

A TECHNIQUE FOR ZOOM TRANSFORM AND
LONG-TIME SIGNAL ANALYSIS

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ABSTRACT

This paper reviews a useful technique for performing zoom transform. The method involves recording a long-time signal and transforming it in parts, using a smaller transform. Its ability to handle long-time signals renders the method attractive to both acousticians and vibration engineers. Examples and the listing of a computer program are given to demonstrate the technique.

SOMMAIRE

Une technique pratique pour réaliser la transformation avec zoom nécessite l'enregistrement d'un signal de longue durée, pour ensuite le transposer en parties en utilisant une plus petite transformation. La possibilité de traiter des signaux de longue durée accroît l'intérêt de cette technique pour les spécialistes de l'acoustique et des vibrations. Des exemples ainsi qu'une liste imprimée d'un programme informatique sont donnés pour illustrer cette technique.

The discrete Fourier transform (DFT) has become a very important tool for analysis because of the availability of efficient computer algorithms and low priced mini- and micro-computers with fast array processors. The efficient method for computing DFT is called the fast Fourier transform (FFT). Most machines, however, are usually limited to a 1 k or 2 k point transform ($k = 1024$). This limitation often presents problems to acousticians who have to analyse very long time signals and to vibration engineers who need fine resolution spectra for modal analysis. Although FFT instruments with zoom features are readily available, software can be difficult to find. For example, no such programs are listed in "Programs for Digital Signal Processing" by the IEEE Press.¹

Techniques for zoom transform and for long-time signal analysis are available in the literature,²⁻⁴ but they involve fairly complicated procedures. There is, however, a straightforward method used by the Bruel and Kjaer Type 2033 "High Resolution Signal Analyser" to solve both problems. Unfortunately, neither the Bruel and Kjaer manual nor Thrane⁵ give enough information for other researchers to implement the technique with their own computers. The procedure involves taking smaller transforms on selected data points from different segments of a pre-recorded long-time signal. Although Yip⁴ did not promote this procedure in his paper, the mathematical foundation can be gained from his analysis. This paper reviews this technique and presents some examples.

ANALYSIS

Although both the discrete and fast Fourier transform algorithms are, in general, considered for complex variables, the following discussion is restricted to real input data since all experimental time functions are real. Consider a time function $x(t)$ sampled at interval Δt and starting at $t = 0$. If N is the total number of contiguous data points, the finite discrete Fourier transform of $x(t)$ is given by the following equation:⁶

$$X(n\Delta f) = \sum_{k=0}^{N-1} x(k\Delta t) e^{-j2\pi nk/N} \quad (1)$$

where $n = 0, 1, \dots, N-1$, and $\Delta f = 1/N\Delta t$ is the spectral resolution.

In practice, the sampling frequency $f_s (= 1/\Delta t)$ is governed by the highest frequency of interest, f_m . The sampling theorem requires that $f_s = 2f_m$. Thus,

$$f_m = \frac{1}{2\Delta t} \quad (2)$$

As a result, N has to be large for fine resolution analysis at high frequencies. If the hardware limits N to 1 or 2 k, the usual zoom technique by frequency shift has to be used. This paper offers a different solution.

Suppose the computer is limited to a P ($= 1$ k for example) point transform and the requirement for N is M ($= 10$ for example) times P . For some machines these N data points have to be stored either in a disk or a tape file first. The proposed technique involves performing an ordinary 1 k transform ten times using data selected from different parts of the 10 k samples. After the results from the ten smaller transforms have been properly combined, the solution is equal to a 10 k transform. It is also possible to compute, with the same resolution, only a selected portion of the whole spectrum. Thus, this technique can be considered as a zoom transform also. The mathematical background is given in the following paragraphs.

Let the N data points be divided into P blocks of M points each, such that $N = MP$. Using the following index transformation,⁴

$$k = rM + s \quad (3)$$

where $r = 0, 1, \dots, (P-1)$
 $s = 0, 1, \dots, (M-1)$.

Equation (1) can be rewritten as

$$\begin{aligned} X(n\Delta f) &= \sum_{r=0}^{P-1} \sum_{s=0}^{M-1} x[(rM + s)\Delta t] e^{-j2\pi n(rM+s)/N} \\ &= \sum_{s=0}^{M-1} e^{-j2\pi ns/N} \sum_{r=0}^{P-1} x[(rM + s)\Delta t] e^{-j2\pi nr/P} \end{aligned} \quad (4)$$

Another index transformation,

$$n = \alpha P + \beta \quad (5)$$

with $\alpha = 0, 1, \dots, (M-1)$
 $\beta = 0, 1, \dots, (P-1)$

will recast Eq. (4) into

$$X[(\alpha P + \beta)\Delta f] = \sum_{s=0}^{M-1} e^{-j2\pi\alpha s/M} e^{-j2\pi\beta s/N} \sum_{r=0}^{P-1} e^{-j2\pi\beta r/P} x[(rM + s)\Delta t] \quad (6)$$

The first summation on the index r of Eq. (6) represents a DFT on the P sampled points, $x(s)$, $x(M+s)$, $x(2M+s)$, etc., to give P complex spectral components. The DFT can be carried out by the usual FFT algorithm and this operation has to be repeated M times as s changes from 0 to $M-1$. These intermediate results are termed partial spectra. That is

$$X_s(\beta\Delta f) = \sum_{r=0}^{P-1} e^{-j2\pi\beta r/P} x[(rM+s)\Delta t] \quad (7)$$

where $\beta = 0, 1, \dots, (P-1)$.

As pointed out by Thrane,⁵ the term $\exp(-j2\pi\beta s/N)$ represents a phase shift correction to the frequency components of each of the M partial spectra. This is governed by the index s and is required to compensate for the time shift between the M sets of data used in the transform. Thus, Eq. (6) can be rewritten as

$$X[(\alpha P + \beta)\Delta f] = \sum_{s=0}^{M-1} e^{-j2\pi\alpha s/M} X'_s(\beta\Delta f) \quad (8)$$

where $X'_s(\beta\Delta f) = X_s(\beta\Delta f) e^{-j2\pi\beta s/N}$ are the M compensated partial spectra.

There is no mention of the other term, $\exp(-j2\pi\alpha s/M)$, in Thrane's paper, but it may be thought of as a weighting function of the M compensated partial spectra. For a given value of α , Eq. (8) generates up to P spectral lines. The number selected within the range $\alpha P\Delta f$ to $[(\alpha+1)P-1]\Delta f$ is determined by the choice of the range for β . This procedure provides a form of zoom transform. By allowing α and β to take on all values from 0 to $M-1$ and to $P-1$, respectively, the full spectrum of N lines can be generated if necessary.

To minimize storage space and computing time, Yip⁴ chose to re-order the summation procedure so that data could be used in chronological order. He had to treat the phase shift factor $\exp(-j2\pi\beta s/N)$ as unity, however, and to correct the results after the zoom spectrum had been computed. As the correction factor depends on the type of signals being analysed, his zoom scheme is less attractive than the method presented here.

PROGRAMMING HINTS

The M partial spectra as defined by Eq. (7) can be performed with any available FFT program. It is important to note, however, that there are FFT programs for complex input data and other programs for real input data only. The two types will have different input and output format.

For real input data the frequency spectrum obtained is a conjugate even function; that is,

$$X_s[(P-\beta)\Delta f] = X_s^*(\beta\Delta f) \quad (9)$$

where $*$ denotes complex conjugate. Some FFT programs intended for use only with real input may return only $P/2$ points. As Eq. (8) requires the complete P

lines of the partial spectra, they must be recreated using Eq. (9). In general, P is set by the available software or hardware and is much larger than M . The second summation over the index s can be performed in the straightforward manner.

It is important to realize that the DFT is just an approximation of the continuous Fourier transform, and that there are problems associated with its usage, for example, aliasing, leakage and picket-fence effect. These problems have been dealt with in the literature.⁷ If it is necessary to apply windowing such as Hanning to the data, it should be applied to the original N data points. For the full spectrum, only the first $N/2$ frequency lines are independent.

EXAMPLES

To verify the proposed technique, 512 data points are generated using an analytical function. First, an ordinary transform on the 512 data points was performed to give 256 complex frequency results. The same 512 data points were then divided into 128 blocks of four data points each to be used in the proposed transform procedure. No significant differences were found between the two results.

To illustrate the zoom capability, a test signal consisting of two sine waves (198 Hz and 200 Hz) and a band-limited random noise was used. The test signal was sampled at a rate of 1 kHz. Initially, an ordinary 512 point transform was used. As the spectral resolution for this transform was only 1.95 Hz, it would not be capable of resolving the two sine waves (see Fig. 1). Increasing the total number of data points to 5120 and using the proposed transform technique with $P = 512$ and $M = 10$, the spectral resolution became 0.195 Hz and it was possible to resolve the two sine waves, as indicated in Fig. 2. The listing of a sample program is given in the Appendix.

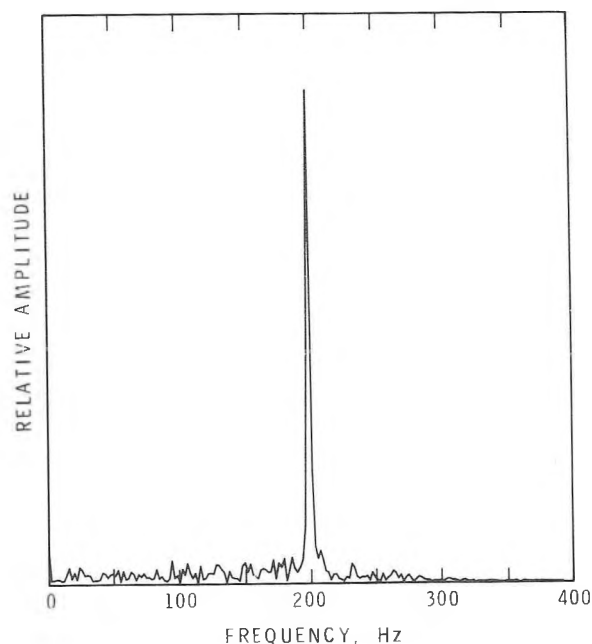


Figure 1. Overlapped spectrum obtained by the ordinary FFT method using 512 points. Spectral resolution = 1.95 Hz

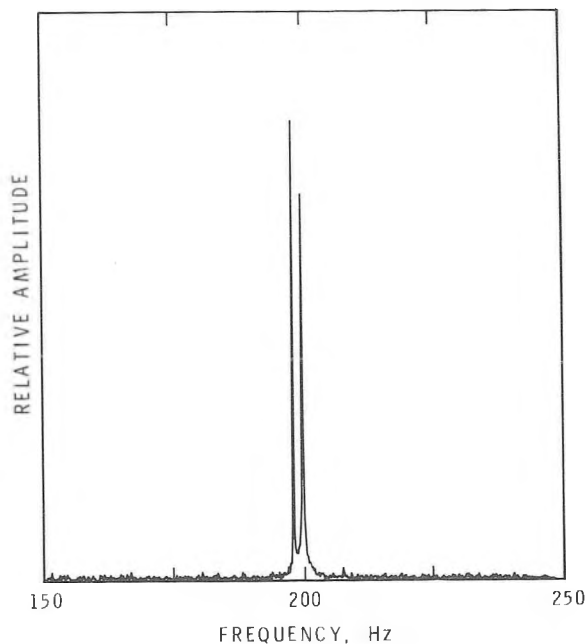


Figure 2. Fine resolution spectrum obtained by the proposed transform method using 5120 points. Spectral resolution = 0.195 Hz

CONCLUSION

A simple technique for zoom transform and long-time signal analysis has been reviewed and examples have been given to illustrate its applications. It is hoped that other acousticians and vibration engineers will find it useful.

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APPENDIX

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C      SAMPLE PROGRAM FOR A 10 k POINT TRANSFORM USING A 1 k FFT
C      SUBROUTINE. THE FFT SUBROUTINE CALLED FAST BY BERGLAND AND DOLAN
C      IS FOR REAL INPUT DATA ONLY AND IS LISTED IN THE IEEE BOOK ON
C      PROGRAMS FOR DIGITAL SIGNAL PROCESSING.
C      COMPLEX C, F, CEXAF, CEXBA
C      DIMENSION P(10240), PI(1026), C(1024), F(1024)
C      COMMON/CONS/PII, P7, P7TWO, C22, S22, PI2
C      SPLIT THE 10240 POINTS INTO 1024 BLOCKS OF 10 DATA POINTS EACH.
C      NP=1024; # OF BLOCKS GOVERNED BY THE FFT SIZE.
C      NM=10; # OF POINTS PER BLOCK.
C      NMH=NM/2
C      NP1=NP+1
C      NPH1=NP/2+1
C      NPH2=NP/2+2
C      FOR 'ZOOM' TRANSFORM, ENTER PARTICULAR NAF VALUE FOR ALPHA.
C      EXAMPLE GIVEN IS FOR THE COMPLETE SPECTRUM.
C      NOTE, A 10 k TRANSFORM GIVES 5 k INDEPENDENT FREQUENCY COMPONENTS
C      ONLY.
C      DO 20 NAF=1, NMH
C      NAFO=NAF-1
C      DO 25 NBE=1, NP
C      F(NBE)=0.0
25  CONTINUE
C      DO 30 NS=1, NM; SET UP EXPONENTIAL ALPHA-S TERM
C      NSO=NS-1
C      ARGAF=2.*3.14159*NAFO*NSO/NM
C      EXAFR=COS(ARGAF)
C      EXAFI=-SIN(ARGAF)
C      CEXAF=CMPLX(EXAFR, EXAFI)
```

```

C   PERFORM 1 k POINT TRANSFORM WITH SELECTED DATA FROM EACH BLOCK
DO 40 NR=1, NP
NRO=NR-1
J=NRO*NM+NS
PI(NR)=P(J)
40  CONTINUE
CALL FAST(PI, NP)
C   SINCE SUBROUTINE FAST GIVES (NP/2+1) FREQUENCY RESULTS ONLY, IT IS
C   NECESSARY TO GENERATE RESULTS FOR THE REST OF THE PARTIAL
C   SPECTRUM
J=1
DO 50 L=1, NP1, 2
LC=L+1
C(J)=CMPLX(PI(L), PI(LC))
J=J+1
50  CONTINUE
J1=1
DO 60 J=NPH2, NP
K=NPH1-J1
C(J)=CONJG(C(K))
J1=J1+1
60  CONTINUE
DO 70 NBE=1, NP; SET UP EXPONENTIAL BETA-S TERM
NBE0=NBE-1
ARGBA=2.*3.14159*NBE0*NSO/N
EXBAR=COS(ARGBA)
EXBAI=-SIN(ARGBA)
CEXBA=CMPLX(EXBAR, EXBAI)
F(NBE)=F(NBE)+C(NBE)*CEXAF*CEXBA
70  CONTINUE
30  CONTINUE
DO 80 I=1, NP
TOR=REAL(F(I))
TOI=AIMAG(F(I))
TYPE TOR, TOI
80  CONTINUE
20  CONTINUE
STOP
END

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