

A Systematics and Phenomenology of Meson Family^{*)}

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(Received May 1, 1966)

Previous studies on a systematics and phenomenology of meson family are reviewed and extended based on the intuitive pictures. In these discussions, the triplet configurations of hadrons are extensively used, from which the meaning of the selection principle is made clear. This principle is applied to the meson-baryon vertices. We further suggest an indirect test of both the u -baryon models and the non-relativistic descriptions of meson nonets, where the $E1$ -transitions, $1^+ - \underline{9} \rightarrow 1^- - \underline{9} + \gamma$, are taken as the test processes.

§1. Introduction

In our series of works I-III,^{1),2)} we presented a consistent description of meson nonet series and their mutual interactions based on the composite model,³⁾ $U(3)$ -symmetry^{4),5)} and the non-relativistic picture^{6),7)} (N. R. P.). We note here that the consistent story in this framework may require the following points: 1) In the mass level discussions, the effects of the involved momenta and the LS force should be small. 2) A measure of the pair effects should be inferable from experimental data and that it should be small. 3) A stronger vertex of meson interactions should correspond to the one with a smaller number of the pair creation-annihilations. 4) There should be appropriate domains for the mass levels and interactions, where the symmetry breakdowns manifest themselves as regular, etc. We hesitate to claim at this moment that our works are consistent with all the above requirements and all the up-to-date available data, though we may say that so far there is no definite data against our story.

In this paper, we wish to discuss the following subjects: in §2, a brief review of the meson mass levels and the related topics, in §3, the selection principle (S. P.) and the related topics, where the utilities of the configuration approach will be emphasized, in §4, a possible test of both the $U(3)$ -tensor character of the electromagnetic interaction (E. M. I.) and the N. R. description of meson nonet series. The importance of this

^{*)} The major part of this work is based on the following papers: J. Iizuka, DPKU-012-66, February, 1966 and the author's "Introductory talk" given in the "Extended colloquium" held at the Department of Physics, Atomic Energy Research Institute, College of Science and Engineering, Nihon University on March 22-23, 1966; Soryushiron Kenkyu (mimeographed circular in Japanese) **33** (1966), 167.

last point is incontestable in view of the following fact, i.e. all of the proposed ur-baryon models are divided into the two classes; $\underline{1} + \underline{8}$ (Sakaton⁸⁾ and its various extended versions⁸⁾) and $\underline{8}$ (quark-ace⁹⁾ and the particular three triplet models¹⁰⁾) in terms of the transformation character of E. M. I. under $U(3)$. We remark in this connection that in N.R.P., the examinations of appropriate $E1$ -transition processes are clearly favorable over any others, since they explicitly preserve the original structures of the charge matrices inherent to the ur-baryon models and that the coupling strength of the given process is the same among the models. This consideration will be applied in the $1^+ - \underline{9} \rightarrow 1^- - \underline{9} + \tau$ adopting our classification for $1^+ - \underline{9}$, where various advantages of the particular choice of this process will be discussed in detail. Finally concluding remarks will be made in the last section.

§2. The meson mass levels and the related topics

Let us summarize the main results of the low lying mass levels of meson nonet series with the $(t\bar{t})$ -structure^{*}) obtained in the previous works.^{1),2)}

1) The mesons are described, in the first approximation, as $(3, 3^*)_a$ of $U_1(3) \otimes U_{\bar{1}}(3)$ with $a = (n, {}^iL_J)$, where $t(\bar{t})$ refers to the ur-baryon triplet (anti-ur-baryon). Their constructive force is superstrong, well-shaped, spin and unitary-spin independent. Concentrating our discussions to the $n=1$ case, the level spacing between the two neighboring nonets with the orbital angular momenta L and $L+1$ mainly comes from the centrifugal effect.

2) The level spacings among the nonets with the same L are mainly due to the unitary spin independent LS force. Furthermore each nonets are splitted by S. B. I., which is almost spin- and momentum-independent and thus its major part may be identified with the ur-baryon mass differences δm ; Ishida¹⁰⁾-Zweig.⁹⁾ From these considerations, we obtain the two "linear" mass relations, those examples being shown below for $L \leq 1$.

A) The equi-distant mass difference relation ($L=1$ system) due to LS force.

$$\begin{aligned} \pi(2^+) - \pi(1'^+) &\cong K(2^+) - K(1'^+) \cong \eta'(2^+) - \eta'(1'^+) \\ &\cong \eta(2^+) - \eta(1'^+) \cong \pi(1'^+) - \pi(1^+) \cong \dots\dots \\ &\cong \pi(1^+) - \pi(0^+) \cong \dots\dots, \end{aligned} \quad (1)$$

where $1'^+ = {}^1P_1$ and the others are 3P_J . η' denotes the heavier $I=0$ state.

B) Ishida-Zweig relations (I. Z. relations).

$$\begin{aligned} \pi(a) &\cong \eta(a), \quad \eta'(a) - \eta(a) \cong 2(K(a) - \pi(a)), \\ \theta(a) &\cong 35^\circ, \quad a = {}^iS_J, {}^iP_J, \text{ etc.} \end{aligned} \quad (2)$$

^{*}) Mesons with $L \neq 0$ were first suggested by Ohnuki et al.¹¹⁾ from the composite model view-point.

For the later purposes, we also give the Ishida-Zweig configurations:

$$\begin{aligned} \pi^+(a) &= (t_1 \bar{t}^2)_a, \quad K^+(a) = (t_1 \bar{t}^3)_a, \quad \eta'(a) \cong - (t_3 \bar{t}^3)_a, \\ \eta(a) &\cong \frac{1}{\sqrt{2}} (t_1 \bar{t}^1 + t_2 \bar{t}^2)_a, \quad \text{etc.} \end{aligned} \tag{3}$$

Now a reasonable correspondence between the available data and Eqs. (1)-(2) has been made in I-II and also in Dalitz's work.¹³⁾ At this stage, the difficulties in the correspondence are the 1S_0 - 3S_1 degeneracy and the problem related with the irregular mass level pattern of 1S_0 - $\underline{9}$. Our opinions expressed in I-III on these points are as follows: Let us consider that the ur-baryon carries a kind of charge called the supercharge. Thus the ur-baryon could have the induced dipole moment μ_i associated with this charge. We may then expect a quite short-range dipole-dipole interaction, which is, for example, of the form $C\mu_i \cdot \mu_j \delta(\mathbf{r})$ in analogy of Fermi's hyperfine interaction. This kind of force can split the above degeneracy without any appreciable effects on $L \neq 0$ states, as is obvious from Fig. 1. Although it may be meaningless to speak on the force in $r \lesssim r_i = 1/M_i$ and the S -state wave function as will be discussed shortly, we note that the 1S_0

wave function is considerably large in $r \sim 0$, while that of 3S_1 is not and it is rather similar to the P -state wave function.

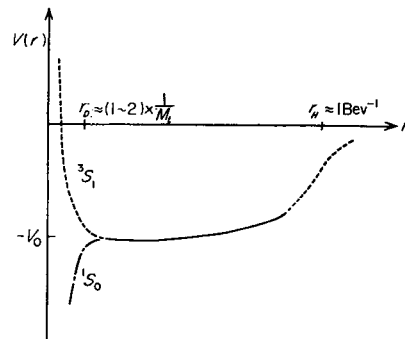


Fig. 1. A schematic illustration of 3S_1 - 1S_0 difference in the deepest region. The scale should not be taken seriously. $V_0 \approx 2M_t \dots$ Matumoto relation.⁶⁾

Next we come to the question of the 1S_0 - $\underline{9}$ irregular pattern. In III, we applied the Taketani approach¹⁴⁾ in nuclear force problem to the $t\bar{t}$ dynamics, which is shown in Table I. We note that in the deepest region, there may be strong pair creation-annihilations, the mass shifts due to them, the complex mechanism of S.B.I. far from δm such as the multi-exchanges of S.B.I.-quantum, the recoil effects, and all of their complicated correlations, etc. These are supposed to occur, in major part, inside the dipole-dipole interaction, which may explain the appearance of the irregularity in the 1S_0 state only.

We shall close this section with the following remarks: i) A possible origin of V_P and LS force may be either massive neutral,¹⁵⁾ nonet-, octet-vector¹⁰⁾ theories or the four-fermion interaction with dominantly V type.¹⁶⁾ All of these have common difficulties in the shape of V_P and LS/V_P ratio,

Table I. A possible application of the Taketani approach to the $(t\bar{t})$ dynamics.

We consider that the mass level patterns of meson nonet series directly reflect the dynamical characteristics of the classified regions.

Regions	Characteristics
Deepest region $r \lesssim (1 \sim 2)(1/M_t) (= r_t)$	Multi-pair effects, S. B. I., dipole-dipole int., and their correlations, something unknown ...
Intermediate region $r_t < r < 1 \text{ BeV}^{-1}$	V_P, LS , other minor forces (spin-dependent, tensor, quadratic LS , ... with appropriate $U(3)$ character)
Hadron region $1 \text{ BeV}^{-1} \lesssim r$	tail of V_P, LS , forces due to hadron exchanges, ...

these being far from the consequences obtainable from simple static approximation of the above theories. ii) As for δm related with S. B. I., we may have the possibilities such as Nagoya model¹⁷⁾ and the vector meson¹⁸⁾ coupled to t_3 or hypercharge, etc. In the latter case, the marked differences between 1S_0 - and 3S_1 - 1P_1 -nonets (these follow the I.Z. pattern in good approximation) may require $m_v \gtrsim (1/2)M_t$, where the lower bound of M_t is $\sim 4.5 \text{ BeV}^{13)}$ and $M_t \approx 10 \text{ BeV}$ is conjectured by several authors.¹⁹⁾ Our conjecture on m_v (assuming its existence) is quite different from that of Ne'eman.¹⁸⁾ We also note that $M_{t_1} = M_{t_2}$, $M_{t_3} \cong M_{t_1} + \mu$ -meson mass. iii) The shifts from I.Z. pattern in various low-lying nonets may be due to the mass shifts due to pair effects, the δm change by recoils, effects of hadron clouds and the minor forces in the intermediate region, etc., these being of course different from nonet to nonet. We consider all of these as the hyperfine effects and thus the deviations of $\underline{1}-\underline{8}$ mixing angles for low-lying nonets should be measured as the ones from $\theta = 35^\circ$. iv) Considerations of mesons with the $(t\bar{t}\bar{t})$ -configuration are beyond our present investigations. The experimental confirmations of these states may give useful informations on the inner ur-baryon dynamics as was remarked in II.

§3. The selection principle and the related topics

An extension of the N.R.P. discussed in the preceding section to the domain of the hadron interactions should be of the following form: For a given process, the dominant characteristics is generally understood in terms of the minimum number of ur-baryon pair creation-annihilations, which is required for the existence of the relevant process. Stated in another way, the suppression of the pair effects or the configuration approach should be useful, even in this domain, as the reasonable interpretations of their dominant features. This kind of idea was suggested by Ogawa et al.²⁰⁾ and by Nambu,⁷⁾ who made the conjecture that the transitions among hadrons may be classified as the first forbidden, the second forbidden and so on according to the

number of the pair effects.

Disregarding the complicated processes, let us now apply the above consideration to the effective trilinear interaction among the meson nonets a , b and c . Our argument leads in this case that the dominant effective vertex should be of the single pair creation or annihilation type for the positive Q -value decays $a \rightarrow b+c$ or the reaction $b+c \rightarrow a$. From this conjecture and the I.Z. configuration, Eq. (3), we immediately obtain the results:

$$\begin{aligned}
 1)^{21)} \quad \eta'(2^+) \sim f'(1500) &\leftrightarrow K + \bar{K}, K + \bar{K}^*, \bar{K} + K^*, \text{ etc.}, \\
 &\leftrightarrow \pi + \pi, [2\rho] \rightarrow 4\pi. \\
 \eta(2^+) \sim f^o(1250) &\leftrightarrow \pi + \pi, K + \bar{K}. \text{ etc.} \\
 2)^{22)} \quad \eta'(1^-) \sim \varphi(1020) &\leftrightarrow K + \bar{K}, \\
 &\leftrightarrow \rho + \pi. \\
 \eta(1^-) \sim \omega(783) &\leftrightarrow [\rho\pi] \rightarrow 3\pi. \\
 3) \text{ conjecture}^*) & \\
 \eta'(1^+) \sim E(1410) &\leftrightarrow K + \bar{K}^*, \bar{K} + K^*, \text{ etc.}, \\
 &\leftrightarrow \rho + \pi. \\
 \eta(1^+) [\sim 1140 \pm 80] &\leftrightarrow \rho + \pi. \\
 \eta'(0^+) [\sim 1170 \pm 80] &\leftrightarrow K + \bar{K}, \\
 &\leftrightarrow \pi + \pi. \\
 \eta(0^+) [\sim 920 \pm 80] &\leftrightarrow \pi + \pi, \\
 \text{etc.} &
 \end{aligned}$$

Leaving the detailed discussions in the later part of this section, we extend the requirement for the vertices into the process, which may include low virtualities such as one of the participating mesons being in off-the-mass-shell. Although the extension has been made in III in terms of the somewhat “*sophistic*” combination of the N.R.P. and the field characteristics of hadrons, i.e. by the selection principle, it has a quite simple meaning from the view point of composite model diagram.

Selection principle (S.P.):

Among all of possible effective vertices of a given process, the dominant one corresponds to the connected diagram viewed from the composite model.

In order to apply S.P. for a given process, we must first express the participating hadrons in terms of the triplet configurations and then connect each

*) In I-III, we adopted the notations α , β , γ and δ for $\eta(1^+)$, $\eta(1^-)$, $\eta'(0^+)$ and $\eta(0^-)$, respectively. As these are not adequate, we change their notations as is used in this paper.

other by introducing the minimum number of the required pair effect for the existence of the process. Among the vertices obtained in this manner, the connected (disconnected) ones are argued to be the major (minor) part of the given process. If the given process corresponds to certain disconnected diagrams only, it should be quite suppressed compared to the ones, where the participating hadrons have similar structures. It should be noted that, under the restricted crossing symmetries (R. C. S.), disconnected diagrams are generally transformed into the forbidden types in N.R.P., even if they look like allowed types viewed from a certain time direction. These examples are shown in Figs. 2 and 3.

From S.P. and Eq. (3), we obtain various allowed transitions (real or virtual) of the familiar types and the following forbidden ones:

$$\eta'(a) \leftrightarrow \pi(b) + \pi(c), \quad (4a)$$

$$\pi(a) \leftrightarrow \eta'(b) + \pi(c), \quad (4b)$$

$$\eta'(a) \leftrightarrow \eta(b) + \eta(c), \quad (4c)$$

$$\eta(a) \leftrightarrow \eta'(b) + \eta(c), \quad (4d)$$

$$\eta(a) \leftrightarrow \eta'(b) + \eta'(c), \quad (4e)$$

$$\eta'(a) \leftrightarrow \eta'(b) + \eta(c), \quad (4f)$$

where all the possible permutations of the characters a , b and c are also understood. Note that the pairs of reactions (a–b), (c–d) and (e–f) are connected by R.C.S. Besides 1)–3) and their R.C. reactions, we give here additional examples of Eq. (4) (Decay processes):

$$\begin{aligned} 4b)^{2b)} \quad \pi(1^{+}) \sim B(1220) &\rightarrow \pi + \omega, \\ &\rightarrow \pi + \varphi. \\ \pi(2^{+}) \sim A_2(1320) &\rightarrow K + \bar{K}, \pi + X^0, \rho + \pi, \\ &\rightarrow \pi + \eta. \\ \pi(0^{+}) \sim X^{-}(962)? &\rightarrow \pi + \eta. \\ 4e-f) \quad \eta(2^{+}) \sim f^0(1250) &\rightarrow 2\eta. \\ \eta'(1^{+}) \sim E(1410) &\rightarrow \omega + \eta, \text{ etc.} \end{aligned}$$

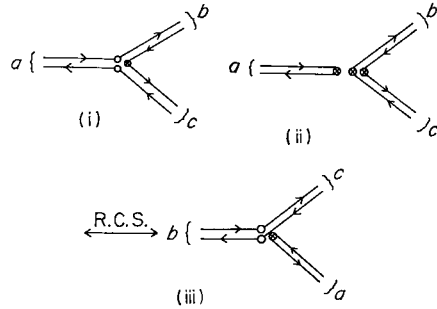


Fig. 2. Composite model diagrams of the trilinear vertices of the meson nonets a , b and c . \otimes denotes the pair effect, while \circ means the triplet or anti-triplet conserving part. (i) is the connected diagram, so it represents the dominant vertex made of a , b and c . (ii) and (iii) are the disconnected ones. These diagrams are transformed into each other under the restricted crossing symmetry.

In these reactions, $\eta(0^-)$ is assumed²⁾ to have much $(t_3 \bar{t}^3)$ component, i.e. $\delta\theta (\equiv 35^\circ - \theta_{\eta\pi 0}) \sim +10^\circ$ rather than the conventional²⁵⁾ $\delta\theta \sim +25^\circ$ or 45° . Those processes are indicated by dot lines in the examples. We note that the lack of examples for Eqs. (4b)–(4f) is partly because 1S_0 - $\underline{9}$ has the irregular patterns and partly because the $I=0$ states are relatively heavier compared to the others and the relative level spacings of the known mesons. Thus the S.P. will be more clearly tested, if the higher excited systems such as the $L=2$ mesons³⁾ are found in future.

In the preceding discussions, we have extensively utilized the Ishida-Zweig configurations, the single pair effect and S.P. without explicitly emphasizing the symmetry considerations. If one wishes to phrase the above results by the broken $U_i(3) \otimes U_i(3) [\rightarrow U(3)]$, the dominant part of the effective trilinear vertices of meson nonets a, b and c is the singlet component of $(3, 3^*) + (3^*, 3)$ followed by the rearrangement, i.e. it is $\text{Tr}(\phi_a \phi_b \phi_c)$ -type in the $U(3)$ -limit, as is clear from Fig. 2. This form was first suggested by Okubo²⁴⁾ in his 1^- - $\underline{9}$ study and then applied to 2^+ - $\underline{9}$ by Glashow and Socolov.²⁶⁾

Next let us make a brief comment on the effective coupling between the meson nonet $M_j^i(a)$ and the baryon octet $B_i^k [\equiv (1/\sqrt{2})\epsilon^{ijk} \cdot \Psi_{[ij]l}]$ is assumed, as far as the triplet configuration is concerned]. From S.P. and Fig. 3, we have the two possible connected vertices, $\bar{\Psi}^{[ij]\alpha} \Psi_{[ij;\beta]} M_\alpha^\beta$ and $\bar{\Psi}^{[\alpha i]j} \Psi_{[\beta i]j} M_\alpha^\beta$. We thus obtain the following effective coupling as the dominant one:

$$\begin{aligned}
 H(\bar{B}BM) &= g_f \{ \text{Tr}(\bar{B}O_M [B, M]) - \text{Tr}(M) \text{Tr}(\bar{B}O_M B) \} \\
 &+ g_D \{ \text{Tr}(\bar{B}O'_M \{B, M\}) - \text{Tr}(M) \text{Tr}(\bar{B}O'_M B) \},
 \end{aligned}
 \tag{5}$$

where O_M is the appropriate operator depending upon the space-time character of $M(a)$. As for the determination of F/D ratio, we need the detailed knowledge of more than the triplet configuration for the baryon octet as is assumed, for example, in $SU(6)$ ²⁰⁾ and $\tilde{U}(12)$,²⁷⁾ etc.

Substituting Eqs. (2) and (3) into Eq. (5), we have:

$$g_{\eta'(a)NN} = 0,
 \tag{6}$$

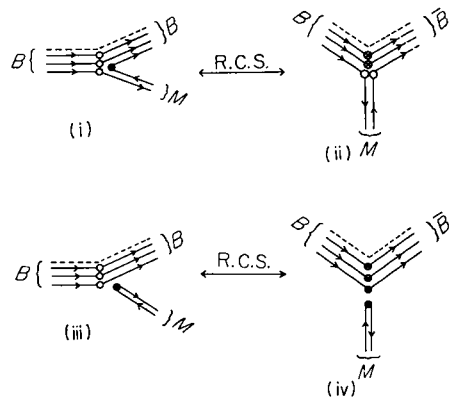


Fig. 3. The diagrams of the meson-baryon vertices. The dot line indicates a possible existence of new matter in baryons. (i) and (ii) are the connected diagrams, while (iii) and (iv) are disconnected ones.

where $g_{\eta'(a)NN}$ denote the effective coupling constant of $\eta'(a)$ and nucleon. Equation (6) is of course a natural consequence of our approach. From this fact and the form of the dominant trilinear meson vertices, we may generally expect the low production rates of $\varphi^{28)}$, $\eta'(0^+)$, D , E and f' , etc., compared to those of ω , $\eta(1^{++})$ and f^0 , etc., in the peripheral- and baryon exchange-type πN collisions. $\eta'(a)$ production will be associated with non-peripheral processes, i.e. the ones with considerable excitations in the internal state of the recoil nucleon.²⁹⁾

Finally we wish to give a few remarks: i) From the extreme non-relativistic picture and its minimum extension, i.e. the inclusion of the one pair effect only, we are directly led to the reasonable interpretations for both the mass levels and some of strong decays such as 1)-3) and 4f), although 3) and 4f) are in the level of conjecture. It may be said that the utilities of the configuration approach first emphasized by Ogawa et al.²⁰⁾ are promising even in strong decays of the relatively simple composite systems. In view of these results, let us take the non-relativistic picture (N. R. P.) seriously and then consider the pair creation-annihilation processes, a characteristics of the quantized field theory. In this situation, we think it worthwhile to unite the two features, if not satisfactory at all. The best possibility that we can offer at the moment is the selection principle. Thus S. P. should be understood in a sense of the correspondence principle and it means no more than that, i.e. it could be understood, when we could have the concrete idea or mechanism of superstrong interaction. ii) The forbidden processes predicted by S. P. such as the ones indicated in 1)-4) and Eq. (6) occur through the $\delta\theta$ deviations and various higher order pair effects. Their coupling strengths are of the order of $g\delta\theta$, where g denotes a typical strength of the allowed processes. Noting that pair effects themselves are possible origins of $\delta\theta$, we must include, in general, minor vertices rejected by S. P. for the better descriptions of these processes. This remark may be particularly relevant to the forbidden processes with η and X^0 . iii) S. P. cannot give any informations for the relative strengths among the different decays of the same parent particle, where the particles with different structures participate. For example, the dominance³⁰⁾ of $2^+ \rightarrow 1^- + 0^-$ over $2^+ \rightarrow 0^- + 0^-$ is not explained by S. P. alone. Noting the wave function difference between 1^- and 0^- [see §2], the above fact may be understood by the overlapping difference between $2^+ - 1^-$ and $2^+ - 0^-$. iv) If we set $M(a) = 1^- - \underline{9}$, $O_M = \gamma_\mu + K\sigma_{\mu\nu}q_\nu$ and assume $g_D O'_M \approx K'\sigma_{\mu\nu}q_\nu$ in Eq. (5), we obtain $9(g_{\rho NN}^2)/4\pi \approx (g_{\omega NN}^2)/4\pi$ from Eqs. (2) and (3), a reasonable agreement with the result obtained from the nuclear force analysis.³¹⁾

§4. The electromagnetic interaction and the triplet models

Generally speaking, a final choice of the ur-baryon models is rather

hard for various reasons. However we could discuss whether or not a given model is adequate for the particular problems set up under the physically meaningful circumstances. Based on this view point, we wish to present an indirect test of the ur-baryon models, where we shall concentrate our discussions to the electromagnetic interaction (E. M. I.) of the involved triplet (or triplets) only.

Let us assume that the non-relativistic picture developed in the preceding sections is acceptable, particularly for the meson nonets that follow the Ishida-Zweig pattern. As was mentioned in the Introduction, we may safely conclude from this assumption and the well-known properties of E. M. multipole transitions that one of the best processes is the experimental studies of the appropriate $E1$ -transitions between the two meson nonets rather than $M1$ such as $1^- - \underline{9} \rightarrow 0^- - \underline{8} + \gamma$. To make our argument clear, let us consider the electric dipole operator (e. d. o.) of the meson system, in analogy of the atomic spectra. From Fig. 4, we obtain:

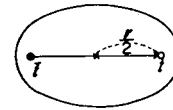


Fig. 4. A schematic representation of the meson nonets.

$$\begin{aligned} \text{e. d. o.} &= \frac{\mathbf{r}}{2}(eQ_i) + \left(-\frac{\mathbf{r}}{2}\right)(-eQ_j), \\ Q_i &= Q_i^T, \end{aligned} \tag{7}$$

where $Q(Q^T)$ denotes the charge operator (transposed) of the relevant ur-baryon triplets. We consider the following three cases for Q :

A) A massive Sakaton⁸⁾

$$Q_s = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}.$$

B) A modified massive Sakaton⁸⁾

$$Q_s' = \begin{pmatrix} 0 & & \\ & -1 & \\ & & -1 \end{pmatrix}.$$

C) Quark-Ace⁹⁾ and the specific three triplet models¹⁰⁾

$$Q_s = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}.$$

Now in the non-relativistic limit, the amplitudes of the $E1$ -transitions between the two nonets a and b are proportional to the expectation values of

e. d. o. taken between the given nonets, where the proportionality constant is common among the three cases A), B) and C). Let us identify $a=1^+-\underline{9}$ and $b=1^--\underline{9}$, where we adopt the $1^+-\underline{9}$ classification studied in our works. From these identifications, Eqs. (3) and (7), we can readily obtain the expectation values of e. d. o. apart from the common factor, those examples being shown in Table II for our later purposes.

Table II. Radiative decays of the neutral axial vector mesons.

$F_{ab} = \langle b | \langle \text{e. d. o.} \rangle | a \rangle / (e/2) \langle b | r | a \rangle$, Γ/G^2 are measured in Mev unit. The particle masses are respectively $D(1285)$, $\varphi(1020)$, $\omega(783)$, $\rho(763)$ and $\eta(1^+)(1050)$ [a tentative value]. Note $D \rightarrow \rho + \gamma$, $\omega + \gamma$ and $\eta(1^+) \rightarrow \varphi + \gamma$ in our approximation.

Processes	A)		B)		C)	
	F	Γ/G^2	F	Γ/G^2	F	Γ/G^2
$\eta(1^+) \rightarrow \rho^0 + \gamma$	1	.759	1	.759	1	.759
$\eta(1^+) \rightarrow \omega + \gamma$	1	.621	-1	.621	1/3	.069
$D \rightarrow \varphi + \gamma$	0	0	-2	1.65	-2/3	.184

At this stage, we remark that the relativistic extension of the above picture may uniquely lead us to the following effective vertex as the dominant one:

$$H(Q) = iG e \epsilon_{\alpha\beta\gamma\delta} \text{Tr}(\phi_\alpha(1^+) \{Q, \phi_\beta(1^-)\}) F_{\gamma\delta}, \quad (8)$$

where G denotes the Q -structure-independent strength. The decay rate is from Eq. (8)

$$\Gamma(1^+(a) \rightarrow 1^-(b) + \gamma) = G^2 \frac{e^2}{4\pi} \frac{m_a}{3} \left[1 + \left(\frac{m_a}{m_b} \right)^2 \right] \left[1 - \left(\frac{m_b}{m_a} \right)^2 \right]^3 F_{ab}^2, \quad (9)$$

$$F_{ab} = \langle b | Q + Q^T | a \rangle,$$

where $a(b)$ is an appropriate state of $1^+-\underline{9}(1^--\underline{9})$. The calculated results are shown in Table II for the limited decay processes.

As is clear from Table II, there are considerable differences among A), B) and C). In order that these differences are physically meaningful, i.e. their experimental detections are feasible, the following requirements have to be satisfied: 1) The strong decay widths of D and $\eta(1^+)$ mesons should be small. 2) A large number of the clean samples of these mesons have to be prepared. As for the first requirement, we have made the conjecture in III that the full widths of these mesons are likely of the order of or less than 10 Mev. In fact, this conjecture is valid, as far as no drastic change occurs in the decay width of the A_1 meson (one of the input data in III). Although $\eta(1^+)$ is not directly connected with the A_1 -meson data, we can expect from S.P. that its full width is also small. Next for the

second point, we may tentatively consider the following processes.

$$\begin{aligned} &K^- + p \rightarrow D(\eta(1^+)) + Y, D(\eta(1^+)) + Y + \pi + \dots; \\ &r + p \rightarrow \eta(1^+) + p; K^- + d \rightarrow D(\eta(1^+)) + p + \Sigma^-, \\ &D(\eta(1^+)) + p + \Sigma^- + \pi + \dots, \text{ etc.} \end{aligned}$$

From the above considerations, we now propose to measure the following quantities:

$$R \equiv \left. \begin{aligned} &\frac{\Gamma(D \rightarrow 1^- + r)}{\Gamma(\eta(1^+) \rightarrow 1^- + r)} \cong \begin{matrix} 0 & \dots\dots & A) \\ 1.2 & \dots\dots & B) \\ .2 & \dots\dots & C) \end{matrix} \end{aligned} \right\}, \quad (10)$$

$$r \equiv \left. \begin{aligned} &\frac{\Gamma(\eta(1^+) \rightarrow \omega + r)}{\Gamma(\eta(1^+) \rightarrow \rho^0 + r)} \cong \begin{matrix} .82 & \dots\dots & A), B) \\ .09 & \dots\dots\dots & C) \end{matrix} \end{aligned} \right\}. \quad (11)$$

Finally we give again a few remarks: i) As is demonstrated in the non-relativistic picture, the $E1$ -transitions directly reflect the original charge structure of the ur -baryon triplet (or triplets). This feature does not necessarily hold for the $M1$ -transitions, i.e. the ones in A) and B), which may make the comparisons of the models rather obscure. In the $M1$ -transitions, the magnitudes as well as the number of the coupling strength are model-dependent. In this respect, our opinion is that none of present theories of hadron interaction is developed enough to calculate the coupling strengths appearing in these transitions for arbitrary models. ii) Our results are obtained in the ideal limit, i.e. using Eqs. (3), (8) and (9). We must actually consider the effects of $\delta\theta$, iso-impurity, minor vertices and various clouds such as the $N\bar{N}$ dissociation, etc. Our expectation is that possible changes of our results due to these effects may be of the order of or less than 20%. iii) As a matter of principle, the mesons with $I=1/2$ and 1 can be used as a test material of the ur -baryon models. Unfortunately these states have considerably large widths as the general tendency, this being quite unfavourable for our purpose.

§5. Concluding remarks

In this paper, we have tried to clarify our basic attitude by explicitly mentioning the involved meanings and the consistencies of the non-relativistic approach developed in I-III on the meson mass levels, their strong and electromagnetic interactions and also on the meson-baryon interactions. We divided all of these phenomena into the two levels, i.e. the dominant and the minor parts. Then we attempted to give reasonable interpretations for the former based on the N.R.P. and the selection principle (S.P.), while the latter was left for future studies as they are mainly induced by various

hyperfine effects. As for the better treatments of the latter, we need both the accurate data and the developments of the reliable theories on hadron as well as ur-baryon dynamics.

Among our results, we emphasize the importance of S.P., since it seems likely one of the direct reflections of the inner ur-baryon dynamics. It should be noted that S.P. is not only consistent with the $U(3)$ symmetry, but also it gives a definite pattern of its break-downs in the domain of hadron interactions when it is united with the configurations of physical hadrons. We also argue*) that S.P. is better than the A -selection rule³²⁾ in its wide applicabilities, its intuitive meaning and also in the sense mentioned above. For example, the former can be applied to any system made from hadrons, while it is not the case with the latter. We also note a possible violation of the A parity in the f' decays, i.e. it predicts $f' \rightarrow 2\pi$, if $f' \rightarrow K + \bar{K}$; the data shows $f' \rightarrow K + \bar{K}$, but $f' \rightarrow 2\pi$ or quite suppressed.

Next to S.P., we come to the indirect test of both the ur-baryon models and the N.R.P. We emphasize that our arguments presented in §4 are different from simple parameter adjustments. From the N.R.P., we are naturally led to the view that the real E.M. transitions between the two hadrons are directly connected with the changes of the inner ur-baryon states of the given hadrons rather than those of their surrounding clouds and thus the energy difference of the two systems goes directly to r ray, a familiar mechanism in atomic spectra. Let us wait how this analogy works in the level of new matter.

Finally we admit the applicability limit of the N.R.P. as well as the N R. dynamics. For example, we could not treat the processes with multi-pair effects nearly at one space-time point in any reasonable ways. For the descriptions of these processes, a certain new dynamical principle will be required.

Acknowledgements

It is my greatest pleasure to dedicate this paper to Professor S. Tomonaga in the celebration of his sixtieth birthday.

I would like to thank Professor T. Tati and members of Kanazawa University for their kind interest in this work, suggestive discussions and encouragement. I am also grateful to the members of the research group "On the Models and the Structures of Elementary Particles" for their suggestive discussions and helpful comments. In particular, I wish to thank Professor O. Hara and Dr. S. Ishida for providing me a stimulating circumstance and encouragement. Finally I am indebted to the Sakukokai Foundation for financial support.

*) The author is indebted to Professor S. Ogawa for various discussions and also pointing out the possible differences between the two rules.

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