L. H. Macedo, C. V. Montes, J. F. Franco, M. J. Rider, and R. Romero, "MILP Branch Flow Model for Concurrent AC Multistage Transmission Expansion and Reactive Power Planning with Security Constraints," *IET Generation, Transmission & Distribution*, vol. 10, pp. 3023–3032, September 2016.

[313] Z. Yuan and M. R. Hesamzadeh, "Improving the Accuracy of Second-Order Cone AC Optimal Power Flow by Convex Approximations," in *IEEE Innovative Smart Grid Technologies–Asia (ISGT Asia)*, (Singapore), pp. 172–177, May 2018.

[314] S. Bolognani and F. Dörfler, "Fast Power System Analysis via Implicit Linearization of the Power Flow Manifold," in *53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 402–409, September 2015.

[315] D. Deka, S. Backhaus, and M. Chertkov, "Structure Learning and Statistical Estimation in Distribution Networks – Part I," *arXiv:1501.04131*, February 2015.

[316] J. Yang, N. Zhang, C. Kang, and Q. Xia, "A State-Independent Linear Power Flow Model With Accurate Estimation of Voltage Magnitude," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3607–3617, September 2017.

[317] A. Moser, *Langfristig optimale Struktur und Betriebsmittelwahl für 110-kV-Überlandnetze.* dissertation, RWTH Aachen University, 1995.

[318] A. Braun, *Anlagen- und Strukturoptimierung von 110-kkV-Netzen.* dissertation, RWTH Aachen University, 2001.

[319] A. M. C. A. Koster and S. Lemkens, *Designing AC Power Grids Using Integer Linear Programming*, pp. 478–483. Hamburg, Germany: Springer Berlin Heidelberg, 2011.

[320] H. Zhang, V. Vittal, G. T. Heydt, and J. Quintero, "A Relaxed AC Optimal Power Flow Model based on a Taylor Series," in *IEEE Innovative Smart Grid Technologies–Asia (ISGT Asia)*, (Bangalore, India), pp. 1–5, November 2013.

[321] H. Zhang, G. T. Heydt, V. Vittal, and J. Quintero, "An Improved Network Model for Transmission Expansion Planning Considering Reactive Power and Network Losses," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 3471–3479, August 2013.

[322] T. Akbari and M. T. Bina, "Linear Approximated Formulation of AC Optimal Power Flow using Binary Discretisation," *IET Generation, Transmission & Distribution*, vol. 10, pp. 1117–1123, April 2016.

[323] Z. Yang, H. Zhong, A. Bose, T. Zheng, Q. Xia, and C. Kang, "A Linearized OPF Model with Reactive Power and Voltage Magnitude: A Pathway to Improve the MW-Only DC OPF," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1734–1745, March 2018.

[324] Z. Yang, H. Zhong, Q. Xia, and C. Kang, "A Novel Network Model for Optimal Power Flow with Reactive Power and Network Losses," *Electric Power Systems Research*, vol. 144, pp. 63–71, 2017.

[325] Z. Yang, H. Zhong, Q. Xia, A. Bose, and C. Kang, "Optimal Power Flow based on Successive Linear Approximation of Power Flow Equations," *IET Generation, Transmission & Distribution*, vol. 10, no. 14, pp. 3654–3662, 2016.

[326] Z. Yang, A. Bose, H. Zhong, N. Zhang, Q. Xia, and C. Kang, "Optimal Reactive Power Dispatch With Accurately Modeled Discrete Control Devices: A Successive Linear Approximation Approach," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 2435–2444, May 2017.

[327] S. M. Fatemi, S. Abedi, G. B. Gharehpetian, S. H. Hosseinian, and M. Abedi, "Introducing a Novel DC Power Flow Method With Reactive Power Considerations," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3012–3023, November 2015.

[328] Z. Li, J. Yu, and Q. H. Wu, "Approximate Linear Power Flow Using Logarithmic Transform of Voltage Magnitudes With Reactive Power and Transmission Loss Consideration," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 4593–4603, July 2018.

[329] K. Dvijotham, E. Mallada, and J. W. Simpson-Porco, "High-Voltage Solution in Radial Power Networks: Existence, Properties, and Equivalent Algorithms," *IEEE Control Systems Letters*, vol. 1, no. 2, pp. 322–327, October 2017.

[330] Z. Yang, H. Zhong, Q. Xia, and C. Kang, "Solving OPF using Linear Approximations: Fundamental Analysis and Numerical Demonstration," *IET Generation, Transmission & Distribution*, vol. 11, pp. 4115–4125, November 2017.

[331] Z. Yang, K. Xie, J. Yu, H. Zhong, N. Zhang, and Q. Xia, "A General Formulation of Linear Power Flow Models: Basic Theory and Error Analysis," to appear in *IEEE Transactions on Power Systems*, 2019.

[332] S. Bolognani and S. Zampieri, "On the Existence and Linear Approximation of the Power Flow Solution in Power Distribution Networks," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 163–172, January 2016.

[333] S. V. Dhople, S. S. Guggilam, and Y. C. Chen, "Linear Approximations to AC Power Flow in Rectangular Coordinates," in *53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (Monticello, IL, USA), pp. 211–217, September 2015.

[334] S. S. Guggilam, E. Dall'Anese, Y. C. Chen, S. V. Dhople, and G. B. Giannakis, "Scalable Optimization Methods for Distribution Networks With High PV Integration," *IEEE Transactions on Smart Grid*, vol. 7, no. 4, pp. 2061–2070, July 2016.

[335] A. Bernstein, C. Wang, E. Dall'Anese, J.-Y. Le Boudec, and C. Zhao, "Load-Flow in Multiphase Distribution Networks: Existence, Uniqueness, and Linear Models," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 5832–5843, November 2018.

[336] A. Bernstein and E. Dall'Anese, "Linear Power-Flow Models in Multiphase Distribution Networks," in *7th IEEE International Conference on Innovative Smart Grid Technologies (ISGT Europe)*, (Torino, Italy), pp. 1–6, September 2017.

[337] B. Stott and O. Alsaç, "Fast Decoupled Load Flow," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-93, no. 3, pp. 859–869, May 1974.

[338] R. A. M. van Amerongen, "A General-Purpose Version of the Fast Decoupled Load Flow," *IEEE Transactions on Power Systems*, vol. 4, no. 2, pp. 760–770, May 1989.

[339] F. F. Wu, "Theoretical Study of the Convergence of the Fast Decoupled Load Flow," *IEEE Transactions on Power Apparatus and Systems*, vol. 96, no. 1, pp. 268–275, January 1977.

[340] J. Nanda, D. P. Kothari, and S. C. Srivastava, "Some Important Observations on Fast Decoupled Load Flow Algorithm," *Proceedings of the IEEE*, vol. 75, no. 5, pp. 732–733, May 1987.

[341] R. D. Zimmerman and C. E. Murillo-Sánchez, "MATPOWER 6.0 User's Manual," tech. rep., Power Systems Engineering Research Center, December 2016.

[342] P. Yan and A. Sekar, "Study of Linear Models in Steady State Load Flow Analysis of Power Systems," in *IEEE Power & Energy Society Winter Meeting*, vol. 1, (New York, NY, USA), pp. 666–671, 2002.

[343] M. Liu and G. Gross, "Effectiveness of the Distribution Factor Approximations Used in Congestion Modeling," in *14th Power Systems Computation Conference (PSCC)*, (Seville, Spain), June 2002.

[344] T. J. Overbye, X. Cheng, and Y. Sun, "A Comparison of the AC and DC Power Flow Models for LMP Calculations," in *37th Hawaii International Conference on System Sciences (HICSS)*, (Big Island, HI, USA), pp. 1–9, January 2004.

[345] R. Baldick, K. Dixit, and T. J. Overbye, "Empirical Analysis of the Variation of Distribution Factors with Loading," in *IEEE Power & Energy Society General Meeting*, (San Francisco, CA, USA), pp. 221–229, June 2005.

[346] K. Purchala, L. Meeus, D. Van Dommelen, and R. Belmans, "Usefulness of DC Power Flow for Active Power Flow Analysis," in *IEEE Power & Energy Society General Meeting*, (San Francisco, CA, USA), pp. 454–459, June 2005.

[347] D. Van Hertem, J. Verboomen, K. Purchala, R. Belmans, and W. L. Kling, "Usefulness of DC Power Flow for Active Power Flow Analysis with Flow Controlling Devices," in *8th IEE International Conference on AC and DC Power Transmission (ACDC)*, (London, England, UK), pp. 58–62, March 2006.

[348] F. Li and R. Bo, "DCOPF-Based LMP Simulation: Algorithm, Comparison With ACOPF, and Sensitivity," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1475–1485, November 2007.

[349] C. Duthaler, M. Emery, G. Andersson, and M. Kurzidem, "Analysis of the Use of Power Transfer Distribution Factors (PTDF) in the UCTE Transmission Grid," in *16th Power Systems Computation Conference (PSCC)*, (Glasgow, Scotland, UK), July 2008.

[350] B. Stott, J. Jardim, and O. Alsaç, "DC Power Flow Revisited," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1290–1300, August 2009.

[351] Y. Qi, D. Shi, and D. Tylavsky, "Impact of Assumptions on DC Power Flow Model Accuracy," in *North American Power Symposium (NAPS)*, (Champaign, IL, USA), pp. 1–6, September 2012.

[352] C. Coffrin, P. Van Hentenryck, and R. Bent, "Accurate Load and Generation Scheduling for Linearized DC Models with Contingencies," in *IEEE Power & Energy Society General Meeting*, (San Diego, CA, USA), pp. 1–8, July 2012.

[353] C. Coffrin and P. Van Hentenryck, "Transmission System Restoration with Co-Optimization of Repairs, Load Pickups, and Generation Dispatch," *International Journal of Electrical Power & Energy Systems*, vol. 72, pp. 144–154, 2015. Special Issue for the 18th Power Systems Computation Conference (PSCC).

[354] H. Cetinay, S. Soltan, F. A. Kuipers, G. Zussman, and P. Van Mieghem, "Comparing the Effects of Failures in Power Grids under the AC and DC Power Flow Models," *IEEE Transactions on Network Science and Engineering*, vol. 5, no. 4, pp. 301–312, October 2018.

[355] R. Kaye and F. F. Wu, "Analysis of Linearized Decoupled Power Flow Approximations for Steady-State Security Assessment," *IEEE Transations on Circuits and Systems*, vol. 31, no. 7, pp. 623–636, July 1984.

[356] R. Baldick, "Variation of Distribution Factors with Loading," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1316–1323, November 2003.

[357] K. Dvijotham and D. K. Molzahn, "Error Bounds on Linear Power Flow Approximations: A Convex Relaxation Approach," in *IEEE 55th Annual Conference on Decision and Control (CDC)*, (Las Vegas, NV, USA), pp. 2411–2418, December 2016.

[358] C. Coffrin, P. Van Hentenryck, and R. Bent, "Approximating Line Losses and Apparent Power in AC Power Flow Linearizations," in *IEEE Power & Energy Society General Meeting*, (San Diego, CA, USA), pp. 1–8, July 2012.

[359] F. Dörfler and F. Bullo, "Novel Insights into Lossless AC and DC Power Flow," in *IEEE Power & Energy Society General Meeting*, (Vancouver, BC, Canada), pp. 1–5, July 2013.

[360] F. Dörfler, M. Chertkov, and F. Bullo, "Synchronization in Complex Oscillator Networks and Smart Grids," *Procedings of the National Academy of Sciences of the United States of America*, vol. 110, no. 6, pp. 2005–2010, 2013.

[361] J. W. Simpson-Porco, "Lossy DC Power Flow," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2477–2485, May 2018.

[362] B. Gentile, J. W. Simpson-Porco, F. Dörfler, S. Zampieri, and F. Bullo, "On Reactive Power Flow and Voltage Stability in Microgrids," in *American Control Conference (ACC)*, (Portland, OR, USA), pp. 759–764, June 2014.

[363] J. W. Simpson-Porco, F. Dörfler, and F. Bullo, "Voltage Collapse in Complex Power Grids," *Nature Communications*, vol. 7, no. 10790, February 2016.

[364] D. B. Arnold, M. D. Sankur, R. Dobbe, K. Brady, D. S. Callaway, and A. Von Meier, "Optimal Dispatch of Reactive Power for Voltage Regulation and Balancing in Unbalanced Distribution Systems," in *IEEE Power & Energy Society General Meeting*, (Boston, MA, USA), pp. 1–5, 2016.

[365] M. D. Sankur, R. Dobbe, E. Stewart, D. S. Callaway, and D. B. Arnold, "A Linearized Power Flow Model for Optimization in Unbalanced Distribution Systems," *arXiv:1606.04492*, November 2016.

[366] J. Franco, L. Ochoa, and R. Romero, "AC OPF for Smart Distribution Networks: An Efficient and Robust Quadratic Approach," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4613–4623, September 2018.

[367] A. R. D. Fazio, M. Russo, S. Valeri, and M. D. Santis, "Linear Method for Steady-State Analysis of Radial Distribution Systems," *International Journal of Electrical Power & Energy Systems*, vol. 99, pp. 744–755, 2018.

[368] C. Coffrin and P. Van Hentenryck, "A Linear-Programming Approximation of AC Power Flows," *INFORMS Journal on Computing*, vol. 26, no. 4, pp. 718–734, 2014.

[369] J. R. Martí, H. Ahmadi, and L. Bashualdo, "Linear Power-Flow Formulation Based on a Voltage-Dependent Load Model," *IEEE Transactions on Power Delivery*, vol. 28, no. 3, pp. 1682–1690, July 2013.

[370] H. Ahmadi, J. R. Martí, and A. von Meier, "A Linear Power Flow Formulation for Three-Phase Distribution Systems," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 5012–5021, November 2016.

[371] R. S. Ferreira, C. L. T. Borges, and M. V. F. Pereira, "A Flexible Mixed-Integer Linear Programming Approach to the AC Optimal Power Flow in Distribution Systems," *IEEE Transactions on Power Systems*, vol. 29, no. 5, pp. 2447–2459, September 2014.

[372] S. Misra, D. K. Molzahn, and K. Dvijotham, "Optimal Adaptive Linearizations of the AC Power Flow Equations," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[373] M. Hohmann, J. Warrington, and J. Lygeros, "Optimal Linearizations of Power Systems with Uncertain Supply and Demand," to appear in *IEEE Transactions on Power Systems*, 2019.

[374] P. S. Martin, *Mejoras en la Eficacia Computacional de Modelos Probablistas de Explotacón Generación/Red a Medio Plazo.* dissertation, University Pontifica de Comillas, Madrid, Spain, 1998.

[375] A. L. Motto, F. D. Galiana, A. J. Conejo, and J. M. Arroyo, "Network-Constrained Multiperiod Auction for a Pool-Based Electricity Market," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 646–653, August 2002.

[376] M. R. Almassalkhi and I. A. Hiskens, "Model-Predictive Cascade Mitigation in Electric Power Systems With Storage and Renewables–Part I: Theory and Implementation," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 67–77, January 2015.

[377] P. Fortenbacher and T. Demiray, "Linear/Quadratic Programming-Based Optimal Power Flow using Linear Power Flow and Absolute Loss Approximations," *arXiv:1711.00317*, December 2017.

[378] J. A. Martin and I. A. Hiskens, "Generalized Line Loss Relaxation in Polar Voltage Coordinates," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 1980–1189, 2017.

[379] J. S. Thorp and S. A. Naqavi, "Load Flow Fractals," in *28th IEEE Conference on Decision and Control,*, (Tampa, FL, USA), pp. 1822–1827, December 1989.

[380] J. S. Thorp, S. A. Naqavi, and H. Chiang, "More Load Flow Fractals," in *29th IEEE Conference on Decision and Control*, (Honolulu, HI, USA), pp. 3028–3030, December 1990.

[381] J.-J. Deng and H.-D. Chiang, "Convergence Region of Newton Iterative Power Flow Method: Numerical Studies," *Journal of Applied Mathematics*, October 2013.

[382] J. E. Tate and T. J. Overbye, "A Comparison of the Optimal Multiplier in Polar and Rectangular Coordinates," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1667–1674, November 2005.

[383] V. H. Quintana and N. Müller, "Studies of Load Flow Methods in Polar and Rectangular Coordinates," *Electric Power Systems Research*, vol. 20, no. 3, pp. 225–235, 1991.

[384] D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo, "A Compensation-based Power Flow Method for Weakly Meshed Distribution and Transmission Networks," *IEEE Transactions on Power Systems*, vol. 3, no. 2, pp. 753–762, May 1988.

[385] E. Bompard, E. Carpaneto, G. Chicco, and R. Napoli, "Convergence of the Backward-Forward Sweep Method for the Load Flow Analysis of Radial Distribution Systems," *International Journal of Electrical Power & Energy Systems*, vol. 22, no. 7, pp. 521–530, October 2000.

[386] W. H. Kersting, *Distribution System Modeling and Analysis, Third Edition.* Boca Raton, FL: CRC Press, 2012.

[387] P. Fortenbacher, M. Zellner, and G. Andersson, "Optimal Sizing and Placement of Distributed Storage in Low Voltage Networks," in *19th Power Systems Computation Conference (PSCC)*, (Genoa, Italy), pp. 1–7, June 2016.

[388] C. B. Garcia and W. I. Zangwill, *Pathways to Solutions, Fixed Points and Equilibria.* Englewood Cliffs, NJ: Prentice Hall, 1981.

[389] G. B. Price, "A Generalized Circle Diagram Approach for Global Analysis of Transmission System Performance," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-103, no. 10, pp. 2881–2890, October 1984.

[390] V. Ajjarapu and C. Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis," *IEEE Transactions on Power Systems*, vol. 7, no. 1, pp. 416–423, February 1992.

[391] C. A. Cañizares and F. L. Alvarado, "Point of Collapse and Continuation Methods for Large AC/DC Systems," *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 1–8, February 1993.

[392] A. Wächter, "Nonlinear Optimization Algorithms," in *Advances and Trends in Optimization with Engineering Applications* (T. Terlaky, M. F. Anjos, and S. Ahmed, eds.), ch. 17, pp. 221–235, SIAM, 2017.

[393] A. Forsgren, P. E. Gill, and M. H. Wright, "Interior Methods for Nonlinear Optimization," *SIAM Review*, vol. 44, no. 4, pp. 525–597, 2002.

[394] N. Gould, D. Orban, and P. Toint, "Numerical Methods for Large-Scale Nonlinear Optimization," *Acta Numerica*, vol. 14, pp. 299–361, 2005.

[395] A. Wächter and L. T. Biegler, "On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, 2006.

[396] R. H. Byrd, J. Nocedal, and R. A. Waltz, "Knitro: An Integrated Package for Nonlinear Optimization," in *Large-Scale Nonlinear Optimization*, pp. 35–59, Springer, 2006.

[397] R. J. Vanderbei, "LOQO: An Interior Point Code for Quadratic Programming," *Optimization Methods and Software*, vol. 11, no. 1-4, pp. 451–484, 1999.

[398] K. A. Clements, P. W. Davis, and K. D. Frey, "An Interior Point Algorithm for Weighted Least Absolute Value Power System State Estimation," in *IEEE Power & Energy Society Winter Meeting*, (New York, NY, USA), February 1991.

[399] K. Ponnambalam, V. H. Quintana, and A. Vannelli, "A Fast Algorithm for Power System Optimization Problems using an Interior Point Method," *IEEE Transactions on Power Systems*, vol. 7, no. 2, pp. 892–899, May 1992.

[400] H. Wang, C. E. Murillo-Sanchez, R. D. Zimmerman, and R. J. Thomas, "On Computational Issues of Market-Based Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1185–1193, August 2007.

[401] F. Capitanescu and L. Wehenkel, "Experiments with the Interior-Point Method for Solving Large Scale Optimal Power Flow Problems," *Electric Power Systems Research*, vol. 95, pp. 276–283, 2013.

[402] A. Forsgren, "On Warm Starts for Interior Methods," in *22nd IFIP TC7 Conference on System Modeling and Optimization* (F. Ceragioli, A. Dontchev, H. Futura, K. Marti, and L. Pandolfi, eds.), (Torino, Italy), pp. 51–66, July 2005.

[403] R. C. Burchett, H. H. Happ, and D. R. Vierath, "Quadratically Convergent Optimal Power Flow," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-103, no. 11, pp. 3267–3275, November 1984.

[404] P. Gill, W. Murray, and M. Saunders, "SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization," *SIAM Review*, vol. 47, no. 1, pp. 99–131, 2005.

[405] R. Fletcher and S. Leyffer, "Nonlinear Programming without a Penalty Function," *Mathematical Programming*, vol. 91, no. 2, pp. 239–269, January 2002.

[406] M. Fazel, *Matrix Rank Minimization with Applications*. dissertation, Stanford University, March 2002.

[407] B. Recht, M. Fazel, and P. A. Parrilo, "Guaranteed Minimum Rank Solutions to Linear Matrix Equations via Nuclear Norm Minimization," *SIAM Review*, vol. 52, pp. 471–501, 2010.

[408] R. Madani, J. Lavaei, and R. Baldick, "Convexification of Power Flow Problem over Arbitrary Networks," in *IEEE 54th Annual Conference on Decision and Control (CDC)*, (Osaka, Japan), pp. 1–8, December 2015.

[409] Y. Zhang, R. Madani, and J. Lavaei, "Conic Relaxations for Power System State Estimation with Line Measurements," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 1193–1205, September 2018.

[410] S. You and Q. Peng, "A Non-Convex Alternating Direction Method of Multipliers Heuristic for Optimal Power Flow," in *IEEE International Conference on Smart Grid Communications (SmartGridComm)*, (Venice, Italy), pp. 788–793, November 2014.

[411] T. Liu, B. Sun, and D. H. K. Tsang, "Rank-one Solutions for SDP Relaxation of QCQPs in Power Systems," to appear in *IEEE Transactions on Smart Grid*, 2019.

[412] Y. Shi, H. D. Tuan, H. Tuy, and S. Su, "Global Optimization for Optimal Power Flow over Transmission Networks," *Journal of Global Optimization*, vol. 69, no. 3, pp. 745–760, November 2017.

[413] Y. Shi, H. D. Tuan, and A. V. Savkin, "Three-Phase Optimal Power Flow for Smart Grids by Iterative Nonsmooth Optimization," in *6th International Conference on Smart Cities and Green ICT Systems – Volume 1: SMARTGREENS*, (Porto, Portugal), pp. 323–328, April 2017.

[414] Y. Shi, H. D. Tuan, P. Apkarian, and A. V. Savkin, "Global Optimal Power Flow over Large-Scale Power Transmission Network," *Systems & Control Letters*, vol. 118, pp. 16–21, August 2018.

[415] A. S. Zamzam, N. D. Sidiropoulos, and E. Dall'Anese, "Beyond Relaxations and Newton-Raphson: Solving AC OPF for Multi-Phase Systems with Renewables," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 3966–3975, September 2018.

[416] R. Louca and E. Bitar, "Stochastic AC Optimal Power Flow with Affine Recourse," in *IEEE 55th Annual Conference on Decision and Control (CDC)*, (Las Vegas, NV, USA), pp. 2431–2436, December 2016.

[417] W. Wei, J. Wang, N. Li, and S. Mei, "Optimal Power Flow of Radial Networks and Its Variations: A Sequential Convex Optimization Approach," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2974–2987, November 2017.

[418] A. S. Zamzam, C. Zhao, E. Dall'Anese, and N. D. Sidiropoulos, "A QCQP Approach for OPF In Multiphase Radial Networks with Wye and Delta Connections," in *10th IREP Symposium on Bulk Power System Dynamics and Control*, (Espinho, Portugal), August 2017.

[419] A. A. Ahmadi and G. Hall, "DC Decomposition of Nonconvex Polynomials with Algebraic Techniques," *Mathematical Programming*, April 2017.

[420] Z. Tian and W. Wu, "Recover Feasible Solutions for SOCP Relaxation of Optimal Power Flow Problems in Mesh Networks," *arXiv:1708.06504*, August 2017.

[421] C. Wang, A. Bernstein, J.-Y. Le Boudec, and M. Paolone, "Explicit Conditions on Existence and Uniqueness of Load-Flow Solutions in Distribution Networks," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 953–962, March 2018.

[422] K. Dvijotham and K. Turitsyn, "Construction of Power Flow Feasibility Sets," *arXiv:1506.07191*, July 2015.

[423] S. Yu, H. D. Nguyen, and K. Turitsyn, "Simple Certificate of Solvability of Power Flow Equations for Distribution Systems," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[424] J. W. Simpson-Porco, "A Theory of Solvability for Lossless Power Flow Equations – Part II: Conditions for Radial Networks," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 1373–1385, September 2018.

[425] C. Wang, A. Bernstein, J.-Y. Le Boudec, and M. Paolone, "Existence and Uniqueness of Load-Flow Solutions in Three-Phase Distribution Networks," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3319–3320, 2017.

[426] K. Dvijotham, H. D. Nguyen, and K. Turitsyn, "Solvability Regions of Affinely Parameterized Quadratic Equations," *IEEE Control Systems Letters*, vol. 2, no. 1, pp. 25–30, January 2018.

[427] H. D. Nguyen, K. Dvijotham, S. Yu, and K. Turitsyn, "A Framework for Robust Steady-State Voltage Stability of Distribution Systems," *arXiv:1705.05774*, May 2017.

[428] H. D. Nguyen, K. Dvijotham, and K. Turitsyn, "Constructing Convex Inner Approximations of Steady-State Security Regions," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 257–267, January 2019.

[429] C. Wang, J.-Y. Le Boudec, and M. Paolone, "Controlling the Electrical State via Uncertain Power Injections in Three-Phase Distribution Networks," to appear in *IEEE Transactions on Smart Grid*, 2019.

[430] D. Lee, H. D. Nguyen, K. Dvijotham, and K. Turitsyn, "Convex Restriction of Power Flow Feasibility Set," *arXiv:1803.00818*, March 2018.

[431] L. Debnath and P. Mikusiński, *Hilbert Spaces with Applications*. Academic Press, 2005.

[432] T. H. Chen, M. S. Chen, K. J. Hwang, P. Kotas, and E. A. Chebli, "Distribution System Power Flow Analysis–A Rigid Approach," *IEEE Transactions on Power Delivery*, vol. 6, no. 3, pp. 1146–1152, July 1991.

[433] Z. Wang, B. Cui, and J. Wang, "A Necessary Condition for Power Flow Insolvability in Power Distribution Systems With Distributed Generators," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1440–1450, March 2017.

[434] G. Teschl, "Topics in Real and Functional Analysis." http://www.mat.univie.ac.at/~gerald/ftp/book-fa/fa.pdf, April 2017.

[435] J. W. Simpson-Porco, "A Theory of Solvability for Lossless Power Flow Equations – Part I: Fixed-Point Power Flow," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 1361–1372, September 2018.

[436] K. Dvijotham, S. H. Low, and M. Chertkov, "Convexity of Energy-Like Functions: Theoretical Results and Applications to Power System Operations," *arXiv:1501.04052*, February 2015.

[437] C. Wang, E. Stai, and J.-Y. Le Boudec, "A Polynomial-Time Method for Testing Admissibility of Uncertain Power Injections in Microgrids," *arXiv:1810.06256*, October 2018.

[438] D. Shchetinin, T. T. De Rubira, and G. Hug, "Conservative Linear Line Flow Constraints for AC Optimal Power Flow," in *IEEE Manchester PowerTech*, (Manchester, England, UK), pp. 1–6, June 2017.

[439] D. Shchetinin, T. T. De Rubira, and G. Hug, "On the Construction of Linear Approximations of Line Flow Constraints for AC Optimal Power Flow," to appear in *IEEE Transactions on Power Systems*, 2019.

[440] D. K. Molzahn and L. A. Roald, "Grid-Aware versus Grid-Agnostic Distribution System Control: A Method for Certifying Engineering Constraint Satisfaction," in *52nd Hawaii International Conference on Systems Sciences (HICSS)*, (Wailea, Hawaii, USA), January 2019.

[441] D. K. Molzahn and L. A. Roald, "Towards an AC Optimal Power Flow Algorithm with Robust Feasibility Guarantees," in *20th Power Systems Computation Conference (PSCC)*, (Dublin, Ireland), June 2018.

[442] R. Louca and E. Bitar, "Robust AC Optimal Power Flow," to appear in *IEEE Transactions on Power Systems*, 2019.

[443] N. A. Ruhi, K. Dvijotham, N. Chen, and A. Wierman, "Opportunities for Price Manipulation by Aggregators in Electricity Markets," *IEEE Transactions on Smart Grid*, vol. 9, no. 6, pp. 5687–5698, November 2018.

[444] S. Emiroglu, G. Ozdemir, and M. Baran, "Assessment of Linear Distribution Feeder Models used in Optimization Methods," in *IEEE Power Energy Society Innovative Smart Grid Technologies Conference (ISGT)*, (Minneapolis, MN, USA), pp. 1–5, September 2016.

[445] H. Nagarajan, R. Bent, P. Van Hentenryck, S. Backhaus, and E. Yamangil, "Resilient Transmission Grid Design: AC Relaxation vs. DC Approximation," *arXiv:1703.05893*, March 2017.

[446] D. Bienstock, M. Chertkov, and S. Harnett, "Chance-Constrained Optimal Power Flow: Risk-Aware Network Control Under Uncertainty," *SIAM Review*, vol. 56, no. 3, pp. 461–495, 2014.

[447] J. Schmidli, L. A. Roald, S. Chatzivasileiadis, and G. Andersson, "Stochastic AC Optimal Power Flow with Approximate Chance-Constraints," in *IEEE Power & Energy Society General Meeting*, (Boston, MA, USA), pp. 1–5, July 2016.

[448] L. A. Roald, D. K. Molzahn, and A. F. Tobler, "Power System Optimization with Uncertainty and AC Power Flow: Analysis of an Iterative Algorithm," in *10th IREP Symposium on Bulk Power System Dynamics and Control*, (Espinho, Portugal), August 2017.

[449] L. A. Roald and G. Andersson, "Chance-Constrained AC Optimal Power Flow: Reformulations and Efficient Algorithms," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2906–2918, May 2018.

[450] L. Halilbasic, P. Pinson, and S. Chatzivasileiadis, "Convex Relaxations and Approximations of Chance-Constrained AC-OPF Problems," to appear in *IEEE Transactions on Power Systems*, 2019.

[451] B. Li, M. Vrakopoulou, and J. L. Mathieu, "Chance Constrained Reserve Scheduling Using Uncertain Controllable Loads–Part II: Analytical Reformulation," to appear in *IEEE Transactions on Smart Grid*, 2019.

[452] M. Vrakopoulou, M. Katsampani, K. Margellos, J. Lygeros, and G. Andersson, "Probabilistic Security-Constrained AC Optimal Power Flow," in *IEEE Grenoble PowerTech*, (Grenoble, France), pp. 1–6, June 2013.

[453] A. Venzke, L. Halilbasic, U. Markovic, G. Hug, and S. Chatzivasileiadis, "Convex Relaxations of Chance Constrained AC Optimal Power Flow," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2829–2841, May 2018.

[454] J. F. Marley, M. Vrakopoulou, and I. A. Hiskens, "Towards the Maximization of Renewable Energy Integration Using a Stochastic AC-QP Optimal Power Flow Algorithm," in *10th IREP Symposium on Bulk Power System Dynamics and Control*, (Espinho, Portugal), August 2017.

[455] M. Vrakopoulou, B. Li, and J. L. Mathieu, "Chance Constrained Reserve Scheduling Using Uncertain Controllable Loads–Part I: Formulation and Scenario-based Analysis," to appear in *IEEE Transactions on Smart Grid*, 2019.

[456] Y. Weng, Q. Li, R. Negi, and M. Ilic, "Semidefinite Programming for Power System State Estimation," in *IEEE Power & Energy Society General Meeting*, (San Diego, CA, USA), pp. 1–8, July 2012.

[457] J. F. Marley, D. K. Molzahn, and I. A. Hiskens, "Solving Multiperiod OPF Problems using an AC-QP Algorithm Initialized with an SOCP Relaxation," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3538–3548, September 2017.

[458] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A Tutorial on Geometric Programming," *Optimization and Engineering*, vol. 8, no. 1, p. 67, 2007.

[459] S. Misra, M. W. Fisher, S. Backhaus, R. Bent, M. Chertkov, and F. Pan, "Optimal Compression in Natural Gas Networks: A Geometric Programming Approach," *IEEE Transactions on Control of Network Systems*, vol. 2, no. 1, pp. 47–56, March 2015.

[460] H. Zhu and G. B. Giannakis, "Power System Nonlinear State Estimation Using Distributed Semidefinite Programming," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 6, pp. 1039–1050, December 2014.

[461] H. G. Aghamolki, Z. Miao, and L. Fan, "SOCP Convex Relaxation-Based Simultaneous State Estimation and Bad Data Identification," *arXiv:1804.05130*, April 2018.

[462] M. E. Baran and F. F. Wu, "Network Reconfiguration in Distribution Systems for Loss Reduction and Load Balancing," *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1401–1407, April 1989.

[463] R. A. Jabr, R. Singh, and B. C. Pal, "Minimum Loss Network Reconfiguration Using Mixed-Integer Convex Programming," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 1106–1115, May 2012.

[464] J. A. Taylor and F. S. Hover, "Convex Models of Distribution System Reconfiguration," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1407–1413, August 2012.

[465] C. Coffrin, H. L. Hijazi, K. Lehmann, and P. Van Hentenryck, "Primal and Dual Bounds for Optimal Transmission Switching," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–8, August 2014.

[466] H. L. Hijazi and S. Thiébaux, "Optimal AC Distribution Systems Reconfiguration," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), August 2014.

[467] M. Lu, H. Nagarajan, E. Yamangil, R. Bent, S. Backhaus, and A. Barnes, "Optimal Transmission Line Switching under Geomagnetic Disturbances," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2539–2550, May 2018.

[468] D. Gayme and U. Topcu, "Optimal Power Flow with Large-Scale Storage Integration," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 709–717, May 2013.

[469] E. Davoodi, E. Babaei, and B. Mohammadi-ivatloo, "An Efficient Covexified SDP Model for Multi-Objective Optimal Power Flow," *International Journal of Electrical Power & Energy Systems*, vol. 102, pp. 254–264, 2018.

[470] R. Bent, C. Coffrin, R. R. E. Gumucio, and P. Van Hentenryck, "Transmission Network Expansion Planning: Bridging the Gap between AC Heuristics and DC Approximations," in *18th Power Systems Computation Conference (PSCC)*, (Wroclaw, Poland), pp. 1–8, August 2014.

[471] S. Merkli, A. Domahidi, J. Jerez, and R. S. Smith, "Globally Optimal AC Power System Upgrade Planning under Operational Policy Constraints," in *European Control Conference (ECC)*, (Limassol, Cyprus), June 2018.

[472] B. Ghaddar and R. A. Jabr, "Power Transmission Network Expansion Planning: A Semidefinite Programming Branch-and-Bound Approach," to appear in *European Journal of Operational Research*, 2019.

[473] A. Lorca and X. A. Sun, "The Adaptive Robust Multi-Period Alternating Current Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1993–2003, March 2018.

[474] Y. Liu and M. Ferris, "Security-Constrained Economic Dispatch using Semidefinite Programming," in *IEEE Power & Energy Society General Meeting*, (Denver, CO, USA), pp. 1–5, July 2015.

[475] K. Sundar, C. Coffrin, H. Nagarajan, and R. Bent, "Probabilistic N-k Failure-Identification for Power Systems," *arXiv:1704.05391*, July 2018.

[476] X. Wu, A. J. Conejo, and N. Amjady, "Robust Security Constrained ACOPF via Conic Programming: Identifying the Worst Contingencies," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 5884–5891, November 2018.

[477] A. Lorca and X. A. Sun, "Multistage Robust Unit Commitment with Dynamic Uncertainty Sets and Energy Storage," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 1678–1688, May 2017.

[478] A. Nasri, S. J. Kazempour, A. J. Conejo, and M. Ghandhari, "Network-Constrained AC Unit Commitment Under Uncertainty: A Benders' Decomposition Approach," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 412–422, January 2016.

[479] C. Gambella, J. Marecek, M. Mevissen, J. M. F. Ortega, S. P. Djukic, and M. Pezic, "Transmission-Constrained Unit Commitment," *arXiv:1806.09408*, June 2018.

[480] R. Madani, A. Atamtürk, and A. Davoudi, "A Scalable Semidefinite Relaxation Approach to Grid Scheduling," tech. rep., Industrial Engineering & Operations Research, University of California, Berkeley, July 2017. BCOL Research Report 17.03, *arXiv:1707:03541*.

[481] E. Salgado, A. Scozzari, F. Tardella, and L. Liberti, "Alternating Current Optimal Power Flow with Generator Selection," in *International Symposium on Combinatorial Optimization (ISCO)*, (Marrakesh, Morocco), April 2018.

[482] N. C. Koutsoukis, P. A. Karafotis, P. S. Georgilakis, and N. D. Hatziargyriou, "Optimal Service Restoration of Power Distribution Networks Considering Voltage Regulation," in *IEEE Manchester PowerTech*, (Manchester, England, UK), pp. 1–6, June 2017.

[483] L. Gan and S. H. Low, "An Online Gradient Algorithm for Optimal Power Flow on Radial Networks," *IEEE Journal on Selected Areas in Communication*, vol. 34, no. 3, pp. 625–638, March 2016.

[484] E. Dall'Anese, S. V. Dhople, and G. B. Giannakis, "Photovoltaic Inverter Controllers Seeking AC Optimal Power Flow Solutions," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 2809–2823, 2016.

[485] E. Dall'Anese and A. Simonetto, "Optimal Power Flow Pursuit," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 942–952, March 2018.

[486] G. Wang, V. Kekatos, A. J. Conejo, and G. B. Giannakis, "Ergodic Energy Management Leveraging Resource Variability in Distribution Grids," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 4765–4775, 2016.

[487] Z. Yuan, M. R. Hesamzadeh, and D. R. Biggar, "Distribution Locational Marginal Pricing by Convexified ACOPF and Hierarchical Dispatch," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 3133–3142, July 2018.

[488] D. K. Molzahn, B. C. Lesieutre, and C. L. DeMarco, "A Sufficient Condition for Power Flow Insolvability With Applications to Voltage Stability Margins," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2592–2601, August 2013.

[489] D. K. Molzahn, V. Dawar, B. C. Lesieutre, and C. L. DeMarco, "Sufficient Conditions for Power Flow Insolvability Considering Reactive Power Limited Generators with Applications to Voltage Stability Margins," in *9th IREP Symposium on Bulk Power System Dynamics and Control*, (Rethymno, Greece), August 2013.

[490] D. K. Molzahn, I. A. Hiskens, and B. C. Lesieutre, "Calculation of Voltage Stability Margins and Certification of Power Flow Insolvability using Second-Order Cone Programming," in *49th Hawaii International Conference on Systems Sciences (HICSS)*, (Koloa, HI, USA), pp. 2307–2316, January 2016.

[491] B. Cui and X. A. Sun, "A New Voltage Stability-Constrained Optimal Power Flow Model: Sufficient Condition, SOCP Representation, and Relaxation," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 5092–5102, September 2018.