

A survey on pickup and delivery problems Part II: Transportation between pickup and delivery locations

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Abstract This paper is the second part of a comprehensive survey on routing problems involving pickups and deliveries. Basically, two problem classes can be distinguished. The first part dealt with the transportation of goods from the depot to linehaul customers and from backhaul customers to the depot. The second part now considers all those problems where goods are transported between pickup and delivery locations, denoted as Vehicle Routing Problems with Pickups and Deliveries (VRPPD). These are the Pickup and Delivery Vehicle Routing Problem (PDVRP – unpaired pickup and delivery points), the classical Pickup and Delivery Problem (PDP – paired pickup and delivery points), and the Dial-A-Ride Problem (DARP – passenger transportation between paired pickup and delivery points and user inconvenience taken into consideration). Single as well as multi vehicle mathematical problem formulations for all three VRPPD types are given, and the respective exact, heuristic, and metaheuristic solution methods are discussed.

Keywords Pickup and delivery vehicle routing · Pickup and delivery problem · Dial-a-ride problem · Transportation · Survey

Zusammenfassung Der vorliegende Artikel ist Teil II einer umfassenden Überblicksarbeit in zwei Teilen über *pickup and delivery* Probleme. Grundsätzlich können zwei Problemklassen unterschieden werden. Der erste Teil dieser Arbeit beschäftigte sich mit dem Transport von Gütern von einem Depot zu Auslieferungs-Kunden (*linehaul customers*) und von Rückladungs-Kunden (*backhaul customers*) zu einem Depot. Der zweite Teil behandelt nun all jene Probleme, die sich mit dem

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Transport zwischen Abhol- und Auslieferungsorten befassen, im Folgenden als *Vehicle Routing Problems with Pickups and Deliveries (VRPPD)* bezeichnet. Diese Problemklasse umfasst das *Pickup and Delivery Vehicle Routing Problem (PDVRP)*, ungepaarte Abhol- und Auslieferungsorte, das klassische *Pickup and Delivery Problem (PDP)*, gepaarte Abhol- und Auslieferungsorte, und das *Dial-A-Ride Problem (DARP)*, Personentransport zwischen gepaarten Abhol- und Ablieferungsorten unter Berücksichtigung von serviceorientierten Kriterien. Die jeweiligen Problemtypen werden zuerst anhand von mathematischen Einfahrzeug- und Mehrfahrzeugproblemformulierungen definiert. Im Anschluss werden die in der Literatur beschriebenen Lösungsmethoden diskutiert.

Schlüsselwörter Tourenplanung · Vehicle Routing mit Pickups und Deliveries · Dial-a-Ride Problem · Überblicksartikel

1 Basic definitions

The aim of this paper is to present a classification scheme as well as a comprehensive survey on pickup and delivery problems and their variants. Part I of this survey presented all problem types belonging to the class of Vehicle Routing Problems with Backhauls (VRPB), where goods are transported from a depot to linehaul customers and from backhaul customers to a depot. Furthermore, the motivation for this survey was given. Part II covers all problem types where goods are transported between pickup and delivery locations, referred to as Vehicle Routing Problems with Pickups and Deliveries (VRPPD).

The two pickup and delivery problem classes as well as their subclasses are depicted in Fig. 1. The gray part was discussed in part I of this survey. The numbers indicated in the boxes specify the sections covering the respective problems. The first two indicators refer to the modeling part while the last refer to the sections on solution methods.

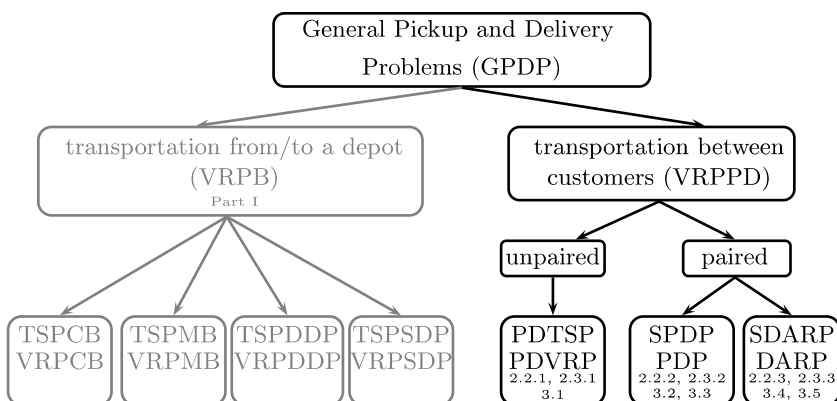


Fig. 1 Pickup and delivery problems. The numbers indicated refer to the sections covering the respective problems

1.1 VRPPD subclass definitions

The class we denote VRPPD refers to problems where goods are transported from pickup to delivery points. It can be further divided into two subclasses. The first subclass refers to situations where pickup and delivery locations are unpaired. A homogeneous good is considered. Each unit picked up can be used to fulfill the demand of any delivery customer. In the literature mostly the single vehicle case is tackled, denoted as capacitated traveling salesman problem with pickups and deliveries (Anily and Bramel 1999), One-Commodity Pickup and Delivery Traveling Salesman Problem (1-PDTSP) (Hernández-Pérez and Salazar-González 2003), and traveling salesman problem with pickup and delivery (Hernández-Pérez and Salazar-González 2004a). Since also a multi vehicle application has been reported in the literature (see Dror et al. 1998) we will denote this problem class as Pickup and Delivery Vehicle Routing Problem (PDVRP) and Pickup and Delivery Traveling Salesman Problem (PDTSP), in the multi and in the single vehicle case, respectively.

The second VRPPD subclass comprises the classical Pickup and Delivery Problem (PDP) and the Dial-A-Ride Problem (DARP). Both types consider transportation requests, each associated with an origin and a destination, resulting in paired pickup and delivery points. The PDP deals with the transportation of goods while the DARP deals with passenger transportation. This difference is usually expressed in terms of additional constraints or objectives that explicitly take user (in)convenience into account. The single vehicle variant of the PDP has also been referred to as pickup-delivery traveling salesman problem (Kalantari et al. 1985) and the multi vehicle case as vehicle routing problem with pickup and delivery (Derigs and Döhmer 2008). However, a majority of the work published denotes this problem class as Pickup and Delivery Problem (PDP) (see e. g. Dumas et al. 1991; van der Bruggen et al. 1993). We will follow this naming. Dial-a-ride problems are also mostly referred to as such. However, some authors, such as Toth and Vigo (1996), denote the same problem as the handicapped persons transportation problem. The dynamic version is also referred to as demand responsive transport (compare e. g. Mageean and Nelson 2003). We denote the single vehicle case of the PDP as SPDP, the single vehicle case of the DARP as SDARP.

1.2 Limitations

In the field of transportation two problem classes can be distinguished: full-truck-load problems and less-than-truck-load problems. Full-truck-load problems deal with vehicles of unit capacity as well as unit demand or supply at every customer location. In case of paired pickup and delivery locations, no other customer can be visited between a pickup and its associated delivery location (see Archetti and Speranza 2004; Coslovich et al. 2006a; Currie and Salhi 2003,2004; de Meulemeester et al. 1997; Doerner et al. 2001; Fleischmann et al. 2004; Gronalt et al. 2003; Gronalt and Hirsch 2007; Imai et al. 2007; Jordan and Burns 1984; Jordan 1987; Powell et al. 1995,2000a,2000b; Regan et al. 1996a,1996b,1998; Russell and Challinor 1988; Wang and Regan 2002; Yang et al. 1998;2004; etc.).

Furthermore, there exists a problem class that is by definition not part of the full-truck-load class, since vehicles with capacity two are considered. However, these problems are strongly related to the full-truck-load category since they originate from the same application area (container dispatching). The aim of this paper is to present a concise classification scheme and survey limited to vehicle routing problems involving pickup and delivery operations where the routing and not the scheduling aspect is of predominant importance. Therefore, full-truck-load dispatching approaches as well as approaches strongly related to this class (see e. g. Archetti et al. 2005; le Blanc et al. 2006) will not be covered in this article. Also literature on sequencing problems, e. g. arising at intermodal stations, involving gantry crane pickup and delivery operations (compare e. g. Böse et al. 1999,2000; Gutenschwager et al. 2003) will not be part of this survey.

1.3 Structure of the survey

The remainder of this paper is organized as follows. First, in order to clearly define the different VRPPD types, consistent mathematical problem formulations for the single and for the multi vehicle case are given. Then, solution methods are discussed in subsections devoted to the corresponding problems. In each of these subsections exact, heuristic, and metaheuristic solution methods are presented. Also related work will be mentioned. Section 4 gives an overview of existing benchmark instances for the different problem classes as well as some information regarding best known solutions. Concluding remarks and possible directions for future research are provided at the very end of this survey.

2 Mathematical problem formulation

In the following section consistent mathematical problem formulations will be presented. First, the notation used throughout the survey is given. After that two basic problem formulations are introduced, one for the single and one for the multi vehicle case. These are adjusted to the unpaired pickup and delivery problem, the classical pickup and delivery problem, and the dial-a-ride problem, respectively.

Note that the mathematical models are only given for definition purposes, regardless the strength of their LP relaxations or whether they were used in the context of additive bounding, branch and cut algorithms, or Lagrange relaxations. We refer the interested reader to the work summarized in the sections on exact solution procedures for additional information concerning these aspects. All problems defined are NP-hard as they generalize the well-known Traveling Salesman Problem (TSP) (Garey and Johnson 1979).

2.1 Notation

n ... number of pickup vertices

\tilde{n} ... number of delivery vertices; in case of paired pickups and deliveries $n = \tilde{n}$

- P ... set of backhauls or pickup vertices, $P = \{1, \dots, n\}$
- D ... set of linehauls or delivery vertices, $D = \{n + 1, \dots, n + \tilde{n}\}$
- K ... set of vehicles
- q_i ... demand/supply at vertex i ; pickup vertices are associated with a positive value, delivery vertices with a negative value; at the start depot 0 and the end depot $n + \tilde{n} + 1$ the demand/supply is zero, $q_0 = q_{n+\tilde{n}+1} = 0$
- e_i ... earliest time to begin service at vertex i
- l_i ... latest time to begin service at vertex i
- d_i ... service duration at vertex i
- L_i ... maximum ride time of user i
- c_{ij}^k ... cost to traverse arc or edge (i, j) with vehicle k
- t_{ij}^k ... travel time from vertex i to vertex j with vehicle k
- C^k ... capacity of vehicle k
- T^k ... maximum route duration of vehicle/route k

Note that this notation is valid for the symmetric and for the asymmetric case. In the symmetric case $t_{ij}^k = t_{ji}^k$ and $c_{ij}^k = c_{ji}^k$, arc (i, j) and arc (j, i) could thus be modeled by one edge. Consequently, fewer variables would be needed. However, since we focus on problem definition and not on computational efficiency, we refrain from presenting these variants here. VRPPD are modeled on complete graphs $G = (V, A)$ where V is the set of all vertices $V = \{0, n + \tilde{n} + 1\} \cup P \cup D$, and A the set of all arcs. For practical reasons the arc set can be reduced to $A = \{(i, j) : i, j \in V, i \neq n + \tilde{n} + 1, j \neq 0, i \neq j\}$.

During the optimization process some or all of the following decision variables are determined, depending on the problem considered.

- $x_{ij}^k \dots = \begin{cases} 1, & \text{if arc } (i, j) \text{ is traversed by vehicle } k \\ 0, & \text{else} \end{cases}$
- Q_i^k ... load of vehicle k when leaving vertex i
- B_i^k ... beginning of service of vehicle k at vertex i

Note that vehicle dependent start as well as end vertices can easily be introduced into the model. However, for the sake of simplicity we will not consider this extension in our formulation.

In the single vehicle problem formulation the superscript k can be omitted, resulting in the parameter coefficients c_{ij} , t_{ij} , C , T and the decision variables x_{ij} , Q_i , B_i .

2.2 Single vehicle pickup and delivery problem formulations

The single vehicle model for the different VRPPD is based on an open TSP formulation. For any subset $S \subseteq V$, let $A(S, \bar{S}) = \{(i, j) \in A : i \in S, j \notin S\}$. Then, the open TSP formulation is

$$\min \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{i:(i,j) \in A} x_{ij} = 1 \quad \forall j \in V \setminus \{0\}, \quad (2)$$

$$\sum_{j:(i,j) \in A} x_{ij} = 1 \quad \forall i \in V \setminus \{n + \tilde{n} + 1\}, \quad (3)$$

$$\sum_{(i,j) \in A(S,\bar{S})} x_{ij} \geq 1 \quad \forall S \subseteq V \setminus \{n + \tilde{n} + 1\}, S \neq \emptyset, \quad (4)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A. \quad (5)$$

The objective function (1) minimizes total routing cost. Equalities (2) and (3) ensure that each vertex is visited exactly once. Constraints (4) present one of several possibilities to ensure route-connectivity. For other options we refer to part I of this survey.

2.2.1 PDTSP

Here it is assumed that every unit picked up can be used to satisfy every delivery customer's demand. Similar to the TSPMB, described in part I of this survey, vehicle loading constraints have to be amended to formulation (1)–(5),

$$x_{ij} = 1 \Rightarrow Q_j = Q_i + q_j \quad \forall (i, j) \in A, \quad (6a)$$

$$\max\{0, q_i\} \leq Q_i \leq \min\{C, C + q_i\} \quad \forall i \in V. \quad (6b)$$

In difference to the TSPMB the initial load of the vehicle is free here (see Hernández-Pérez and Salazar-González 2004a). Furthermore, the PDTSP (PDVRP) is the only VRPPD class where equality is needed in (6a), for all other problem types a “ \geq ” sign would suffice.

2.2.2 SPDP

The SPDP considers situations where pickup and delivery vertices are paired, i. e. $n = \tilde{n}$. In the literature it is common to refer to such a vertex pair as a *request*, indexed by $i = 1, \dots, n$ with i being the origin or pickup point and $n + i$ the corresponding destination or delivery point. To ensure that every destination is only visited after its origin, in addition to (1)–(3), (5), and (6), precedence constraints are needed which are usually modeled via time variables,

$$B_i \leq B_{n+i} \quad \forall i \in P, \quad (7)$$

$$x_{ij} = 1 \Rightarrow B_j \geq B_i + d_i + t_{ij} \quad \forall (i, j) \in A. \quad (8)$$

Constraints (7) state that every origin is to be visited before its destination and (8) ensure that time variables are consistent with travel and service times. Note that (8) also guarantee that short cycles are avoided and therefore inequalities (4) are not needed, given that $(t_{ij} + d_i) > 0$ for all $(i, j) \in A$.

2.2.3 SDARP

The SDARP deals with the transportation of people. Problems of this kind arise, e. g., in connection with the transportation of handicapped or elderly persons. Another possible application is, however, the transportation of perishable goods, that also requires maximum ride time limits. In addition to the basic model, given in (1)–(3) and (5)–(8), user inconvenience must be considered. This can be handled by another term in the objective function or by additional constraints. Here we choose to formulate a maximum user ride time limit,

$$B_{n+i} - (B_i + d_i) \leq L_i \quad \forall i \in P. \tag{9}$$

2.2.4 Time window constraints

Constraints referring to the compliance with Time Windows (TW),

$$e_i \leq B_i \leq l_i \quad \forall i \in V, \tag{10}$$

can be added to all of the above formulations. In the context of passenger transportation TW are usually present.

Note that the above formulations are not linear due to constraints (6a) and (8). These can be linearized using a big M formulation (cf. Cordeau 2006).

2.3 Multi vehicle pickup and delivery problem formulations

The basic model for multi vehicle pickup and delivery problems is an adapted three index vehicle routing problem formulation of the one proposed in Cordeau et al. (2002, p. 158f.) for the VRPTW.

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{11}$$

subject to:

$$\sum_{k \in K} \sum_{j:(i,j) \in A} x_{ij}^k = 1 \quad \forall i \in P \cup D, \tag{12}$$

$$\sum_{j:(0,j) \in A} x_{0j}^k = 1 \quad \forall k \in K, \tag{13}$$

$$\sum_{i:(i,n+\bar{n}+1) \in A} x_{i,n+\bar{n}+1}^k = 1 \quad \forall k \in K, \tag{14}$$

$$\sum_{i:(i,j) \in A} x_{ij}^k - \sum_{i:(j,i) \in A} x_{ji}^k = 0 \quad \forall j \in P \cup D, k \in K, \tag{15}$$

$$x_{ij}^k = 1 \Rightarrow B_j^k \geq B_i^k + d_i + t_{ij}^k \quad \forall (i, j) \in A, k \in K, \tag{16}$$

$$x_{ij}^k = 1 \Rightarrow Q_j^k = Q_i^k + q_j \quad \forall (i, j) \in A, k \in K, \tag{17}$$

$$\max \{0, q_i\} \leq Q_i^k \leq \min \{C^k, C^k + q_i\} \quad \forall i \in V, k \in K, \tag{18}$$

$$x_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in A, k \in K. \tag{19}$$

The objective function (11) minimizes total routing cost. Constraints (12) state that every vertex has to be served exactly once. Equalities (13) and (14) guarantee that every vehicle starts at the depot and returns to the depot at the end of its route. Note that this does not mean that every vehicle has to be used. A vehicle may only use the arc $(0, n + \tilde{n} + 1)$, i. e. it does not leave the depot. Flow conservation is ensured by (15). Time variables are used to eliminate subtours in (16), given that $(t_{ij}^k + d_i) > 0$ for all $(i, j) \in A$. Constraints (17) and (18) guarantee that a vehicle's capacity is not exceeded throughout its tour. It should be noted that this formulation requires the introduction of additional decision variables, Q_i^k , corresponding to the total load of vehicle k at vertex i . These variables are not needed to formulate the basic VRP but are essential for its extension to a pickup and delivery problem. In VRP formulations inequalities similar to (17) are sometimes used to ensure route connectivity (accomplished by (16) in the above formulation). Furthermore, in (17) equality is only needed for the PDVRP (PDTSP). For all other VRPPD types it can be replaced by " \geq ".

Non-linear constraints, given in (16) and (17), can be linearized using a big M formulation (cf. Cordeau 2006).

2.3.1 PDVRP

The special characteristic of the PDVRP is that every unit picked up can be used to satisfy every customer's demand. The above formulation (11)–(19) already is an appropriate formulation for the PDVRP. No additional adaptations are needed.

2.3.2 PDP

Here every pickup point is associated with a delivery point and therefore $n = \tilde{n}$. In addition to (11)–(19), two more sets of constraints are needed. First, both origin and destination of a request must be served by the same vehicle,

$$\sum_{j:(i,j) \in A} x_{ij}^k - \sum_{j:(n+i,j) \in A} x_{n+i,j}^k = 0 \quad \forall i \in P, k \in K. \quad (20)$$

Furthermore, delivery can only occur after pickup, i. e.

$$B_i^k \leq B_{n+i}^k \quad \forall i \in P, k \in K. \quad (21)$$

2.3.3 DARP

To extend the multi vehicle PDP to the multi vehicle DARP, again, as in the single vehicle case, constraints related to user inconvenience need to be added. As for the single vehicle version, we will restrict this requirement to adding maximum user ride time constraints to formulation (11)–(21):

$$B_{n+i}^k - (B_i^k + d_i) \leq L_i \quad \forall i \in P, k \in K. \quad (22)$$

2.3.4 Additional constraints

Two more sets of constraints can be added to all of the above problem classes. These are time window and maximum route duration restrictions,

$$e_i \leq B_i^k \leq l_i \quad \forall i \in V, k \in K, \quad (23)$$

$$B_{n+\bar{n}+1}^k - B_0^k \leq T^k \quad \forall k \in K. \quad (24)$$

3 Solution methods for VRPPD

In the following section an overview of the different solution methods for the PDVRP, the PDP, and the DARP are presented. Solution methods classify into exact, heuristic, and metaheuristic approaches. A description of the benchmark instances used is given in Section 4. For references to general information on the solution concepts applied, we refer to the first part of this paper. Another survey on different solution methods can be found in Berbeglia et al. (2007). Because of maximum paper length restrictions, it was not possible to describe all the work published in detail. An overview of the proposed methods is given in tabular form. Only contributions we considered especially important, for recency or originality reasons, are described in further detail. These are marked by an asterisk in the different tables. Solution methods are ordered chronologically within the tables, according to single and multi vehicle approaches. Further details on solution algorithms are given in the same order.

3.1 Unpaired pickups and deliveries

The PDVRP, i. e. the problem class where every good can be picked up and transported anywhere, did not receive as much attention in the literature as the other problem classes. Moreover, most of the literature is restricted to the PDTSP. Therefore, with the exception of Dror et al. (1998), all the solution methods presented are only applicable to the one vehicle case. To the authors' knowledge no metaheuristic approach for the PDTSP has been proposed until today.

3.1.1 Exact methods

The only exact method proposed for the problem at hand was introduced in Hernández-Pérez and Salazar-González (2003,2004a). It is a branch and cut algorithm. To speed up the algorithm construction and improvement heuristics are applied. They are used to obtain feasible solutions from the solution of the LP relaxation at the current node. The construction heuristic is an adaptation of the nearest insertion algorithm. It is improved by 2-opt and 3-opt exchanges (Lin 1965). The test instances solved are adaptations of the ones used in Mosheiov (1994) and Gendreau et al. (1999), containing up to 75 customers.

3.1.2 Heuristics

A special case of the PDTSP is considered in Chalasani and Motwani (1999); the number of goods to be picked up is equal to the number of goods to be delivered; the demand (supply) at every delivery (pickup) location is equal to one. This problem is an extension of the swapping problem where the vehicle's capacity is also set to one. Chalasani and Motwani propose an approximation algorithm with a worst case bound of 9.5. They use Christofides' heuristic (Christofides 1975) to construct two traveling salesman tours, one containing only pickup locations, and one containing only delivery locations. These two tours are then combined by means of decomposition and matching. Anily and Bramel (1999) devise a polynomial time iterated tour matching algorithm for the same problem.

Two heuristic methods can be found in Hernández-Pérez and Salazar-González (2004b). The first algorithm is of the construction-improvement type, using a greedy construction procedure, improved by 2-opt and 3-opt exchanges. The second heuristic is based on incomplete optimization; the branch and cut procedure, described in Hernández-Pérez and Salazar-González (2004a), is used on a reduced number of variables associated with promising edges. Random instances with up to 500 customers are solved.

An approximation algorithm on a tree graph is developed in Lim et al. (2005). It is based on a recurrent construction process and has a worst case bound of 2.

The PDTSP on a tree and on a line is also subject to investigation in Wang et al. (2006). They propose an $O(|V|^2 / \min\{C, |V|\})$ algorithm for the line case. The unit capacity as well as the uncapacitated version can be solved in linear time. On a tree an $O(|V|)$ algorithm is devised for the case of unit capacity and an $O(|V|^2)$ algorithm for the uncapacitated case.

Finally, Dror et al. (1998) propose a heuristic algorithm for the application of the multi vehicle PDVRP to the redistribution of self-service cars. It is related to Dijkstra's algorithm (Dijkstra 1959). Also other solution approaches are briefly discussed.

3.2 The static pickup and delivery problem

Solution methods for the classical pickup and delivery problem (PDP), where every transportation request is associated with a pickup and a delivery point, are presented in this section. Lokin (1978) was the first to discuss the incorporation of precedence constraints into the traditional TSP, needed to formulate the PDP. The first attempt to generalize the PDP in unified notation was proposed in Savelsbergh and Sol (1995), covering all possible versions of the PDP, including the dial-a-ride problem. They also provide a brief overview of existing solution methods until 1995. Mitrović-Minić (1998) presents a survey on the PDP with Time Windows (PDPTW). An early survey on vehicle routing problems already including the PDP is given in Desrochers et al. (1988). Cordeau et al. (2004) review demand responsive transport, covering PDP and DARP. A review of recent work focusing on exact approaches is given in Cordeau et al. (2007). Further surveys on solution methods can be found in Assad (1988), Desaulniers et al. (2002), and Desrochers et al. (1988). In the following paragraphs the various solution techniques for the

static PDP are summarized according to exact, heuristic, and metaheuristic approaches.

3.2.1 Exact methods

A number of exact solution procedures have been presented for the static PDP. In Table 1 the different techniques are briefly described. The literature reference, objective(s), and additional constraints considered, are given. Furthermore, either the benchmark instances used, or the size of the largest instance solved to optimality, in terms of number of requests, are provided. Entries marked by an asterisk are described in further detail below.

Ruland and Rodin (1997) present a branch and cut algorithm to solve the SPDP. It includes cutting plane generation in the bounding phase as well as a greedy route construction procedure to generate the first upper bound.

A branch and bound algorithm for the single vehicle PDPTW with Last-In-First-Out (LIFO) loading, i. e. goods have to be delivered in the reverse order they were picked up, is reported in Cordeau et al. (2006). They propose three mathematical

Table 1 Exact methods for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
* Ruland and Rodin (1997)	–	min. RC	–	branch and cut algorithm, greedy route construction	up to 15 req.
* Cordeau et al. (2006)	–	min. RC	TW, LIFO	branch and bound algorithms	up to 51 req.
The multi vehicle case					
* Kalantari et al. (1985)	–	min. RC	–	extension of branch and bound algorithm of Little et al. (1963)	up to 15 req.
Desrosiers and Dumas (1988)	–	min. RC	TW	study of constr. shortest path problem, column generation, heur.	–
* Dumas et al. (1991)	HF, MD	min. RC	TW	column generation, constr. shortest path subproblems	up to 19 req.
Sigurd et al. (2004)	MD	min. RC	TW	column generation	up to 205 req.
Lu and Dessouky (2004)	–	min. RC	soft TW	branch and bound, valid inequalities	up to 25 req.
* Ropke and Cordeau (2006)	–	min. RC	TW	branch and cut and price algorithm	RCL07, partly LL01
* Ropke et al. (2007)	–	min. RC	TW	2 branch and cut algorithms	RCL07

Bench. = Benchmark, Con. = Constraints, constr. = constrained, heur. = heuristic(s), HF = Heterogeneous Fleet, LIFO = Last-In-First-Out, MD = Multi depot, Obj. = Objective(s), RC = Routing Cost, req. = requests, TW = Time Windows; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

problem formulations and various valid inequalities. These are based on existing inequalities for PDP and new problem specific ones.

The first exact solution method applicable to both the single as well as the multi vehicle case dates back to Kalantari et al. (1985). The branch and bound algorithm described is an extension of the one developed by Little et al. (1963).

A column generation approach is elaborated in Dumas et al. (1991). They consider heterogeneous vehicles, time windows as well as multiple depots. The constrained shortest path problems are solved by means of a forward dynamic programming algorithm.

Ropke and Cordeau (2006) introduce a branch and pricing algorithm with additional valid inequalities. The elementary constrained shortest path problem with time windows, capacity, and pickup and delivery restrictions is the natural pricing problem of the PDPTW. It is solved by means of a label setting shortest path algorithm. Valid inequalities are added in a branch and cut fashion; their impact on the structure of the pricing problem is discussed.

A branch and cut algorithm departing from two different 2-index PDPTW formulations is studied in Ropke et al. (2007). Formulation one makes use of time variables. In formulation two time related constraints are modelled by means of infeasible path inequalities. The latter formulation proves more efficient when used as a basis for the branch and cut algorithm. New valid inequalities to strengthen the proposed formulations are also discussed.

3.2.2 Heuristics

Heuristics for the static PDP have first been proposed in the 1980s. Table 2 gives an overview of the various heuristic solution techniques developed since then. Information concerning the type of problem considered, objective(s) used, additional constraints, the type of algorithm proposed as well as either the benchmark instances solved or the largest instance considered for testing purposes is given.

Sexton and Choi (1986) use Bender's decomposition procedure to solve the static SPDP approximately. Initial solutions are constructed using a space-time heuristic. The route improvement phase is based on Bender's decomposition. As soft time windows are considered, the objective function takes into account the total operating time as well as time window violation penalties.

A construction-improvement heuristic algorithm for the static SPDP is discussed in van der Bruggen et al. (1993). First, a feasible initial solution is constructed. Then, this solution is improved by exchange procedures maintaining feasibility at all times.

Renaud et al. (2000) also propose a construction-improvement algorithm for the same problem. They use a double insertion construction heuristic improved by deletion and re-insertion (4-opt* (Renaud et al. 1996)).

Renaud et al. (2002) present seven different perturbation heuristics to generate near optimal solutions for the static SPDP. In all seven implementations, first, an initial solution is computed which is improved by a 4-opt** heuristic, an adaptation of the 4-opt* heuristic, such that precedence constraints are respected at all times. Then, a perturbation scheme is applied (instance, algorithmic or solution perturbation), followed by a post-optimization phase. The last two steps are repeated until

Table 2 Heuristics for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
* Sexton and Choi (1986)	–	min. RC, min. TWV	soft TW	heur. algorithm using Bender's decomposition	up to 17 req.
* Van der Bruggen et al. (1993)	–	min. RC	TW	variable depth-search based algorithm	up to 50 req.
* Renaud et al. (2000)	–	min. RC	–	2-phase algorithm. (1) double insertion (2) deletion and re-insertion (4-opt*)	RBO00
* Renaud et al. (2002)	–	min. RC	–	7 perturbation heur. (instance, algorithmic, solution perturbation)	RBO00
The multi vehicle case					
Shang and Cuff (1996)	transf.	min. UC	soft TW	concurrent insertion, mini-clustering algorithm	SC96
Lim et al. (2002)	–	min. NV, min. RC, min. RD	TW	squeaky wheel optimization ¹	LL01
* Xu et al. (2003)	HF, HG	min. RC	mTW, RD, LIFO	column generation based heur.	up to 500 req.
Mitrović-Minić and Laporte (2006)	trans-shipment	min. RC	TW	cheapest insertion, local search improvement	up to 100 req.
* Lu and Dessouky (2006)	–	min. RC	TW	construction heur. based on distance increase, TW slack reduction, visual attractiveness	LL01
Thangiah and Awan (2006)	transf., split	min. UC	TW	(Shang and Cuff 1996) plus improvement heur.	SC96

Bench. = Benchmark, Con. = Constraints, heur. = heuristic(s), HF = Heterogeneous Fleet, HG = Heterogeneous Goods, LIFO = Last-In-First-Out, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), req. = requests, RC = Routing Cost, RD = Route Duration, split = split deliveries, (m)TW = (multiple) Time Windows, TWV = TW Violation, transf. = transfers between vehicles, UC = Unsatisfied Customers; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

¹ i. e. an iterative procedure based on an insertion algorithm. It prioritizes requests that caused difficulties in previous iterations.

a stopping criterion is met. The best results are obtained with the solution perturbation scheme.

Xu et al. (2003) propose a column generation based heuristic algorithm for the multi vehicle case. They consider several additional constraints, such as multiple time windows at pickup and delivery locations, loading restrictions, compatibility of goods and vehicles as well as driver working hours. The column generation master problem can be solved using a commercial LP solver. The resulting subproblems are solved by

means of two heuristics, called merge and two-phase, i. e. merging trips and greedy deletion and insertion of requests.

Lu and Dessouky (2006) elaborate on a construction heuristic. It does not only incorporate distance increase into the evaluation criterion but also time window slack reduction as well as visual attractiveness (referred to as crossing length percentage).

3.2.3 *Metaheuristics*

Table 3 gives an overview of the different metaheuristic solution methods proposed for the static PDP. For each reference the same information as in the previous tables is provided.

A variable neighborhood search for an extension of the SPDP is discussed in Carrabs et al. (2007). The problem tackled includes a further constraint regarding rear loading, i. e. items can only be delivered in a LIFO manner. Eight different construction techniques are used to generate an initial solution. The neighborhoods are defined by couple exchange, block exchange, relocate-block, 2-opt-L (2-opt considering only LIFO-feasible arc exchanges) and multi-relocate operators.

Li and Lim (2001) develop a tabu embedded simulated annealing approach for the PDP. Pickup and delivery pair swap neighborhoods are defined. These are based on a shift, an exchange, and a rearrange operator. The first two define the neighborhoods searched by the metaheuristic, the third is used for postoptimization purposes.

Pankratz (2005b) proposes a grouping genetic algorithm for the PDP. It differs from traditional genetic algorithms in that a group-oriented genetic encoding is used. The encoding used by Pankratz (2005b) corresponds to the cluster of requests forming a route. The routing aspect, not comprised in the encoding, is added while decoding the chromosome.

Ropke and Pisinger (2006a) present an adaptive large neighborhood search algorithm for the PDPTW. Multiple depots as well as the existence of service times can be handled by the approach at hand. In Pisinger and Ropke (2007) and Ropke and Pisinger (2006b) the proposed method is used to solve VRP and VRPB instances by transforming them into rich PDPTW. We refer to part I of this survey for further details on this solution procedure.

A two-stage hybrid algorithm for the static PDPTW is presented in Bent and van Hentenryck (2006). The first phase uses simulated annealing to decrease the number of vehicles needed. The second phase consists of a large neighborhood search algorithm in order to reduce total travel cost.

Derigs and Döhmer (2008) discuss an indirect (evolutionary) local search heuristic for the same problem. In indirect search solutions are encoded such that the problem of securing feasibility is separated from the metaheuristic search process. Here a greedy decoding is used.

3.2.4 *Summary*

Summarizing, the largest static PDP problem instance solved to optimality with a state-of-the-art method comprises 205 requests (Sigurd et al. 2004). However, the size of the largest instance solved is not always a good indicator; tightly constrained

Table 3 Metaheuristics for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
Landrieu et al. (2001)	S	min. RC	TW	(probabilistic) tabu search	up to 40 req.
* Carrabs et al. (2007)	–	min. RC	LIFO	variable neighborhood search	up to 375 req.
The multi vehicle case					
Nanry and Barnes (2000)	S	min. RC, min. TWV, min. LV	TW	reactive tabu search, insertion and swap neighborhoods	NB00
Jung and Haghani (2000)	–	min. RC, min. TWV	TW	genetic algorithm	up to 30 req.
Lau and Liang (2001, 2002)	–	min. RC	TW	tabu search, adapted (Nanry and Barnes 200) neighborhoods	adapted (Solomon, 1987), NB00
* Li and Lim (2001)	S	min. NV, min. RC, min. RD, min. WT	TW	tabu embedded simulated annealing	NB00, LL01
Schönberger et al. (2003)	reject req.	max. profit	TW	hybrid genetic algorithm	adapted NB00
Caricato et al. (2003)	MD	min. mRD	track contention	parallel tabu search	up to 50 req.
Ambrosini et al. (2004)	–	min. RC	LIFO	greedy randomized adaptive search (GRASP)	up to 100 req.
Creput et al. (2004)	–	min. NV, min. RC	TW	genetic algorithm combined with local search	LL01
* Pankratz (2005b)	–	min. NV, min. RC	TW	grouping genetic algorithm	NB00, LL01
* Ropke and Pisinger (2006a)	MD, S	min. RC	TW	adaptive large neighborhood search	LL01+
* Bent and van Hentenreycck (2006)	–	min. NV, min. RC	TW	hybrid algorithm. (1) simulated annealing (2) large neighborhood search	LL01+
* Derigs and Döhmer (2008)	–	min. NV, min. RC	TW	indirect search	LL01

Bench. = Benchmark, Con. = Constraints, LIFO = Last-In-First-Out, LV = Load Violation, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, (m)RD = (maximum) Route Duration, S = Service Time, TW = Time Windows, TWV = TW Violation, WT = Waiting Time; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

problems are easier to solve than less tightly constrained ones. The benchmark data set most often used to assess the performance of (meta)heuristic methods for the static PDPTW is the one described in Li and Lim (2001) (LL01, LL01+, see Table 6). Recent new best results have been presented in Ropke and Pisinger (2006a) and Bent

and van Hentenryck (2006), two metaheuristic solution procedures. The metaheuristic of Li and Lim (2001) still holds the best results for a part of the smaller instances. However, also exact methods advanced; the more tightly constrained part of the LL01 data set has recently been solved by a state-of-the-art branch and cut and price algorithm (Ropke and Cordeau 2006).

3.2.5 Related work

Pickup and delivery problems do not only arise in the context of vehicle routing but also in ocean borne transportation. Christiansen and Nygreen (1998a) propose a combined PDPTW and multi inventory model arising in the context of ship routing. It is solved by means of column generation. Similar solution approaches are presented in Christiansen and Nygreen (1998b) and Christiansen (1999). Column generation is also used in Christiansen and Nygreen (2005), assuming uncertainties in sailing times and considering inventory constraints to be soft. Another exact solution method for the traditional PDPTW in ship routing is reported in Christiansen and Fagerholt (2002). Brønmo et al. (2007) propose a multi-start local search heuristic. A relaxed version of the multi-ship PDPTW, considering soft time windows, is tackled in Fagerholt (2001). Fagerholt and Christiansen (2000a) study a combined ship scheduling and allocation problem. Optimal solutions are computed for several real life cases. The subproblem, a TSPTW with allocation and precedence constraints, of the combined problem is studied by Fagerholt and Christiansen (2000b). For an extensive survey on ship routing problems we refer to Christiansen et al. (2004).

An interesting extension of the PDPTW was proposed by Recker (1995), namely the household activity pattern problem. It involves ridesharing as well as vehicle-switching options. The objective is the minimization of household travel disutility. Recker solved the problem defined by means of a genetic algorithm.

Research dedicated to polyhedral analysis is presented in Dumitrescu (2005) for the SPDP. New valid inequalities are discussed. Combinatorially simple pickup and delivery paths are studied in Lübbecke (2004). These are concatenations of several short patterns. Each pattern consists of visiting at most two requests.

Gambardella and Dorigo (2000) elaborate on another problem related to the PDP, namely the sequential ordering problem. Its objective is to determine a minimum weight Hamiltonian path in a directed graph, with weights on arcs and vertices, respecting precedence constraints between vertices. In contrast to the PDPTW, one vertex can have multiple predecessors. Gambardella and Dorigo (2000) propose an ant colony optimization based approach to solve this problem. Other solution methods involve, e. g. those of Escudero (1988) and Ascheuer et al. (1993).

3.3 Dynamic and stochastic pickup and delivery problems

Although many real world PDP are inherently dynamic, the dynamic version of the PDP has not received as much attention as its static counterpart. The term *dynamic* usually indicates that the routing and scheduling of requests has to be done in real time; new requests come in dynamically during the planning horizon and have to be inserted into existing partial routes. In general, the same objectives as for the static

PDP are applied, i. e. the minimization of the total operating costs. Surveys on dynamic routing can be found in Ghiani et al. (2003) and Psaraftis (1988). In stochastic variants of the PDP, information regarding a certain part of the data (e. g. vehicle travel times) are only available in terms of probability distributions. So far, exact procedures have not been used to solve dynamic or stochastic PDP. In the following the different heuristic and metaheuristic methods proposed for the dynamic and stochastic versions of the PDP are discussed.

3.3.1 Heuristics

Swihart and Papstavrou (1999) solve a stochastic SPDP. They assume that new requests arrive according to a poisson process; pickup and delivery locations are uniformly distributed in the service region; and a single vehicle travels at constant speed in the Euclidean plane. The objective minimized is the expected time the requests remain in the system. They test three routing policies, a sectoring, a nearest neighbor and a stacker crane policy. The stacker crane policy refers to grouping arriving demands into contiguous sets of equal size and serving them according to the first-come-first-serve rule. Lower bounds on the expected time a request remains in the system under light and heavy traffic conditions are computed.

Three *online* algorithms (REPLAN, IGNORE, and SMARTSTART) for a single server PDP are investigated in Ascheuer et al. (2000). The objective considered is the completion time. In the literature the problem at hand is called online dial-a-ride problem. However, the denotation *dial-a-ride problem* is not used in the same way as defined in this survey. The term *online* is used whenever no request is known in advance. By means of *competitive analysis*, i. e. the online (no knowledge about the future) algorithm is compared to its offline (complete knowledge about the future) counterpart (Van Hentenryck and Bent 2006; Jaillet and Stafford 2001), it can be shown that REPLAN and IGNORE are $5/2$ competitive, while SMARTSTART has a competitive ratio of 2. Hauptmeier et al. (2000) discuss the performance of REPLAN and IGNORE under reasonable load. Feuerstein and Stougie (2001) devise another 2-competitive algorithm for the same problem. A probabilistic version is investigated in Coja-Oghlan et al. (2005), the online problem minimizing maximum flow time in Krumke et al. (2005).

The first heuristic for the dynamic multi vehicle PDP is proposed in Savelsbergh and Sol (1998). Their solution methodology called DRIVE (Dynamic Routing of Independent VEHicles) incorporates a branch and price algorithm based on a set partitioning problem formulation. It generates approximate solutions via incomplete optimization. The problem tackled is a ten days real life simulation. Up to 354 active requests are considered at the various re-optimization runs.

Lipmann et al. (2004) study the influence of restricted information on the multi server online PDP, i. e. the destination of a request is only revealed after the object has been picked up. Competitive ratios of two deterministic strategies (REPLAN and SMARTCHOICE) for the time window case are computed in Yi and Tian (2005).

An insertion based heuristic for the dynamic multi vehicle PDP is discussed in Popken (2006). It is combined with different types of order circuitry control in order

to increase the efficient utilization of the vehicle's capacity. Results for test instances with up to 2500 initial orders are reported.

Fabri and Recht (2006) present an adaptation of a heuristic algorithm to the dynamic PDPTW. It was initially designed for the dynamic DARP by Caramia et al. (2002). In contrast to Caramia et al., they allow vehicle waiting times. To enhance the procedure an additional local search phase is introduced. This phase is initiated whenever a new request has been inserted and ends when the next request comes in. Fabri and Recht (2006) report solutions to instances with up to 1000 requests, arriving at an average rate of one request per minute.

Thangiah and Awan (2006) test their algorithm (see Table 2) also in a real-time setting, considering up to 159 requests.

3.3.2 *Metaheuristics*

A population based metaheuristic approach, namely a hybrid genetic algorithm, for the dynamic SPDP with TW is proposed in Jih and Hsu (1999). It is called hybrid, as the genetic algorithm is combined with a dynamic programming algorithm. The data sets used for testing purposes consist of up to 50 requests.

Early research on dynamic multi vehicle PDP is conducted in Shen et al. (1995) and Potvin et al. (1995). Both articles focus on neural networks with learning capabilities to support vehicle dispatchers in real-time. In Potvin et al. (1995) the neural network based learning techniques are compared to a linear programming based method. Real life data sets with 200 and 140 requests are used in Potvin et al. (1995) and Shen et al. (1995), respectively, to assess the performance of their algorithms. Potvin et al. (1995) conclude that the neural network performed better than the linear programming model.

The first neighborhood based metaheuristic solution method for the dynamic multi vehicle PDPTW is presented in Malca and Semet (2004). The neighborhood used is of the request-to-vehicle-assignment type. To accelerate the search an elimination matrix is used. It memorizes the compatibility of two requests. Thus, only promising moves are considered. The proposed procedure is tested on some adapted instances of Li and Lim (2001).

A two-phase solution procedure for the dynamic PDPTW is described in Mitrović-Minić and Laporte (2004). In the first phase an initial solution is constructed via cheapest insertion. This solution is improved by means of a tabu search algorithm. In the second phase different waiting strategies are used to schedule the requests. The waiting strategies tried are referred to as drive first, wait first, dynamic waiting, and advanced dynamic waiting. They differ regarding the vehicle's location when waiting occurs. In the drive first strategy, the vehicle leaves every vertex as early as possible. If it arrives too early at the subsequent stop it waits there until service is viable. When using the wait first strategy, the vehicle leaves every vertex as late as possible with respect to time windows of subsequent vertices. Dynamic waiting refers to a strategy where customers are clustered according to time windows. The vehicle waits as long as possible before moving on to the first customer of the next cluster. Advanced dynamic waiting denotes a strategy where waiting time before visiting the first cluster depends on the latest possible time to begin service at the last cluster, without violat-

ing time windows at intermediate clusters. Mitrović-Minić and Laporte (2004) report solutions to problem instances with a total of up to 1000 requests, all occurring during the service period. The results indicate that the advanced dynamic waiting strategy is the most efficient.

Advanced dynamic waiting is also used by Mitrović-Minić et al. (2004) in a double horizon based heuristic. The routing part is solved by means of a construction heuristic improved by tabu search. Scheduling is conducted according to advanced dynamic waiting. Routes are segmented. The first segment corresponds to the short term horizon, the remainder of the route to the long term horizon. Also the case of several depots is considered. Again instances with a total of up to 1000 requests are solved.

Gutenschwager et al. (2004) compare a steepest descent, a reactive tabu search, and a simulated annealing algorithm. They solve the dynamic PDP on an electric monorail system. The best results are obtained with the tabu search algorithm.

In Pankratz (2005a) a grouping genetic algorithm for the static PDP is embedded in a rolling horizon framework in order to solve the dynamic PDPTW. It is tested on data sets with different degrees of dynamism.

Another tabu search algorithm for the dynamic multi vehicle PDPTW was proposed by Gendreau et al. (2006). It uses an ejection chain neighborhood (Glover 1996), i. e. one request is removed from its current route and inserted into another route, forcing the ejection of a request on this route to a third route, and so on. A lateness criterion is incorporated into the objective function. To speed up the optimization procedure a parallel implementation is conducted. The resulting program is tested on simulations over 7.5 hours with 20 vehicles and 23 requests per hour, and over four hours with ten vehicles and 33 requests per hour.

3.3.3 Summary

To summarize, over the last decades, in the field of dynamic PDP, a number of solution procedures have been developed. However, the proposed algorithms cannot be directly compared since, so far, no standardized simulation environment has been used by more than one group of authors. Benchmark instances are available; e. g. those used in Mitrović-Minić and Laporte (2004) and Mitrović-Minić et al. (2004), see Section 4.

3.4 The static dial-a-ride problem

The dial-a-ride problem class has received considerable attention in the literature. The first publications in this area date back to the late 1960s and early 1970s (cf. Rebibio 1974; Wilson and Weissberg 1967; Wilson et al. 1971; Wilson and Colvin 1977). Surveys on solution methods can be found in Cordeau and Laporte (2003a,2007), Cordeau et al. (2004), and Gendreau and Potvin (1998). Kubo and Kasugai (1990) compare different local search based heuristic methods for the SDARP.

3.4.1 Exact methods

An early exact dynamic programming algorithm for the single vehicle DARP is introduced in Psaraftis (1980). Service quality is taken care of by means of maximal

position shift constraints compared to a first-come-first-serve visiting policy. The largest instance solved comprises nine requests. In Psaraftis (1983b) a modified version of the above algorithm is presented. Instead of backward recursion forward recursion is used. Also time windows are considered.

A forward dynamic programming algorithm for the static SDARP is discussed in Desrosiers et al. (1986). Possible states are reduced by eliminating those that are incompatible with respect to vehicle capacity, precedence, and time window restrictions. User inconvenience in terms of ride times is incorporated into time window construction, resulting in tight time windows at both, origin and destination of the transportation request. The largest instance solved contains 40 transportation requests.

Kikuchi (1984) develops a balanced LP transportation problem for the multi vehicle case, minimizing empty vehicle travel as well as idle times and thus fleet size. In a preprocessing step the service area is divided into zones, the time horizon into time periods. Every request is classified according to an origin and a destination zone as well as a departure and an arrival time period. An example with four zones is presented.

Cordeau (2006) proposes a branch and cut algorithm for the static DARP. The algorithm is based on a 3-index mixed-integer problem formulation. New valid inequalities as well as previously developed ones for the PDP and the VRP are employed. The largest instance solved to optimality comprises 36 requests. Two branch and cut algorithms are described in Ropke et al. (2007). Instead of a 3-index formulation, two more efficient 2-index problem formulations and additional valid inequalities are used. A 96-request instance is the largest instance solved to optimality. Extensions of the 3-index and the two 2-index based branch and cut algorithms (Cordeau 2006; Ropke et al. 2007) to the DARP with heterogeneous fleet and passengers are discussed in Parragh et al. (2008).

3.4.2 Heuristics

Over the last decades, a large number of heuristic algorithms have been proposed for the static DARP. Table 4 gives an overview of the various solution methods reported in chronological order, divided into single and multi vehicle approaches.

A heuristic routing and scheduling algorithm for the SDARP using Bender's decomposition is described in Sexton and Bodin (1985a,b). The scheduling problem can be solved optimally; the routing problem is solved with a heuristic algorithm.

One of the first heuristic solution procedures for static multi vehicle DARP is discussed in Cullen et al. (1981). They develop an interactive algorithm that follows the cluster first route second approach. It is based on a set partitioning formulation solved by means of column generation. The location-allocation subproblem is only solved approximately. However, user related constraints or objectives are not explicitly considered. The same applies to the work of Healy and Moll (1995).

Jaw et al. (1986) propose a sequential insertion procedure. First, customers are ordered by increasing earliest time for pickup. Then, they are inserted according to the cheapest feasible insertion criterion. The notion of active vehicle periods used.

Table 4 Heuristics for the static DARP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
Psaraftis (1983a)	–	min. RC	–	MST heur., local interchanges	up to 50 req.
Psaraftis (1983c)	–	min. RC	–	adapted 2-opt and 3-opt	up to 30 req.
* Sexton and Bodin (1985a, b)	–	min. CI	DDT	routing and scheduling algorithm based on Bender's decomposition	up to 20 req.
Healy and Moll (1995)	–	min. RC	–	2-opt improvement, optimizing/sacrificing phases	up to 100 req.
The multi vehicle case					
Stein (1978a, b)	transfers	min. RC	TW	cluster first route second	–
* Cullen et al. (1981)	–	min. RC	–	cluster first route second, column generation	up to 50 req.
Roy et al. (1985a, b)	HF	min. RC, min. CI	TW	parallel insertion	up to 578 req.
Bodin and Sexton (1986)	–	min. CI	DDT	cluster first route second	up to 85 req.
* Jaw et al. (1986)	–	min. RC, min. CI	TW, RT	sequential feasible insertion algorithm	up to 2617 req.
Alfa (1986)	HF	min. RC	TW, RT	adapted heur. of Jaw et al. (1986)	up to 49 req.
Psaraftis (1986)	–	min. RC, min. CI	TW, RT	comparison of Jaw's heur. and grouping-clustering-routing heur.	–
Desrosiers et al. (1988), Dumas et al. (1989)	MD	min. RC	TW	mini-clustering algorithm, column generation	up to 200 req.
Kikuchi and Rhee (1989)	–	max. NCS	TW	sequential insertion	up to 200
Desrosiers et al. (1991)	HF	min. RC	TW, RD	improved mini-clustering algorithm of Desrosiers et al. (1988)	up to 2411 req.
Potvin and Rousseau (1992)	–	min. RC	TW, RT	constraint directed search (beam search)	up to 90 req.
* Ioachim et al. (1995)	HF, MD, S	min. NV, min. RC	TW	mini-clustering, column generation	up to 2545 req.
* Madsen et al. (1995)	HF, S	min. RC, NV, TWT, DPS	TW, RD, RT	REBUS. insertion based algorithm	up to 300 req.

Table 4 continued

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
Toth and Vigo (1996)	HF	min. RC	TW, RT	parallel insertion, improved by trip insertion, exchange, double insertion, moves	TV96a
* Borndörfer et al. (1997)	HF, MD, S	min. RC	TW, (RT), RD	cluster first route second, set partitioning, branch and bound	up to 1771 req.
Fu (2002a)	HF, S	min. RC, min. CI	TW, RT	parallel insertion, stochastic travel times	up to 2800 req.
* Diana and Dessouky (2004)	–	min. RC, min CI, idle times	TW, RT	parallel regret insertion heur.	up to 1000 req.
Xiang et al. (2006)	HF, S	min. RC	TW, RT, RD, BR	construction-improvement; clustering by TW, ideas of sweep heur.; local search improvement	up to 2000 req.
Wong and Bell (2006)	HF, S	min. RC, min. CI	TW, RT, RD	parallel insertion, improved by trip insertion	up to 150 req.
Wolfler et al. (2007)	–	max. NCS, max. SL	TW	cluster first route second, assignment heur., vertex reinsertions	up to 180 req.

Bench. = Benchmark, BR = BReak Time between two trips, CI = Customer Inconvenience, Constr. = Constraints, DDT = Desired Delivery Time, DPS = Deviation from Promised Service, HF = Heterogeneous Fleet, heur. = heuristic(s), MD = Multi Depot, NCS = Number of Customers Served, NV = Number of Vehicles, Obj. = Objective(s), PoC = Position of Customer, RC = Routing Cost, RD = Route Duration, req. = requests, TW = Time Windows, TWT = Total Waiting Time, RT = Ride Time, S = Service time, SL = Service Level; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

An optimization based mini-clustering algorithm is presented in Ioachim et al. (1995). It uses column generation to obtain mini-clusters and an enhanced initialization procedure to decrease processing times. As in Desrosiers et al. (1988) also the case of multiple depots is considered.

A multi-objective approach is followed in Madsen et al. (1995). They discuss an insertion based algorithm called REBUS. The objectives considered are the total driving time, the number of vehicles, the total waiting time, the deviation from promised service times as well as cost.

A classical cluster first route second algorithm is proposed in Borndörfer et al. (1997). The clustering as well as the routing problem are modeled as set partitioning problems. The clustering problem can be solved optimally while the routing subproblems are solved approximately by a branch and bound algorithm (only a subset of all possible tours is used). Customer satisfaction is taken into account in terms of punctual service. Customer ride times are implicitly considered by means of time windows.

A regret insertion algorithm for the static DARP is developed in Diana and Dessouky (2004). First, all requests are ranked according to ascending pickup times. Some swaps in this order are allowed, giving preference to requests that might be difficult to insert later on, regarding their spatial location. The first m requests are used as seed customers, with m being the number of vehicles. All the remaining requests are inserted following a regret insertion strategy (Potvin and Rousseau 1993). The regret insertion based process is also subject to analysis in a study by Diana (2004); in order to determine why the performance of this heuristic is superior to that of other insertion rules.

3.4.3 Metaheuristics

Also metaheuristic solution methods have been developed for the static DARP. Table 5 provides an overview of the different algorithms proposed. For each reference the same information as in the previous tables is given. Toth and Vigo (1997) describe a local search based metaheuristic, i. e. a tabu thresholding algorithm, for the static multi vehicle DARP. It employs the neighborhoods defined in Toth and Vigo (1996). Initial solutions are constructed using parallel insertion.

Cordeau and Laporte (2003b) develop a tabu search algorithm. Time windows are considered at either origin or destination depending on the type of request (inbound or outbound). The neighborhood used is defined by moving one request to another route. The best possible move serves to generate a new incumbent solution. Reverse moves are declared tabu. However, an aspiration criterion is defined, such that tabu moves that provide a better solution, with respect to all other solutions already constructed by the same move, can constitute a new incumbent solution.

3.4.4 Summary

State-of-the-art exact methods for the static DARP solve some instances with up to 96 requests to optimality (Ropke et al. 2007). However, the same limitation applies as stated in Sect. 3.2.4, i.e. the size of the test instance is not a very meaningful indicator; tightly constrained instances are easier to solve than those with less tight constraints and no standardized data set has been solved by the proposed approaches. In case of heuristic and metaheuristic methods, comparison becomes even harder since a large part of the solution procedures developed are motivated by real world problem situations. They differ regarding problem type (single and multi depot, homogeneous and heterogeneous fleet), constraints, and objective(s). Moreover, even when the same data sets are used different objectives are considered (Cordeau and Laporte 2003a; Jørgensen et al. 2007). Consequently, we can only state that in general heuristic methods run faster whereas metaheuristics usually outperform basic heuristic procedures with respect to solution quality.

3.4.5 Related work

Dealing with the transportation of people, especially handicapped or elderly, research has also been dedicated to the comparison of dial-a-ride systems with other modes

Table 5 Metaheuristics for the static DARP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The multi vehicle case					
Calorni et al. (1996)	–	max. NCS, min. CI	RD	simulated annealing	up to 100 req.
* Toth and Vigo (1997)	HF, MD	min. RC	TW, RT	parallel insertion algorithm, tabu thresholding	TV96a
Baugh et al. (1998)	–	min. NV, min. RC, min. CI	TW	simulated annealing	up to 300 req.
Uchimura et al. (1999)	–	min. RC	RT, RD	genetic algorithm	10 req.
* Cordeau and Laporte (2003b)	S	min. RC	TW, RT, RD	tabu search algorithm	CL03, up to 295 req.
Aldaihani and Dessouky (2003)	mix with FRT	min. RC, min. CI	TW	tabu search	up to 155 req.
Ho and Haughland (2004)	probabilistic	min. RC	TW, RT	tabu search, hybrid GRASP-tabu search	adapted CL03
Melachrinoudis et al. (2007)	HF, MD	min. RC, min CI	TW	tabu search	up to 8 req.
Rekiek et al. (2006)	S, MD, HF	min. NV, min. CI	TW, RT, VA	grouping genetic algorithm	up to 164 req.
Jørgensen et al. (2007)	S	min. RC, min. RT, min. TWV, ...	TW, RT, RD	genetic algorithm, space-time nearest neighbor heur.	CL03
Parragh et al. (2007)	S	min. RC, min. mRT	TW, RT	variable neighborhood search, path relinking	Cor06+

Bench. = Benchmark, CI = Customer Inconvenience, Con. = Constraints, DDT = Desired Delivery Time, FRT = Fixed Route Transit, HF = Heterogeneous Fleet, MD = Multi Depot, mRT = mean RT, NV = Number of Vehicles, Obj. = Objective(s), RD = Route Duration, RT = Ride Time, RC = Routing Cost, S = Service time, TW = Time Windows, TWV = TW Violation, VA = Vehicle Availability; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

of transportation. Early studies of dial-a-ride transportation systems are reported in Carlson (1976) and Teixeira and Karash (1975). ElMBERG (1978) tests a robot dispatcher dial-a-ride system in Sweden. Daganzo (1984) compares fixed route transit systems with checkpoint dial-a-ride and door-to-door dial-a-ride systems. He concludes that most of the time either a fixed route system or door-to-door transportation is the appropriate choice. Belisle et al. (1986) investigate the impact of different operating scenarios on the quality of transportation systems for the handicapped. More recent studies comparing dial-a-ride and traditional bus systems by means of simulation are reported in Noda et al. (2003) and Noda (2005). Shinoda et al. (2003) study the usability of dial-a-ride systems in urban areas. Mageean and Nelson (2003) eval-

uate telematics based demand responsive transport services in Europe. Palmer et al. (2004) analyze the impact of management practices and advanced technologies in the context of demand responsive transport systems. The impact of information flows, e. g. the percentage of real time requests or the length of the interval between the arrival of a new request and its requested pickup TW, is investigated in Diana (2006). Diana et al. (2006) study optimal fleet sizes with respect to predetermined service quality.

Research has also been dedicated to possible ways of computation time reduction. Hunsaker and Savelsbergh (2002), e. g., propose a fast feasibility check for the DARP. The proposed procedure can deal with time window, waiting time as well as ride time restrictions. Castelli et al. (2002) discuss three algorithms granting 2-opt-improvement feasibility.

A problem class related to the DARP is the *car pooling* problem. It consists of finding subsets of employees that share a car, the path the driver should follow, and possibly also who should be the driver. In contrast to the DARP either origin or destination are the same for all users depending on whether the trip is from home to the office or back. Two variants can be investigated, either one car pool for both ways or differing to-work and from-work problems. Baldacci et al. (2004) propose an exact as well as a heuristic procedure for this problem. A real life application is reported in Wolfler Calvo et al. (2004). Maniezzo et al. (2004) propose an ant colony optimization algorithm for the long-term problem.

3.5 Dynamic dial-a-ride problems

Less research has been dedicated to the domain of dynamic DARP. Again, the term *dynamic* indicates that routing is done in real time; new requests pop up dynamically during the day and have to be scheduled into existing routes. The different solution techniques developed are depicted in the following paragraphs. Predominantly heuristic methods have been used to solve dynamic versions of the DARP.

3.5.1 Exact methods

Exact methods have not been explicitly developed for the dynamic DARP. A possible reason is that, in the context of dynamic routing, the concept of “optimal solutions” becomes debatable. However, in Psaraftis (1980) the static version of his algorithm is adapted to the dynamic case.

3.5.2 Heuristics

Early heuristic algorithms for the dynamic DARP are discussed in Daganzo (1978). Daganzo analyzes three different insertion algorithms. In the first the closest stop is visited next. The second consists in visiting the closest origin or the closest destination in alternating order. The third only allows the insertion of delivery locations after a fixed number of passengers have been picked up. Algorithm two results in higher

waiting to ride time ratios than algorithm one. However, it will, in general, generate lower waiting times for equal ride times when compared to algorithm three. The dynamic case of the multi vehicle DARP is also studied in Dial (1995). New transportation requests are assigned to clusters according to least cost insertion. Routes are then optimized using dynamic programming. Results for a real life problem instance are reported.

Teodorovic and Radivojevic (2000) propose a two-stage fuzzy logic based heuristic algorithm. The first approximate reasoning heuristic decides which vehicle a new request is assigned to. The second heuristic handles the adjustment of the vehicle's route. Results for instances with up to 900 requests are reported.

Another cluster first route second algorithm is proposed in Colorni and Righini (2001). Considering only the most urgent requests, the routing subproblems can be solved to optimality with a branch and bound algorithm.

Caramia et al. (2002) use a dynamic programming algorithm to iteratively solve the single vehicle subproblems to optimality. Results for instances with up to 50 clients per hour are reported.

Horn (2002a) provide a software environment for fleet scheduling and dispatching of demand responsive services. The system can handle advance as well as immediate requests. New incoming requests are inserted into existing routes according to least cost insertion. A steepest descent improvement phase is run periodically. Also automated vehicle dispatching procedures, to achieve a good combination of efficient vehicle deployment and customer service, are included. The system was tested in the modeling framework LITRES-2 (Horn 2002b), using a 24-hours real life data set of taxi operations with 4282 customer requests. Another simulation environment to test solution methods for the dynamic DARP can be found in Fu (2002b).

Coslovich et al. (2006b) propose a two-phase insertion heuristic. A simple insertion procedure allows for quick answers with respect to inclusion or rejection of a new customer. The initial solution is improved by means of local search using 2-opt arc swaps. Results for instances with a total of up to 50 (unexpected and expected) requests are reported.

Xiang et al. (2008) propose a fast heuristic for the dynamic DARP. They consider travel time fluctuations, absent customers, vehicle breakdowns, cancellation of requests, traffic jams etc. To solve this complex problem situation the heuristic proposed in Xiang et al. (2006) is adapted to the dynamic case. For testing purposes several simulations under varying parameter settings are conducted. Up to 610 requests are considered with different proportions of them already known in advance.

3.5.3 Metaheuristics

Metaheuristic solution methods have not been explicitly developed for the dynamic DARP, since short response times are necessary in dynamic settings. Only one algorithm developed for the static version has been used to solve the dynamic case. The tabu search of Cordeau and Laporte (2003a) is adapted to the dynamic DARP by means of parallelization (Attanasio et al. 2004). Different parallelization strategies are tested on instances with up to 144 requests.

3.5.4 Summary

To summarize, only a limited number of solution algorithms have been proposed for the dynamic DARP. Most of them are based on repeated calls of static solution rou-

Table 6 Benchmark Instances for VRPPD

Literature Ref.	Type	Req.	#	Characteristics	Abbr.
Schang and Cuff (1996)	PDPTW, transfers	159	1	real world data	SC96
Nanry and Barnes (2000)	PDPTW	13-50	43	based on VRPTW instances (Solomon 1987), optimal solution schedules by procedure of Carlton (1995), customers randomly paired	NB00
Renaud et al. (2000)	SPDP	25-249	108	based on 36 TSPLIB instances (Reinelt 1991), for each pickup a delivery chosen among the 5 (10) closest or all unselected neighbors.	RBO00
Li and Lim (2001)	PDPTW	50	56	based on those of Solomon (1987), customers appearing on the same route in a solution of the VRPTW, using the solution procedure of Li et al. (2001), were randomly paired;	LL01
		100-500	298	extended data set	LL01+
Toth and Vigo (1996)	DARP	276-312	5	real life data, Municipality of Bologna	TV96a
Cordeau and Laporte (2003b)	DARP	24-144	20	randomly generated around seed points, half of the requests have a tight TW at the origin, half a tight TW at the destination, 10 instances with narrow, 10 with wider TW.	CL03
Mitrović-Minić and Laporte (2004)	dyn. PDPTW	100-1000	40	ten hours service period, 60 × 60 km ² area, vehicle move at 60 km/h, requests occur according to a continuous uniform distribution, no requests are known in advance	–
Cordeau (2006)	DARP	16-48	24	randomly generated; 12 instances with $C = 3$, unit user demand and $L = 30$; 12 instances with $C = 6$, varying user demand and $L = 60$.	–
		16-96	42	extended data set	Cor06+
Ropke et al. (2007)	PDPTW	30-75	40	randomly generated as described in Savelsberg and Sol (1998).	RCL07

= number of instances, Abbr. = abbreviation used, bh. = backhaul customers, MD = Multi Depot, Req. = approximate number of requests of each instance, SD = Single Depot, TW = Time Window

tines. Comparison across the different methods proposed becomes a difficult task since a majority of the work presented is motivated by real life applications. Consequently, each solution methodology was tested on different data sets with varying problem specific characteristics. Whenever a solution method originally developed for the static case was used we refer to Sect. 3.4 of this survey for further details on its performance in a static setting.

4 Benchmark Instances for VRPPD

Table 6 provides the interested reader with information on available benchmark instances used in the literature. Column one gives the literature reference. Column two through four state the problem type, the size of the different instances in terms of number of requests per instance, and the number of instances provided. Then, a brief description of the data set as well as the abbreviations used in this article are given.

The data set predominantly used to assess the performance of PDPTW is the one proposed by Li and Lim (2001). The latest new best results for both the primary and the extended data sets can be found in Ropke and Pisinger (2006a) and Bent and van Hentenryck (2006).

In contrast to the field of PDP, solution methods developed for the DARP have not been tested on standardized benchmark instances. This might be due to the fact that most methods vary considerably with respect to the constraints considered as well as the objectives minimized. However, since data sets for rather standard problem settings do exist now, this might change in the near future.

5 Conclusion

The solution methods presented in this paper are state-of-the-art in the field of VRPPD. In line with the first part of this survey, we believe that future research will involve the incorporation of additional real life constraints, the effects of dynamism, and knowledge about future events, in terms of probability distributions. Moreover, the DARP is, in contrast to many other routing problems, a natural multi-objective problem. This aspect will also be part of future investigations.

We hope and trust that this survey will lead to future research in the area of vehicle routing involving pickups and deliveries.

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