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A Tesselated Probabilistic Representation for Spatial Robot Perception and Navigation

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Abstract

The ability to recover robust spatial descriptions from sensory information and to efficiently utilize these descriptions in appropriate planning and problem-solving activities are crucial requirements for the development of more powerful robotic systems. Traditional approaches to sensor interpretation, with their emphasis on geometric models, are of limited use for autonomous mobile robots operating in and exploring unknown and unstructured environments. In this paper, we present a new approach to robot perception that addresses such scenarios using a probabilistic tesselated representation of spatial information called the Occupancy Grid. *The Occupancy Grid is a multi-dimensional random field that maintains stochastic estimates of the occupancy state of each cell in the grid. The cell estimates are obtained by interpreting incoming range readings using probabilistic models that capture the uncertainty in the spatial information provided by the sensor. A Bayesian estimation procedure allows the incremental updating of the map using readings taken from several sensors over multiple points of view. We provide an overview of the Occupancy Grid framework and illustrate its application to a number of problems in mobile robot mapping and navigation. We argue that a number of robotic problem-solving activities can be performed directly on the Occupancy Grid representation, and draw some parallels between operations on Occupancy Grids and related image processing operations.*

1 Introduction

Two crucial requirements for the development of more flexible and powerful robotic systems are the ability to recover robust spatial descriptions of the surrounding world using sensory information, and the ability to efficiently utilize these descriptions in appropriate planning and problem-solving activities. Traditional approaches to sensor interpretation in Robotics and Computer Vision have largely relied on the recovery and manipulation of geometric world models [6]. "Low-level" sensing procedures extract geometric features such as line segments or surface patches from the sensor data, while "high-level" sensor modules use prior geometric models and heuristic assumptions about the environment to constrain the sensor interpretation process. The resulting deterministic geometric descriptions of the environment of the robot are subsequently used as the basis for other robotic activities, such as obstacle avoidance, path-planning and navigation, or planning of grasping and assembly operations. These approaches, which we characterize as part of the *Geometric Paradigm* in Computer Vision, have, however, several shortcomings [6]. Generally speaking, the Geometric Paradigm leads to sparse and brittle world models; it requires early decisions in the interpretation of the sensor data for the instantiation of specific model primitives; it does not provide appropriate mechanisms for handling the uncertainty and errors intrinsic in the sensory information; and it relies heavily on the accurateness and adequacy of the prior world models and heuristic assumptions used. As a result, these geometric approaches are of limited use for complex scenarios such as those that arise in the use of autonomous or semi-autonomous vehicles for planetary exploration. Such mobile robots have to be able to operate in and explore unknown and unstructured environments, while coping with unforeseen conditions.

More recently, a number of other methodologies have started to be applied to robot perception tasks, with encouraging preliminary results. We have discussed elsewhere [6, 4] the role of stochastic sensor models and representation schemes in the development of robust robot systems operating in unstructured real-world environments.

In this paper, we review a new approach to robot perception and world modelling that uses a probabilistic tesselated representation of spatial information called the *Occupancy Grid* [6, 4]. The Occupancy Grid is a multi-dimensional random field that maintains stochastic estimates of the occupancy state of each cell in the grid. The cell estimates are obtained by interpreting incoming range readings using probabilistic models that capture the uncertainty in the spatial information provided by the sensors. Bayesian estimation procedures allow the incremental updating of the Occupancy Grid using readings taken from several sensors over multiple points of view. As a result, the disambiguation of sensor data is performed not through heuristics or prior models, but by higher sensing rates and the use of appropriate sensing strategies.

In subsequent sections, we will provide an overview of the Occupancy Grid formulation and discuss how the Occupancy Grid framework provides a unified approach to a number of tasks in mobile robot perception and navigation. These tasks include range-based mapping, multiple sensor integration, path-planning and obstacle avoidance, handling of robot position uncertainty and other related problems. We suggest that a number of robotic problem-solving activities can be performed directly on the Occupancy Grid representation, precluding the need for the recovery of deterministic geometric descriptions. We also draw some parallels between operations on Occupancy Grids and related image processing operations.

2 The Occupancy Grid Approach

The scenario under consideration in this paper involves a mobile robot operating in unknown and unstructured environments, and carrying a complement of sensors that provide range information directly (sonar, scanning laser rangefinders) or indirectly (stereo systems). We will be mainly concerned with the development of robust mechanisms for robot perception and navigation. In this section, we provide a brief outline of the Occupancy Grid formulation, while in the succeeding sections we discuss several applications of Occupancy Grids to mobile robot mapping and navigation. More details can be found in [6, 4]; preliminary experimental results were reported in [8, 5, 9, 3, 13].

2.1 The Occupancy Grid Representation

The Occupancy Grid is a multi-dimensional (typically 2D or 3D) tesselation of space into cells, where each cell stores a probabilistic estimate of its state. Formally, an *Occupancy Field* $O(\mathbf{x})$ can be defined as a discrete-state stochastic process defined over a set of continuous spatial coordinates $\mathbf{x} = (x_1, x_2, \ldots)$, while the *Occupancy Grid* is defined over a discrete spatial lattice. Consequently, the Occupancy Grid corresponds to a discrete-state (binary) random field [19]. A *realization* of the Occupancy Grid is obtained by estimating the state of each cell from sensor data.

More generally, the cell state could encompass a number of properties, described using a random vector associated with each lattice point of the random field, and estimated accordingly. We refer to such general world model representations, which are again instances of random fields, as *Inference Grids* [6]. Since in our current discussion we are mainly interested in *spatial* models for robot perception, we will restrict ourselves to the estimation of a single property, the *occupancy state* of each cell.

In the Occupancy Grid, the state variable $s(C)$ associated to a cell C is defined as a discrete random variable with two states, *occupied* and *empty*, denoted OCC and EMP. Since the states are exclusive and exhaustive, $P[s(C) = \text{OCC}] + P[s(C) = \text{EMP}] = 1$. Each cell has, therefore, an associated probability mass function that is estimated by the sensing process.

2.2 Estimating the Occupancy Grid

To construct a map of the robot's environment, two processing stages are involved. First, a sensor range measurement r is interpreted using a stochastic sensor model. This model is defined by a probability density function (p.d.f.) of the form $p(r \mid z)$, where z is the actual distance to the object being detected. Secondly, the sensor reading is used in the updating of the cell state estimates of the Occupancy Grid. For simplicity, we will derive the interpretation and updating steps for an Occupancy Grid defined over a single spatial coordinate, and outline the generalization to more dimensions.

In the continuous case, the random field $O(x)$ is described by a probability mass function defined for every x and is written as $O(x) = P[s(x) = \text{OCC}](x)$, the probability of the state of x being *occupied*. The probability of x being *empty* is obviously given by $P[s(x) = \text{EMP}](x) = 1 - P[s(x) = \text{OCC}](x)$. The conditional probability of the state of x being occupied given a sensor reading r will be written as $O(x \mid r) = P[s(x) = \text{OCC} \mid r](x)$. For the discrete case, the Occupancy Grid corresponds to a sampling of the random field over a spatial lattice. We will represent the probability of a cell C_i being occupied as $O(C_i) = P[s(C_i) = \text{OCC}](C_i)$, and the conditional probability given a sensor reading r as $O(C_i \mid r) = P[s(C_i) = \text{OCC} \mid r](C_i)$. When only a single cell C_i is being referenced, we will use the more succinct notation $P[s(C_i) = \text{OCC}]$.

We now consider a range sensor characterized by a sensor model defined by the p.d.f. $p(r \mid z)$, which relates the reading r to the true parameter space range value z. Determining an optimal estimate $\hat{z}$ for the parameter z is a straightforward estimation step, and can be done using Bayes' formula and MAP estimates [2, 18]. Recovering a model of the environment as a whole, however, leads to a more complex estimation problem. In general, obtaining an optimal estimate of the occupancy grid $O(C_i \mid r)$ would require determining the conditional probabilities of all possible world configurations. For the two-dimensional case of a map with $m \times m$ cells, a total of 2^{m^2} alternatives are possible, leading to a non-trivial estimation problem. To avoid this combinatorial explosion of grid configurations, the cell states are estimated as *independent* random variables. This is equivalent to assuming that the Occupancy Grid is a Markov Random Field (MRF) of order 0 [19], and can be relaxed using estimation procedures for higher order MRFs [10, 12].

To determine how a sensor reading is used in estimating the state of the cells of the Occupancy Grid, we start by applying Bayes' theorem to a single cell C_i:

$$P[s(C_i) = \text{OCC} \mid r] = \frac{p[r \mid s(C_i) = \text{OCC}]\, P[s(C_i) = \text{OCC}]}{\sum_{s(C_i)} p[r \mid s(C_i)]\, P[s(C_i)]} \quad (1)$$

Notice that the $p[r \mid s(C_i)]$ terms that are required in this equation do not correspond directly to the sensor model

$p(r \mid z)$, since the latter implicitly relates the range reading to the detection of a single object surface. In other words, the sensor model can be rewritten as:

$$p(r \mid z) = p[r \mid s(C_i) = \text{OCC} \wedge s(C_k) = \text{EMP}, k < i] \qquad (2)$$

To derive the distributions for $p[r \mid s(C_i)]$, it is necessary to perform an estimation step over all possible world configurations. This can be done using Kolmogoroff's theorem [15]:

$$p[r \mid s(C_i) = \text{OCC}] = \sum_{\{G^O_{s(C_i)}\}} \left(p[r \mid s(C_i) = \text{OCC}, G^O_{s(C_i)}] \times P[G^O_{s(C_i)} \mid s(C_i) = \text{OCC}]\right) \qquad (3)$$

where $G^O_{s(C_i)} = (s(C_1) = s_1, \cdots, s(C_{i-1}) = s_{i-1}, s(C_{i+1}) = s_{i+1}, \cdots, s(C_n) = s_n)$ stands for a specific grid configuration with $s(C_i) = \text{OCC}$, and $\{G^O_{s(C_i)}\}$ represents all possible grid configurations under that constraint. In the same manner, $p[r \mid s(C_i) = \text{EMP}]$ can be computed as:

$$p[r \mid s(C_i) = \text{EMP}] = \sum_{\{G^E_{s(C_i)}\}} \left(p[r \mid s(C_i) = \text{EMP}, G^E_{s(C_i)}] \times P[G^E_{s(C_i)} \mid s(C_i) = \text{EMP}]\right) \qquad (4)$$

where $G^E_{s(C_i)}$ is defined in a manner similar to $G^O_{s(C_i)}$, above.

The configuration probabilities $P[G_{s(C_i)} \mid s(C_i)]$ are determined from the individual prior cell state probabilities. These, in turn, can be obtained from experimental measurements for the areas of interest, or derived from other considerations about likelihoods of cell states. We have opted for the use of non-informative or maximum entropy priors [1], which in this case reduce to equal probability assignments for the two possible states:

$$P[s(C_i) = \text{OCC}] = P[s(C_i) = \text{EMP}] = 1/2 \qquad (5)$$

Finally, Eq. 2 is used in the computation of the distributions $p[r \mid s(C_i)]$. The full derivation of these terms is found in [6]; we only remark that because there are subsets of configurations that are *indistinguishable* under a single sensor observation r, it is possible to derive closed form solutions of these equations for certain sensor models, and to compute numerical solutions in other cases.

To illustrate the approach, consider the case of an ideal sensor, characterized by the p.d.f. $p(r \mid z) = \delta(r - z)$, where δ is the Kronecker delta. For this case, the following closed form solution of Eq. 1 results (Fig. 1):

$$P[s(C_i) = \text{OCC} \mid r] = \begin{cases} 0 & \text{for } x < r, x \in C_i \\ 1 & \text{for } x, r \in C_i \\ 1/2 & \text{for } x > r, x \in C_i \end{cases} \qquad (6)$$

which is an intuitively appealing result: if an ideal sensor measures a range value r, the corresponding cell has occupancy probability 1; the preceding cells are empty and have occupancy probability 0; and the succeeding cells have not been observed and are therefore unknown, having occupancy probability 1/2.

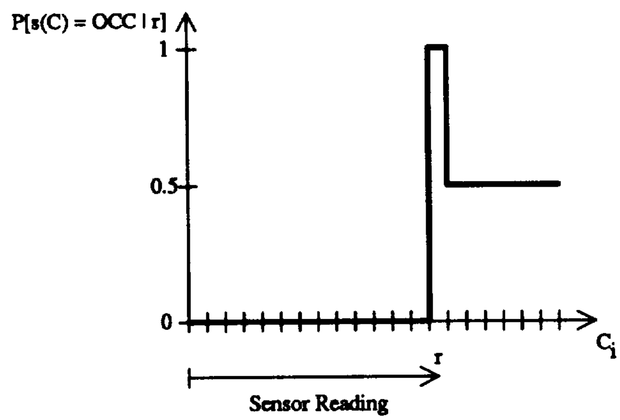


Figure 1: Occupancy Probability Profile for an ideal sensor, given a range measurement r.

As another example, consider a range sensor whose measurements are corrupted by Gaussian noise of zero mean and variance σ^2. The corresponding sensor p.d.f. is given by:

$$p(r \mid z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(r-z)^2}{2\sigma^2}\right) \qquad (7)$$

This equation can be used in the numerical evaluation of Eqs. 3 and 4. A plot of a typical cell occupancy profile obtained for this sensor from Eq. 1 is shown in Fig. 2.

To extend the derivation to two spatial dimensions, consider the example of a range sensor characterized by Gaussian uncertainty in both range and angle, given by the variances σ_r^2 and σ_θ^2. In this case, the sensor p.d.f. can be represented in polar coordinates as:

$$p(r \mid z, \theta) = \frac{1}{2\pi\sigma_r\sigma_\theta} \exp\left[-\frac{1}{2}\left(\frac{(r-z)^2}{\sigma_r^2} + \frac{\theta^2}{\sigma_\theta^2}\right)\right] \qquad (8)$$

In this formula, the dependency of the random variable r on z and θ is decoupled, a reasonable assumption for a first-order model of certain types of range sensors. Consequently, the estimation of the two-dimensional Occupancy Grid can be performed conveniently in polar coordinates (ρ, φ), using fundamentally the same formulation as above (Eqs. 3 and 4) and applying Eq. 8 to recover the distributions $p[r \mid s(C_{\rho_i\varphi_j})]$. These in turn are used to obtain the polar Occupancy Grid $P[s(C_{\rho_i\varphi_j}) \mid r]$. To generate the corresponding two-dimensional cartesian Occupancy Grid, the polar grid can be scanned and resampled. The results are similar to the 2D cartesian Occupancy Grid shown in Fig. 3, obtained from a single sonar reading. Similar derivations can be performed for 3D Occupancy Grids.

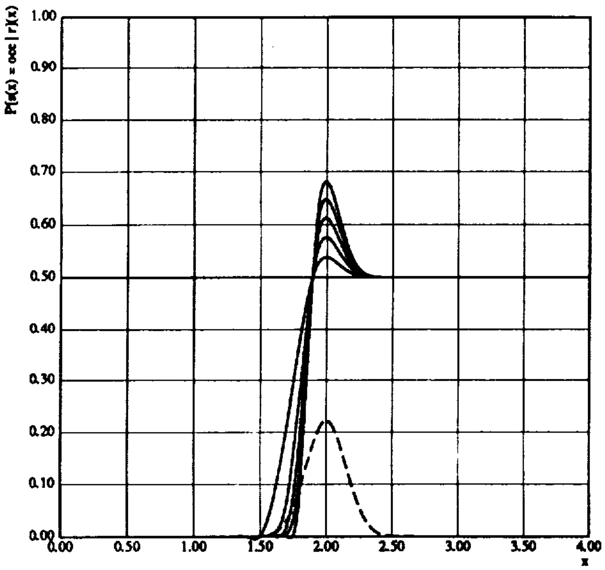


Figure 2: Occupancy Probability Profiles obtained from a sensor with Gaussian distribution. The sensor model $p(r \mid z)$ is shown superimposed (dashed line). Several successive updates of the cell occupancy probabilities are plotted, with the sensor positioned at $x = 0.0$ and with $r = 2.0$. The grid was initialized with $P[s(x) = \text{OCC}] = 0.5$. The profiles show that the Occupancy Grid converges towards the behaviour of the ideal sensor.

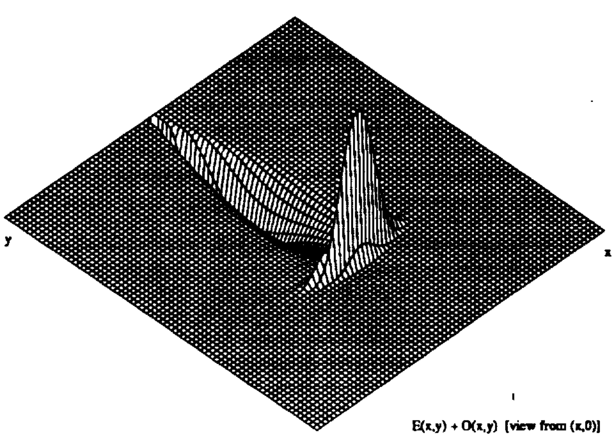


Figure 3: Two-Dimensional Sonar Occupancy Grid. The occupancy profile shown corresponds to a range measurement taken by a sonar sensor positioned at the upper left, pointing to the lower right. The plane shows the UNKNOWN level.

2.3 Updating the Occupancy Grid

Due to the intrinsic limitations of sensor systems, recovering a description of the world from sensory information is fundamentally an underconstrained problem. As mentioned previously, this has historically been addressed by the heavy use of prior models and simplifying heuristic assumptions about the robot's environment. Within the Occupancy Grid framework, this problem is handled instead by the use of additional sensing to resolve sensor ambiguity and uncertainty. Rather than relying on a single observation to obtain an estimate of the Occupancy Grid, information from multiple sensor readings taken from different viewpoints is composed to improve the sensor-derived map. This leads naturally to an emphasis on higher sensing rates and on the development of adequate sensing strategies.

To allow the incremental composition of sensory information, we use the sequential updating formulation of Bayes' theorem [6]. Given the current estimate of the state of a cell $s(C)$, $P[s(C_i) = \text{OCC} \mid \{r\}_t]$, based on observations $\{r\}_t = \{r_1, \cdots, r_t\}$, and given a new observation r_{t+1}, we can write:

$$P[s(C_i) = \text{OCC} \mid \{r\}_{t+1}] = \frac{p[r_{t+1} \mid s(C_i) = \text{OCC}]\, P[s(C_i) = \text{OCC} \mid \{r\}_t]}{\sum_{s(C_i)} p[r_{t+1} \mid s(C_i)]\, P[s(C_i) \mid \{r\}_t]} \qquad (9)$$

In this formula, the previous estimate of the cell state, $P[s(C_i) = \text{OCC} \mid \{r\}_t]$, serves as the prior and is obtained directly from the Occupancy Grid. Tables for the sensor model-derived terms $p[r_{t+1} \mid s(C_i)]$ can be computed offline and used in the updating procedure. The new cell state estimate $P[s(C_i) = \text{OCC} \mid \{r\}_{t+1}]$ is subsequently stored again in the map. An example of this Bayesian updating procedure is shown in Fig. 2.

2.4 Sensor Integration

To increase the capabilities and the performance of robotic systems in general, a variety of sensing devices are necessary to support the different kinds of tasks to be performed. This is particularly important for mobile robots, where multiple sensor systems can provide higher levels of fault-tolerance and safety. Additionally, qualitatively different sensors have different operational characteristics and failure modes, and can therefore complement each other.

Within the Occupancy Grid framework, sensor integration can be performed using a formula similar to Eq. 9 for the combination of estimates provided by different sensors [6]. This allows the updating of the *same* Occupancy Grid by multiple sensors operating independently. Consider two independent sensors S_1 and S_2, characterized by sensor models $p_1(r \mid z)$ and $p_2(r \mid z)$. In this case, the integration of readings r_{S_1} and r_{S_2}, measured by sensors S_1 and S_2, respectively, can be done using:

$$P[s(C_i) = \text{OCC} \mid r_{S_1}, r_{S_2}] =$$

$$= \frac{p[r_{S_2} \mid s(C_i) = \text{OCC}]\; P[s(C_i) = \text{OCC} \mid r_{S_1}]}{\sum_{s(C_i)} p[r_{S_2} \mid s(C_i)]\; P[s(C_i) \mid r_{S_1}]} \tag{10}$$

A different estimation problem occurs when separate Occupancy Grids are maintained for each sensor system, and integration of these sensor maps is performed at a later stage by composing the corresponding cell probability estimates. This requires the combination of probabilistic evidence from different sources [1]. Consider the two cell occupancy probabilities $P_1 = P_{S_1}[s(C_i) = \text{OCC} \mid \{r\}_{t_1}]$ and $P_2 = P_{S_2}[s(C_i) = \text{OCC} \mid \{r\}_{t_2}]$, obtained from separate Occupancy Grids built using sensors S_1 and S_2. The general solution to this problem involves the use of a *Superbayesian* approach [1]. It requires the definition of probabilistic models of the form $f_{S_i}(P_{S_i}[s(C_i)] \mid s(C_i))$ for each sensor, which serve to provide an evaluation of the sensor performance. It can be shown [6] that for simple linear models, the Superbayesian estimation procedure is reduced to a probabilistic evidence combination method known as the *Independent Opinion Pool* [1]. This method, when applied to the combination of the two sensor-derived estimates, P_1 and P_2, yields the simple formula [6]:

$$P[s(C_i) = \text{OCC} \mid P_1, P_2] = \frac{P_1\, P_2}{P_1\, P_2 + (1 - P_1)\,(1 - P_2)} \tag{11}$$

Though this method is suboptimal in a Bayesian sense, it provides a computationally simple updating procedure. In previous work, described in [9, 13], the Independent Opinion Pool approach was used to integrate Occupancy Grids derived separately from two sensor systems, a sonar array and a single-scanline stereo module, mounted on a mobile robot. An example of the resulting maps is presented in Section 3.2.

2.5 Incorporation of User-Provided Maps

Throughout this paper we are mainly concerned with scenarios where the robot is operating in unknown environments, so that no prior maps can be used. There are other contexts, however, where such information *is* available. For example, mobile robots operating inside nuclear facilities could access detailed and substantially accurate maps derived from blueprints, while planetary rovers could take advantage of global terrain maps obtained from orbiting platforms. Such information can be represented using symbolic and geometric models such as those described in [11]. The incorporation of these high-level user-provided maps can be done within the Occupancy Grid framework using the same methodology outlined in the previous sections. To provide a common representation, the geometric maps are scan-converted into an Occupancy Grid, with occupied and empty areas being assigned the corresponding probabilities. These user maps can subsequently be used as priors for sensor maps, or can be treated simply as another source of information to be integrated with sensor-derived maps.

2.6 Decision-Making

In certain contexts, it may be necessary to make discrete choices concerning the state of a cell C. For that, the optimal estimate is provided by the *maximum a posteriori* (MAP) decision rule [2], which can be written in terms of occupancy probabilities as:

$$\begin{cases} C \text{ is OCCUPIED} & \text{if } P(s(C) = \text{OCC}) > P(s(C) = \text{EMP}) \\ C \text{ is EMPTY} & \text{if } P(s(C) = \text{OCC}) < P(s(C) = \text{EMP}) \\ C \text{ is UNKNOWN} & \text{if } P(s(C) = \text{OCC}) = P(s(C) = \text{EMP}) \end{cases} \tag{12}$$

Additional factors, such as the cost involved in making different choices, can be taken into account by using other decision criteria, such as minimum-cost estimates [18]. Depending on the specific application, it may also be of interest to define an UNKNOWN band, as opposed to a single thresholding value. As argued in [6], however, many robotic tasks can be performed directly on the Occupancy Grid, obviating the need to make discrete choices concerning the state of individual cells. In path-planning, for example, the cost of a path can be defined by a risk factor directly related to the corresponding cell probabilities [8].

3 Using Occupancy Grids for Mobile Robot Mapping

We will now proceed to illustrate the Occupancy Grid approach by discussing some applications of Occupancy Grids to autonomous mobile robots. In this section, we summarize the use of Occupancy Grids in sensor-based mobile robot *Mapping*, while in Section 4 we provide an overview of the use of Occupancy Grids in mobile robot *Navigation*. The experimental results shown here have been mostly obtained in operating environments that can be adequately described by two-dimensional maps. We have recently started to extend our work to the generation and manipulation of 3D Occupancy Grids.

One possible flow of processing for sensor-derived mobile robot mapping applications is outlined below and summarized in Fig. 4. As the mobile robot explores and maps its environment, the incoming sensor readings are interpreted using probabilistic sensor models.The map of the world that the robot acquires from a single sensor reading is called a *Sensor View*. Various Sensor Views taken from a single robot position can be composed into *Local Sensor Maps*, which can be maintained separately for each sensor type. A composite description of the robot's surroundings is obtained through sensor integration of separate Local Sensor Maps into a *Robot View* (as mentioned previously, Robot Views can be generated directly from the different sensors). As a result, the Robot View encapsulates the information recovered at a single mapping location. As the robot explores its surroundings, Robot Views taken from multiple data-gathering positions are composed into a *Global Map* of the environment. This requires relative registration of the Robot Views, an issue that is addressed in Section 4.

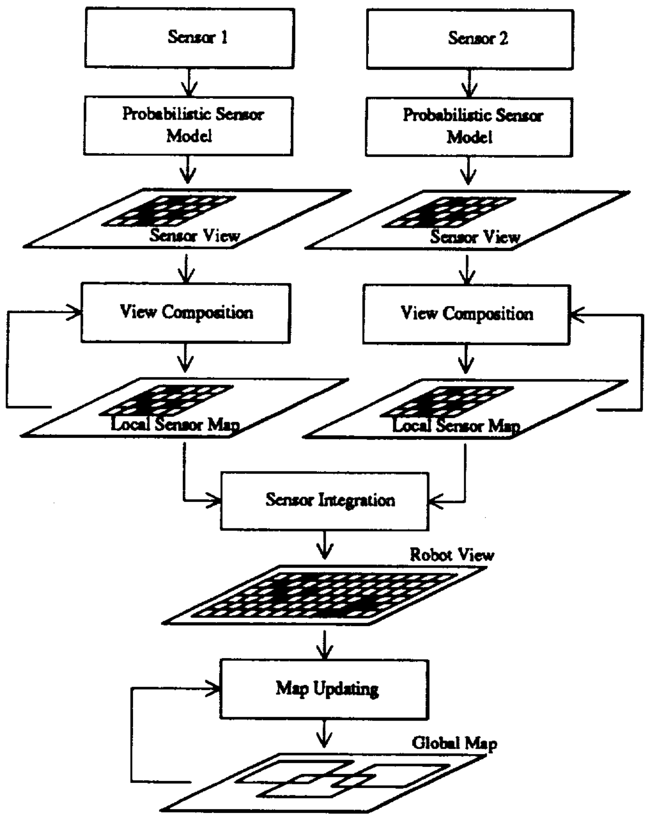


Figure 4: A Framework for Occupancy Grid Based Robot Mapping.

3.1 Sonar-Based Mapping

The Occupancy Grid representation was first developed in the context of sonar-based mapping experiments [14, 8]. The specific limitations of sonar sensors and the desire to recover robust and dense maps of the robot's environment precluded simple geometric interpretation methods [8] and led to the investigation of tesselated probabilistic representations. We developed an experimental system for sonar-based mapping and navigation for autonomous mobile robots called *Dolphin* [7, 8], and performed a number of indoor and outdoor experiments [6]. Fig. 5 shows a sonar map obtained during navigation down a corridor. Preliminary results were encouraging: the resulting sonar maps were robust and very useful for navigation. The cell updating mechanisms are computationally fast, allowing a high sensing to computation ratio. This led us to develop the Occupancy Grid formulation further and to apply it to other domains [6, 9, 13, 4].

3.2 Sensor Integration of Sonar and Scanline Stereo

The Occupancy Grid framework provides a straightforward approach to sensor integration. Range measurements from each sensor are converted directly to the Occupancy

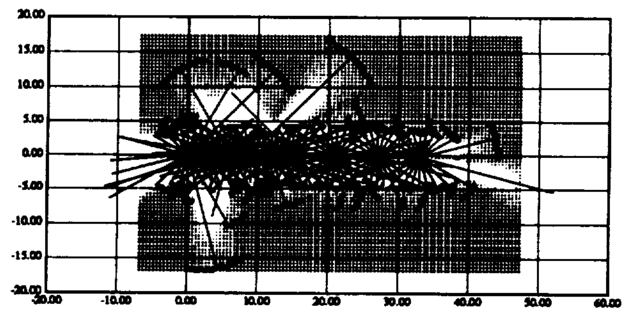


Figure 5: Sonar Mapping and Navigation Along a Corridor. Walls and open doors can be distinguished and enough resolution is present that even wall niches can be noticed in the map. The range readings taken from each robot stop are drawn superimposed on the map.

Grid representation, where data taken from multiple views and different sensors can be combined naturally. Sensors are treated modularly, and separate sensor maps can be maintained concomitantly with integrated maps, allowing independent or joint sensor operation. We have performed experiments in the integration of data from two sensor systems: a *sonar sensor array* and a *single-scanline stereo module* that provides horizontal depth profiles, both mounted on a mobile robot. This allows the generation of improved maps, taking advantage of the complementarity of the sensors [9, 13]. A typical set of maps is shown in Fig. 6.

4 Using Occupancy Grids for Robot Navigation

For autonomous robot navigation, a number of concerns have to be addressed. In this section, we briefly outline the use of Occupancy Grids in path-planning and obstacle avoidance, estimating and updating the robot position, and incorporating the positional uncertainty of the robot into the mapping process (Fig. 7).

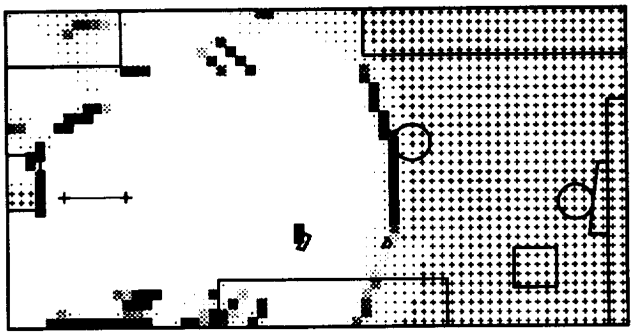
4.1 Path-Planning and Obstacle Avoidance

In the *Dolphin* system, path-planning and obstacle avoidance are performed using potential functions and an A* search algorithm. The latter operates directly on the Occupancy Grid, optimizing a path cost function that takes into account both the distance to the goal and the occupancy probabilities of the cells being traversed [8, 6].

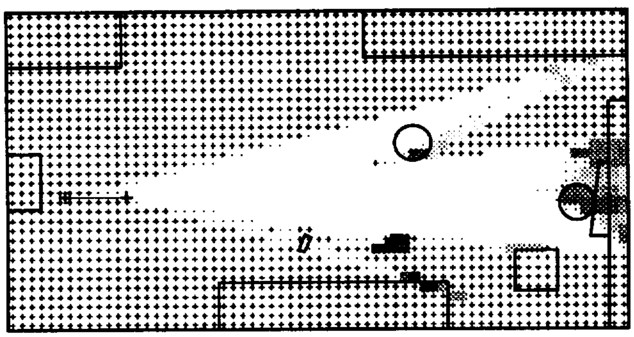
4.2 Handling Robot Position Uncertainty

To desambiguate sensor information and recover accurate and complete descriptions of the environment of operation of a robot, it is necessary to integrate sensor data acquired from multiple viewing positions. To allow the

Sonar Map:



Scanline Stereo Map:



Integrated Sonar and Scanline Stereo Map:

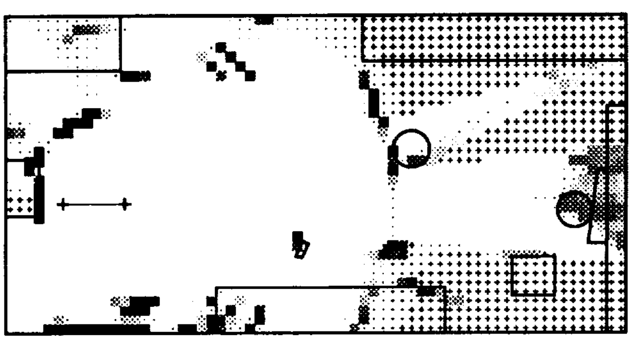


Figure 6: Sensor Integration of Sonar and Scanline Stereo. Occupancy Grids generated separately for sonar and scanline stereo, and jointly through sensor integration are shown. *Occupied* regions are marked by shaded squares, *empty* areas by dots fading to white space, and *unknown* spaces by + signs.

composition of these multiple views into a coherent model of the world, accurate information concerning the relative transformations between data-gathering positions is necessary to allow precise registration of the views for subsequent integration. For mobile robots that move around in unstructured environments, recovering precise position information poses major problems. Over longer distances, dead-reckoning estimates are not sufficiently reliable; consequently, motion-solving methods that use landmark tracking or map matching approaches are usually applied to reduce the registration imprecision due to motion. Additionally, the positional error is compounded over sequences of movements as the robot traverses its environment. This leads to the need for explicitly handling positional uncertainty and taking it into account when composing sensor information.

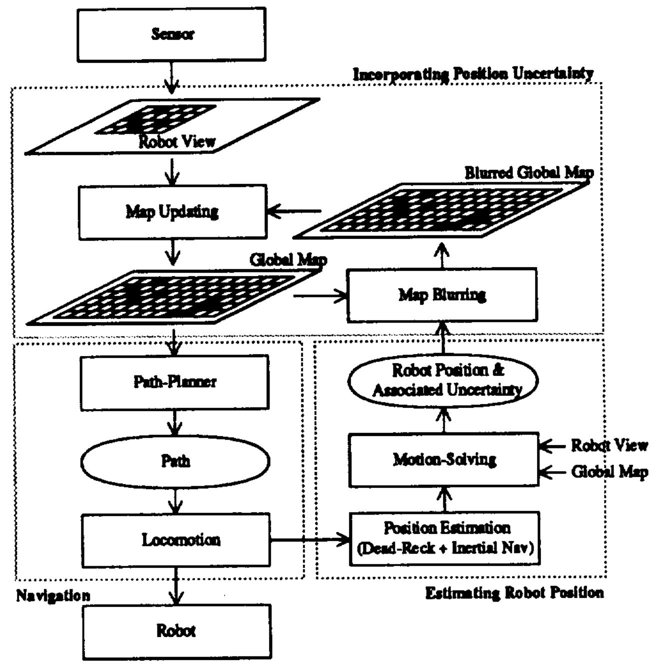


Figure 7: A Framework for Occupancy Grid-Based Robot Navigation.

To represent and estimate the robot position as the vehicle explores its environment, we use the *Approximate Transformation* (AT) framework [16]. A robot motion M, defined with respect to some coordinate frame, is represented as $\widetilde{M} = <\widehat{M}, \Sigma_M>$, where $\widehat{M}$ is the estimated (nominal) position, and Σ_M is the associated covariance matrix that captures the positional uncertainty. The parameters of the robot motion are determined from dead-reckoning and inertial navigation estimates, which can be composed using the AT *merging* operation, while the updating of the robot position uncertainty over several moves is done using the AT *composition* operation [16].

4.3 Motion-Solving

For more precise position estimation, a multi-resolution correlation-based motion-solving procedure is employed. It searches for an optimal registration between the new Robot View and the current Global Map, by matching the corresponding Occupancy Grids before map composition [14].

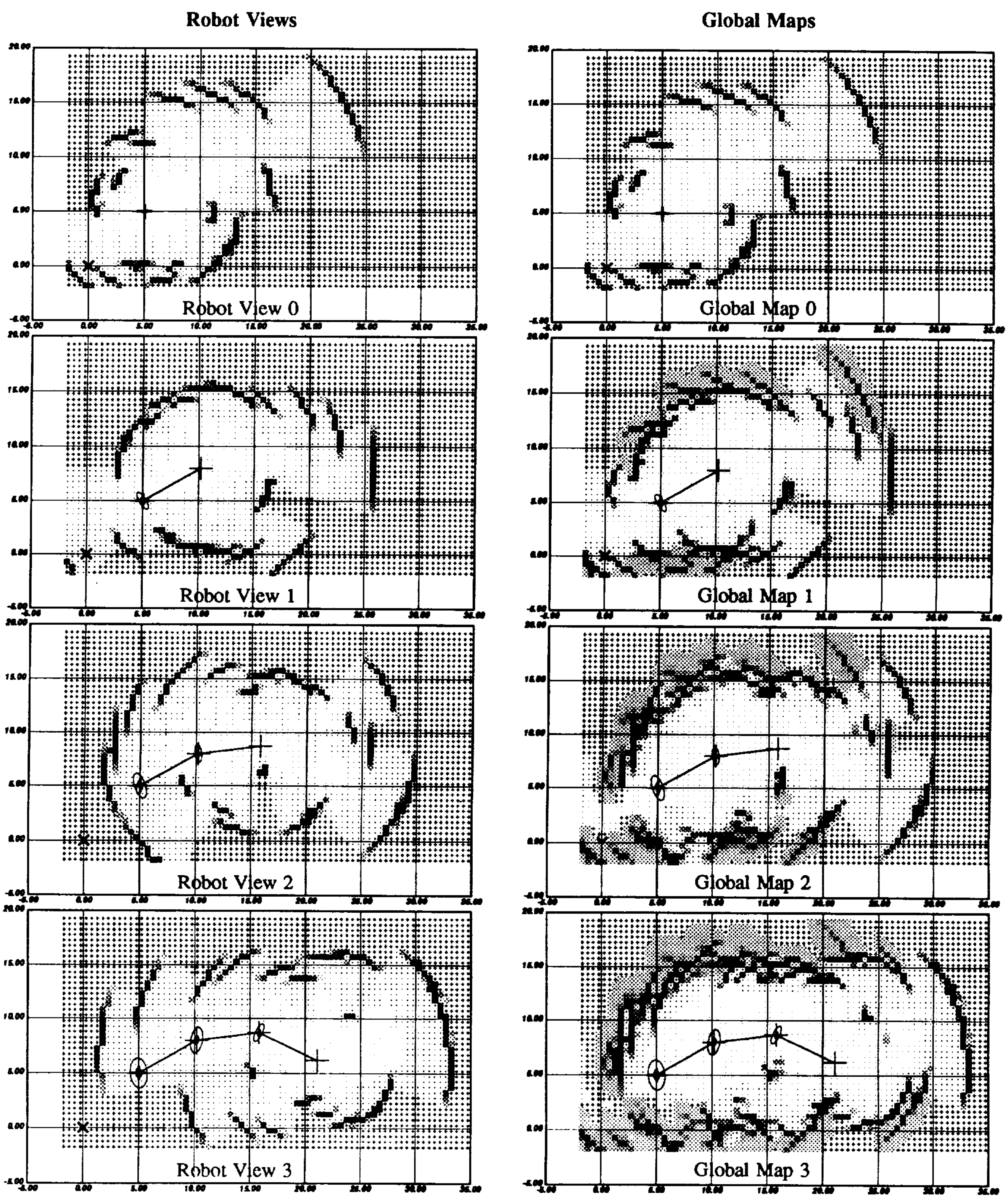


Figure 8: Incorporating Motion Uncertainty into the Mapping Process. For robot-centered mapping, the Global Map is blurred by the robot position uncertainty (shown using the corresponding covariance ellipses) prior to composition with the Robot View.

4.4 Incorporating Positional Uncertainty into the Mapping Process

After estimating the registration between the new Robot View and the Global Map, the associated uncertainty is incorporated into the map updating process as a blurring or convolution operation performed on the Occupancy Grid. We distinguish between *World-Based Mapping* and *Robot-Based Mapping* [6, 4].

In *World-Based Mapping*, the motion of the robot is related to the observer or world coordinate frame, and the current Robot View is blurred by the robot's positional uncertainty prior to composition with the Global Map. If we represent the Global Map by M_G, the current Robot View by V_R, the robot position by the AT $\tilde{R} = < \hat{R}, \Sigma_R >$, the blurring operation by the symbol $\tilde{\otimes}$ and the composition of maps by the symbol $\tilde{\oplus}$, we can express the world-based mapping procedure as:

$$M_G \leftarrow M_G \tilde{\oplus} (V_R \tilde{\otimes} \tilde{R}) \tag{13}$$

Since the global robot position uncertainty increases with every move, the effect of this updating procedure is that the new Views become progressively more blurred, adding less and less useful information to the Global Map. Observations seen at the beginning of the exploration are "sharp", while recent observations are "fuzzy". From the point of view of the inertial observer, the robot eventually "dissolves" in a cloud of probabilistic smoke.

For *Robot-Based Mapping* (Fig. 8), the registration uncertainty of the Global Map due to the recent movement of the robot is estimated, and the Global Map is blurred by this uncertainty prior to composition with the current Robot View. This mapping procedure can be expressed as:

$$M_G \leftarrow V_R \tilde{\oplus} (M_G \tilde{\otimes} \tilde{R}) \tag{14}$$

A consequence of this method is that observations performed in the remote past become increasingly uncertain, while recent observations have suffered little blurring. From the point of view of the robot, the immediate surroundings (which are of relevance to its current navigational tasks) are "sharp". The robot is leaving, so to speak, an expanding "probabilistic trail" of weakening observations behind it (see Fig. 8).

It should be noted, however, that the local spatial relationships observed within a Robot View still hold. So as not to lose this information, we use a two-level spatial representation, incorporating Occupancy Grids and Approximate Transformations. On one level, the individual Views are stored attached to the nodes of an AT graph (a *stochastic map* [17]) that describes the movements of the robot. Coupled to this, a Global Map is maintained that represents the robot's current overall knowledge of the world (Fig. 9).

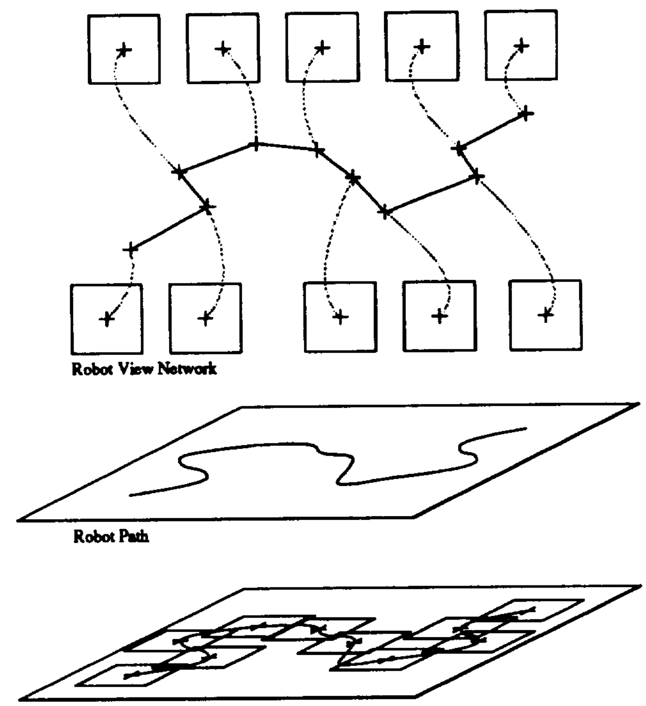


Figure 9: Maintaining a Dual Representation. A stochastic graph with the individual Robot Views is maintained in conjunction with the Global Map.

5 Other Applications

In the previous sections, we have seen that Occupancy Grids provide a unified approach to a number of issues in Robotics and Computer Vision. Additional tasks that can be addressed include the recovery of geometric descriptions from Occupancy Grids [7, 8], incorporation of user-provided maps, landmark recognition [8], prediction of sensor readings from Occupancy Grids, detection of moving objects using space-time filtering techniques, and other problems. In our own work, we are starting to explore two issues: the generation of 3D Occupancy Grids from depth profiles derived from laser scanners or stereo systems, and the development of mapping and navigation strategies that incorporate high-level user-provided maps when these are available.

It should be noticed that several robotic tasks can be performed on Occupancy Grids using operations that are similar or equivalent to computations performed in the image processing domain. Table 10 provides a qualitative overview and comparison of some of these operations.

We finalize our discussion with an observation concerning low-level versus high-level representations. It is interesting to observe that in Robotics and Computer Vision there has been historically a slow move from very high-level (stylized) representations of blocks-world objects to

Comparison of Operations on Occupancy Grids and on Images	
Occupancy Grids	**Images**
Labelling cells as Occupied, Empty or Unknown	Thresholding
Handling Position Uncertainty	Blurring/Convolution
Removing Spurious Spatial Readings	Low-Pass Filtering
Motion Solving/Map Matching	Correlation
Obstacle Growing for Path-Planning	Region Growing
Path-Planning	Edge Tracking
Determining Object Boundaries	Edge Detection
Extracting and Labelling Occupied and Empty Areas	Segmentation/Region Colouring/Labelling
Prediction of Sensor Readings from User-Provided Maps	Convolution
Incorporating User-Provided Maps	Scan-Conversion
Object Motion Detection over Map Sequences	Space-Time Filtering

Figure 10: An Overview of Operations on Occupancy Grids and the Corresponding Image Processing Operations.

the recovery of simple spatial features in very constrained real images; from there to the recovery of surface patches; and recently towards "denser", tesselated representations of spatial information. A parallel evolution from sparse, high-level or exact descriptions to denser, lower-level and sometimes approximate descriptions can be seen in some other computational fields, such as Computer Graphics and Finite Element Analysis.

6 Conclusions

We have reviewed in this paper the Occupancy Grid framework and presented results from its application to mobile robot mapping and navigation in unknown and unstructured environments. The Occupancy Grid approach supports agile and robust sensor interpretation methods, incremental discovery procedures, composition of information from multiple sensors and over multiple positions of the robot, and explicit handling of uncertainty. Furthermore, the world models recovered using sensor data can be used efficiently in robotic planning and problem-solving activities. The results lead us to suggest that the Occupancy Grid framework provides an intermediate-level spatial representation that has the characteristics of robustness and generality necessary for real-world robotic applications.

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