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A SURVEY OF SIMULATION OPTIMIZATION TECHNIQUES AND PROCEDURES

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ABSTRACT

Discrete-event simulation optimization is a problem of significant interest to practitioners interested in extracting useful information about an actual (or yet to be designed) system that can be modeled using discrete-event simulation. This paper presents a brief survey of the literature on discrete-event simulation optimization over the past decade (1988 to the present). Swisher et al. (2000) provides a more comprehensive review of this topic while Jacobson and Schruben (1989) covers the literature preceding 1988. Optimization of both discrete and continuous input parameters are examined herein. The continuous input parameter case is separated into gradient and non-gradient based optimization procedures. The discrete input parameter case differentiates techniques appropriate for small and for large numbers of feasible input parameter values.

1 INTRODUCTION

Simulation optimization provides a structured approach to determine optimal input parameter values, where optimal is measured by a function of output variables (steady state or transient) associated with a simulation model. Several excellent surveys have been written on this topic. Azadivar (1999) provides a survey of issues specific to simulation optimization. Andradottir (1998a,b) also presents a review of simulation optimization techniques, focusing on both gradient estimation techniques (for continuous input parameters) and random search methods (for discrete input parameters). Carson and Maria (1997) present a general summary of simulation optimization. Fu (1994a,b) provides a comprehensive review of simulation optimization and simulation gradient estimation techniques. Vysypkov et al. (1994) classify and analyze different situations in simulation model optimization and suggest appropriate search algorithms. Gaivoronski (1992) offers a survey of recent results on the optimization of stochastic discrete-event dynamic systems. Safizadeh (1990) contributes a general survey of simulation optimization techniques and procedures. Park (1990) provides an overview of simulation optimization techniques, including a discussion of methods appropriate to uni-modal and multi-modal objective functions. Glynn (1989) discusses research issues associated with optimizing simulated systems, including convergence rates for different gradient estimators and stochastic approximation (SA) algorithms.

The objective of this paper is to survey the simulation optimization literature since 1988. At that time, most simulation optimization procedures were path search based, involving some type of gradient estimation technique (e.g., perturbation analysis) imbedded in a SA algorithm. Other techniques that were reported included pattern search methods (e.g., method of Hooke and Jeeves), random methods, and integral methods. This survey covers several new advances that have surfaced through the course of the past decade, including multiple comparisons with the best and metaheuristics such as tabu search and simulated annealing.

The paper is organized as follows: Section 2 provides a description of the problem. Section 3 looks at simulation optimization techniques designed for continuous input parameters, while Section 4 looks at techniques for discrete input parameters. Section 5 concludes the paper.

2 PROBLEM DESCRIPTION

Consider a discrete-event simulation model with p deterministic input parameters $\psi \equiv (\psi_1,\psi_2,\ldots,\psi_p)$ and q stochastic output variables $Y \equiv (Y_1,Y_2,\ldots,Y_q)$, where Y is a

function of ψ (i.e., $Y = Y(\psi)$). Suppose that the input parameters are defined over a *feasible region* $\mathbf{\Psi}$. Define a real function of Y, C(Y), that combines the q output variables into a single stochastic output variable. The goal is to determine values for ψ such that $F(\psi)$, the simulation response function, is optimized. Typically, $F(\psi) \equiv E[C(Y(\psi))]$.

The p deterministic input parameters ψ can be either continuous or discrete (or both). For now, assume that the p input parameters are either all continuous or all discrete. If the p input parameters are continuous, assume that $F(\psi)$ is continuously differentiable, where ψ is defined over the feasible region Ψ. If the p input parameters are discrete, the feasible region Ψ can be either countably finite (and small), countably finite (and large) or countably infinite (the last two classifications will be grouped together). The simulation optimization techniques that have been applied depend on whether the input parameters are continuous or discrete. Each of these cases will be treated separately.

The challenge associated with determining $\psi^* = \arg \text{opt}_{\psi \in \mathbf{\Psi}} F(\psi)$ is that $F(\psi)$ cannot be observed directly, but rather, must be estimated. This may require multiple simulation run replications or long simulation runs, with $F(\psi)$ estimated by $\hat{F}(\psi)$ using $Y^i(X)$ as the i^{th} output from n simulation replications. The stochastic nature of the output from a simulation run complicates the optimization problem. Note that this survey focuses on single response optimization. See Evans et al. (1991) for a description of techniques that can be used for the multi-criteria simulation optimization problem.

3 CONTINUOUS INPUT PARAMETER METHODS

The feasible region, $\mathbf{\Psi}$, is uncountable and infinite when the set of input parameters are continuous. An extensive body of research exists for simulation optimization problems of this type. Such continuous input parameter methods may be classified as either gradient-based or nongradient-based.

3.1 Gradient Approaches

Gradient-based simulation optimization procedures have attracted a great deal of research attention over the past decade, due largely to the enormous amount of research attention given to gradient estimation techniques. Perturbation analysis (Bettonvil 1989, Glasserman 1991, Ho and Cao 1991), likelihood ratios or score functions (Glynn 1987, Rubinstein 1991, Rubinstein and Shapiro 1993), harmonic analysis (Jacobson and Schruben 1999), and phantom rare perturbation analysis estimation (Bremaud and Vazquez-Abad 1992) have been studied with the objective of developing efficient gradient estimators applicable to a broad class of discrete-event simulation models. These gradient estimators are then imbedded in optimization algorithms which control the step size taken in the gradient direction at each iteration. SA (Robbins and Monro 1951, Kiefer and Wolfowitz 1952) is the most popular and widely used optimization algorithm used for this purpose.

SA has been applied to numerous types of simulation models, using several different gradient estimation procedures. Suri and Leung (1987) present a single run optimization procedure that uses SA with perturbation analysis gradient estimates. Their results suggest that even with multiple input parameters, their procedure obtains good solutions in a reasonable amount of computing time. In contrast, Arsham (1996) presents a single run optimization technique that uses the score function for gradient estimation. He shows the method to be particularly effective and efficient for the M/M/1 queueing problem. Fu and Ho (1988) provide an empirical study of SA also using the single server queue. They conclude that the performance of SA can be improved based on the quality of the search direction estimator used. Leung and Suri (1990) compare the finite-time behavior of the Robbins-Monro SA algorithm and a modified (single run) SA algorithm that uses projection operators on an AR(1) process. Suri and Leung (1991) further illustrate these results on a M/M/1 queue simulation model. Wardi and Lee (1991) propose and apply a steepest descent algorithm with Armijo step sizes and gradient estimation via finite perturbation analysis to queueing networks. The algorithm does not require an *a priori* rate of reduction for the step sizes, allowing them to remain large and provide substantial descent until the iterates approach optimal (suboptimal, stationary) points. L'Ecuyer and Glynn (1994) prove that for the SA algorithm to converge (when applied to at GI/G/1 queue), the gradient estimates must converge, hence the simulation run length must approach infinity. Fu (1990) presents (and proves the convergence of) a SA algorithm using infinitesimal perturbation analysis for a GI/G/1 queue. Fu and Hu (1997) discuss numerous examples of applying different variations of perturbation analysis with SA algorithms. Hill and Fu (1994a, b) introduce simultaneous perturbation stochastic approximation (SPSA) that combines simultaneous perturbation gradient estimation (Spall 1992) with SA. SPSA requires only two sample estimates to compute a gradient estimate, regardless of the number of input parameters of interest, thereby significantly reducing the number of simulation runs for multi-parameter problems. As these papers demonstrate, SA has been applied in wide and varied ways over the last decade.

Research has also been undertaken to identify ways to accelerate the convergence of SA. Andradottir (1990) presents a scaling modification to SA that is provably convergent on a large class of problems, and in many instances converges faster to the optimal input parameter

values. Andradottir (1991) obtains similar results with a projected SA algorithm. She notes that if the feasible region is unbounded, and the simulation expected responses function grows faster than a quadratic function in ψ, then the stochastic approximation algorithm may not converge. To overcome this and establish convergence, Andradottir (1995) presents a projection modification to SA, while Andradottir (1996b) presents a scaling modification to SA. Andradottir (1996c) presents results with a Robbins-Monro SA algorithm using likelihood ratio gradient estimates. Yakowitz (1993), motivated by the machine learning problem, describes a globally convergent SA algorithm that assumes the objective function to be smooth. This restriction on the objective function yields an accelerated convergence rate that could be applied to a discrete-event simulation model.

Research on gradient-based simulation optimization techniques other than SA has also been reported. For example, Joshi et al. (1995) present an enhanced response surface methodology algorithm that employs conjugate gradient search techniques, second order models, and common random numbers. In contrast to SA, Shapiro (1996) investigates stochastic counterpart (SC) methods in which $F(\psi)$ is approximated by an average function and then solved using a deterministic optimization algorithm. SC methods are meant to overcome the slow convergence rates, absence of robust stopping rules, and constraint handling difficulties exhibited by SA. Shapiro suggests that the use of steepest descent algorithms with Armijo step sizes (Wardi 1990) are appropriate for use in SC methods. Rubinstein (1997) develops an SC algorithm for optimizing simulation models with rare events.

3.2 Non-Gradient Approaches

Non-gradient approaches provide an alternative to SA and the gradient estimation-based procedures described in Section 3.1. These methods include the Nelder-Mead (simplex) method and the Hooke and Jeeves method.

Haddock and Bengu (1987) present an unconstrained simulation optimization procedure that incorporates the Nelder-Mead method, the Hooke and Jeeves method, and a modified integer parameter search. Barton and Ivey (1991, 1996) present a modification to the Nelder-Mead method for simulation optimization that reduces the risk of false convergence. Humphrey and Wilson (1998) also present a variant of the Nelder-Mead method that is designed to avoid excessive sensitivity to starting values, premature termination at a local optimum, lack of robustness against noisy responses, and lack of computational efficiency. The authors demonstrate that their method provides improved performance, but requires slightly higher computational effort than either Barton and Ivey or a traditional Nelder-Mead approach. Tomick et al. (1995) present guidelines on the number of simulation replications that reduce the frequency of inappropriate (non-optimal) termination.

Other approaches have also been considered. Gurkan et al. (1994) present the sample path method for simulation optimization. Their approach determines the optimal solution for each simulation sample path, and iteratively moves towards an optimal solution with each successive sample path. Robinson (1996) shows that under certain assumptions sample path optimization will almost surely find a point that is sufficiently close (i.e., user-specified) to the set of optimizers of the limit function. Dolgui and Ofitserov (1997) propose a method that uses information from the previous simulation to find a new starting point which is closer to the global optimum. Their method is applicable to both discrete and continuous optimization problems. Schruben (1997) introduces a simulation optimization procedure that uses event time dilation and simultaneous run replications. Lee et al. (1997) discuss the reverse-simulation method for simulation optimization.

4 DISCRETE INPUT PARAMETER METHODS

Several techniques have been developed for simulation optimization when the number of input parameter values (i.e., the size of $\boldsymbol{\Psi}$) is discrete. If the set $\boldsymbol{\Psi}$ is finite and small, ranking and selection and multiple comparison procedures are appropriate (Goldsman and Nelson 1994, 1998a, Bechhofer et al. 1995, Hsu 1996). If the set $\boldsymbol{\Psi}$ is infinite or very large, then techniques such as ordinal optimization (Ho et al. 1992), simulated annealing (Fleischer 1995), tabu search (Glover and Laguna 1997), and genetic algorithms (Liepins and Hilliard 1989) have been adapted for the simulation environment.

4.1 Small Number of Feasible Solutions

Ranking and Selection (R&S) focuses on selecting the optimal input parameter values over a finite set $\boldsymbol{\Psi} = \{\psi^1,\psi^2,\ldots,\psi^k\}$, $k < +\infty$, where k is small (i.e., 2 to 20). The objective is to determine which of the k input parameter values minimizes $F(\psi)$. By applying a two-stage procedure (Dudewicz and Dalal 1975, Rinott 1978), the output from simulation runs at the k input parameter values can be used to determine the most likely input parameter values that minimize $F(\psi)$. By defining differences in $F(\psi)$ that are less than $\delta > 0$ to be insignificant, we can assure the probability of making the correct selection P^* by choosing the length of our simulation run carefully. In general, as P^* approaches one, or alternatively, as δ approaches zero, the length of the k simulation runs must approach infinity. Procedures of this type are referred to as *indifference zone* ranking and selection (R&S) procedures. To apply these procedures, the simulation runs must be independently seeded to ensure that the simulation outputs

from each run are independent. Koenig and Law (1985) extend the indifference zone approach for use as a screening procedure. They present a method for selecting a subset of size *m* (user-specified) of the *k* systems so that with probability at least P^*, the selected subset will contain the best system. Morrice et al. (1998, 1999) propose an indifference zone R&S procedure that allows multiple performance measures through the use of a multiple attribute utility function. A second type of R&S procedure, known as *subset selection*, aims to produce a subset of random size that contains the best system with probability P^* without specification of an indifference zone (i.e., $\delta = 0$). Originally these procedures saw little usage, since they required equal and known variances among alternatives, though Sullivan and Wilson (1989) have made subset selection more feasible with a procedure that allows for unknown and unequal variances.

Like R&S, multiple comparison procedures (MCPs) attempt to identify the optimal input parameter values over a finite set $\mathbf{\Psi} = \{\psi^1,\psi^2,\ldots,\psi^k\}$, $k < +\infty$. MCPs approach the optimization problem as a statistical inference problem and, unlike R&S procedures, do not guarantee a decision. Three main classes of MCPs are used in practice: all pairwise multiple comparisons (MCA) (Tukey 1953), multiple comparisons with the best (MCB) (Hsu 1984, Hsu and Nelson 1988), and multiple comparisons with a control (MCC) (Dunnett 1955, Bofinger and Lewis 1992, Damerji and Nakayama 1999). The most popular approach among these is MCB. In particular, MCB looks at $F(\psi^j) - \text{opt}_{i\neq j} F(\psi^i)$, $j = 1,2,\ldots,k$, to determine that j^* with $F(\psi^{j^*}) - \text{opt}_{i\neq j^*} F(\psi^i) > 0$. Simultaneous confidence intervals for $F(\psi^j) - \text{opt}_{i\neq j} F(\psi^i)$, $j = 1,2,\ldots,k$, can be used to determine j^*, by looking for the confidence interval with lower confidence limit that is zero. To apply this procedure, the simulation runs must be independently seeded and the simulation output must be normally distributed (or averaged so that the estimators used are approximately normally distributed). Yang and Nelson (1989, 1991) present modifications to the MCB procedure (and MCA and MCC) that incorporates two variance reduction techniques (common random numbers and control variates). Their results suggest that using variance reduction can lead to correct decisions with higher probabilities. Goldsman and Nelson (1990) present a MCB procedure for steady state simulation experiments. Nakayama (1995) generalizes these results on applying MCB using batch means in steady-state simulations. Matejcik and Nelson (1993) establish a fundamental connection between indifference zone procedures and MCB. They present three procedures that incorporate MCB based on whether independent or common random numbers are used to obtain the simulation outputs. Matejcik and Nelson (1995) extend their earlier work by showing that most indifference zone procedures can simultaneously provide MCB confidence intervals. Nelson and Matejcik (1995) show how common random numbers can be incorporated into a combined R&S-MCB procedure. Yuan and Nelson (1993) discuss MCB procedures for steady-state simulations, where the simulation outputs are assumed to follow an auto-regressive process with normally distributed residuals. Nakayama (1996, 1997a) presents a single-stage MCB procedure that is asymptotically valid for steady-state simulations, hence extending the domain of applicability of previous MCB procedures. Nakayama (1997b, 2000) presents a two-stage MCB procedure using common random numbers for steady-state simulations, and shows that it is asymptotically valid.

Retrospective optimization uses several simulation output runs to generate a performance function. This performance function is then optimized using standard deterministic techniques. Healy and Schruben (1991) illustrate retrospective optimization on a yield management simulation model to determine optimal production lot sizes, on a tandem queuing network simulation model to determine optimal buffer allocations, and on an inventory management simulation model, to determine optimal inventory policies. Fu and Healy (1992) present a comparison of retrospective optimization with a gradient-based SA algorithm and conclude that no meaningful comparisons can be made since the two approaches are so different. Fu and Healy (1997) extend their earlier work and present a hybrid gradient-based, retrospective algorithm that attempts to build on the strengths and mitigate the shortcomings of each approach. Healy (1994) presents a simulation optimization technique where the idea is to retrospectively solve a related deterministic optimization problem with respect to realizations of the stochastic effects as if the outcomes of all uncertainties were known in advance.

4.2 Large Number of Feasible Solutions

When the set of possible input parameter values is discrete, but very large, different methods than those presented in Section 4.1 must be applied. *Ordinal optimization* (Ho et al. 1992, Ho 1994) focuses on finding good solutions, rather than trying to find the very best solution (i.e., goal softening) (Lee et al. 1999). In doing this, ordinal optimization reduces the search for an optimal solution from sampling over a very large set of solutions to sampling over a smaller, more manageable set of good solutions. Ho and Deng (1994) show that goal-softening can be effective for problems with greater than 10,000 alternatives. Dai (1995) and Xie (1997) address the convergence properties of ordinal optimization. Xie shows that for a regenerative system, the probability of obtaining a desired solution using ordinal optimization converges at an exponential rate while the variance of the performance measures converge at rate $O(1/t^2)$, where t is the simulation time. Deng et al. (1992) study the impact of correlation on

ordinal optimization. They show that correlation between the output data results in ordinal optimization selecting a larger number of good solutions in the selected set. Lau and Ho (1997) examine how the choice of subset selection rules affect alignment probability (i.e., the probability of the intersection between the "good enough" subset and the selected subset) and provide recommendations for the subset size given certain system parameters. Chen (1996) establishes two methods for establishing a lower bound on the probability that the selected subset contains at least one good design and the probability that the best of the selected subset is very close to the true best design. Deng and Ho (1997) introduce an iterative ordinal optimization procedure, analogous to traditional hill climbing procedures, that moves from one subset or search representation to another (rather than between individual solutions in a solution space). Ho and Larson (1995) introduce a new approach to rare event simulation using ordinal optimization. Chen (1995) and Chen et al. (1996) present an extension of ordinal optimization for smartly allocating computing budgets for discrete-event simulation and suggest a possible tenfold speedup in simulation time through its application.

General search strategies such as *simulated annealing* (Eglese 1990, Fleischer 1995), *genetic algorithms* (Liepins and Hilliard 1989, Muhlenbein 1997), and *tabu search* (Glover and Laguna 1997) have been adapted for the stochastic environment associated with discrete-event simulation optimization. Simulated annealing mimics the annealing process for crystalline solids, where a solid is cooled very slowly from an elevated temperature, with the hope of relaxing towards a low-energy (objective function value) state. Genetic algorithms emulate the evolutionary behavior of biological systems to create subsequent generations that guide the search towards optimal/near-optimal solutions. Tabu search uses memory to guide the search towards optimal/near-optimal solutions, by dynamically managing a list of forbidden moves.

Haddock and Mittenhall (1992) discuss how to apply simulated annealing to discrete-event simulation models. Zeng and Wu (1993) present a strategy for introducing perturbation analysis techniques into a simulated annealing algorithm for simulation optimization. Andradottir (1998a) presents a modified simulated annealing algorithm that can be applied to discrete-event simulations and provides conditions under which the algorithm converges to a global optimum. Tompkins and Azadivar (1995) utilize a genetic algorithm for a class of manufacturing system simulation models, where the algorithm is capable of optimizing over qualitative input parameters. Hall and Bowden (1997) compare evolutionary strategies and tabu search with the Nelder-Mead algorithm for simulation optimization. For smooth convex response surfaces, they observe that the Nelder-Mead method obtains better solutions. These articles illustrate that general search strategies conceived for discrete optimization problems can be successfully adapted for simulation optimization.

Andradottir (1996a) uses a random walk approach to develop a simulation optimization algorithm over a large discrete set of input parameter values. Alrefaei and. Alrefaei and Andradottir (1998b) present a modified stochastic ruler algorithm and show that it converges almost surely to a global optimum. Cassandras and Strickland (1989) and Cassandras (1993) display a technique termed "rapid learning" that aims at enumerating all possible sample paths under different input parameter values based on the observed sample path under nominal input parameter values. Rapid learning relies on two necessary conditions: *observability*, which asserts that every state observed in the nominal path is always richer in terms of feasible events than the states observed in constructed paths, and *constructability*, which requires that the lifetime of the events has the same distribution as their residual lives. Yücesan and Jacobson (1997) show that verification of the observability condition is an NP-hard search problem and thereby encourage the development of heuristic procedures to validate the applicability of rapid learning.

Shi and Ólafsson (1997) present the nested partitions method for simulation optimization. Nested partitions combines partitioning, random sampling, a selection of a promising index, and backtracking to create a Markov chain that converges to a global optimum. Ólafsson and Shi (1998, 1999) show that the Markov chain generated by nested partitions converges geometrically fast to the true stationary distribution and use these results to derive a stopping criterion for the method. Olafsson (1999) links this work with iterative ranking and selection methods. Shi et al. (1999) discuss the nested partitions method as applied to a simultaneous simulation environment.

5 CONCLUSIONS

Research and applications in the field of simulation optimization has advanced significantly in the past decade. The 1980s were dominated by gradient estimation research over continuous input parameters. The 1990s has seen a shift towards discrete sets of input parameter values. The unification of ranking and selection and multiple comparison procedures has been a significant advancement for those problems with discrete input parameters and a small number of feasible solutions. Moreover, the cross-fertilization between discrete optimization heuristics and discrete-event simulation has provided a rich avenue for research breakthroughs and new hybrids in the field of optimization.

Research that generalizes the domain of application of all these techniques, as well as the incorporation of variance reduction and other efficiency enhancement techniques, will serve to broaden the application and appeal of simulation optimization procedures. Such enhancements may also serve to build bridges, for example, between R&S/MCB

procedures and the techniques used for discrete input parameters with a large number of feasible solutions. The next decade promises to provide many new breakthroughs that build upon such research foundations.

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