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A Tale of Two Countries: A Directed Technical Change Approach

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A tale of two countries: a directed technical change approach

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Abstract

It is widely recognized that scientific research has a dramatic impact on economies since it is crucial to foster technological knowledge. Today's migratory movements and concentration of highly educated population and population with high scientific potential in developed countries play an essential role in enhancing research and boosting economic growth. We propose a North-South model that encompasses these empirical facts and proposes explanatory mechanisms. We show how the technological-knowledge gap is hard to reverse, namely when, due to higher returns, the majority of scientists are concentrated in the North. The implications of having either perfect- or no-labour mobility between countries are studied. In addition, it is showed the effect of complementarity or substitutability of goods on scientists' incentives may allow countries to avoid a poverty trap. The calibrated model provides consistent dynamics with actual data. Scientists' incentives are highlighted as the main source of divergence or convergence between countries.

Keywords: Direct Technical Change; Economic Growth; Inequality; Migration; Trade.
Jel classification: O31; O33; O47; F16; F22; J31.

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1. Introduction

We live in a period consensually identified in growth and development economics as the modern growth era (Galor, 2011), with specific features concerning the behaviour of macroeconomic and demographic variables. The migratory movements and concentration of scientific community and highly educated population in the developed countries (North) rather than in the underdeveloped countries (South) are one of the features. By North and South we mean two stylised countries that operate in the same economic environment, but the North has initial higher levels of technological knowledge and more access to novelties (Acemoglu and Zilibotti, 2001; Afonso, 2012). These features play an essential role in the persistent economic divergence of most of the world countries to the western ones. This divergence is observable mainly on the increasing technological gap, income per capita gap and on the movements of Southern population towards the North. Hence, we propose a North-South model that accommodates these empirical facts by exploring the intrinsic explanatory mechanisms that operate underneath. The main structure of the model and the mechanism involved, suggests that the present divergence between countries relies on the already established conditions of each country that come from the Industrial Revolution. Since developed countries are already in the fore front of technology and they concentrate most of worlds' scientific community, they create incentives for scientists to keep themselves in these developed countries. Thus, no scientist or high skilled person would want to be in the South and, instead, would want to move to the North. Then, this study aims to show how the power of these incentives is reinforced by the usual elements in international economics as it is migratory movements, trade and innovation. The main conclusions obtained point to five main conclusions: (i) divergence persists when goods traded are substitutable; (ii) migratory movements lead to the stretching of the gaps (iii) with substitutable goods innovation only occurs in the North; (iv) trade of complementary goods leads to a catching up process of the South and (v) with complementary goods innovation occurs in both countries.

Migratory movements from the South towards the North have emerged from the population's need to seek higher quality of life and higher returns. Today, a strong intensification of skilled-labour migration (researchers and scientists) has been taking place. According to Özden *et al.* (2011), the world faced an increase in the stock of migrants from 92 million in 1960 to 165 million in 2000. Between the 1960s and the 2000s, the increase in the number of migrants was mostly due to Southern to Northern movements. Docquier and Marfouk (2006) conclude that almost 40% of total emigration to OECD countries comes from low and low-middle income countries. The smallest and poorest countries have high rates of emigration, notably for skilled emigrants, and the USA, Canada, Australia, the UK, France and Germany absorb about 85 per cent of these skilled emigrants. Many of the skilled individuals flee towards Northern countries (namely OECD countries), where they can benefit from higher returns on their discoveries (Weinberg, 2011). These findings support the main insights provided by the extensive literature on brain drain, which traditionally assumes a damaging impact on home countries due to the loss of skilled population needed for production efficiency and to endure the innovation process (e.g., Gibson and Mckenzie, 2011, for a concise analysis of this literature).

Some new literature on this topic (Stark, 2004; Dumond and Lemaître, 2005) shows evidence against the usually claimed negative impacts for home countries, by stating that there may be some gains on welfare and growth deriving from this brain drain process. However, there is still controversy regarding these more recent findings as other studies have concluded that these gains may not exist or are too small (e.g., Schiff, 2006).

Apart from the above discussion, it is undisputable that skilled labour and, more precisely, the scientific community, positively affect economic development by generating technological knowledge, as stressed by seminal endogenous growth models (e.g., Lucas, 1988, Romer, 1990, Aghion and Howitt, 1992). Thus, in order to foster growth, it would be beneficial for the South to educate and maintain its scientific community. However, in spite of some Southern countries being able to produce high skilled population and potential future scientists and researchers, they fail to keep them in the country. In fact, not only high skilled population tend to move to the North but also most of the persons from the South who conduct their research in North have already acquired their education in this last region. Therefore, we observe a high concentration of scientists and researchers in the North while the South appears to be deprived from this specific population. Looking at the available data,[2] we can find that, despite some dispersion, there is quite a significant correlation between the number of scientists and income per capita (Figure 1). Incentives for education and scientific research in the North (UNESCO, 2007) as well as the brain drain phenomenon lead to an accumulation of scientists in this area and a shortage in the South. Thus, the pace of innovation in the South is negatively affected, leading to a higher North-South technological-knowledge gap. In turn, from the number of patents issued in the last two decades (Figure 2), Middle and Upper-Middle-Income countries have been catching up with high income countries in the last five years (mostly due to the BRIC's – Brazil, Russia, India and China).[3] Lower-Middle-Income countries have few patents. Low-Income countries have no available data; we assume the extremely low level to be zero throughout the entire period.

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[2] Data was available for 64 countries, for both variables (income per capita and researchers), gathered from World Development Indicators (WDI) dataset.

[3] Although patents have some drawbacks because they do not completely cover all innovative outcomes, we will assume they are "interpreted as indicators of invention (a precursor to innovation), and there is a positive relationship between patent counts and other indicators of inventive performance such as productivity…" (OECD, 2010, pp. 47).

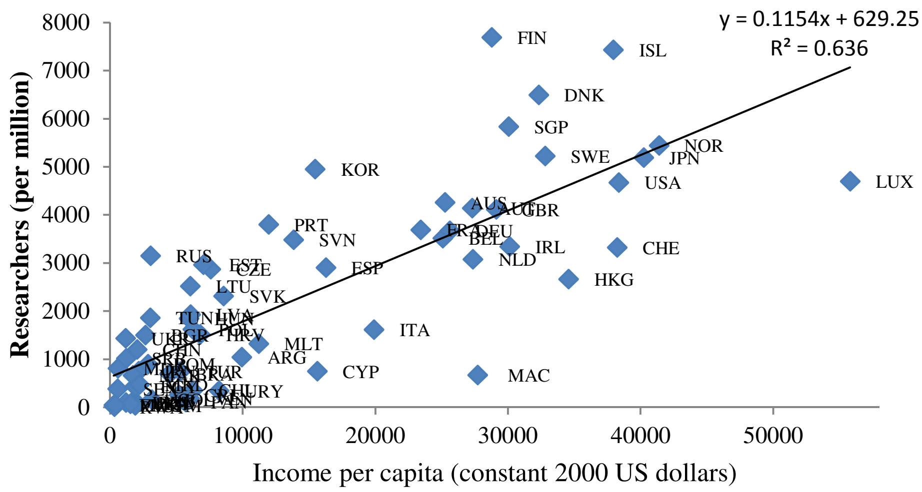

Figure 1 - Correlation between researchers and income per capita - 2008 (authors' own elaboration)

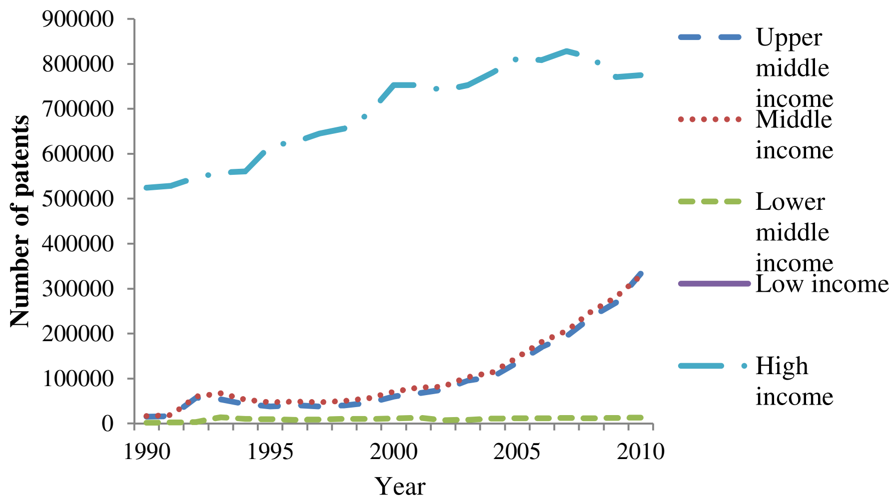

Figure 2 - Number of patents issued each year (authors' own computations from WDI database)

The gap has increased for the latter two groups of countries, which shows that growth is very low in such regions. Moreover, several models and empirical studies confirm that the technological-knowledge gap may affect the growth rate of countries, which can lead them to diverge in relation to higher developed countries (Acemoglu and Zilibotti, 2001; Afonso, 2012).

Another specific feature of our modern growth era is related to inter-country distribution of income. Some authors claim that inequality is increasing (Milanovic, 2009 and Zanden *et al.*, 2011), while others (e.g., Sala-i-Martin, 2006) infer a reduction on global poverty rates and a progressive decline of inequality albeit with some exceptions, such as Africa. Nevertheless, since the emergence of the Industrial Revolution, the world has experienced a process of divergence between the richest and the poorest regions (e.g., Maddison, 2008) that has persisted through time (Maddison, 2003). Today, the gap measured in terms of GDP per capita is considerable, having attained a ratio of 18:1 in 2000 (Galor, 2011). Moreover, contrary to the evidence available for the

19th century, inequality now occurs mainly between countries rather than within countries (Milanovic, 2009).

As already referred, our North-South model explains this growth divergence through scientists' incentives. We replicate the persistency of divergence through the existence of incentives to scientists to keep researching in the North. This will remove the only source of growth in the South, causing it to economically stagnate and, hence, increase divergence between them. In a nutshell, the proposed North-South model encompasses the empirical facts identified above such as why the divergence process has been persistent, why the absolute majority of scientific research is concentrated in the developed countries of North, while the less developed countries of South produce almost no research at all, and the patterns of unskilled labour movement. It suggests that technology and growth depend mostly on research, particularly on the allocation pattern of scientists between countries. The willingness of scientists to research in a country relies on their profits which, according to our model, are affected by the type of trade (complementary or substitute goods) and the level of population mobility between the two countries. By considering two levels of labour mobility and international trade, we are indeed able to reproduce the observed paths of income, population and innovation during the modern economic era. We adapt the framework of Acemoglu *et al*. (2012) for the decentralized economy.[4]

Although the literature proposes several different approaches for these research questions (Matsuyama, 2000; Galor and Mountford, 2008; Borgy *et al*, 2010; Docquier *et al*, 2010), we try to accommodate in just one model the specific features referred above (scientific research, level of substitutability of goods and population mobility) to understand how they may affect the economic fate of countries. Thus, the model shows which incentives the scientific community seeks to promote innovation and move to a country, and, hence, shows some of the mechanisms (e.g., incentives, type of traded goods) that influence countries' growth dynamics.

The paper is organized as follows. In section 2, we define the set-up of the model and the main assumptions. In section 3, we provide an analysis of the main results of the model. In section 4, we develop a quantitative exercise to verify the empirical consistency of our theoretical model. Finally, a discussion is drawn in the closing section.

2. Model setup

We consider two blocks of countries – North, d (developed), and South, u (underdeveloped) – with mobility of capital and free trade of goods (final goods are assumed to be non-tradable). Regarding labour mobility, we consider two hypotheses: perfect mobility and immobility. The population of both countries is defined as a continuum of workers and scientists.

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[4] Also in a directed technical change context, Acemoglu *et al*. (2012) have studied the impacts of dirty and clean technologies on the environment.

Following Acemoglu *et al.* (2012) we consider an infinite-horizon discrete-time economy where a continuum of households comprising workers, entrepreneurs and scientists lives. The economy's household is a representative one with preferences $\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} U(C_t)$, where $\rho > 0$ is the discount rate[5]. The final good is a composite, non-tradable good, competitively produced, using goods produced in each country. For the sake of simplicity, each country only produces one good, which enforces trade between countries. The aggregator, for each country, is given by:

$$Y_t^i = \left(\left(Y_{it}^N\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(Y_{-it}^F\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where: Y_t^i is the country i final good production at time t and Y_{it}^N (Y_{it}^F) is the good produced by country i at time t to be consumed internally (exported), i.e., $i \in \{d,u\}$, $-i \in \{u,d\}$. The subscript *N* stands for national and the subscript *F* stands for foreign. Since we are dismissing any international lending or borrowing, and since free trade exists in the world, each country must run a balanced international trade; thus, we must guarantee $p_{ut}Y_{ut}^F = p_{dt}Y_{dt}^F$. Where p_{it} is the intermediate good i price. Furthermore, $\varepsilon \in (0,+\infty)$ is the elasticity of substitution between goods of the two countries; they are substitutable (complementary) if $\varepsilon > 1$ ($\varepsilon < 1$).

The two goods are produced using a continuum of labour and country-specific machines supplied by monopolistically competitive firms led by scientists:

$$Y_{it} = L_{it}^{1-\alpha} \int_0^1 A_{iat}^{1-\alpha} x_{iat}^{\alpha}\, da, \tag{2}$$

where: $\alpha \in (0,1)$; A_{iat} is the technology value of each type *a* machine used in country $i \in \{d,u\}$ at time *t*; x_{iat} is the machinery used in each country. This aggregate production is divided between national consumption and exports: $Y_{it} = Y_{it}^N + Y_{it}^F$.

The market clearing condition for the final good is given by:

$$C_t{}^i = Y_t{}^i - \psi\left(\int_0^1 x_{iat} da\right) - p_{it}Y_{it}^F + p_{-it}Y_{-it}^F, \tag{3}$$

where ψ is the unit cost of machines (equal in both countries), normalized to $\psi \equiv \alpha^2$.

The model, following Acemoglu *et al* (2012), works by considering that, at the beginning of every period, scientists decide whether to be in one or another country and do research on machines in that country. Scientists are independent of machines that exist in both countries, being randomly allocated to one type of machine. Each scientist is the only one working on it. The probability of success in innovation on that machine is $\varrho_i \in (0,1)$, $i \in \{d,u\}$, which increases its technological knowledge by $(1+\gamma)$, with $\gamma > 0$.[6] Each scientist may become a monopolist if s/he achieves a better version of a machine, obtaining monopoly rights for one period. A successful scientist, who has invented a better version of machine, obtains a one-period patent and becomes the entrepreneur for the current period in the production of machine in country i. When innovation is not achieved, a

[5] The model will focus the analysis on the decentralized economy where household preferences are not directly implemented, so any consideration on households' side is not fully extended here.

[6] The value of γ has to be sufficiently high to avoid any older machine, of lower quality, to be viable in the market.

random scientist is chosen and the monopoly rights are allocated randomly to the potential entrepreneurs who then use the old technology. We normalise the quantity of scientists to 1 and there are s_{it} located at *i* at each *t*. The market clearing condition for this case is:

$$s_{dt} + s_{ut} \leq 1, \tag{4}$$

As for labour any particular difference between countries is defined as:

$$L_{dt} + L_{ut} \leq 1 \qquad \text{or} \qquad L_{it+1} = (1 + n_i)L_{it}\ , \tag{5}$$

for the cases where there is either perfect- or no-labour mobility between countries, respectively. We assume that $n_u > n_d$, which is in line with real data.

On the technology used, we can define the aggregate level for each country at time *t*:

$$A_{it} \equiv \int_0^1 A_{iat} da, \tag{6}$$

which evolves according to a rule expressed by the following difference equation:

$$A_{it} = (1 + \gamma \varrho_i s_{it}) A_{it-1}. \tag{7}$$

Note that scientists can either be only in one country ($s_{it} = 1$) or be allocated to both countries ($s_{it} = s$). As we will see below, unless there is equilibrium in the scientists' profit ratio (complementary goods case) they will be allocated only to one of the countries.

3. The two countries model

Following Acemoglu *et al*. (2012), we develop the model for the decentralized economy. Agents and firms act according to their own interests and following the incentives provided by the market. The idea is to emphasize the struggle the South faces to get on to the development track. The final objective is to observe the behaviour of agents, namely of workers and scientists, that affect the economic growth rate in both countries and which may lead economies either to stagnancy or to continuous growth and development.

The equilibrium is given by sequences of wages, w_t, prices for inputs, p_{it}, prices for machines, p_{iat}, demands for machines, x_{iat}, demands for each good, Y_{it}, labour demands, L_{it}, research allocations, s_{it}, and technological knowledge A_{it}. In each *t*: (i) the pair (p_{ait}; x_{ait}) maximizes profits by the producer of machine *a* in country *i*; (ii) L_{it} maximizes profits by producers of input of country *i*; (iii) Y_{it} maximizes the profits of final good producers; (iv) s_{it} maximizes the expected profit of a researcher at *t*; (v) w_{it} and p_{it} clear the labour and input markets, respectively. From the maximization problem of the competitive final-good market for the world economy results:

$$\frac{p_{ut}}{p_{dt}} = \left(\frac{Y_{dt}}{Y_{ut}}\right)^{\frac{1}{\varepsilon}}, \tag{8}$$

and for each country, in a free market situation, we obtain the relative prices of the two inputs for the North and South, respectively:[7] $\frac{p_{ut}}{p_{dt}}=\left(\frac{Y_{ut}^{N}}{Y_{dt}^{F}}\right)^{-\frac{1}{\varepsilon}}$ and $\frac{p_{dt}}{p_{ut}}=\left(\frac{Y_{dt}^{N}}{Y_{ut}^{F}}\right)^{-\frac{1}{\varepsilon}}$. As expected, from (8), relative prices are decreasing with relative supply. Furthermore, the price of the final good is normalized to one in both countries, from (1):

$$\left[p_{ut}^{1-\varepsilon}+p_{dt}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1, \tag{9}$$

To study the incentives scientists have for conducting research into machines for a specific country, we need first to find the expected profits. This helps defining what the decisions of scientists will be and, hence, the direction of technological change. Thus, we need to determine the demand for machines in each *t*, which is achieved through profit maximization of producers of each input, yielding the demand for machines:

$$x_{iat}=\left[\frac{\alpha p_{it}}{p_{iat}}\right]^{\frac{1}{1-\alpha}}L_{it}A_{iat}, \tag{10}$$

We can maximize profits of machine producers using this inverse demand curve as a constraint. Taking profit maximization as $\pi_{iat}=(p_{iat}-\psi)x_{iat}$, where, for $\psi=\alpha^2$, we get $p_{iat}=\alpha$, which yields $x_{iat}=[p_{it}]^{\frac{1}{1-\alpha}}L_{it}A_{iat}$. The total production for each country is:

$$Y_{it}=p_{it}^{\frac{\alpha}{1-\alpha}}L_{it}A_{it}. \tag{11}$$

We are able to determine explicitly the profit of machine producers, too:

$$\pi_{iat}=(1-\alpha)\alpha p_{it}^{\frac{1}{1-\alpha}}L_{it}A_{iat}. \tag{12}$$

Using (12), the average profit of any scientist that decides to do research in *i* at *t* is:

$$\Pi_{it}=\varrho_i(1+\gamma)(1-\alpha)\alpha p_{it}^{\frac{1}{1-\alpha}}L_{it}A_{it-1}. \tag{13}$$

A self-reinforcing effect on research may occur in one of the countries since when there is innovation in one of them profits for scientists working there increase. Indeed, from (13), scientists expect higher profits if they pursue working in that country and, from the process of allocation of scientists, more innovation occurs there, see (7). This means more produced goods, see (11), and allows more consumption. Thus, there is a self-reinforcing effect, increasing the attractiveness of the country for scientists and creating conditions for growth and development. The relation of scientists' profits with the decline or rise of growth and development in a country is better understood by looking at the ratio of scientists' profits:

$$\frac{\Pi_{ut}}{\Pi_{dt}}=\frac{\varrho_u}{\varrho_d}\left(\frac{p_{ut}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}\frac{L_{ut}}{L_{dt}}\frac{A_{ut-1}}{A_{dt-1}}. \tag{14}$$

We detect three main factors affecting the ratio of profits (14). The first is the price (of inputs) effect, $\left(\frac{p_{ut}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}$: the higher the relative price in the South the higher the profits of scientists

[7] By equalizing these two equations and considering total production equal to $Y_{jt}=Y_{jt}^{N}+Y_{jt}^{F}$, we arrive to expression (8), which will be used afterwards, as it helps find comprehensive expressions.

choosing that country. From computations, relative prices are inversely related with the technological-knowledge gap. Hence, technology research would direct towards the South, favouring its catch up process. The second factor is the market size effect, $\frac{L_{ut}}{L_{dt}}$: the higher the employment is in one country, the wider the available market and, thus, the higher the profits. The third is the productivity effect, $\frac{A_{ut-1}}{A_{dt-1}}$: it benefits the country with higher innovation; for instance, a higher innovation ratio makes scientists better off in the South. The analysis below will consider goods as substitute, except for the last subsection where we will study the effects on the South of producing goods that are complementary to goods produced in the North.[8]

3.1. No-labour mobility hypothesis

Considering first the hypothesis that only scientists can move from one country to another, in (5), we assume an exogenous labour ratio $\frac{L_{ut}}{L_{dt}} = l$ that grows at a rate $(n_u - n_d) > 0$. Then, we solve the profit ratio by applying conditions obtained from maximization of profit functions of firms, scientists and final good producers. Departing from (14), we reach the final profit ratio:

$$\frac{\Pi_{ut}}{\Pi_{dt}} = \frac{\varrho_u}{\varrho_d} l^{\frac{\varphi}{\varphi-1}} \left(\frac{1+\gamma\varrho_u s_{ut}}{1+\gamma\varrho_d s_{dt}}\right)^{\frac{1}{\varphi-1}} \left(\frac{A_{ut-1}}{A_{dt-1}}\right)^{\frac{\varphi}{\varphi-1}}, \tag{15}$$

where $\varphi = (1-\varepsilon)(1-\alpha)$. From this expression we can already infer that not only technology but also exogenous population growth (market-size effect) have a role in determining the decisions of scientists. The market-size effect favours the South.

***Condition* 1:** For the sake of simplicity, if only technology in the North grows, the growth of the technological-knowledge gap overcomes the growth in the population ratio: $\left|\widehat{\left(\frac{A_{ut-1}}{A_{dt-1}}\right)}\right| = |-\gamma\varrho_d| \geq |(n_u - n_d)| = |\hat{l}|$.

Moreover, we assume that initially the North is technologically more advanced, $A_{d0} > A_{u0}$,[9] and more profitable for scientists $\frac{\Pi_{ut}}{\Pi_{dt}} < 1$. Thus, since $\frac{\Pi_{ut}}{\Pi_{dt}}(s)$, with $s_{ut} = s$, is increasing in *s,* then we need to guarantee that $\frac{\Pi_{ut}}{\Pi_{dt}}(1) < 1$ for $s_{ut} = 1$, $\left(\frac{\varrho_u}{\varrho_d}\right)^{\frac{1-\varphi}{\varphi}} l^{-1}(1+\gamma\varrho_u)^{-\left(\frac{1}{\varphi}\right)} > \frac{A_{ut-1}}{A_{dt-1}}$. Thus:

***Condition* 2:** $\left(\frac{\varrho_u}{\varrho_d}\right)^{\frac{1-\varphi}{\varphi}} l^{-1}(1+\gamma\varrho_u)^{-\left(\frac{1}{\varphi}\right)} > \frac{A_{ut-1}}{A_{dt-1}}$.

These conditions affect the path of the economies. *Ceteris paribus*, under substitutability, the higher the technological-knowledge gap, the lower is the profit ratio and thus the more profitable is

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[8] Note that our goal is mostly to understand the dynamics of the model concerning the effects on workers and scientists movements, as well as the effect on technological knowledge, output, prices and trade. Hence, the household intertemporal problem is not treated in our derivations.

[9] This is a trivial assumption since it is what is empirically verified (Hall and Jones, 1999).

for scientists to stay in the North ($s_{dt} = 1$), note that $\frac{\varphi}{\varphi-1} > 0$ and Condition 1 – see (15). Moreover, with an initial technological ratio obeying Condition 2, the profit ratio decreases even more because scientists choose the North. This reinforces the cumulative effect of technology in the North, see (7), so that technology remains stagnant in the South. From another perspective, we can draw the conclusion that the price and the market effects are not sufficiently strong to overcome the productivity effect, see (14). More specifically, productivity gains in the North are stronger than increasing relative prices and than increasing population in the South (Condition 1), yielding higher returns for intermediate firms and, consequently, for scientists that sell machines to the North. The expectation that the South will attract scientists diminishes over time. We can draw a Lemma from the conditions above and the analysis of (15):

Lemma* 1*: *In a decentralized economy, it is equilibrium for innovation occurring in*:

(i) *the South at time t only when* $\varrho_u l^{\frac{\varphi}{\varphi-1}}(1+\gamma\varrho_u)^{\frac{1}{\varphi-1}}(A_{ut-1})^{\frac{\varphi}{\varphi-1}} > \varrho_d (A_{dt-1})^{\frac{\varphi}{\varphi-1}}$,

(ii) *in the North when* $\varrho_u l^{\frac{\varphi}{\varphi-1}}(A_{ut-1})^{\frac{\varphi}{\varphi-1}} < \varrho_d(1+\gamma\varrho_d)^{\frac{1}{\varphi-1}}(A_{dt-1})^{\frac{\varphi}{\varphi-1}}$ *and*

(iii) *in both countries if* $\varrho_u l^{\frac{\varphi}{\varphi-1}}(1+\gamma\varrho_u s_{ut})^{\frac{1}{\varphi-1}}(A_{ut-1})^{\frac{\varphi}{\varphi-1}} = \varrho_d(1+\gamma\varrho_d s_{dt})^{\frac{1}{\varphi-1}}(A_{dt-1})^{\frac{\varphi}{\varphi-1}}$, *for* $s_{it} \neq 0$ *and* $s_{ut} + s_{dt} = 1$

Adding condition 2, this leads to the following proposition:

Proposition* 1*: *If we have substitutable goods* ($\varepsilon > 1$) *and Condition* 2 *applies, then there exists a unique decentralized equilibrium where innovation always occurs in the developed country* (*see explanation above*).

Regarding the dynamics of the technological-knowledge gap since A_{ut} has a zero growth rate, while A_{dt} grows at a rate of $\gamma\varrho_d$, the ratio $\frac{A_{ut}}{A_{dt}}$ constantly falls at the same rate. We usually observe the stretching of the technological-knowledge gap since the North remains in the frontier, while the South cannot catch up with this technological knowledge, often adopting imitation activities (substitute goods) to try to overcome the technological-knowledge gap (Afonso (2012)).

The paths of prices and workers evolve over time depending on the growth rate of the population ratio and the technological-knowledge gap. Relative prices can be defined by $\frac{p_{ut}}{p_{dt}} = \left(\frac{W_{ut}}{W_{dt}}\right)^{1-\alpha}\left(\frac{A_{dt}}{A_{ut}}\right)^{1-\alpha} = \left(\frac{A_{ut}}{A_{dt}}\right)^{\frac{1-\alpha}{\varphi-1}} l^{\frac{1-\alpha}{\varphi-1}}$, where the left part is computed using the labour maximization process on intermediate firms' profits, yielding $\frac{W_{ut}}{W_{dt}} = \left(\frac{A_{ut}}{A_{dt}}\right)^{\frac{\varphi}{\varphi-1}} l^{\frac{1}{\varphi-1}}$. Relative wages rely on the technological-knowledge gap and population growth. Both ratios, for $\varphi < 0$, entail a decreasing wage ratio. More advanced technology implies more labour productivity and thus higher wages in the North, whereas a higher population gap implies a lower wage ratio. As for relative prices, there are two opposite effects. Wages increase in the North, which leads to greater purchasing power of its population leading to pressure on demand. However, technological improvements happen in the North, which reduces marginal costs of production, allowing for prices in the North to be lower than in the South. The overall outcome, from Condition 1, is an increase of relative prices.

On the output side, the pace of relative output follows the pace of prices (using 8). As long as relative prices increase, relative output $\left(\frac{Y_{ut}}{Y_{dt}}\right)$ decreases, but at a higher rate than relative prices since $\left(\widehat{\frac{Y_{ut}}{Y_{dt}}}\right) = -\varepsilon\left(\widehat{\frac{p_{ut}}{p_{dt}}}\right)$. Yet, both countries increase their outputs although at different rates. In fact, in the long run, $\hat{Y}_{dt} = \hat{A}_d + \hat{L}_d$ and $\hat{Y}_{ut} = \frac{\alpha}{1-\varphi}\hat{A}_d + \hat{L}_u - \frac{\alpha}{1-\varphi}\hat{l}$ are positive: (i) Y_{dt} grows due to technological knowledge and population growth; (ii) Y_{ut} grows because there is population that demands Southern goods and there is international trade between countries (L_u for internal demand and A_d for external demand due to international trade), which means that for not too high levels of substitutability between goods, the high Northern demand of goods will lead to an increase in Southern exports. More specifically, Y_{ut}^F grows more than Y_{ut}^N due to the difference in prices between the two goods. The North produces high quantities of its good efficiently, while the South produces its own good less efficiently. However, even so, with no mobility of population and little substitutability between goods, trade between countries occurs.

Concerning output per worker, y_{it}, in the very long run, and using (11), it becomes $y_{it} = p_{it}^{\frac{\alpha}{1-\alpha}}A_{it}$ and its growth rate is $\hat{y}_{it} = \frac{\alpha}{1-\alpha}\hat{p}_i + \hat{A}_i$ where, by the maximization problem of intermediate producers, we reach for the North $\hat{y}_d = \hat{A}_d$ and for the South $\hat{y}_u = \frac{\alpha}{1-\varphi}\hat{A}_d - \frac{\alpha}{1-\varphi}\hat{l}$. This means, for high substitutability levels, that $\lim_{\varepsilon\to\infty}\hat{y}_u = 0$. Whatever the level of ε, $\hat{y}_{ut}$ would be significantly inferior compared to $\hat{y}_{dt}$. Thus, we conclude that output per worker grows more in the North stretching the gap between countries. For some extreme conditions, we observe stagnation in the South.

3.2. Perfect-labour mobility hypothesis

Considering now perfect-labour mobility – workers as well as scientists can freely move between countries –, we obtain the solution for the price ratio (here (8) will be employed) with respect to technology and we also solve the market effect with respect to the technological-knowledge gap – see (5). Provided that we assume perfect-labour mobility, wages now equalize, $\frac{w_{ut}}{w_{dt}} = 1$. Therefore, we will obtain the following expression:

$$\frac{\Pi_{ut}}{\Pi_{dt}} = \frac{\varrho_u}{\varrho_d}\left(\frac{1+\gamma\varrho_d s_{dt}}{1+\gamma\varrho_u s_{ut}}\right)^{\varphi+1}\left(\frac{A_{ut-1}}{A_{dt-1}}\right)^{-\varphi} \tag{16}$$

We assume again $A_{d0} > A_{u0}$ as well as conditions to ensure that initially $\frac{\Pi_{ut}}{\Pi_{dt}} < 1$. Depending on $\varphi + 1 < 0\ (> 0)$, $\frac{\Pi_{ut}}{\Pi_{dt}}(s)$ is decreasing (increasing) in *s* and so we need the following condition to allow for the same effects on the profit ratio observed in the previous case:

$$\textbf{\textit{Condition 3:}}\ min\left\{\left(\frac{\varrho_u}{\varrho_d}\right)^{\frac{1}{\varphi}}(1+\gamma\varrho_u)^{-\left(\frac{\varphi+1}{\varphi}\right)}, \left(\frac{\varrho_u}{\varrho_d}\right)^{\frac{1}{\varphi}}(1+\gamma\varrho_d)^{\frac{\varphi+1}{\varphi}}\right\} > \frac{A_{ut-1}}{A_{dt-1}}.$$

Note that the higher the technological-knowledge gap, the lower is the profit ratio and, thus, the more profitable is for scientists to stay in the North, with $\varphi < 0$ (see 16). The reinforcement of the cumulative effect of profits in the North due to technology (see 7) will again keep the South stagnant. Nevertheless, now the dynamics of both economies will rely on the elasticity of substitution of the two countries' goods. The outcomes of both hypotheses will differ depending on whether we have weak or strong substitutability.

Keeping the same method of analysis, the price effect is now the only one that positively affects the profit ratio. The market effect is now endogenous and because the technological ratio implies a decreasing population ratio, both market and technological effects offset the price effect. Indeed, the productivity and market-size gains in the North are stronger than increasing prices in the South, yielding higher returns for intermediate firms in the North and, thus, for scientists that sell machines there. Hence, from the conditions above and the analysis of (16), we can state:

***Lemma* 2:** *In a decentralized economy, it is equilibrium for innovation occurring in*:
(i) *the South at time t only when* $\varrho_u(1+\gamma\varrho_u)^{\varphi+1}(A_{ut-1})^{-\varphi} > \varrho_d(A_{dt-1})^{-\varphi}$,
(ii) *the North when* $\varrho_u(A_{ut-1})^{-\varphi} < \varrho_d(1+\gamma\varrho_d)^{\varphi+1}(A_{dt-1})^{-\varphi}$ *and*
(iii) *both countries if* $\varrho_u(1+\gamma\varrho_u s_{ut})^{\varphi+1}(A_{ut-1})^{-\varphi} = \varrho_d(1+\gamma\varrho_d s_{dt})^{\varphi+1}(A_{dt-1})^{-\varphi}$, for $s_{it} \neq 0$ and $s_{ut} + s_{dt} = 1$

Considering also condition 3, this leads to the following proposition:

***Proposition* 2:** *If we have substitutable goods* ($\varepsilon > 1$) *and condition* 3 *applies, then there exists a unique decentralized equilibrium where innovation always occurs in the developed country (see explanation above).*

On the dynamics of the economic variables, and knowing that, as before, $\hat{A}_u = 0$ and $\hat{A}_d = \gamma\varrho_d$, we can analyse the path of prices and workers over time. The relative prices gap is now given by $\frac{p_{ut}}{p_{dt}} = \left(\frac{A_{dt}}{A_{ut}}\right)^{1-\alpha}$, meaning that it increases on the technological-knowledge gap (more efficient production in the North means better terms of trade for the North itself). The endogenous employment ratio is now given by $\frac{L_{ut}}{L_{dt}} = \left(\frac{A_{ut}}{A_{dt}}\right)^{-\varphi}$. For $\varphi < 0$, the ratio falls so that we observe a flow of immigrants from the South to the North.

This movement, jointly with the concentration of scientists in the North, can be matched with the referred brain drain and immigration theses, which advocate that the most productive agents will move to the North along with workers in order to improve their quality of life. It replicates the migratory trends observed on the borders of Northern countries (North Africa and Southern Europe, or Mexico and the USA). The empirical verification of the brain drain phenomenon is also sketched by researchers' movements and permanence in developed areas (as referred to in the introduction). This is exactly what is simulated here. Scientists searching for higher profits will move to and stay in the North. By the same token, since employment increases in the North, workers will move there to fill the open vacancies. There is a clear population movement to the North, depleting the South of population and, particularly, scientists who are the source of innovations and, thus, growth and development.

Concerning the output produced in each country, we use again optimizing equations for intermediate goods in each country to determine its dynamics. We now distinguish between weak ($\alpha + \varphi > 0$) and strong substitutes ($\alpha + \varphi < 0$). Each case yields different dynamics of output and, hence, different facts regarding the behaviour of both economies.

For strong substitutes we know that Y_{ut} (Y_{dt}) will grow negatively (positively) in the long run, $|(\alpha + \varphi)\gamma\varrho_d|$ and $\gamma\varrho_d$.[10] These rates mean a clear fall of the South into a poverty trap. Strong substitutability will cause the substitution of goods produced in the South for goods produced in the North in the final good production function ($\hat{Y}_{ut}^N < 0$). Depending on the parameterization, this may cause an increase in the share of imported goods in the South, Y_{dt}^F, in a first stage, but then it will also converge to zero in the long run since production will cease in the country. This is understandable if we think of the trade-off consumers face when buying goods; they can choose between cheaper Northern goods and more expensive Southern goods to achieve the same utility. They will choose the cheapest so that production in the South will fall through time.

Finally, to measure the performance of each economy, we compute the output per worker by using the maximization problem of goods producers $\hat{p}_{it} = (1-\alpha)\hat{w}_t - (1-\alpha)\hat{A}_{it}$.[11] The result shows again that $\hat{y}_{dt} = \gamma\varrho_d$ and that $\hat{y}_{ut}$ grows slowly over time since innovation does not occur in the South; thus, $\hat{y}_{ut} = \alpha\,\gamma\varrho_d$. The output per worker now grows since population in the South is falling faster than the output. Still, the gap between countries increases.

In the case of weak substitutes, the effects are not so straightforward. Innovation still occurs only in the North, but there is not a complete switch of production to that country. Looking closely at the growth rates, in the long run, output in the North grows at the same rate as before, $\gamma\varrho_d$, while in the South grows at a lower rate: $(\alpha + \varphi)\gamma\varrho_d > 0$. Hence, output in the South increases slowly;

[10] Thus, its level will reach zero in the long run and its growth rate gets bounded to zero; we are admitting no population growth.

[11] Where $\hat{w}_t$=$\hat{A}_{dt}$ from the relations obtained between output per capita and growth rates of labour and output. Wages then increase in both countries since there is free inter-country labour mobility.

but the country still ends up in a poverty trap, as in the strong substitution case, since the output gap increases. Substitutability causes the substitution of Southern goods by Northern ones in final-good production: $\widehat{\left(\frac{Y_{ut}^N}{Y_{ut}}\right)} < 0$. At the same time, Y_{ut}^F increases at decreasing rates. The latter grows asymptotically at a rate equal to $\hat{Y}_{ut}$. Again, the trade-off faced by consumers when deciding which goods to buy leads to this behaviour. Similar to the strong substitutes' case, output per worker shows the same performance, but in the present situation it increases since population is decreasing at a lower rate than in the latter case and, also, since output is increasing.

3.3. Complementary goods

We now examine the effects of having complementary goods. This study provides different insights on the behaviour of scientists and countries and may offer a different perspective on how some technologically backward countries may benefit from international trade. More specifically, the study discusses in which situation they produce goods that complement the Northern ones in final good production, $0 < \varepsilon < 1$, instead of competing with Northern goods. Looking at (15) or (16), by initially having a higher technological level in the North (taking Condition 2 or Condition 3 for initial values), turns research more profitable in the South. According to the no-labour mobility hypothesis, with substitute goods, both price and market effects are combined making returns in the South higher than in the North. With perfect mobility, both market and price effects, $\left(\frac{A_{ut}}{A_{dt}}\right)^{-\varphi}$ and $\left(\frac{A_{ut}}{A_{dt}}\right)^{-1}$, respectively, tend in the same direction when the technology ratio varies.

According to the two hypotheses the technological effect, see (15) and (16), is supplanted by the other two effects since, with the new conditions, the higher the technological ratio $\left(\frac{A_{dt}}{A_{ut}}\right)$, the more profitable is researching in the South. Indeed, as goods are complementary, there will be demand for both. A greater demand for the North's goods will also mean higher demand for the South's goods. This effect increases profits and creates incentives for scientists to extract these profits in the South. Following the analysis above we establish the proposition:

***Proposition* 3:** *If we have complementary goods* ($\varepsilon < 1$) *and conditions* 2 *or* 3 *apply, then in a decentralized economy beginning with a superior technology in the North, innovations will occur first in the South until there is a catching up process with the North. From then on, innovation will occur in both countries* (*see explanation in the text*).

Thus, from above, at the beginning of the process scientists are located in the South where profits are higher so that, for $\varphi > 0$, s_{ut} is 1. As this happens, innovation initially occurs in the South. Then, $\frac{A_{dt}}{A_{ut}}$ decreases and so profits for scientists increase in the North. As the process continues, the profit ratio comes to one. Once the profit ratio approaches one, there is a distribution of scientists between countries, as they become increasingly indifferent to location.[12] The world economy reaches a stabilized solution, where $s_{dt}, s_{ut} > 0$ and $s \in (0,1)$ such that $s_{dt} + s_{ut} = 1$. The growth rate of each technology is roughly given by $\gamma \varrho_i \bar{s}_i$, where $\bar{s}_i$ is the allocation of scientists in each country i. $\frac{A_{ut}}{A_{dt}}$ forever remains in a steady state under perfect-labour mobility. In the case of no-labour mobility it decreases. Nevertheless, the economy is in a "steady state" given that the profit ratio does not change because the growth of the population ratio compensates for the increase in the technological ratio.

Regarding the transition phase, A_{ut} will grow at rate $\gamma \varrho_u$ until the South catches up to the North. For both hypotheses, in the transition phase, relative prices, $\frac{p_{ut}}{p_{dt}}$, decrease over time. The moment the profit ratio equals 1, so that the economies attain the "steady state", relative prices remain stable over time since innovation and population ratios compensate each other (under no-labour mobility) or innovation rates in both countries offset each other (under perfect-labour mobility). During the transition process, the employment ratio (only in the case of perfect-labour mobility) is higher in the South (since we assume a technological ratio lower than one), leading afterwards to a migratory phenomenon towards the North - note assumption (6) – until both economies stabilize around the equilibrium value $\frac{L_{ut}}{L_{dt}} = \left(\frac{\varrho_d}{\varrho_u}\right)(1 + \gamma \varrho_u)^{\varphi+1} > 1$.

Population migrates during the transition period for different reasons than those in the substitute goods case. More innovation in the South now leads to more efficient production and, hence, producers need fewer workers. In the North, there is an increase in production because demand increases as goods are complementary and there is no innovation. This case differs from the substitute - goods case since, despite innovation occurring in the North, there is also a huge increase in production - all demand is directed to the good produced in the North, while in the South production tends to decrease (see perfect–labour mobility hypothesis) or increase very slowly. Nevertheless, notice that population at the end of the process is still higher in the South since the technological-knowledge gap will not be completely closed (given the previous assumptions). The North is still in the technological frontier while the other country has the same rate of technological growth, but remains behind the frontier. Although its scientists allow for the same level of technological innovation, the initial lag inhibits the complete catch-up for the South.

On the output side (assuming no-labour mobility), output grows in both countries. In the South, it grows faster since, in the transition phase, innovation only takes place there. In the North, we verify that output growth relies mostly on population growth (and hence on demand) as well as on innovation in the South, and the subsequent increase in its output that will boost demand for the North's goods in order to produce the final good. As for the South, it has a growth rate for the intermediate sector dependent on innovation and population growth although its growth is higher as

[12] The number of periods will depend on the parameters and initial values.

the technological effects have direct effects on its production. This allows both economies to grow even in the transition period since output per worker will be positive in both. Depending on the parameterization, the country that has higher output per worker growth can be either of the two.

Under perfect-labour mobility, we observe positive growth for production in the North but very close to zero, while the South has a growth rate for the intermediate sector close to $\varepsilon(1-\alpha)\gamma\varrho_u$.[13] Subsequently, $\hat{y}_{dt} \approx 0$ is observed in the North (albeit still positive), while in the South $\hat{y}_{ut} \approx (1-\alpha)\gamma\varrho_u$, until it catches up.

After the transition phase, under no-labour mobility, $\frac{A_{ut}}{A_{dt}}$ decreases over time so that the technological-knowledge gap will rise over time. This offsets the increasing population ratio in the profit ratio, keeping it stable. The larger Southern population must be offset with more innovation in the North to enhance production since the North needs to cope with a large volume of production for itself and the South, but with fewer workers. Hence, they will need to rely more on capital and technology than on labour. That is why the technological-knowledge gap stretches. This increase in the gap does not have a direct effect on the price ratio, which stabilizes due to the countervailing effects of population and technology. Moreover, the output ratio and the exports/imports ratio also remain stable over time. Thus, although each variable rises over time, the relation between them remains the same. In equilibrium, more demand for one leads to the increase in demand for the other. In fact, prices of both countries increase as well as output and exports/imports. Output per worker also increases, but the gap between countries widens, which means the South always lags behind the North, mostly because population increases faster in the South than in the North.

The solution under perfect-labour mobility is more elegant. As population movements are endogenous, innovation is crucial for scientists' profits. Both countries have positive levels of innovation, which means that output per worker grows in both, while what is more striking, the rate of innovation is the same. Prices and population remain stable in each country although there is a bias towards the South, which comprises a larger population and higher prices. This bias stems from the technological-knowledge bias given by $\frac{A_{ut}}{A_{dt}} = \left(\frac{\varrho_d}{\varrho_u}\right)^{-\frac{1}{\varphi}}(1+\gamma\varrho_u)^{-\frac{\varphi+1}{\varphi}}$, when the profit ratio is stable. Hence, one of the countries is at the frontier, depending on the parameterization. If $\varrho_d > \varrho_u(1+\gamma\varrho_u)^{-(\varphi+1)}$ then the North will remain at the frontier. Since the probability of success on innovation in the North, ϱ_d, is at least equal to or higher than that in the South (by assumption), then we can assume that the North will indeed be at the frontier even after being stagnant for some periods, while the South was catching up. From here and from the expressions

[13] $\hat{Y}_{ut} = \left[1-(\alpha+\varphi)\frac{1}{1+K}\right]\hat{A}_u$, where $K = \left(\frac{A_{dt}}{A_{ut}}\right)^{\varphi}$ is falling on A_{ut} and so the ratio $\frac{1}{1+K}$ will increase till $\frac{A_{dt}}{A_{ut}}$ reaches the threshold (still higher than 1), so that, as time passes, $1-(\alpha+\varphi)\frac{1}{1+K}$ gets closer to $\varepsilon(1-\alpha)$. The same procedure makes us reach the conclusion to the growth rate of Y_{dt}.

for price and population ratios, prices are higher in the South and the population is located there, as production needs more workers to compensate for the lower technological knowledge level.

As in the other hypothesis, the output ratio and exports/imports ratios are stable over time, albeit each single variable is increasing. As for the output per worker ratio, it is constant although output per worker in both countries tends to increase. Thus, as under no-labour mobility, there is not a complete catching up process, but the gap remains stable. Furthermore, the North has a higher output per worker if the condition on the probability of innovation remains true.

The South benefits from this since in the substitutable goods scenario, it would be stuck in a poverty trap with stagnant output per worker. While with no-labour mobility, as the gap in technology and output per worker rises, leading to divergence between countries, with perfect mobility, the gap remains constant. So, despite the difference in absolute values, there is a leader and a follower that have the same innovative pace and the follower is able to keep track of the leader. Still, in both cases, we observe growth of output per worker meaning an improvement in each person's quality of life, lifting them from the chains of poverty. There is also a clear specialization of productive systems. In the South, production relies on the work force and less on technology - improved capital, whilst the North, because it has fewer workers (exogenous or endogenously), relies on technology - improved capital. This is the reason wages increase more in the North for labour immobility and wages increase in both countries when labour can freely move.

Supporting investment on complementary products instead of imitation of goods produced in the North seems to be the best strategy to follow for governments in the South. In fact, without any fiscal intervention, these countries may have the needed dynamics to avoid a poverty trap situation. In this case, more balanced economic forces are at work, allowing balanced productions in each country, beneficial to less developed countries.

4. A Quantitative exercise

From the above model, we could expect some patterns on the path of an economy that is diverging or converging to the levels of developed economies. To verify whether the model has empirical consistency in the real world, we undertake a quantitative exercise with actual data from the industry sector, for some countries around the world. The main goal is to compare actual behaviour of these sector's economic variables with the estimated path of the variables related with goods in our model. We use the USA data as the benchmark country and, to apply the dynamics of the model, we also consider Mexico, Cuba, Japan and China. The choice of these countries relies on the need for countries with different features that may relate to the models we covered above. For instance, Cuba is a good example of the no-mobility hypothesis, while Mexico is a better example of perfect mobility, although we know that there are many constraints to migration. As for China and Japan, there are migratory movements between these countries and the US, and they represent catching-up examples. We apply the same calibration procedure to aggregates of Low-Income countries and Upper-Middle-Income countries, taking as a benchmark High-Income countries aggregate data. All the data was obtained from the WDI database for the years 1980 to 2011.[14]

In this calibration, t = 0 corresponds to 1980; thus, we cover a period of 32 years. We use as initial values the technological-knowledge gap of the industry sector from which we estimate the values for each one of the necessary variables – labour, price, output produced and output per worker gaps. To obtain the initial values for technological knowledge, we adopted the standard methodology to measure total factor productivity (TFP) and thus technological-knowledge growth rates (Hall and Jones, 1999; Ghosh and Kraay, 2000). We use equation $Y_{it} = K_{it}^{\alpha}(A_{it}L_{it})^{1-\alpha}$ as a proxy to measure TFP as Solow residuals. The proxy for Y_{it} is GDP (constant 2000, US dollars), L_{it} corresponds to employment. As for physical capital stocks, we use the perpetual inventory method.[15] This data for the entire economy is sufficient to determine the TFP in each year.[16]

Still, we need to define the parameters of the model. We assume standard values for the share of capital in production, α = 0.3, and for the depreciation rate, δ = 0.06 (Barro and Sala-i-Martin, 2004). The increasing technology factor is given by γ = 0.1 – we consider that a machine improvement is not, on average, too high (see footnote 4). The average growth rate and the probability of success in innovation are country specific. We compute the latter by comparing the actual technological-knowledge growth rate and the growth rate in the model $\gamma\varrho_i$ in (7), for s equal to 1 and γ = 0.1. Thus, we have a value for ϱ_d for the USA and the High-Income countries. The value for ϱ_u will be given for Mexico, Japan, China, the Low-Income countries, and the Upper-Middle-Income countries. Finally, we have to calibrate the elasticity of substitution between goods of both countries, which is a free parameter and depends on the kind of behaviour we are defining for each country. If it is converging, we assume an elasticity less than 1, which initially is calibrated to ε = 0.8. Nevertheless, if we observe divergent behaviour, we assume this elasticity to be higher than 1, ε = 1.2. These values for elasticity are assumed close to one to avoid extreme behaviour of estimates. Thus the economy is fully characterized by 6 parameters (see Table 1).

Having defined these parameters and possessing the initial value for technological knowledge, we can determine the other necessary initial values. We use the equations in section 3 to define the values for the price ratio $\frac{P_{u0}}{P_{d0}}$, labour ratio $\frac{L_{u0}}{L_{d0}}$, profit ratio $\frac{\Pi_{u0}}{\Pi_{d0}}$ and the exports ratio $\frac{Y_{d0}^F}{Y_{u0}^F}$. Then, from (9), we obtain the output ratio $\frac{Y_{u0}}{Y_{d0}}$ and thus, jointly with the labour ratio, we compute

[14] Imports and exports values were obtained from the US Department of Commerce, Bureau of the Census, Foreign Trade Division. We do not provide values for Cuba due to the economic and political conflict that makes data almost inexistent.

[15] The initial capital-stock value is determined by: $K_0 = \frac{I_0}{(g+\delta)}$ where I_0 is gross-capital-formation level as the proxy for investment, δ is the investment rate that we assume equal to 0.06 and g is the average investment growth rate.

[16] To avoid a tautological exercise, we use aggregate data from the entire economy to measure TFP levels and rates. Then, we use these outcomes to estimate some model' parameters. Thus, we use a set of data to construct TFP measures and a different one (industry sector data) to compare actual differences in economic variables with our model estimations.

the output per worker ratio $\frac{y_{u0}}{y_{d0}}$. The effect on each of these variables through time will depend on the parameters, as we will see below, and on the type of model considered for each country.

Table 1 – Parameter calibration

Parameter		Values
Capital Sharing (α)		0.3
Depreciation rate (δ)		0.06
Elasticity of substitution (ε)		0.8 or 1.2
Technology factor (γ)		0.1
Probability of success	USA (ϱ_d)	0.147
	Mexico (ϱ_u)	0.096
	Cuba (ϱ_u)	0.304
	Japan (ϱ_u)	0.214
	China (ϱ_u)	0.734
	High-Income countries (ϱ_d)	0.167
	Upper and Middle Income countries (ϱ_u)	0.347
	Low-Income countries (ϱ_u)	0.034

Note: Standard literature values and computed ones from the WDI database.

To approach this empirical exercise we analyse actual data and verify if there is convergence of the technological-knowledge and output per worker gaps. If so, we assume the complementary good case in section 3.3; otherwise, we consider the substitution good case in section 3.2. The path of the main variables will then depend on these features. According to our model, the lengths of time for convergence relies mainly on the initial technological-knowledge gap, on the probability of success, and on the elasticity of substitution. Using the calibrated parameters from above and the initial values for the technological-knowledge gap, we can compute the number of years expected to be required for each country to reach an equilibrium with the USA or, in case of aggregate data, with the High-Income countries for different elasticity values (Table 2).

Table 2 – Number of convergence periods

Elasticity of substitution	Cuba	Japan	China	Upper & Middle Income countries
0.1	215	29	91	96
0.2	220	33	95	100
0.4	234	45	108	112
0.6	263	68	134	136
0.8	348	138	211	209

From Table 2, the convergence means that the profit ratio is equal to one so that scientists are divided into both countries. We conclude that Cuba is an extreme case, for $\varepsilon = 0.8$, where 348 years would be needed to converge, since we are assuming a different model for the autarky Cuba case, which, indeed, is a closed economy as far as population is concerned, and technological-knowledge rates are also small compared to other countries. In the other exercises we assume free mobility of population, which fastens the process of convergence.

We can state that the model fits well some of the patterns verified in reality. By assuming the tendencies of each country and applying these to the type of model that translates the same tendencies, we obtain interesting conclusions on the path of the economic variables - see Figure 3 and Figure 4. We will focus mainly on the technological-knowledge gap, output per worker gap and the exports ratio for two of the countries above Japan and Mexico, as an example. Since the hypothesis concerning labour presupposes extreme cases, which are unusual to find, we will not make many considerations about it. As for the price gap, it is not our main concern and has many features in reality that cannot easily be replicated by the model.

There are features worth noting between the model estimations and the data. Taking the example of Japan (Figure 4), we show how the model mimics the pattern of the technological-knowledge gap, as well as of the output per worker.

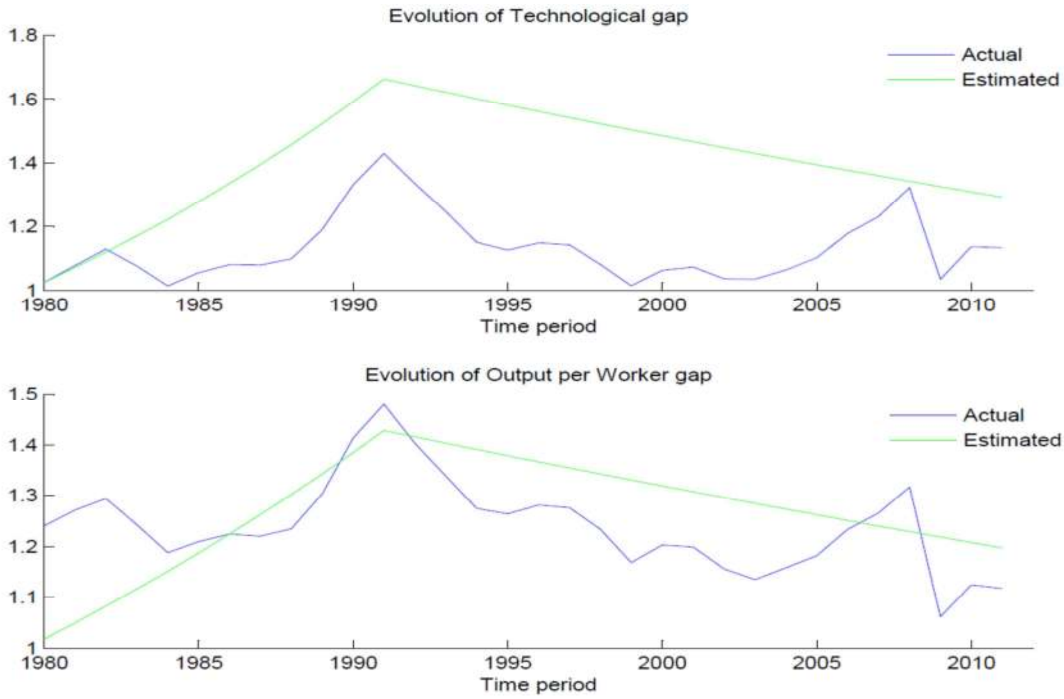

Figure 3 - Evolution through time of technological knowledge and output per worker gaps for Japan.

As we can see, the dynamics of the model show the same trend as the original data and closely match the data until the middle 1990s, by applying the complementary goods case. There is a break in the data that the model does replicate by applying the substitute goods case, since, due to the assumptions of the model, implies a constant growth rate of technological knowledge so that the estimated values tend to increase or decrease at a steady rate over time. For the Mexico case, where, instead, we have divergence, the model also makes a good match with the data. As shown in Figure 5, the evolution of the technological-knowledge gap is very close to the data although it tends to diverge in later years; as for output per worker, the model tends to overestimate its value, but both the trend and value levels are consistently close to each other.

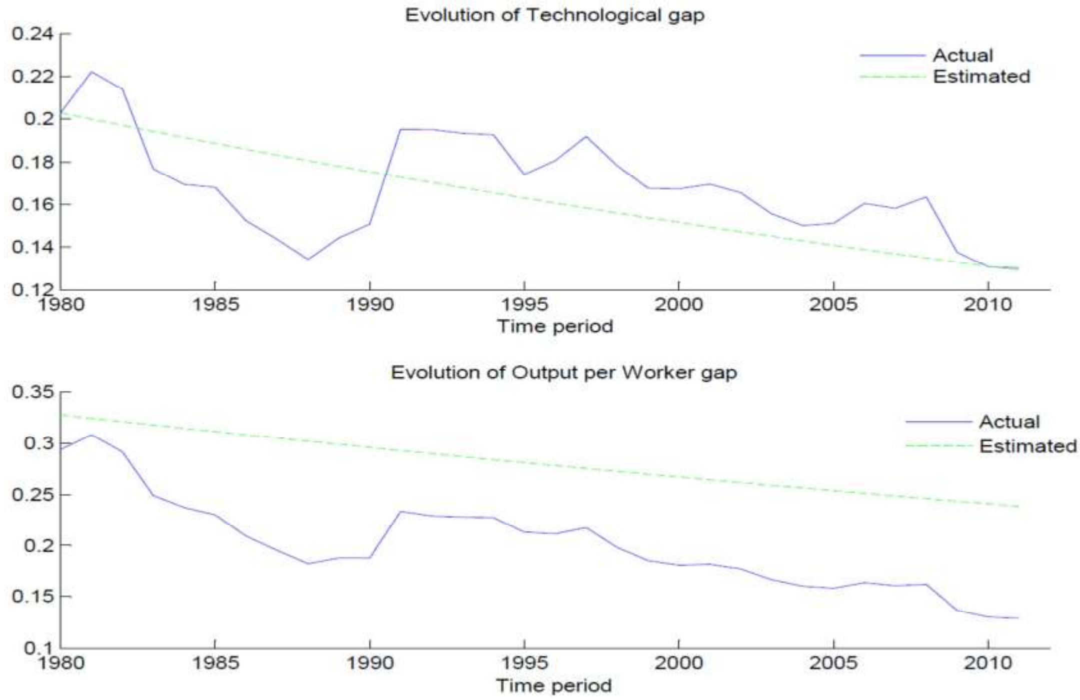

Figure 4 - Evolution through time of technological and output per worker gaps for Mexico

These patterns are further illustrated in Table 3, which shows actual data and the estimates implied by our benchmark calibration between 1980 and 2011.

From Table 3, we must distinguish three cases. Firstly, the Cuba estimates are computed according to the model with complementary goods and no-labour mobility population. The model implies similar levels for the technological-knowledge gap, but higher values for the output per worker. This match regarding output per worker is mainly due to the significant impact of the labour gap in the model. The labour gap is quite smaller than the technological-knowledge gap and, thus, when the output per worker gap is computed, the latter is much higher than in reality. The exports ratio cannot be computed since data on trade between the US and Cuba is almost null. The variation rates are also higher than the actual ones. This feature is common to almost all estimates; usually the model overpredicts the evolution of each variable.

Concerning Mexico, although there is a high restriction on mobility of Mexican citizens to the US, there is a significant rate of illegal immigration.[17] Hence, here we apply the perfect-labour mobility of population model with substitute goods. The model has a good fit for the technological-knowledge gap and output per worker gap. It overpredicts the exports ratio as it assumes a ratio higher than 1. This happens because there is over trade between the two countries favouring the US. However, the trends on the path are the same and predict a higher decline. Regarding the mobility of population, the model predicts a flow of migrants to the US and, from data, we observe

[17] This contrasts with Cuba where the restrictions on mobility are much accentuated due to its geographical conditions.

an inflow of migrants from Mexico to the US. Our model predicts a fall in the population ratio and actual data shows that the level of Mexican migrants tends to increase through time in the US. It increased from about 6.5 millions to about 11.7 millions between 1994 and 2010.[18]

Table 3 – Data and quantitative results

Countries	Variables	Actual Data		Estimated Values		Variation (%)	
		1980	2011	1980	2011	Real Data	Estimation
Cuba	$\frac{A_u}{A_d}$	0.06	0.11	0.06	0.15	94%	153%
	$\frac{y_u}{y_d}$	0.10	0.12	0.47	0.86	16%	84%
	$\frac{Y_d^F}{Y_u^F}$	n.a.	n.a.	-	-	n.a.	-
Mexico	$\frac{A_u}{A_d}$	0.20	0.13	0.20	0.13	-36%	-36%
	$\frac{y_u}{y_d}$	0.29	0.13	0.33	0.24	-56%	-27%
	$\frac{Y_d^F}{Y_u^F}$	0.71	0.78	3.05	4.19	9%	37%
China	$\frac{A_u}{A_d}$	0.02	0.10	0.02	0.16	446%	799%
	$\frac{y_u}{y_d}$	0.02	0.09	0.06	0.27	475%	365%
	$\frac{Y_d^F}{Y_u^F}$	1	0.26	16.92	3.64	-74%	-78%

Regarding China, we assume the case of perfect-labour mobility with complementary goods. It presents gap estimations that follow the trend and are higher, but not too far from the actual estimates. For instance, for 2011, the values are quite consistent: an estimated 0.16 technological-knowledge gap against an actual value of 0.10 for China. As for output per worker values, we observe 0.27 against 0.09 for China. Finally, the exports ratio again overestimates the real value, but its fall is consistent in actual data. Nevertheless, caution has to be taken in the analysis of these variables since there is a strong variation. The reasons are due to the extraordinary economic boom in China, mostly in the last decade. On population mobility, we have observed, in line with our model, movements from China to the US. The level of Chinese migrants in the US has increased from about 575 thousands in 1994 to 1.6 millions in 2010.

Concerning Japan, there is a break in data affecting the analysis. The economic downturn in the 1990s, deepened during the Asian crisis and afterwards from its own internal crisis, prevented

[18] Data from OECD International Migration Database.

Japan from keeping the pace it was following in the 1970s/80s. The break in the middle 1990s needs to be treated carefully (Figure 4). Thus, our approach consisted of computing two estimations: one until 1991 and other afterwards. The first applied the complementary goods case and the latter used the assumption of substitute goods. This approach fits well the data (Table 4).

Table 4 – Data and quantitative results – Japan

Period	Variables	Actual Data		Estimated Values		Variation (%)	
		Begin	End	Begin	End	Real Data	Estimation
1980 to 1991	$\frac{A_u}{A_d}$	1.03	1.43	1.03	1.66	39%	62%
	$\frac{y_u}{y_d}$	1.24	1.48	1.05	1.43	19%	36%
	$\frac{Y_d^F}{Y_u^F}$	0.21	0.53	0.98	0.74	156%	-24%
1991 to 2011	$\frac{A_u}{A_d}$	1.43	1.13	1.43	1.11	-21%	-22%
	$\frac{y_u}{y_d}$	1.48	1.12	1.27	1.08	-24%	-15%
	$\frac{Y_d^F}{Y_u^F}$	0.53	0.51	0.70	0.84	-3%	19%

As we can observe above, actual data has a complete different behaviour from the first (1980-1991) to the second period of time under study. In the first, we have convergence, while in the second the two countries diverge. The variable levels for each period are quite close to the original ones. For the 1980 to 1991 period, estimations fluctuate more than the original data, but the actual levels and the estimates tend not to be far away from each other. The only caveat is the exports ratio since the estimated tendency tends to be contrary to the one observed in reality. In both periods of time the estimated exports ratios drift in an opposite direction from actual data. As for output per worker and technological knowledge, both behave consistently with actual data, growing in the first period and decreasing in the second one. As for population mobility we verify that the model and the data have dissimilar behaviour since for the period of 1994 to 2010 the stock of Japanese migrants in the US has decreased, while our model would predict an increase.

If we depart from the analysis of countries and use aggregates, which can be identified as our North/South counterparts in the model, we reach the empirical results presented in Table 5. We observe similar features regarding the evolution of both gaps. The model tends to predict higher variations than in reality, although some of the predictions on levels are just slightly higher than the actual ones. We can then assume that, even using aggregate variables, our model estimates match well the evolution of these economic variables, similar to the ones using countries' data. In fact, as a general rule, our quantitative exercise shows that the proposed model can produce changes in the technological-knowledge gap and output per worker that are generally comparable with changes in real data. Although it does not resemble all the characteristics in the real world, it provides some important trends that are confirmed by real data.

Table 5 – Data and quantitative results – aggregates

Countries	Variables	Actual Data		Estimated Values		Variation (%)	
		1980	2011	1980	2011	Real Data	Estimation
Low Income	$\frac{A_u}{A_d}$	0.02	0.01	0.02	0.01	-47%	-40%
	$\frac{y_u}{y_d}$	0.02	0.01	0.07	0.05	-46%	-30%
Upper & Middle Income	$\frac{A_u}{A_d}$	0.11	0.16	0.11	0.32	47%	188%
	$\frac{y_u}{y_d}$	0.12	0.13	0.22	0.45	12%	110%

Note: The exports ratios are not available due to the lack of aggregate data on these variables.

5. Discussion

First of all, using the hypothesis of free enterprise between countries for a sufficient differential on technological-knowledge and with substitutable goods, the behaviour of output per worker is similar to the actual behaviour shown by the data. Indeed, the output per worker gap increases in all scenarios for substitutable goods, and even when there is no-labour mobility and complementary goods, an increasing gap exists, mainly due to the technological-knowledge gaps that still emerge. The one fundamental cause for this phenomenon is economic incentives. The main argument we want to highlight is how important incentives are for research and how they may decisively influence the fate of countries. The incentive here is the profit each scientist earns for doing research in one specific country. The rising profits in the North, because of its initial technological-knowledge advantage and subsequent cumulative effect, provide the incentive for scientists to move and maintain themselves there. This implies a thin possibility of recovery for the South since without any kind of change agents have no incentive to move back to the South. Moreover, population tends to move to the North to find more jobs and to earn higher wages, which encompasses some of the empirical facts referred to in the introduction.

We can discuss many perspectives on this change. An intervention through taxes and subsidies or with a change of policy and of the productive structure, can lead to a recovery. For instance, the comparison between both scenarios – substitutable and complementary goods –, which mimics the performance of some actual countries in the world, can be a hypothesis of policy. It shows how imitation processes may not be the best solution for the South. Since the North is closer to the technological-knowledge frontier, it has an advantage to keeping in the fore, pushing countries that try to compete with it towards stagnation. The option for complementary goods may indeed help overcome poverty situations. The implementation of a plan to change the productive structure to produce complementary goods, instead of trying to compete in the international market with similar goods, could be an appropriate solution for underdeveloped countries. The reformulation of the problem from one where no entity manages the relationship between both countries to one where a supranational entity manages that relationship and entertains a broader view of both countries, leads to the overcoming of economic failures. For instance, having the social gains ratio as a decision element to possibly invert the flow of scientists to the North and the correction of the monopoly power of scientists, when producing new machines, ameliorates the economic outcomes of the entire world. The usage of subsidies is just a means to address this issue, but we could more subjectively think of different and non-fiscal measures for solving the problem. As long as they fix the incentives problem, we can have different measures, such as improved facilities, the availability of new and competitive infrastructures, or by implementing a plan to change the productive structure to produce complementary goods, instead of trying to compete in the international market with similar goods.

Nevertheless we may think of the role of private firms and private research institutions in mitigating and independently shaping the "productive structure" of national economies. Since much of research happens in these private enterprises, instead of having the state or a national entity managing this change of production processes, or infra-structures, we may think of these firms as the enhancers of technology in the South. Namely, since they would also weigh the same incentives as scientists in the model. However, the change of type of goods stills depends on their motivation and their sight on the productive structure in order to benefit from the gains the change on production may yield.

In the same line of thought, we could also ask the impact of having different marginal productivities in each country. Although this mechanism is not considered in the model, it would lead to different conclusions, since now firms would have incentives to base themselves in the South as they could hire workers with lower wages. This would add to the profit ratio a positive effect on the South, contributing, even in the case of substitutable goods, to a positive force towards the South. Nevertheless, in the same perspective, if we take into account different goods not only in the level of substitutability, but also on the human capital intensity, we would now have a negative effect for the South (Bartel and Lichtenberg, 1987; Noorbakhsh *et al*, 2001; Vandenbussche *et al*, 2006). Indeed, we know the North is in the technology frontier, and the goods produced are usually human capital intensive. These goods, from one side, need skilled people and scientists that promote innovation which favours the North, where there is a higher concentration of skilled people. From the other side, firms profit more from these goods which induce them to settle in the North where there are more skilled people. This means a force is leading to a concentration of skilled and innovative people as well as firms in the North, which contributes to a stretch in the divergence between countries. In a contrasting view, Mountford (1997) presents a model where the scope of uncertain migration and the assumption of human capital accumulation can conduce to the increase of average productivity in the South and hence contribute to the escape from the poverty trap.

Another perspective that would change the outcomes is the access to credit markets which constrains firms and innovation mostly in developing countries (Ayyagari *et al*, 2008; Bloom *et al*, 2010). Credit markets can mine research by not providing the necessary funds to firms for them to invest and leverage their R&D projects. The absence of these funds and flow of money in developing countries represents a significant constraint/disincentive. In our model this would mean lower incentives for researchers to be in the South where they would probably be prevented from having the means to research and build new machines and innovations as they would in the North. This, again, leads to a concentration of scientists in the North as our model also predicts, but now even with complementary goods the forces against investments may be enough strong to avoid a catching up process.

Back to our model, even considering both hypotheses – free and no-labour mobility – as opposite and extreme poles, we must not regard these effects as a linear and objective function, but rather should look at them as picturing tendencies that play a significant role explaining the empirical evidence. By observing the extremes, we know how variables behave and what adaptation to economic reality is. For example, there is no such thing as mass movements of population and total desertification of the South, but, from the free-labour movement case, we can infer that people tend to move to the North. In addition, from the no-labour mobility case, output grows in the South albeit at a noteworthy lower level than in the North, causing the rise in the output per worker gap.

These features provided by the model have been tested fruitfully during our quantitative exercise. We may conclude that some of the trends in real data can be captured by our model. The model estimates, for each specific case, divergence/convergence, were quite close to the real values and, at a lower degree, to the variations. The major contribution of this paper is to show this empirical consistency of the model which replicates the examples given above, as well as other examples we can retain from the explanations on the behaviour of the economic variables in previous sections.

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