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A survey on two model equations for compressible viscous fluid

By

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In this paper we shall discuss again the temporally global problem of the two model equations treated in [5] and [6]. The notation to be used here is similar to that in [5] and [6].

§ 1. On the generalized Burgers' equation

For the initial-value problem of the generalized Burgers' equation

$$(1.1)\quad \begin{cases} \dfrac{\partial}{\partial t}v(x,t)=\dfrac{\mu}{\rho(x,t)}\dfrac{\partial^2}{\partial x^2}v(x,t)-v\cdot\dfrac{\partial}{\partial x}v, \\ \dfrac{\partial}{\partial t}\rho(x,t)+\dfrac{\partial}{\partial x}(\rho v)=0, \ (v, \text{ one-dimensional velocity}; \rho, \text{ density (scalar quantity)}; \\ x\in R^1, \text{ spatial variable}; t, \text{ temporal variable } (\geqq 0); \mu \text{ positive constant}), \end{cases}$$

$$(1.2)\quad \begin{cases} v(x,0)=v_0(x)\in H^{2+\alpha}, \\ \rho(x,0)=\rho_0(x)\in H^{1+\alpha}, \ (0<\underline{\rho}_0\equiv\inf\rho_0\leqq\rho_0\leqq\bar{\rho}_0\equiv|\rho_0|^{(0)}<+\infty), \end{cases}$$

we have already obtained ([4], [5], [6]):

Theorem 1. *For some* $T\in(0,+\infty)$, *there exists a unique solution* (v,ρ) *of* (1.1)–(1.2) *belonging to* $H_T^{2+\alpha}\times B_T^{1+\alpha}$.

Moreover, we have a result that there is a unique temporally global solution (v,ρ) of (1.1)–(1.2) belonging to $H_T^{2+\alpha}\times B_T^{1+\alpha}$ for an arbitrary $T\in(0,+\infty)$ under the condition that v_0 is to be represented as

$$(1.3)\quad v_0=v_{01}+v_{02}(v_{01}, v_{02}\in H^{2+\alpha}; v_{01}'\leqq 0, v_{02}\in L_1(R^1)).$$

We shall show here the existence of a unique temporally global solution of (1.1)–(1.2) such as mentioned above, without any additional conditions. For this purpose it is required to have a priori estimates for (v, ρ) in $H_T^{2+\alpha}\times B_T^{1+\alpha}$.

Now, if (v, ρ) is a solution of (1.1)–(1.2) belonging to $H_T^{2+\alpha}\times B_T^{1+\alpha}$ for $T\in(0, +\infty)$, then

$$\rho(x, t)=\rho_0(x_0(x, t))\exp\left\{-\int_0^t v_x(\bar{x}(\tau; x, t), \tau)d\tau\right\}, \tag{1.4}$$

where $\bar{x}(\tau; x, t)$ satisfies

$$\begin{cases}\dfrac{d}{d\tau}\bar{x}(\tau; x, t)=v(\bar{x}(\tau; x, t), \tau),\\ \bar{x}(t; x, t)=x(\tau\in[0, T])\end{cases} \tag{1.5}$$

and

$$x_0(x, t)=\bar{x}(0; x, t). \tag{1.6}$$

The transformation $(x_0=x_0(x, t), t_0=t)$ from $R^1\times[0, T]$ into itself is obviously one-to-one and onto. We call (x_0, t_0) the (v-)charcteristic co-ordinates ([4], [7]). We define by use of the above co-ordinates

$$\hat{v}(x_0, t_0)=v(\bar{x}(t_0; x_0, 0), t=t_0), \qquad \text{etc.} \tag{1.7}$$

Then, (1.1)–(1.2) is transformed into

$$\begin{cases}\hat{v}_{t_0}(x_0, t_0)=\dfrac{\mu}{\rho_0(x_0)}\left(\dfrac{\hat{v}_{x_0}}{1+\omega(x_0, t_0)}\right)_{x_0},\\ \rho(x_0, t_0)=\rho_0(x_0)\dfrac{1}{1+\omega}, \quad \left(\omega=\displaystyle\int_0^{t_0}\hat{v}_{x_0}(x_0, t_0')dt_0'\right).\end{cases} \tag{1.8}$$

$$\hat{v}(0, x_0)=v_0(x_0)\in H^{2+\alpha}, \tag{1.9}$$

where the suffixes t_0 etc. denote differentiation in t_0 etc., respectively. Here, we note that

$$\begin{cases}e^{-t|v_x|_t^{(0)}}\leqq\dfrac{\partial x_0}{\partial x}=\dfrac{1}{1+\omega}=\exp\left\{-\displaystyle\int_0^t v_x(\bar{x}(\tau; x, t), \tau)d\tau\right\},\\ \qquad\leqq e^{t|v_x|_t^{(0)}}, \quad \hat{v}\in H_T^{2+\alpha}, \dfrac{1}{1+\omega}\in B_T^{1+\alpha}\ (\text{thus, } \in H_T^{1+\alpha}).\end{cases} \tag{1.10}$$

It is already known ([4], [5]) that, in order to have a priori estimates for (v, ρ) in $H_T^{2+\alpha}\times B_T^{1+\alpha}$, it suffices to have those for $|v|_T^{(0)}$, $|\rho|_T^{(0)}$, $|\rho^{-1}|_T^{(0)}$. It is obvious from (1.1)–(1.2) that

$$|v|_T^{(0)}\leqq|v_0|^{(0)}. \tag{1.11}$$

Thus, it remains to have a priori estimates for $|\rho|_T^{(0)}$ and $|\rho^{-1}|_T^{(0)}$, accordingly those for $\left|\dfrac{1}{1+\omega}\right|_T^{(0)}$ and $|1+\omega|_T^{(0)}$ (cf. (1.8)). Now, we put

(1.12)
$$\begin{cases} Y^a(x_0, t_0) \equiv \displaystyle\int_a^{x_0} \frac{\rho_0(x_0')}{\mu}\{v_0(x_0') - \hat{v}(x_0', t_0)\}dx_0' \\ -\log(1+\omega(a, t_0)). \end{cases}$$

Then, by (1.8), Y^a satisfies

(1.13)
$$\begin{cases} (Y^a)_{t_0} = -\dfrac{\hat{v}_{x_0}}{1+\omega} = \dfrac{\mu}{1+\omega}\left(\dfrac{(Y^a)_{x_0}}{\rho_0}\right)_{x_0} - \dfrac{v_0'}{1+\omega}, \\ Y^a(x_0, 0) = 0, \quad \left((Y^a)_{x_0} = \dfrac{\rho_0}{\mu}(v_0 - \hat{v})\right). \end{cases}$$

Obviously, Y^a satisfies Täcklind's condition. Therefore, for any a and $a' \in R^1$, $Y^a = Y^{a'}$. Thus, we define

(1.14)
$$Y \equiv Y^a = Y^{a'}.$$

We remark that

(1.15)
$$Y(x_0, t_0) = Y(a, t_0) + \int_a^{x_0} \frac{\rho_0}{\mu}(v_0 - \hat{v})dx_0'$$

and that, by (1.13),

(1.16)
$$Y_{t_0} = (-\log(1+\omega))_{t_0}.$$

Thus, we have

(1.17)
$$\frac{1}{1+\omega} = e^Y.$$

From (1.10) and (1.17) follows that

(1.18)
$$Y(x_0, t_0) = -\int_0^{t_0} v_x(\bar{x}(\tau; x, t), \tau)d\tau \Big|_{\{x = x(x_0, t_0) \atop t = t_0\}}.$$

From above, we have

(1.19)
$$|Y(\cdot, t_0)|^{(0)} \leqq |Y|_T^{(0)} \leqq |v_x|_T^{(0)} \cdot T, \qquad (t_0 \in [0, T]).$$

By (1.17), it suffices for us to have a priori estimates for $|e^Y|_T^{(0)}$ and $|e^{-Y}|_T^{(0)}$ (or $|Y|_T^{(0)}$). By (1.15), it holds that, for an arbitrary number $a \in R^1$,

(1.20)
$$\begin{cases} 1 + \omega(x_0, t_0) = e^{-Y(x_0, t_0)} = e^{-Y(a, t_0)} \\ \qquad\qquad \times \exp\left\{-\displaystyle\int_a^{x_0} \frac{\rho_0}{\mu}(v_0 - \hat{v})dx_0'\right\}. \end{cases}$$

Therefore,

$$(1.21)\quad \begin{aligned}\int_a^{a+l}(1+\omega)dx_0&=l+\int_0^{t_0}\{\hat{v}(a+l,\tau)-\hat{v}(a,\tau)\}d\tau\\&=e^{-Y(a,t_0)}\times\int_a^{a+l}\exp\left\{-\int_a^{x_0}\frac{\rho_0}{\mu}(v_0-\hat{v})dx_0'\right\}dx_0,\qquad (1>0).\end{aligned}$$

First, take $l=1$. Then, we have an inequality

$$(1.22)\qquad 1+2|v_0|^{(0)}\cdot T\geqq e^{-Y(a,t_0)}\times\exp\left\{-\frac{2\rho_0}{\mu}|v_0|^{(0)}\right\}.$$

Hence, it holds that

$$(1.23)\qquad e^{-Y(a,t_0)}\leqq(1+2|v_0|^{(0)}\cdot T)\exp\left\{\frac{2\rho_0}{\mu}|v_0|^{(0)}\right\}.$$

Next, take $l=1+2|v_0|^{(0)}\cdot T$. Then, we have

$$(1.24)\qquad 1\leqq\int_a^{a+1+2|v_0|T}(1+\omega)dx_0.$$

Thus, it holds that

$$(1.25)\qquad e^{Y(a,t_0)}\leqq(1+2|v_0|^{(0)}\cdot T)\exp\left\{(1+2|v_0|^{(0)}\cdot T)\frac{2\mu}{\rho_0}|v_0|^{(0)}\right\}.$$

By (1.23) and (1.25), we have a priori estimates for $|\rho|_T^{(0)}$ and $|\rho^{-1}|_T^{(0)}$. From the discussion made above follows:

Theorem 2. *There exists a unique temporally global solution* (v,ρ) *of* (1.1)–(1.2) *such that it belongs to* $H_T^{2+\alpha}\times B_T^{1+\alpha}$ *for any* $T\in(0,+\infty)$.

§ 2. On the generalized Burgers' equation with a pressure model term

Here, we shall discuss the Gauchy problem of the following system of differential equations

$$(2.1)\qquad \begin{cases}v_t(x,t)=\dfrac{\mu}{\rho(x,t)}v_{xx}(x,t)-v\cdot v_x-K\dfrac{\rho_x}{\rho},\\ \rho_t(x,t)+(\rho v)_x=0,\qquad (K,\ \text{positive constant};\ x\in R^1,\ t\geqq 0),\end{cases}$$

$$(2.2)\qquad \begin{cases}v(x,0)=v_0(x)\in H^{2+\alpha},\quad \rho(x,0)=\rho_0(x)\in H^{1+\alpha}\ (0<\underline{\rho}_0\leqq\rho_0\leqq\bar{\rho}_0<+\infty,\\ \qquad \text{cf. (1.2)}),\ \rho_0'\in L_0/R^1).\end{cases}$$

In order to show that there exists a unique temporally global solution of (2.1)–(2.2) such as discussed in § 1, it suffices to obtain a priori estimates for $|v|_T^{(0)}$, $|\rho|_T^{(0)}$, and $|\rho^{-1}|_T^{(0)}$, where (v,ρ) is assumed to be a solution of (2.1)–(2.2) belonging to $H_T^{2+\alpha}\times B_T^{1+\alpha}$ for $T\in(0,+\infty)$. This is based on reasons analogous to those in § 1 and on the fact that $-\rho^{-1}\cdot\rho_x$ is to be expressed in the following way (cf. [6], [7])

(2.3)
$$\left\{\begin{aligned}&-\frac{\rho_x}{\rho}=\left(\frac{\rho_0'}{\rho_0}\right)(x_0(x,t))\frac{\partial x_0}{\partial x}-\rho\Big[v-v_0(x_0(x,t))\\&\qquad\times\exp\left\{-k\rho\int_0^t\bar{x}_x(\tau;x,t)^{-1}d\tau\right\}-k\int_0^t\exp\left\{-k\rho(x,t)\right.\\&\qquad\left.\times\int_\tau^t\bar{x}_x(\tau';x,t)^{-1}d\tau'\right\}\times\Big\{\rho(x,t)v(\bar{x}(\tau;x,t),\tau)\\&\qquad\times\bar{x}_x(\tau;x,t)^{-1}-\left(\frac{\rho_0'}{\rho_0}\right)(x_0(x,t))\\&\qquad\times\exp\left\{-\int_0^\tau v_x(\bar{x}(\tau';x,t),\tau')d\tau'\right\}\Big\}d\tau\Big],\ \left(k=\frac{K}{\mu}\right),\\&\left(N.B\colon \rho(x,t)=\rho_0(x_0(x,t))\frac{\partial x_0}{\partial x}=\rho(\bar{x}(\tau;x,t),\tau)\times\bar{x}_x(\tau;x,t)\right).\end{aligned}\right.$$

Now, we assume that $(v,\rho)\in H_T^{2+\alpha}\times B_T^{1+\alpha}$ satisfies (2.1)–(2.2). Then, by expressing (2.1)–(2.2) in the (v-)characteristic co-ordinates, we have

(2.4)
$$\left\{\hat{v}_{t_0}(x_0,t_0)=\frac{\mu}{\rho_0(x_0)}\left(\frac{\hat{v}_{x_0}}{1+\omega}\right)_{x_0}-\frac{K}{\rho_0}\left(\frac{\rho_0}{1+\omega}\right)_{x_0},\qquad \hat{\rho}=\frac{\rho_0}{1+\omega},\right.$$

(2.5) $\hat{v}(x_0,0)=v_0(x_0)\in H^{1+\alpha}$, $\quad(\rho_0\in H^{1+\alpha}\ (0<\underline{\rho}_0\leqq\rho_0\leqq\bar{\rho}_0<+\infty;\ \rho_0'\in L_1(R^1))$,

where we note that $(\hat{v},\hat{\rho})$ belongs to $H_T^{2+\alpha}\times B_T^{1+\alpha}$. Hence, $\hat{v}$ is expressed in the following way

(2.6)
$$\left\{\begin{aligned}&\hat{v}(x_0,t_0)=\int_{R^1}G(x_0,t_0;\xi,0)v_0(\xi)d\xi\\&\qquad-\int_0^{t_0}d\tau\int_{R^1}G(x_0,t_0;\xi,\tau)\frac{K}{\rho_0}\left(\frac{\rho_0}{1+\omega}\right)_\xi d\xi,\end{aligned}\right.$$

where $G(x_0,t_0;\xi,\tau)$ is the fundamental solution of (2.4) as a linear equation. Moreover, noting that

(2.7)
$$\left(\frac{\rho_0}{1+\omega}\right)_\xi=\left(\frac{\rho_0}{1+\omega}-\rho_0+\rho_0\right)_\xi=-\left(\frac{\rho_0\omega}{1+\omega}\right)_\xi+\rho_0'(\xi),$$

we have

(2.8)
$$\left\{\begin{aligned}&\int_0^{t_0}d\tau\int_{R^1}G\frac{K}{\rho_0}\left(\frac{\rho_0}{1+\omega}\right)_\xi d\xi=\frac{K}{\mu}\int_0^{t_0}d\tau\int_{R^1}\left(\frac{\mu}{\rho_0}G\right)_\xi\\&\qquad\times\frac{\rho_0\omega}{1+\omega}+d\xi\int_0^{t_0}d\tau\int_{R^1}G\frac{K\rho_0'}{\rho_0}d\xi\equiv I_1+I_2.\end{aligned}\right.$$

As for I_1, it holds that

$$I_1=-\frac{K}{\mu}\int_0^{t_0}d\tau\int_{R^1}d\xi\left(\frac{\mu}{1+\omega}\left(\frac{G}{\rho_0}\right)_\xi\right)_\xi$$

$$\times\left\{\rho_0(\xi)\int_0^\tau\hat{v}(\xi,\tau')d\tau'-\int_0^\tau d\tau'\int_{-\infty}^\xi\hat{v}(\xi',\tau)\rho_0'(\xi')d\xi'\right\}_1$$

$$\left(N.B.:\rho_0\hat{v}_\xi=\left(\rho_0\hat{v}-\int_{-\infty}^\xi\hat{v}\rho_0'(\xi')d\xi'\right)_\xi\right)$$

(2.9)

$$=\frac{K}{\mu}\int_0^{t_0}d\tau\int_{R^1}d\xi\frac{\partial G}{\partial\tau}\left\{\cdots\right\}_1$$

$$=\frac{K}{\mu}\Bigg[\rho_0(x_0)\cdot\int_0^{t_0}\hat{v}(x_0,\tau)d\tau-\int_0^{t_0}d\tau\int_{-\infty}^{x_0}\hat{v}(\xi,\tau)\rho_0'(\xi)d\xi$$

$$-\int_0^{t_0}d\tau\int_{R^1}G(x_0,t_0;\xi,\tau)\left\{\rho_0(\xi)\hat{v}(\xi,\tau)-\int_{-\infty}^\xi\hat{v}(\xi',\tau)\rho_0'(\xi')d\xi'\right\}d\xi\Bigg].$$

Hence, it follows that

$$|v|_{t_0}^{(0)}\leqq|v_0|^{(0)}+K\left|\frac{\rho_0'}{\rho_0}\right|^{(0)}\cdot t_0 \tag{2.10}$$

$$+\frac{2K}{\mu}(|\rho_0|^{(0)}+\|\rho_0'\|_{L_1(R^1)})\cdot\int_0^{t_0}|\hat{v}|_\tau^{(0)}\,d\tau,\ (t_0\in[0,T]).$$

Thus, we have

$$|v|_{t_0}^{(0)}\leqq|v_0|^{(0)}\cdot e^{C_0T}+\frac{K}{C_0}\left|\frac{\rho_0'}{\rho_0}\right|^{(0)}\cdot(e^{C_0T}-1) \tag{2.11}$$

$$\equiv C_1(T)\,(<+\infty),\ (C_1(T)\nearrow\text{as }T\nearrow,\ C_0\equiv\frac{2K}{\mu}\left(\bar{\rho}_0+\|\rho_0'\|_{L_1(R^1)}\right).$$

Taking (2.4) into consideration, we define, for an arbitrary number $a\in R^1$,

$$Y^a(x_0,t_0)\equiv\int_a^{x_0}\frac{\rho_0}{\mu}(v_0-\hat{v})dx_0'-\log(1+\omega(a,t_0))+\int_0^{t_0}\frac{K}{\mu}\frac{\rho_0}{1+\omega(a,t_0')}dt_0' \tag{2.12}$$

Y^a satisfies the relation

$$\begin{cases}(Y^a)_{t_0}=\dfrac{\mu}{1+\omega}\left(\dfrac{(Y^a)_{x_0}}{\rho_0}\right)_{x_0}-\dfrac{v_0'-k\rho_0}{1+\omega}=\dfrac{-\hat{v}_{x_0}+k\rho_0}{1+\omega},\\ Y^a(x_0,0)=0,\qquad\left(k=\dfrac{K}{\mu}\right).\end{cases} \tag{2.13}$$

Y^a satisfies Täcklind's condition. Therefore,

$$Y^a=Y^{a'}\equiv Y\qquad(\text{for any }a\text{ and }a'\in R^1). \tag{2.14}$$

Here, we note that, for an arbitrary $a\in R^1$,

$$Y(x_0, t_0)=Y^a(a, t_0)+\int_a^{x_0}\frac{\rho_0}{\mu}(v_0-\hat{v})dx_0'. \tag{2.15}$$

From (2.13) follows the relation

$$\begin{cases}(1+\omega)_{t_0}+(1+\omega)Y_{t_0}=k\rho_0,\\ (1+\omega)(x_0, 0)=1.\end{cases} \tag{2.16}$$

Hence, we have

$$(1+\omega)(x_0, t_0)=e^{-Y(x_0,t_0)}\left\{1+k\rho_0(x_0)\cdot\int_0^{t_0}e^{Y(x_0,\tau)}d\tau\right\}. \tag{2.17}$$

Thus, for an arbitrary and fixed $a\notin R^1$,

$$\begin{aligned}e^{Y(a,t_0)}\cdot(1+\omega(x_0, t_0))&=\exp\left\{-\int_a^{x_0}\frac{\rho_0}{\mu}(v_0-v)dx_0'\right\}\\ &\times\left[1+k\rho_0(x_0)\cdot\int_0^{t_0}dt_0'e^{Y(a,t_0')}\right.\\ &\left.\times\exp\left\{\int_a^{x_0}\frac{\rho_0}{\mu}(v_0(x_0')-\hat{v}(x_0', t_0'))dx_0'\right\}\right]_A, \qquad (0\leqq t_0\leqq T).\end{aligned} \tag{2.18}$$

Therefore, by integrating in x_0 both sides of (2.18) over a closed interval $[a, a+l]$ $(l>0)$ we have

$$\begin{aligned}&(0<)e^{Y(a,t_0)}\cdot\left\{l+\int_0^{t_0}(\hat{v}(a+l, \tau)-\hat{v}(a, \tau))d\tau\right\}\\ &=\int_a^{a+0}dx_0\exp\left\{-\int_a^{x_0}\frac{\rho_0}{\rho}(v_0-\hat{v})dx_0'\right\}\times[\cdots]_A.\end{aligned} \tag{2.19}$$

Take $l=1+2C_1(T)T\equiv L(T)$. Then, we obtain an inequality

$$\begin{aligned}e^{Y(a,t_0)}&\leqq L(T)\exp\left\{\frac{\bar{\rho}_0L(T)}{\mu}(|v_0|^{(0)}+C_1(T))\right\}\\ &\times\left[1+k\bar{\rho}_0\cdot\exp\left\{\frac{\bar{\rho}_0L(T)}{\mu}(|v_0|^{(0)}+C_1(T))\right\}\cdot\int_0^{t_0}e^{Y(a,t_0')}dt_0'\right], \qquad (0\leqq t_0\leqq T).\end{aligned} \tag{2.20}$$

Thus, for an arbitrary $a\in R^1$, we have

$$\begin{aligned}(1+\omega(a, t_0))^{-1}&=e^{Y(a,t_0)}\left(1+k\rho_0(a)\cdot\int_0^{t_0}e^{Y(a,\tau)}d\tau\right)^{-1}\\ &\leqq e^{Y(a,t_0)}\leqq C_2(T)(<+\infty), \qquad (C_2(T)\nearrow \text{ as } T\nearrow).\end{aligned} \tag{2.21}$$

Next, seeing that, by (2.13), the following equality holds

$$\begin{cases}Y(x_0, t_0)=\int_0^{t_0}d\tau\int_{R^1}\bar{G}(x_0, t_0; \xi, \tau)\dfrac{k\rho_0-v_0'}{1+\omega}d\xi,\\ (\bar{G}\text{ is the fundamental solution of (2.13) as a linear equation}),\end{cases} \tag{2.22}$$

we have

$$|Y|_{t_0}^{(0)} \leqq (k\bar{\rho}_0 + |v_0'|^{(0)}) t_0 \cdot \left| \frac{1}{1+\omega} \right|_{t_0}^{(0)}. \tag{2.23}$$

Thus, it holds that

$$\begin{aligned}(1+\omega) &\leqq \exp(|Y|_{t_0}^{(0)}) \cdot (1 + k\bar{\rho}_0 t_0 \cdot \exp(|Y|_{t_0}^{(0)})) \\ &\leqq C_3(T)(< +\infty), \qquad (C_3(T) \nearrow \text{as } T \nearrow).\end{aligned} \tag{2.24}$$

By the discussion made above, we obtain:

Theorem 3. *There exists a unique temporally global solution* (v, ρ) *of* (2.1)–(2.2) *such that it belongs to* $H_T^{2+\alpha} \times B_T^{1+\alpha}$ *for any* $T \in (0, +\infty)$.

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Finally, we add that Kazhikhov and Shelukhin ([9]) have recently obtained a good result contributing to the study of our related problems.

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