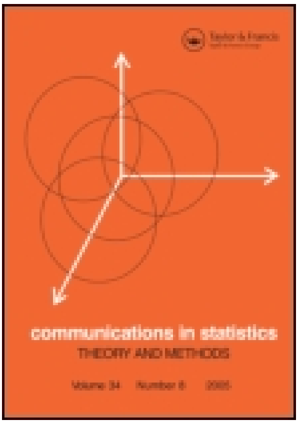

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A survey of tests for exponentiality

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A SURVEY OF TESTS FOR EXPONENTIALITY

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Key Words and Phrases: Exponential distribution; power; simulations

ABSTRACT

A wide selection of tests for exponentiality is discussed and compared. Power computations, using simulations, were done for each procedure. Certain tests (e.g. Gnedenko (1969), Lin and Mudholkar (1980), Harris (1976), Cox and Oakes (1984), and Deshpande (1983)) performed well for alternative distributions with non-monotonic hazard rates, while others (e.g. Deshpande (1983), Gail and Gastwirth (1978), Kolmogorov-Smirnov (Lilliefors (1969)), Hahn and Shapiro (1967), Hollander and Proschan (1972), and Cox and Oakes (1984)) fared well for monotonic hazard rates. Of all the procedures compared, the score test presented in Cox and Oakes (1984) appears to be the best if one does not have a particular alternative in mind.

1. INTRODUCTION

Extensive literature exists on tests for exponentiality. Many procedures have been proposed ranging from Hartley's F Max test (Hartley (1950)) to the score test of Cox and Oakes (1984). There does not appear to be any agreement as to which procedure is the best, or even on how to define best.

Spurrier (1984) offers advice and comments on a vast number of tests for exponentiality, but does *not simultaneously compare* the procedures. Lee, Locke, and Spurrier (1980) discuss several one-sided tests and do power simulations to compare them. Comarisons are also presented *in D'Agostino and Stephens* (1986). The purpose of this paper is to discuss and compare a wide selection of tests for exponentiality, both one-sided and two-sided. Power computations, using simulations, were done for each procedure.

2. DESCRIPTION OF TESTS

Let $X_1, X_2, \ldots, X_N$ be a random sample from a population with density function $f_X(.)$. The null hypothesis under consideration is H_0: $f_X(x) = \lambda\ \text{EXP}\ (-\lambda x)$ (i.e., the random variable X is exponentially distributed with parameter λ) where $x \geq 0$ and $\lambda > 0$. Each of the tests discussed here is scale invariant (i.e., λ does not have to be specified). Normalized spacings, which are used in several tests are defined as: $D_i = (N-i+1)(X_{(i)} - X_{(i-1)})$; where $i = 1,2,\ldots,N$, $X_{(0)} = 0$, and $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(N)}$ are the order statistics. A description of the procedures under consideration follows.

1- Gnedenko's F-test: Q(R) - This procedure is due to Gnedenko (1969) and is discussed by Lin and Mudholkar (1980) and Fercho and Ringer (1972). The N data points are ordered and split into two groups with group one containing the first R points and group two the remaining N-R. The test statistic is:

$$Q(R) = \frac{\sum_{i=1}^{R} D_i/R}{\sum_{i=R+1}^{N} D_i/(N-R)}$$

If the null hypothesis of exponentiality is true, then Q(R) has an F distribution with 2R and 2(N-R) degrees of freedom. The hypothesis is rejected for both small and large values of Q(R). Fercho and Ringer recommend setting R = N/2 and claim the test is well suited for Weibull alternatives and Gammas with monotone hazard rates.

2- Harris' modification of Gnedenko's F-test: $Q'(R)$ - This test was proposed by Harris (1976) and discussed by Lin and Mudholkar (1980). The test statistic is:

$$Q'(R) = \frac{\left(\sum_{i=1}^{R} D_i + \sum_{i=N-R+1}^{N} D_i\right)/2R}{\sum_{i=R+1}^{N-R} D_i/(N-2R)}$$

$Q'(R)$ is distributed as an F with 4R and 2(N-2R) degrees of freedom, given the null hypothesis is true. The hypothesis is rejected for both small and large values of $Q'(R)$. This procedure is claimed to be powerful against the log normal distribution (which has a U shaped hazard) and inferior for monotone hazards. Harris recommends setting $R = N/4$.

3- Lin and Mudholkar's Bivariate F-test: BF(R) - This test, which is essentially a combination of tests one and two above, was proposed by Lin and Mudholkar (1980). Let

$$F_L = \frac{\sum_{i=1}^{R} D_i/R}{\sum_{i=R+1}^{N-R} D_i/(N-2R)} \quad \text{and} \quad F_U = \frac{\sum_{i=N-R+1}^{N} D_i/R}{\sum_{i=R+1}^{N-R} D_i/(N-2R)} .$$

Conditional on the null hypothesis, F_L and F_U jointly follow a bivariate F distribution. Rejection of exponentiality will occur if either F_L or F_U is not within some interval (a,b). This interval is determined by using the following theorem from Hewett and Bulgren (1971): For any $0 \leq a \leq b < +\infty$, $P(a \leq F_L \leq b,\ a \leq F_U \leq b \mid H_0) \leq [P(a \leq F \leq b)]^2$, where F is Snedecor's F random variable with 2R and 2(N-2R) degrees of freedom. The right hand side of the inequality is set equal to $1 - \alpha$ (where α is the desired Type I error) and assuming equal tail probabilities for F, a and b are easily obtained. This procedure is claimed to be powerful against alternatives with non-monotone hazards (e.g. log normal). Lin and Mudholkar (1980) recommend using R = N/10.

4- Skewness and Kurtosis: KUSK - The test statistic proposed here is: $K = (\hat{\beta}_1 + 0.5)/\hat{\beta}_2$, where $\hat{\beta}_1 = \hat{\mu}_3^2/\hat{\mu}_2^3$ (sample skewness coefficient) and $\hat{\beta}_2 = \hat{\mu}_4/\hat{\mu}_2^2$ (sample kurtosis coefficient). When the null hypothesis is true, $(\beta_1 + 0.5)/\beta_2$ assumes a value of 0.5. Lower and upper critical values for K are obtained using simulations. For small sample sizes, this test will be misleading as both $\hat{\beta}_1$ and $\hat{\beta}_2$ are sensitive to outliers.

5- Hollander and Proschan's "New Better Than Used" test: HP - This procedure, which is proposed by Hollander and Proschan (1972), is usually applied to one-sided alternatives (new better than used or new worse than used). In this paper, since no knowledge of the alternative hypothesis was assumed, the test was two-sided. The test statistic is:

$$T = \sum_{i>j>k} G(X_{(i)}, X_{(j)}+X_{(k)}) \text{ where}$$

$$G(a,b) = \begin{cases} 1 \text{ if } a > b \\ 0.5 \text{ if } a = b. \\ 0 \text{ if } a < b \end{cases}$$

The authors provide a table of approximate lower and upper critical values and the following Normal approximation:

$$T^* = \frac{T-E(T|H_0)}{[VAR(T|H_0)]^{1/2}}$$

where $E(T|H_0) = N(N-1)(N-2)/8$ and $VAR(T|H_0) = (1.5(N)(N-1)(N-2)[(5/2592)(N-3)(N-4) + (N-3)(7/432) + (1/48)])$. When the null hypothesis is true and N approaches infinity, T* has an asymptotic Normal distribution with mean 0 and variance 1.

6- The WE test: WE1 - The WE test statistic proposed by Hahn and Shapiro (1967) and discussed by Lee (1980) and Lee, Locke, and

Spurrier (1980) is:

$$WE1 = \sum_{i=1}^{N} (X_i - \bar{X})^2 / (\sum_{i=1}^{N} X_i)^2 = (N-1)S^2/N^2\bar{X}^2$$

where S^2 is the sample variance and $\bar{X}$ is the sample mean. A table of lower and upper critical values may be found in Lee (1980).

7- The Gini statistic: G - This procedure, introduced by Gail and Gastwirth (1978), has the following test statistic:

$$G = [\sum_{i=1}^{N-1} i(N-i)(X_{(i+1)} - X_{(i)})]/[(N-1)\sum_{i=1}^{N} X_i] =$$

$$\sum_{i=1}^{N-1} i\, D_{i+1}/(N-1)\sum_{i=1}^{N} X_i.$$

The authors provide a table of approximate lower and upper critical values and the following Normal approximation:

$$G^* = \frac{G - E(G|H_0)}{[VAR(G|H_0)]^{1/2}}$$

where $E(G|H_0) = 0.5$ and $VAR(G|H_0) = 1/[12(N-1)]$. Under the assumption of exponentiality, G^* has an asymptotic standard Normal distribution even for samples as small as 10. Good power is claimed for Weibull, Uniform, and Gamma alternatives. The Gini statistic may also be adapted to data which is censored at $X_{(R)}$ where $R \leq N$.

8- The Lorenz statistic: L - Gail and Gastwirth (1978) found that the Lorenz statistic yielded a powerful test for exponentiality. The test statistic is:

$$L_N(p) = \sum_{i=1}^{[Np]} X_{(i)}/N\bar{X}$$

where $0 < p < 1$ and $[Np]$ is the largest integer less than or equal to Np. The authors provide lower and upper critical values and recommend setting $p = 0.5$.

9- The Pietra statistic: P - This procedure is discussed by Gail and Gastwirth (1978) who provide the following test statistic:

$$P = \sum_{i=1}^{N} |X_i - \bar{X}|/2N\bar{X}.$$

The authors provide lower and upper critical values.

10- Epstein: EPS - This test is due to Epstein (1960) and is discussed by Fercho and Ringer (1972). The test statistic is:

$$EPS = 2N[Ln(\sum_{i=1}^{N} D_i/N) - N^{-1}\sum_{i=1}^{N} Ln(D_i)]/[1+(N+1)/6N],$$

where Ln is the natural logarithm.

Given the null hypothesis is true, EPS is approximately distributed as a Chi-square with N-1 degrees of freedom. The hypothesis is rejected for large values of EPS. This procedure is claimed to be powerful against Gamma and Weibull alternatives.

11- Kolmogorov-Smirnov test: KSL - The parameter λ was estimated by the inverse of the sample mean and critical values provided by Lilliefors (1969) were used.

12- Deshpande's test: J.b - This procedure was proposed by Deshpande (1983) for testing exponentiality against distributions with increasing failure rates. The test statistic is computed as follows: Multiply X_i, $i = 1,2,\ldots,N$ by b (b = 0.5 or 0.9 here) and arrange $X_1,\ldots, X_N$ and $bX_1,\ldots, bX_N$ together in increasing order of magnitude. Calculate the quantity

$$S = \sum_{i=1}^{N} R_i - 0.5(N)(N+1) - N$$

where R_i is the rank of X_i. One-sided critical values obtained by simulation for this Wilcoxon-type statistic are provided by the author for b = 0.5 and 0.9, when $N \leq 15$. The author recommends using J.5 whenever the alternative distribution is suspected of lying in the larger new better than used class and J.9 when the alternative is the restricted increasing failure rate average class. Since we are assuming no a priori knowledge about the alternate distribution, two-sided critical values for $N = 20$ were obtained by simulation, and used in this study. Deshpande also gives the following Normal approximation to the test: $n^{1/2}\left[J.b - M(F)\right]$ is asymptotically normally distributed with mean 0 and variance 4c where under the assumption of exponentiality, $M(F) = (b+1)^{-1}$ and

$$c = \frac{1}{4}\left[1 + \frac{b}{b+2} + \frac{1}{2b+1} + \frac{2(1-b)}{b+1} - \frac{2b}{b^2+b+1} - \frac{4}{(b+1)^2}\right]$$

13- Hartley's F Max test: HARTF - This test, which was proposed by Hartley (1950) and discussed by Fercho and Ringer (1972), resulted from a test for homogeneity of variances. The test statistic is:

$$\mathrm{HARTF} = \mathrm{Max}(W_i)/\mathrm{Min}(W_i), \text{ where } 1 \le i \le K,$$

$$W_i = \sum_{j=(i-1)R+1}^{iR} D_j,$$

K = the number of groups, and R = the size of each group. Given the null hypothesis is true, HARTF has an F Max distribution with 2R and K degrees of freedom. The hypothesis is rejected for large values of HARTF. When $N = 20$, Fercho and Ringer recommend setting $K = 2$ and $R = 10$.

14- Cox and Oakes Score test: COX - This procedure, which is found in Cox and Oakes (1984), is based on the score function:

$$U = d + \sum \mathrm{Ln}(X_i) - d \sum_{i=1}^{N} X_i \,\mathrm{Ln}(X_i) / \sum_{i=1}^{N} X_i$$

where the first summation is taken over all the uncensored (observable) points and d is the number of uncensored points. In the present case, all the points are observable (i.e. $d = N$). By using the information matrix, an asymptotic standard Normal deviate may be computed. The hypothesis of exponentiality is rejected for both large and small values of the deviate. A pleasing feature of this procedure is the ability to handle censored data. The authors claim the test to be useful against alternative hypotheses which specify monotone hazard functions.

15- Wong and Wong's Extremal Quotient Test: EXQT - This test, which is proposed by Wong and Wong (1979), is based on a quantity known as the extremal quotient: $Q = X_{(n)}/X_{(1)}$, where $X_{(1)}$ and $X_{(n)}$ are the smallest and largest order statistics of the sample, respectively. The authors provide critical values for this test, which rejects the null hypothesis for large values of Q.

When discussing critical regions for rejection of the null hypothesis in the above tests, no knowledge of the alternative hypothesis was assumed. Hence, for tests which could be one-sided or two-sided, the two-sided option was used. Tests with this option included numbers 1, 2, 4-9, and 14.

There are many tests for exponentiality which are not discussed here. Some of these include the use of Cramer-von Mises statistics with censored data (Pettitt (1977) and Sirvanci and Levent (1982)), modifications of Epstein's test to K groups of R items (Epstein (1960)), extensions of the WE1 test (Shapiro and Wilk (1972)), modifications of the Kolmogorov-Smirnov procedure (Margolin and Maurer (1976) and Durbin (1975)), a test based on the empirical characteristic function (Epps and Pulley (1986)), and procedures proposed by Jackson (1967), Moran (1951), Proschan and Pyke (1967), Bickel and Doksum (1969), Chen, Hollander, and Langberg (1983), Koul (1978), Kimber (1985), and Spinelli and Stephens (1987). The work of Spurrier (1984), Lee, Locke and Spurrier (1980), and Stephens (1986) also provide comments and references about other tests for exponentiality not mentioned here.

3. POWER RESULTS

The tests for exponentiality described in section 2 were compared with respect to power against a broad class of alternate distributions (see Table I). This class included three distributions with monotonically decreasing hazard rates (gammas with shape parameters 0.5 and 0.7 and weibull with shape parameter 0.8), nine with monotonically increasing hazard rates (uniform on 0 to 1, gammas with shape parameters 1.5, 2, and 4, weibulls with shape parameters 1.2 and 1.5, betas with shape parameters 1,2 and 2,1, and the triangular distribution), and three whose hazard rates are non-monotonic (log normals with shape parameters 0.6 and 1.0 and beta with shape parameters 0.5, 1.0). In addition, to investigate sensitivity to outliers the following "contaminated" exponential distributions were considered: a.) 18 observations from a negative exponential distribution of mean 1 and 2 observations from a negative exponential of mean 3 (i.e., $\lambda = 1/3$) and b.) 18 from a negative exponential of mean 1 and 2 from a negative exponential of mean 5 (i.e., $\lambda = 1/5$ see Table I). Small deviations from the negative exponential distribution were examined by considering from the above, the two gamma distributions with shape parameters 0.7 and 1.5 and the weibull with shape parameter 1.2 (see Table I). These three distributions are similar in shape to the negative exponential. The density functions for the aforementioned distributions may be found in Patel, Kapadia, and Owen (1976).

The sample size was fixed at 20 and 1000 values of each test statistic were simulated for each alternate distribution. A type I error of 0.05 was utilized. Simulations done with the alternate distribution set equal to the negative exponential ($\lambda = 1$) did not yield any inconsistencies with the preset type I error. Note that each entry of Table I is subject to maximum standard deviation of 0.0158 ($[(0.5^2/1000]^{1/2} = 0.0158$).

The basic assumption underlying the simulations was that the user had no knowledge of the alternate distribution. Hence, critical regions for tests which had a one-sided or two-sided option, were two-sided. The point of discussing different hazard shapes, while still using two-sided rejection regions, was to assess each test in the broadest possible sense. Since an alternate distribution must have a particular hazard shape, we are looking at the consequences of

TABLE I

Power Results (x1000)

Distributions	Q (2)	Q (4)	Q (5)	Q (10)	Q' (2)	Q' (4)	Q' (5)	BF (2)	BF (4)	BF (5)	KUSK	HP	WEI	G	L	P	EPS	KSL	J.5	J.9	HARTF	COX	EXQT
Null Hypothesis																							
EXP(λ = 1)	056	058	047	049	057	055	051	059	054	051	052	058	053	046	044	054	046	046	054	047	051	040	044
Monotonic Increasing Hazard																							
UNIF(0,1)	234	350	401	603	121	115	075	448	565	511	346	670	793	735	564	612	169	544	577	482	576	593	010
GAMMA(1.5)	161	190	151	113	085	080	050	115	126	089	069	198	192	187	173	166	052	171	219	099	102	245	003
GAMMA(2)	424	399	377	237	191	106	088	291	207	182	141	484	490	473	435	454	050	417	483	206	249	576	000
GAMMA(4)	945	946	934	808	721	507	361	867	784	670	301	987	968	985	984	979	231	957	989	714	791	996	000
WEIB(1.2)	164	169	149	128	079	074	058	127	122	106	070	184	165	173	167	161	057	152	190	098	129	139	011
WEIB(1.5)	339	376	363	301	132	114	082	241	230	189	191	463	508	497	468	483	072	397	489	200	315	580	001
BETA(1,2)	115	147	171	215	071	058	055	116	129	146	244	245	344	295	218	247	058	227	206	099	232	251	033
BETA(2,1)	895	960	967	991	627	542	471	941	983	973	237	1000	1000	1000	999	1000	831	992	998	989	991	1000	000
TRIANGULAR	862	938	956	980	575	523	461	798	838	838	741	999	1000	1000	994	996	444	977	995	895	971	1000	000
Monotonic Decreasing Hazard																							
GAMMA(0.5)	544	604	587	449	120	113	108	474	486	458	156	590	376	557	625	559	300	474	602	291	464	705	615
GAMMA(0.7)	199	196	188	187	084	088	068	172	161	143	066	229	132	219	213	194	080	150	218	110	195	276	256
WEIB(0.8)	158	193	178	207	091	081	075	151	169	152	118	208	177	210	233	237	073	161	225	113	185	211	195
Non-Monotonic Hazard																							
LOGN(0.6)	923	840	776	456	644	423	323	819	631	517	119	867	674	784	829	767	135	860	921	460	413	865	000
LOGN(1.0)	089	054	033	070	221	167	151	188	172	139	209	061	172	138	057	139	046	114	089	054	071	070	001
BETA(0.5,1.0)	394	339	287	149	460	415	359	390	383	349	054	135	038	057	192	102	222	145	175	109	152	285	414
Outliers																							
18 EXP(λ=1) and 2 EXP(λ=1/5)	050	075	066	158	212	158	133	257	222	183	333	074	333	281	153	247	093	186	091	076	171	220	095
18 EXP(λ=1) and 2 EXP(λ=1/3)	051	060	049	085	073	079	070	107	102	080	159	057	150	133	090	111	050	100	070	059	079	081	071

assuming no a priori knowledge of its shape. Obviously, if one does have knowledge of the shape, then the more specialized one-sided critical regions should be employed where appropriate as should the two-sided regions.

When the alternate distribution possessed a non-monotonic hazard rate the Gnedenko (Q(2)), Harris (Q'(2)), Lin and Mudholkar (BF(2) and BF(4)), Cox and Oakes, and Deshpande (J.5) tests did relatively well for the set of distributions considered. Lin and Mudholkar claim that their procedure and that of Harris are powerful in detecting non-monotonic hazards. The results of Table I seem to support these claims. Since Harris' test is similar to that of Gnedenko it is not surprising that the Gnedenko test performs well for non-monotonic hazards. These results however are not consistent with the recommendations to use Q(10) and Q'(5) when the sample size is 20, but do appear consistent with advice to use BF(2) and J.5. The Epstein, KUSK, Hartley, Deshpande (J.9), and extremal quotient procedures did relatively poorly for the set of distributions considered.

Many of the tests considered did relatively well when the alternate distribution possessed either a monotonically increasing or monotonically decreasing hazard rate. Cox and Oakes, Deshpande (J.5), Gnedenko, Lin and Mudholkar, and Hollander and Proschan all claim their procedures are powerful for detecting monotonic hazards. The results in Table I seem to support their claims, although the Lin and Mudholkar and Gnedenko procedures appear better suited to alternatives with non-monotonic hazards. The Gini, Lorenz, and Pietra procedures as discussed in Gail and Gastwirth (1978) as well as the Hahn and Shapiro (WE1) and Kolmogorov-Smirnov procedures, also performed relatively well. Harris' procedure, as claimed by Lin and Mudholkar, does not appear to do well for monotonic hazard rates. The Epstein, KUSK, Deshpande (J.9), and extremal quotient procedures also did relatively poorly.

The Hahn and Shapiro (WE1) and KUSK procedures did relatively well in the presence of outliers. This result is not surprising as these procedures are essentially functions of the sample variance which is greatly influenced by outliers. Hence, larger values of the test statistics are generally produced which in turn increases the probability of rejecting the null hypothesis of exponentiality.

When the alternate distribution being considered was nearly exponential, the procedures due to Cox and Oakes, Deshpande (J.5), and Hollander and Proschan performed relatively well.

4. SUMMARY

When a priori nothing is known about the alternate distribution (i.e. hazard shape) the score procedure as described in Cox and Oakes (1984) appears to be the "best" for the class of alternate distributions considered here. This test also did well in rejecting exponentiality for alternate distributions which were nearly exponential in shape. The Cox and Oakes procedure is easy to compute and can also accommodate censored data. Procedures which also performed well were: Deshpande (J.5), Lorenz, Gnedenko (Q(2), Q(4), and Q(5)), Hollander and Proschan, Lin and Mudholkar (BF(2) and BF(4)), Pietra, Gini, Kolmogorov-Smirnov, and Hahn and Shapiro (WE1).

When the alternative distribution possessed a non-monotonic hazard the Gnedenko, Harris, Lin and Mudholkar, Cox and Oakes, and Deshpande (J.5) procedures all fared relatively well. When the hazard was monotonic the Cox and Oakes, Deshpande (J.5), Kolmogorov-Smirnov, Hollander and Proschan, Gini, Lorenz, Pietra, and Hahn and Shaprio (WE1) procedures all did relatively well.

It would be more desireable to tailor the choice of test to specific knowledge about the alternate distribution. If a monotonic hazard is suspected, a more specialized (i.e., one-sided test) procedure would be more appropriate, while the use of a two-sided test may be more appropriate for non-monotonic hazards. As mentioned earlier, this paper examined the consequences of using the more generalized test (i.e., two-sided), when a choice was present.

All of the test procedures analyzed in this paper are easy to compute. Many tests were not considered here as there is a large number of available procedures to test for exponentiality. Please note that the results presented are influenced somewhat by the choice of alternate distributions. An attempt was made to select a fairly representative sample.

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BIBLIOGRAPHY

Bickel, P. and Doksum, K. (1969). Tests on monotone failure rate based on normalized spacings. Annals of Math. Stat. 40, 1216-1235.

Chen, Y. Y., Hollander, M., and Langberg, N. A. (1983). Testing whether new is better than used with randomly censored data. Annals of Statistics 11, 267-274.

Cox, D. R. and Oakes, D. (1984). Analysis of Survival Data. Chapman and Hall.

D'Agostino, R. B. and Stephens, M. A. (1986). Goodness-of-Fit Techniques, Marcel Dekker.

Deshpande, J. V. (1983). A class of tests for exponentiality against increasing failure rate average alternatives. Biometrika 70, 514-518.

Durbin, J. (1975). Kolmogorov-Smirnov tests when parameters are estimated with applications to tests of exponentiality and tests on spacings. Biometrika 62, 5-22.

Epps, T. W. and Pulley, L. B. (1986). A test of exponentiality vs. monotone-hazard alternatives derived from the empirical characteristic function. JRSS B 48, 206-213.

Epstein, B. (1960). Tests for the validity of the assumption that the underlying distribution of life is exponential (part 1). Technometrics 2, 83-101.

Fercho, W. W. and Ringer, L. J. (1972). Small sample power of some tests of the constant failure rate. Technometrics 14, 713-724.

Gail, M. H. and Gastwirth, J. L. (1978). A scale-free goodness-of-fit test for the exponential distribution based on the Gini statistic. JRSS B 40, 350-357.

Gnedenko, B. V., Belyayev, YU. K., and Solovyev, A. D. (1969). Mathematical Models of Reliability Theory. Academic Press.

Hahn, G. J. and Shapiro S. S. (1967). Statistical Models in Engineering. John Wiley and Sons.

Harris, C. M. (1976). A note on testing for exponentiality. Naval Research Logistics Quarterly 23, 169-175.

Hartley, H. O. (1950). The maximum F-ratio as a short-cut test for heterogeneity of variance. Biometrika 37, 308-312.

Hewett, J. E. and Bulgren, W. G. (1971). Inequalities for some multivariate F-distributions with applications. Technometrics 14, 713-724.

Hollander, M. and Proschan, F. (1972). Testing whether new is better than used. Annals of Math. Stat. 43, 1136-1146.

Jackson, O. A. Y. (1967). An analysis of departures from the exponential distribution. JRSS B 29, 540-549.

Kimber, A. C. (1985). Tests for the exponential, weibull and gumbel distributions based on the stabilized probability plot. Biometrika 72, 661-663.

Koul, H. L. (1978). Testing for new is better than used in expectation. Comm. in Stat. A7, 685-701.

Lee, E. T. (1980). Statistical Methods for Survival Data Analysis. Lifetime Learning Publications.

Lee, S. C. S., Locke, C., and Spurrier, J. D. (1980). On a class of tests of exponentiality. Technometrics 22, 547-554.

Lilliefors, H. W. (1969). On the Kolmogorov-Smirnov test for the exponential distribution with mean unknown. JASA 64, 387-389.

Lin, C. C. and Mudholkar, G. S. (1980). A test of exponentiality based on the bivariate F distribution. Technometrics 22, 79-82.

Margolin, B. H. and Maurer, W. (1976). Tests of the Kolmogorov-Smirnov type for exponential data with unknown scale and related problems. Biometrika 63, 149-160.

Moran, P. A. P. (1951). The random division of an interval (part II). JRSS B 13, 147-150.

Patel, J. K., Kapadia, C. H., and Owen, D. B. (1976). Handbook of Statistical Distributions. Marcel Dekker.

Pettitt, A. N. (1977). Tests for the exponential distribution with censored data using Cramer-von Mises statistics. Biometrika 64, 629-632.

Proschan, F. and Pyke, R. (1967). Tests for monotone failure rate. Proceedings of the Fifth Berkeley Symposium, 293-312.

Shapiro, S. S. and Wilk, M. B. (1972). An analysis of variance test for the exponential distribution (complete samples). Technometrics 14, 355-370.

Sirvanci, M. and Levent, I. (1982). Cramer-von Mises statistic for testing exponentiality with censored samples. Biometrika 69, 641-646.

Spinelli, J. J. and Stephens, M. A. (1987). Tests for exponentiality when origin and scale parameters are unknown. Technometrics 29, 471-476.

Spurrier, J.D. (1984). An overview of tests for exponentiality. Communications in Statistics - Theory and Methods 13(13), 1635-1654.

Wong, P. G. and Wong, S. P. (1979). An extremal quotient test for exponential distributions. Metrika 26, 1-4.

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