1990

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ABSTRACT

A systematic procedure is developed to locate and minimize the number of reactive power devices in power systems based on a set of indices and the sensitivity matrix. A new definition of the bus voltage performance index (VPI) is proposed for optimal reactive power planning. It is equal to the root mean square value of the corresponding column of the sensitivity matrix. The VPI's in previous work depend on the norms which can be defined in nonunique ways. In the proposed technique, the three indices are given different weights depending on their values. The load buses are also ordered on the basis of their values depending on the total weights at each bus for the three indices. This ordering of load buses as well as the sensitivity matrix are then used to determine the optimal location and number of reactive power devices required in the system.

INTRODUCTION

The goal of reactive power planning is to determine the minimum investment in reactive power sources (shunt capacitors and inductors) which are necessary to correct unacceptable voltage profiles during anticipated normal and contingency conditions. For today's bulk power systems that include UHV overhead lines and underground cables, the planning has become complicated. Therefore, planning methods such as successive ordinary load flow studies and/or simple gradient methods are not enough to obtain successful solutions.

Reactive power planning is an optimization problem. Solving this problem requires finding an optimal solution that minimizes an objective function, and satisfying the constraints. Traditionally the reactive power planning problem has been handled by a trial-and-error approach utilizing load flow programs. In the past decade, however, more systematic approaches to reactive power planning have been developed. Many of these formulations are, in essence, based on linear programming methods. Examples can be found in the work of Kishore and Hill [1], Maliszewski [2], Happ [3],and Ramalyer [4]. Other researchers have used nonlinear programming methods with penalty functions, e.g.Sachdeva [5] and Burchett [6]. Combinations of the above methods have also been tried, e.g. Hughes [7] and Lebow [8]. A common approach is to assume preassigned locations at certain buses

TH0343-4/90/0000/0283$01.00 © 1990 IEEE

based on engineering judgement, and then try to minimize the number of locations and the amount of reactive power required at each. Savulescu [9] proposed three system-wide indices related to steady-state stability, voltage control and real power loss. Even though the indices are based on heuristic techniques, these indices coupled with engineering judgement can give better locations for the reactive power devices. Carr [10] developed a systematic procedure to locate reactive power devices in a power system using a set of indices, that are based on overall system conditions. This paper proposes a procedure for locating reactive power sources that is based on the indices mentioned above with some modifications. A new definition of the bus voltage performance index (VPI) is proposed to locate reactive power sources for voltage control. It is equal to the root mean square value of the corresponding column of the sensitivity matrix. The VPI's in [10] depend on the norms which can be defined in a nonunique way. In the proposed technique, the three indices are given different weights depending on their values. Whereas in [10] an assumed weight is given equally to all indices. The load buses are also ordered on the basis of their values depending on the total weights at each bus for its three indices. This ordering of load buses as well as the sensitivity matrix are then used to determine the optimal location and number of reactive power devices required in the system.

DEVELOPMENT OF THE REACTIVE POWER INDICES

The first step in optimizing reactive power compensation is to select locations of reactive power devices which have maximum impact on the system performance with respect to steady-state stability, voltage control and power losses. Taking these criteria into consideration, the load buses are ordered for a possible reactive power source installation, as described below.

The Steady-State Stability Index:

The steady state stability of a power system can be estimated by using load flow analysis [11]. For the assessment of the stability limit and the corresponding stability margin, successive changes in initial operating conditions are made and the stability of each change condition is evaluated.

The changes in operating conditions can be obtained by one or a combination of the following[10]:

1. Increasing load and generation at certain nodes of the netwerk.
2. Redistributing generation between nodes.
3. Reducing voltages at specified nodes of the system (deficiency of reactive volt-amperes in a part of the system).

In power systems, the steady-state stability concept corresponds to practical criteria rather than to mathematical ones. This notion refers to the fact that the system steady state is permanently subject to infinitely small changes. The system is said to be unstable if it is unable to insure uninterrupted service. Practical criteria are used for the evaluation of the steady-state stability. One of those criteria consists of calculating the value of dQ/dV; if it passes through zero it is considered that we have to deal with an unstable state. Therefore the term dQ/dV gives usefull stability information and will be used in the proposed concept [9].

The ncdal difference equations for bus powers in a matrix form is given by[12]:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \partial P/\partial\delta & \partial P/\partial V \\ \partial Q/\partial\delta & \partial Q/\partial V \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (1)$$

Generally, it is considered that the dependence of the active power on the voltage magnitudes and that of the reactive power on the angles are neglected ($\partial P/\partial V$ and $\partial Q/\partial\delta=0$); this is the same phenomenon exploited in fast decoupled load flow algorithms [13]. Therefore

$$\begin{bmatrix} \Delta Q \end{bmatrix} = \begin{bmatrix} \partial Q/\partial V \end{bmatrix} \begin{bmatrix} \Delta V \end{bmatrix} \quad (2)$$

For small changes in reactive power and voltage, the diagonal elements $\partial Q_i/\partial V_i$ can be taken to represent steady-state stability indices[9]. If the generator buses are eliminated, then

$$\begin{bmatrix} \Delta Q_1 \end{bmatrix} = \begin{bmatrix} \partial Q_1/\partial V_1 \end{bmatrix} \begin{bmatrix} \Delta V_1 \end{bmatrix} \quad (3)$$

where subscript 1 indicates load buses .

The buses which have higher values of $\partial Q_i/\partial V_i$ can withstand more variation of the induced reactive power without an address effect on the system stablity compared to the buses which have lower values of $\partial Q_i/\partial V_i$. Therefore if load buses are ordered according to the increasing magnitudes of the diagonal Jacobian elements, the result will be a load bus ordering vector S that accounts for the impact on system steady state stability, where

$$S=\begin{bmatrix} K_1, & K_2 \ldots\ldots K_1 \end{bmatrix} \quad (4)$$

and $K_1, K_2, \ldots\ldots K_l$ are load bus numbers. The corresponding weight factors vector W_s is given by

$$W_s = \left[W_{s1},\ W_{s2},\ldots\ldots W_{sl} \right] \tag{5}$$

where $W_{s1} > W_{s2} > \ldots\ldots > W_{sl}$

The Voltage Performance Index:

One of the main objectives of reactive power control is to maintain a desired voltage profile on the system. From equation(2), the expression for the change in the voltages (ΔV) in terms of changes in reactive powers (ΔQ) is given by

$$\left[\Delta V\right] = \left[\partial Q/\partial V \right]^{-1} \left[\Delta Q\right] \tag{6}$$

Eliminating the generator buses in equation (6), the voltage changes due to a changes of reactive power at the load buses is,

$$\left[\Delta V_l\right] = \left[J_V \right] \left[\Delta Q_l\right] \tag{7}$$

In equation (7) the diagonal elements of the sensitivity matrix Jv (J_{Vii}) give information about the voltage variation at bus i, if the reactive power at the same bus is changed. The off diagonal elements J_{Vji} give information about the voltage variation at bus j, if reactive power at bus i is changed.

The location of the reactive power device for voltage control is decided in such a way that as many buses as possible are to be affected. For this a voltage performance index(V.P.I) is defined as follows:

$$(V.P.I)_i = \left[\sum_{j=1}^{N} J_{Vji}^{2} / N \right]^{1/2} \tag{8}$$

The reactive device locations are ordered based on the above index. The higher the value of (V.P.I) the better is its voltage control capability.
The location vector for reactive compensation, taking into account the above voltage performance index, is therefore given by

$$V = \left[m1,\ m2,\ldots\ldots,\ ml \right] \tag{9}$$

and $W_V = \left[W_{m1}, W_{m2}, \ldots\ldots, W_{ml} \right]$ (10)

where m1,m2,....ml are the load buses orders according to decreasing values of (V.P.I) and $W_{m1}, W_{m2} \ldots\ldots W_{ml}$ are the corresponding weight factors.

The Real Power Loss Index:

The complex expression for the power losses is given by

$$P_L + j\, Q_L = E\, I^{*} = E\, Y^{*}\, E^{*} \tag{11}$$

Taking the real part of (11) gives

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} V_i\, V_j\, y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \tag{12}$$

Taking partial derivatives of P_L with respect to V_i and δ_i yields

$$\partial P_L/\partial \delta_i = -2\, V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j\, y_{ij} \cos\theta_{ij} \sin(\delta_i - \delta_j) \tag{13}$$

$$\partial P_L/\partial V_i = 2 \sum_{j=1}^{N} V_j\, y_{ij} \cos\theta_{ij} \cos(\delta_i - \delta_j) \tag{14}$$

Equation (12) is rewritten in the form

$$P_L = P_L\,(V, \delta) \tag{15}$$

Therefore

$$dP_L = \partial P_L/\partial V \;\; dV + \partial P_L/\partial \delta \;\; d\delta$$

$$\begin{aligned} dP_L/dP &= \partial P_L/\partial V \;\; dV/dP + \partial P_L/\partial \delta \;\; d\delta/dP \\ \text{and} \quad & \\ dP_L/dQ &= \partial P_L/\partial V \;\; dV/dQ + \partial P_L/\partial \delta \;\; d\delta/dQ \end{aligned} \tag{16}$$

For small deviations, equation (16) can be written in the form

$$\begin{bmatrix} \partial P_L/\partial P \\ \partial P_L/\partial Q \end{bmatrix} = \begin{bmatrix} J^{T} \end{bmatrix}^{-1} \begin{bmatrix} \partial P_L/\partial \delta \\ \partial P_L/\partial V \end{bmatrix} \tag{17}$$

where J is the system Jacobian given by

$$\begin{bmatrix} J \end{bmatrix} = \begin{bmatrix} \partial P/\partial\delta & \partial P/\partial V \\ \partial Q/\partial\delta & \partial Q/\partial V \end{bmatrix} \tag{18}$$

From (17)

$$\begin{bmatrix} \partial P_L/\partial Q \end{bmatrix} = \begin{bmatrix} J_{L3} & J_{L4} \end{bmatrix} \begin{bmatrix} \partial P_L/\partial\delta \\ \partial P_L/\partial V \end{bmatrix} \tag{19}$$

where J_{L3} and J_{L4} are submatrices of $\begin{bmatrix} J^T \end{bmatrix}^{-1}$

If generator voltages are maintained constant, then

$$\begin{bmatrix} \Delta P_L \end{bmatrix} = \begin{bmatrix} \partial P_L/\partial Q_1 \end{bmatrix} \begin{bmatrix} \Delta Q_1 \end{bmatrix} \tag{20}$$

Accordingly the magnitude of $\partial P_L/\partial Q_1$ influences the real power loss. This results in a third ordering vector L and corresponding index vector W_L for locating reactive power sources that is given by ordering the load buses according to the magnitudes of $\partial P_L/\partial Q_1$. These vectors are given by:

$$L = (l_1, l_2 \ldots\ldots l_n) \tag{21}$$

and

$$W_L = (W_{l1}, W_{l2} \ldots\ldots W_{ln}) \tag{22}$$

where:

$l_1, l_2, \ldots\ldots l_n$ are load buses in order of performances.

and $W_{l1}, W_{l2} \ldots\ldots W_{ln}$ are the corresponding weight factors.

Final Ordering:

For the final ordering of the reactive source locations, the corresponding weights in each index have to be added to get total weights. Bus ordering is then obtained according to the magnitude of the total weights. The final order is given in the form:

$$N_{or} = (N_1, N_2, \ldots\ldots N_n)$$

and the corresponding total weight vector is

$$W_T = (W_1, W_2, \ldots\ldots\ldots W_n)$$

where N_1, $N_2, \ldots\ldots N_n$ are the load buses in their final ordering according to decreasing values of (W_T), and W_1, $W_2, \ldots\ldots W_n$ are the corresponding weight factors.

The number of these locations are determined by using the sensitivity matrix in equation(7). If the value of the off diagonal element Jv_{ji} is greater than the diagonal element Jv_{ii}, then no reactive power source at this bus i is to be considered and then the results is to minimize the number of locations but the size of installation are assumed.

The linear programming technique (LP) is then used to provid the amount of reactive power required at each source for a desired voltage profile. It is to be noted that the number of constraints and variables are not augmented at all in the procedure and all variables are treated as continuous.

EXAMPLES

A 6 bus system:

The proposed method is tested by solving the Ward-Hale 6 bus system. The system is shown in figure 1. It contains 6 buses and 7 transmission lines with bus 1 as the slack bus. The ordering of the system is as follows:

1) The corresponding steady-state stability index for buses 3,4,5,6 are (6.831,10.593,3.982,6.782) respectively.

   S =(5,6,3,4)
   W_S =(1,.645,.64,.376)

2) The value of voltage performence index for buses 3,4,5,6 are (2.876,2.761,2.976,2.869) respectively.

   V =(5,3,6,4)
   W_V =(1,.967,.964,.928)

This order is the same in paper[10] when the norm is defined as the sum of the absolute values of the column elements of Jv.

3) The magnitude of the real power index for load buses are (.529,.514,.544,.525).

   L =(5,3,6,4)
   W_L =(1,.974,.966,.946)

In this example, all indices are given different weights. The value of weigthing coefficient are equal to the per unit value of the indices. Now ordering the buses according to the total weights (depending on the order of the bus, the corresponding weight in each index has to be added to get total weight), the corresponding weighting vector is

W_T =(2.581 ,2.2497 ,3 ,2.575)
=(.86,.7499,1,.858)

The final order of the buses is given by

Norder =(5,3,6,4)

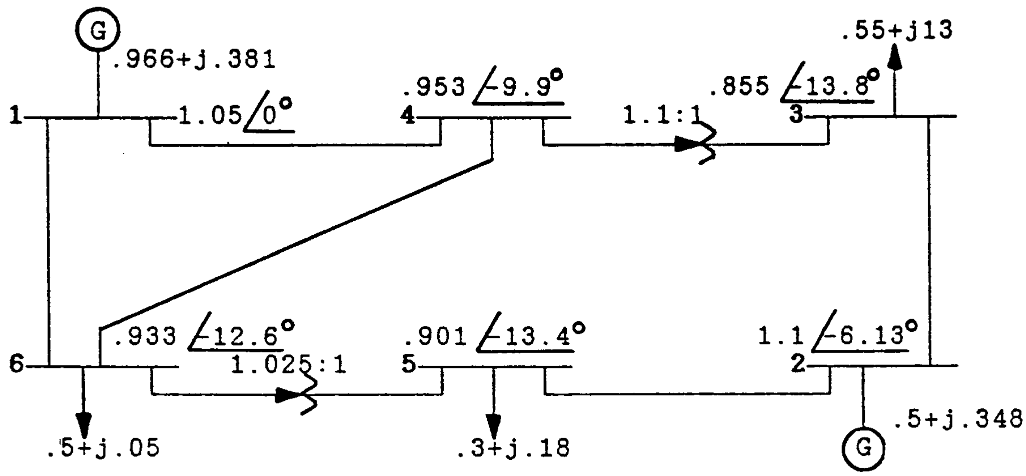


Fig. 1. Base-Case For the 6-Bus System.

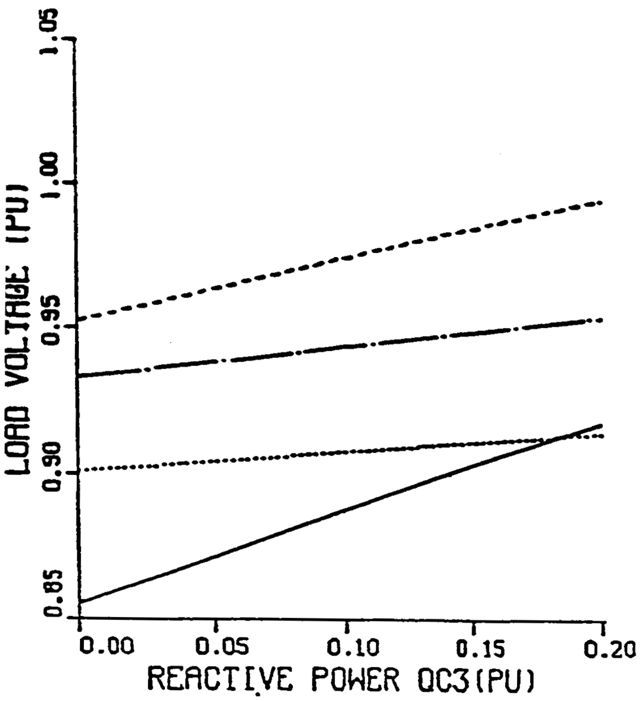
By examining the sensitivity matrix Jv, it is found that the element value of J_{65} is greater than the value J_{66}. Thus, the variation of the reactive power source at bus 5 could affect the highly vars at bus 6. Then, a reactive power source will be required at bus 5. Also, J_{43} is greater than J_{44}. Then, a reactive power source is added at bus 3. Therefore, the optimal location of reactive power sources are at Buses 5 and 3. The validity of these results are also evident from figure 2. This figure show the dependence of the load bus voltages on the possible allocation of reactive power sources at different buses.

A 30 bus system:

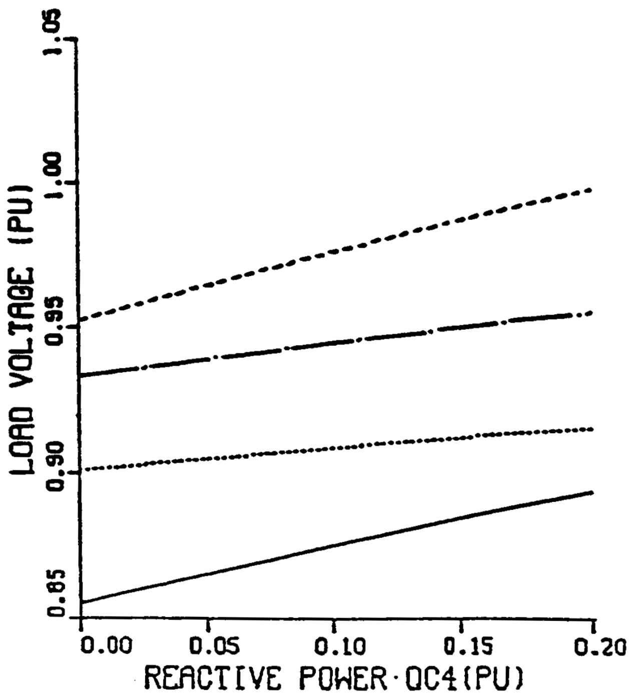
The same approach has also been applied to a 30 bus system [12] to determine the optimal locations and minimum number of reactive power devices in the system. The approach appeared to be very systematic and effective for this system too.

CONCLUSIONS

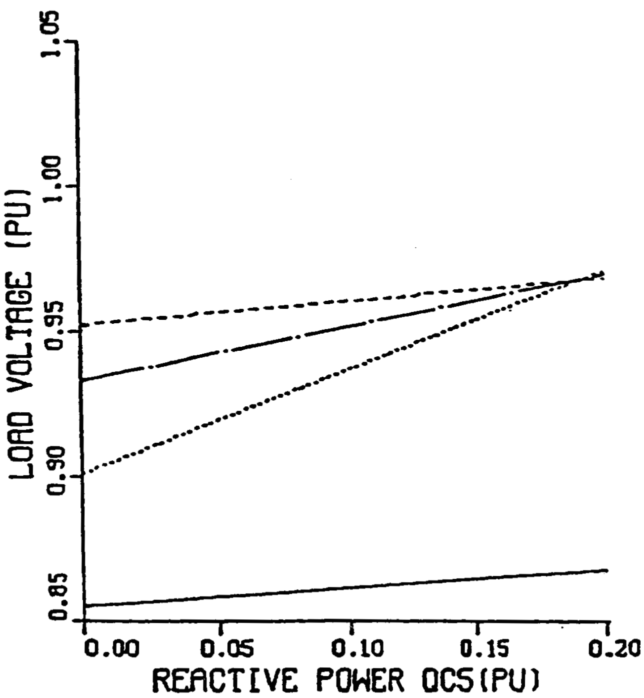
A systematic approach has been developed to determine the loactions and minimum number of reactive power devices required in a power system, based on a set of indices and the sensitivity matrix. This includes a new voltage performance index to locate reactive power sources for voltage control. The approach minimizes the dependence on engineering judgement. It is systematic as well as effective expecially for large-scale systems.



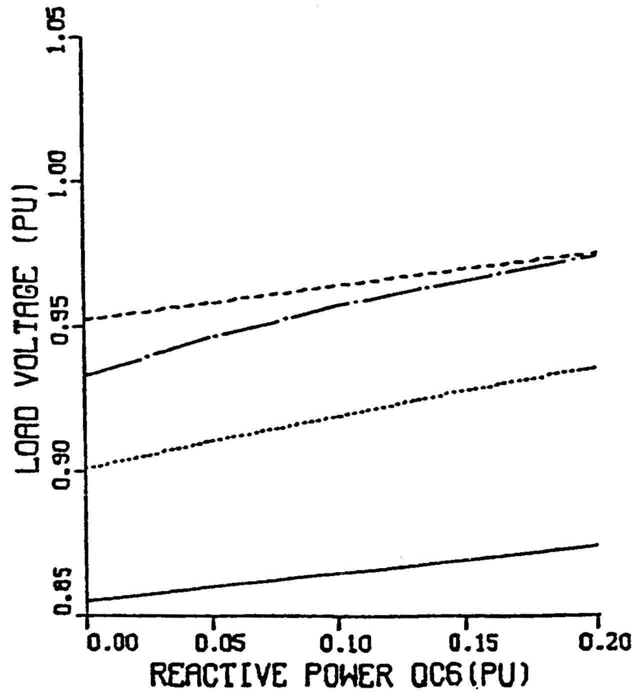
(A) Reactive source 3 vs voltages.



(B) Reactive source 4 vs voltages.



(C) Reactive source 5 vs voltages.



(D) Reactive source 6 vs voltages.

Fig. 2. Reactive Power Sources Versus Load Voltages

———— voltage at bus 3   — — — voltage at bus 4
····· voltage at bus 5   —·— voltage at bus 6

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