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A Theoretical and Empirical Study of Stock Purchase Warrants and Convertible Bonds.

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A THEORETICAL AND EMPIRICAL STUDY OF STOCK PURCHASE WARRANTS AND CONVERTIBLE BONDS

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ABSTRACT

The action of individual securities is characterized by the unexpected. Fortunes are frequently created by investing in unwanted instruments and lost by committing funds to issues in strong demand. Stock purchase warrants and convertible bonds also offer unusual and even erratic performance. This study focuses on the ingredients of fair value in these instruments and develops models to assist in appraising the merits of both warrants and convertible bonds.

An evaluation model for warrants is created by normalizing basic components of warrants in terms of their exercise prices. The price of the common stock is divided by the exercise price, and this ratio is used in predicting normal prices for warrants. Time to expiration is viewed as an important element in valuing individual warrants, and six time categories are created. A mathematical relationship between the common stock and the warrant price is calculated by using a statistical technique frequently referred to as a "power function."

The model's predictive efficiency is tested by comparing its attractively priced warrants to average warrants. Results show the warrant model to select warrants offering

greater upside and downside leverage. Average warrants are shown to provide a higher rate of return over the test period from 1960 through 1969 than those selected by the warrant model. Bear markets in the first and last years of the study lowered the model's performance. Downside leverage in 1969 resulted in huge percentage losses for warrants selected by the model.

The warrant model appears versatile in that it tends to signal generally inflated markets. Percentage-wise, few warrants are deemed attractively priced prior to bear markets. Avoidance of bear markets is shown to significantly increase the overall return from these securities.

An evaluation model is created for convertible bonds. Straight debt value is ascribed as offering the foundation for the model, and it is used to normalize conversion value and conversion feature value. The resultant conversion value ratio is used in predicting the value of the conversion feature. Time to expiration of convertible features is given special attention by structuring three separate time categories. A fourth category is created for long-term conversion features and straight debt values less than $700. Mathematical equations are developed for each category and are used in predicting normal prices for these instruments.

The convertible bond model is tested by comparing its performance to that of one hundred convertible bond observations. The model is shown to select thirty-four of the observations for purchase. Twenty-eight of them appreciate in value while only six decline. Model selections are found to be decidedly superior to average convertibles. Moreover, the model seems to provide a device for signaling generally overpriced markets.

Special trading tactics are employed for warrants and convertible bonds. The models are used to locate candidates for arbitrage-hedge techniques. Actual transactions are simulated. Warrants are found well suited for arbitrage-hedge tactics, but convertible bonds are currently not well suited for these techniques.

The study concludes by discussing other uses of empirically designed models. For example, issuing companies may find the models useful in quantifying the trade off between higher prices for warrants and greater flexibility of the convertible feature. It is concluded that refinements and updating of the designed models should be undertaken as data become available.

CHAPTER I

INTRODUCTION

At one time common stock purchase warrants and convertible bonds were usually associated with highly leveraged or unseasonal business firms. The warrant or convertible feature is often described as being a "sweetner" to a bond issue in making it more palatable to the investing public. By 1969, however, warrants and convertibles were being utilized by some of the most prestigious firms in the United States. More than $4.5 billion in convertible bonds was issued by American Telephone & Telegraph from 1913 through 1958. AT&T recently announced plans to offer an additional $1.57 billion in bonds with 5 year warrants attached. "Mother Bell's" decision to utilize the warrant as a direct source of capital has caused the Board of Governor's of the New York Stock Exchange to consider revoking its rule prohibiting listing of the speculative instrument known as a warrant.[1]

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[1] "Big Board Mulls Listing AT&T Warrants, Wall Street Journal, Vol. 45, No. 16 (January 23, 1970), p. 2.

Historical Use of These Securities

AT&T's proposed financing plan may alter the investment image of warrants. The warrant may be listed on the New York Stock Exchange and could come to be considered as a sophisticated way of participating in the growth of America. Before categorizing the warrant as investment grade, an examination of its historical use as a medium of acquiring capital is required. At the same time financing through the convertible bond witnessed a similar surge. Consequently, data pertaining to convertible bond issues will be reviewed concurrently with data concerned with warrants. Table I represents the correlation of the growth of convertible bond and warrant financing during the late 1920's. After the sharp stock market decline in the early 1930's both warrant and convertible bond financing declined precipitously.

The economic depression that followed "Black Thursday" nearly ended the use of warrants as a financing device. Similarly, convertible bond issues, after reaching a peak of more than 46% of the total bonds issued in 1929, showed a marked decline as a source of capital. Interest in these securities among corporate issuers revived in the 1960's with convertible bond financing reaching an all-time high in 1967. It seems likely that warrant financing will establish a new high in 1970 with the AT&T debt issue.

Table II outlines the growth of convertible and warrant public financing in the 1960's.

TABLE I

VOLUME OF PUBLIC CORPORATE DEBT SECURITIES (1924-1930) (DOLLAR FIGURES IN MILLIONS)

	Total Debt		Convertible Debt		Debt with Warrants	
Year	Number	Amount	Number	Amount	Number	Amount
1924	822	$2,227.0	45	$128.8	25	$ 82.0
1925	773	2,202.4	55	191.4	69	183.6
1926	897	2,724.8	107	329.2	55	187.7
1927	857	3,856.8	88	253.3	79	147.0
1928	810	2,997.0	83	390.5	64	262.7
1929	509	1,957.7	149	909.6	48	365.1
1930	552	2,978.3	95	682.9	18	68.9

Source: W. Braddock Hickman, Statistical Measures of Corporate Bond Financing Since 1900, a study by the National Bureau of Economic Research (Princeton, N.J.: Princeton University Press, 1960), pp. 210-223.

TABLE II

VOLUME OF DOMESTICALLY UNDERWRITTEN PUBLIC FINANCING (1964-1969) (DOLLAR FIGURES IN MILLIONS)

	Total Issues		Convertible Debt		Debt with Warrants	
Year	Number	Amount	Number	Amount	Number	Amount
1964	552	$5,747.7	49	$ 372.0	14	$ 54.0
1965	619	8,971.8	61	1,183.0	8	38.0
1966	630	11,778.1	96	1,761.0	11	122.0
1967	1,034	19,147.1	230	4,062.0	26	304.0
1968	1,548	17,748.6	202	2,699.0	30	381.0
1969	2,242	21,703.5	176	3,022.0	38	657.0

Source: Investment Dealers' Digest, Section II, March 3, 1970, pp. 10-16.

Tables I and II should be viewed as indicators of the importance of warrants and convertible bonds as sources of capital. They are not intended to be vehicles of comparison between the late 1920's and the late 1960's. However, it should be noted that more of these securities were floated in these years than during any previous time periods.

Table II fails to disclose the fastest growing type of warrant usage. Attaching warrants to common stock offerings has soared in recent years. In 1967 only 11 new common stock issues valued at $12,325,000 had warrants accompany the issues. The number of companies issuing warrants with common stock increased to 85 in 1969, and these issues were valued at $644,845,475.[2] Combining warrants with common stock is a relatively new approach to financing. It often enables the issuing company to realize net proceeds in excess of the existing market value of the common stock.

On a conjectural basis it may be stated that warrants and convertible features provide flexible instruments tailored to meet a business climate characterized by huge capital expenditures, thin profit margins, and diminishing equity positions. Because renewed interest in these securities has created a much greater supply than existed at any period in the past, a method for evaluating these securities is more important today than heretofore.

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[2] Investment Dealers' Digest, Section II (March 3, 1970), p. 16.

Purpose of the Study

Alan Abelson, one of Barron's weekly writers, recently devoted the following paragraph to a specific warrant.

> Ever get the feeling that you're missing something? That's the sensation we have about Elgin National Industries warrants. The securities (which expire in 1974) represent an option to buy 0.4 shares of common at $8.75. Viewed another way, they offer the opportunity to purchase a share of Elgin common at $21.875. Okay? Well, on Friday, the common closed at $8.125. But the warrants closed at $2.625. A little arithmetic indicates that anyone buying a warrant at the going price winds up with the right to pay a total of $28.44 ($6.56 plus $21.875) for a stock that's selling at 8.125. That's a premium of around $20, or something over 250%. What's bothering us, simply, is why?[3]

Mr. Abelson's reference to the Elgin warrants may save his readers from commiting money to an overpriced investment, but it does not assist non-subscribers in appraising the investment value of these securities. Stock purchase warrants offer vastly different risk-reward potentials, and the marketplace needs an approach to assist in the evaluation of the attractiveness of any given price.

Convertible bonds also trade at prices that appear highly inconsistent with their risk-reward potentials. The uneven premiums over straight debt value leaves the average investor with the difficult task of selecting convertibles that are priced fairly. In Chapter V these market inconsistencies will be examined in greater depth.

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[3]Alan Abelson, "Up and Down Wall Street," Barron's, Vol. 50, No. 2 (January 12, 1970), p. 23.

The primary purpose of this study is to develop valuation models that will assist the average investor in determining the fair value of stock purchase warrants and convertible debentures. This study is also an attempt to augment the existing knowledge related to these security forms. Another purpose of this study is to assist issuing corporations in determining the optimum investment package to be offered in acquiring new capital through the use of either stock purchase warrants, convertible bonds, or both. Finally, an attempt will be made to analyze various trading techniques that may be used with warrants and convertible bonds in order to generate profits.

Scope of the Study

Although many forms of convertible securities and options are in existence, this study will concentrate on stock purchase warrants and convertible bonds. The results of this study may be extended to other types of similar securities. For example, premiums on warrants with 6 months to expiration may be compared with prices of six month call options.

Empirical data utilized will be taken primarily from the period 1960 through 1969. Prior to that time the supply of listed convertible bonds was too restricted. Furthermore, warrants were in very short supply prior to the

1960's except for a span from the late 1920's to the middle 1930's.

The primary focus of this study will be on investment-oriented goals, with an attempt to enhance the efficicency of investors taking positions in warrants and convertible bonds. Issuing corporations may also benefit since an understanding of investor preferences will be sought.

This research effort is restricted to actively traded securities. Over-the-counter securities are eliminated from the study due to the inconsistencies in reporting bid and asked prices for similar securities. These markets have less-perfect information when compared with listed markets, and, deviations in prices of similar securities may be too great for significant conclusions. For these reasons only the New York and American Stock Exchange securities will be studied.

Methodology

The objectives of this study will be sought in the following four steps: (1) the basic ingredients of warrants and convertible bonds will be specified; (2) empirical models will be designed for warrants and convertible bonds; (3) these models will be compared with existing models and average securities in determining their relative efficiency; and

(4) special trading tactics will be analyzed using these securities.

The first step will require a theoretical presentation of the interaction between the common stock price and the value of the call on the common. Each warrant will be stripped of its individual dignity by reducing it to a common denominator. The convertible bond will also be dissected by analyzing the determinants of its straight debt value apart from its option value.

The second step will be to design an empirical model that will assist in determining representative values for individual securities--all of the warrants listed on the American Stock Exchange from 1960 to 1969 will be analyzed. The relationships between their exercise prices, prices of the common and prices of the warrants will be calculated by utilizing a "power function" of the form:

$$Y_c = a(X)^b$$

The "power function" provides a curvilinear relationship between the independent and dependent variables by transforming these data to natural logarithms. The method of least squares is then applied to these data in obtaining solutions for the two unknown constants a and b.

Each constant requires a separate equation in order to obtain a solution; therefore, two normal equations of the form

$$\text{I} \quad \Sigma \log y = (n)(\log a) + (b) \quad (\Sigma \log x)$$

$$\text{II} \quad \Sigma (\log x)(\log y) = (\log a)(\Sigma \log x) + (b) \quad [\Sigma (\log x)^2]$$

are used in obtaining solutions for a and b. These solutions will be generated through use of the LSU Computer Center.

The resultant values will be plotted for various levels of option price relative to the common stock. These derived values then will become expected or derived fair values of warrants given the ratio between the common stock and the exercise price. Thus, the models are expected to be described as empirically derived models or flexible models based on existing empirical data.

Convertible bonds from the New York Stock Exchange will be analyzed by comparing the debt value of a bond to its conversion value and conversion premium. These empirical relationships will again be determined through using the same least squares program as described for the warrants. The ratio between the convertible bond's straight debt value and its conversion value will be the independent variable used in determining expected or fair values for convertible bonds.

The third step for testing the efficiency of the models will be accomplished by determining the performance of average warrants and convertible bonds. The efficiency

of previously designed models will be tested along with the empirically designed models. Each simulated purchase will assume an equal dollar investment in order to avoid pricing difficulties.

The fourth step, analyzing various trading techniques, will be accomplished by examining specific warrants and convertible bonds. Hedges will be simulated for these securities and the empirical models will be used to locate candidates for hedging. Thus, the empirical models will be tested for their flexibility in locating fair-priced, under-priced, and over-priced securities.

Preview of the Study

This chapter serves as an introduction to the subject of this research project, and defines the approach to be utilized. It serves to introduce the timeliness of the study--both warrants and convertibles have recently reached new heights in the capital market.

Chapter II discusses the basic elements of warrants and convertible bonds. The important variables that determine market valuation are identified.

Chapter III reviews the literature pertaining to the techniques of evaluating these securities. Strengths and weaknesses of the individual models are described.

Chapter IV is concerned with the development of an

empirical model for evaluating warrants. The testing of the model is also included. Chapter V involves the development and testing of an empirical model for convertible bonds.

Various trading tactics that can be applied to these security forms are reviewed in Chapter VI. An attempt is made to simulate actual trading.

General conclusions derived from this study are discussed in Chapter VII along with the merits and limitations of the method of research. Comments are offered on the manner in which the findings may facilitate future research projects.

CHAPTER II

THE NATURE OF WARRANTS AND CONVERTIBLE BONDS

Introduction

This chapter involves an attempt to establish the basic elements related to warrants and convertible bonds. Attention is first focussed on warrants; subsequent comments concern the basic elements of convertible bonds.

The study of warrants begins with a basic definition of the term and then proceeds to an analysis of warrant pricing. The impact of leverage on the speculative values of warrants is discussed concomitant with the methods through which these instruments come into existence. The rationale of Lerner Stores' use of warrants is also analyzed.

Convertible bonds are herein defined according to their basic ingredients, and premiums are properly categorized. Due to its importance in Chapter V, the straight-debt value of a convertible receives special attention. The yield curve concept is briefly reviewed.

The importance of anti-dilution provisions is examined for warrants and convertible bonds. Specific methods of dilution are briefly reviewed. Chapter II is concluded by reviewing the accounting practices relative to these

securities.

Basic Elements Of Stock Purchase Warrants

A common stock purchase warrant is a certificate which a holder may exchange into residual ownership of a corporation. Exchange into residual ownership may be accomplished by surrendering the certificate and a specific amount of money or par value of a specified security. Exercise of the warrant is usually limited by the certificate, and most contain a prearranged expiration date. However, a limited number of these securities are deemed to have a perpetual existence.[1]

The warrant exists as a trading mechanism because most holders are unwilling to pay the necessary price to become residual claimants. If warrants were purely convertible securities, their existence would be short--holders would immediately convert them to common shares. Thus, the

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[1]In 1943, the SEC required that the United Corporation change its capitalization to a single class of common stock and cease to be a holding company with respect to each of its subsidiaries. United's reorganization plan required an exchange of its outstanding perpetual warrants to warrants with a 5 year expiration date. The exchange called for one new warrant to be issued for 5 existing warrants. The new warrants had an exercise price of $7 while the old warrants had a $27.50 exercise price. For further details, see Moody's Manual of Investments, Banks--Insurance Companies--Investment Trusts--Real Estate (New York, N.Y.: Moody's Investor Service, 1950), p. 1276

cost of exchanging a certificate for a common stock gives life to a warrant as a separate instrument and is a critical element in the evaluation of warrants. This cost is typically referred to as the exercise or option price. Through the remainder of this paper these terms will be used interchangably.

Unique Features of Warrants

Complicating the study of warrants are the varieties of contractual provisions offered by individual certificates. The most significant of these differences are: (1) there are various maturity provisions; (2) exercise prices remain constant or increase at pre-specified intervals; (3) the exercise price may be satisfied with cash or the par value of another security type; (4) one warrant may acquire a fraction of a share, one share only, or any number of shares; and (5) the warrant may have different degrees of protection against dilution.

The length of time to expiration appears to be an important variable affecting warrant values as the speculative value of a warrant approaches zero near expiration. Moreover, the warrant becomes worthless if the exercise price exceeds the price of the common upon expiration. Logically, a warrant's speculative value should be enhanced with long expiration dates. Thus, some warrants are issued

with infinite expiration dates and are referred to as perpetual warrants. A major goal of this research is to isolate the impact of the time to expiration as a function of warrant value.

Numerous warrants offer fixed exercise prices; some provide step-ups in exercise price at specified intervals. For example, Alleghany Corporation warrants offer a fixed exercise price to perpetuity while Textron warrants have an exercise price that increases every five years until their expiration in May of 1984. McCrory Warrants, issued in 1961, had a fixed exercise price of $20 to run through 1976. The company decided in 1966, to offer holders the right to exchange the old warrants for new warrants with a final expiration date in 1981. The new warrants had an increase in exercise price to $22.50 from 1976 to 1981. McCrory's decision created an excellent opportunity for studying step-ups in exercise price, since over 300,000 of the warrants were not extended by the holders. Today, McCrory has both an old and a new issue of warrants, the only difference being the additional five years to expiration and a $2.50 increase in exercise price.

An interesting and new feature of structuring exercise prices was designed by the financial innovators of LTV. Warrants issued by them offer a reduction in the exercise price, at the company's option, of as much as one-third--for a minimum period of 21 days. These warrants provide a

potential step-down in exercise price.

A common characteristic of warrants attached to bond issues is that of having their exercise price payable in cash or in par value of the bonds. The exercise price is often effectively lowered by paying with bonds rather than cash, since the bonds frequently trade below par. TWA warrants provide an example of this arrangement as they have recently traded at 65% of par, which means the exercise price could be lowered by 35% through using bonds rather than cash.

Most warrants trade on the basis of one warrant being exercisable into one share of common stock. Numerous warrants trade on the basis of varying amounts of stock acquired through a given warrant. For example, the Rio Algon warrant, traded on the American Exchange from 1961 to 1966, was exercisable into .13 shares of common for $23.07 per share. The mathematics in calculating a logical speculative value was next to impossible. Moreover, some of these relationships change frequently. Tri Continental has instituted a practice of paying periodic stock dividends and the terms of the warrant have changed frequently in recent years.

The Theoretical Value of a Warrant

A warrant, as previously defined, offers value only through its contractual right to be exchanged for common stock.

Theoretically, a high cost of exchange should reduce the value of this right. However, one must recognize the ultimate value of this right is predicated on the value of the common stock. If the market price of the common stock exceeds the exercise price, the warrant will have an arbitrage or intrinsic value. The intrinsic value, common price minus exercise price, will be the minimum price of a warrant due to the availability of instantaneous profits realizable through arbitrage activity at any lower price. For example, the Gulf & Western warrant has an exercise price of $55. If the common stock were trading at $70, the warrant would not sell much below $15 because arbitrageurs will short the common at $70 and cover by buying the warrant and exercising it for a small but quick profit. (Brokerage costs and time to exercise have been ignored in this example.) The practice of arbitrage requires that a warrant sell at its intrinsic value or higher.

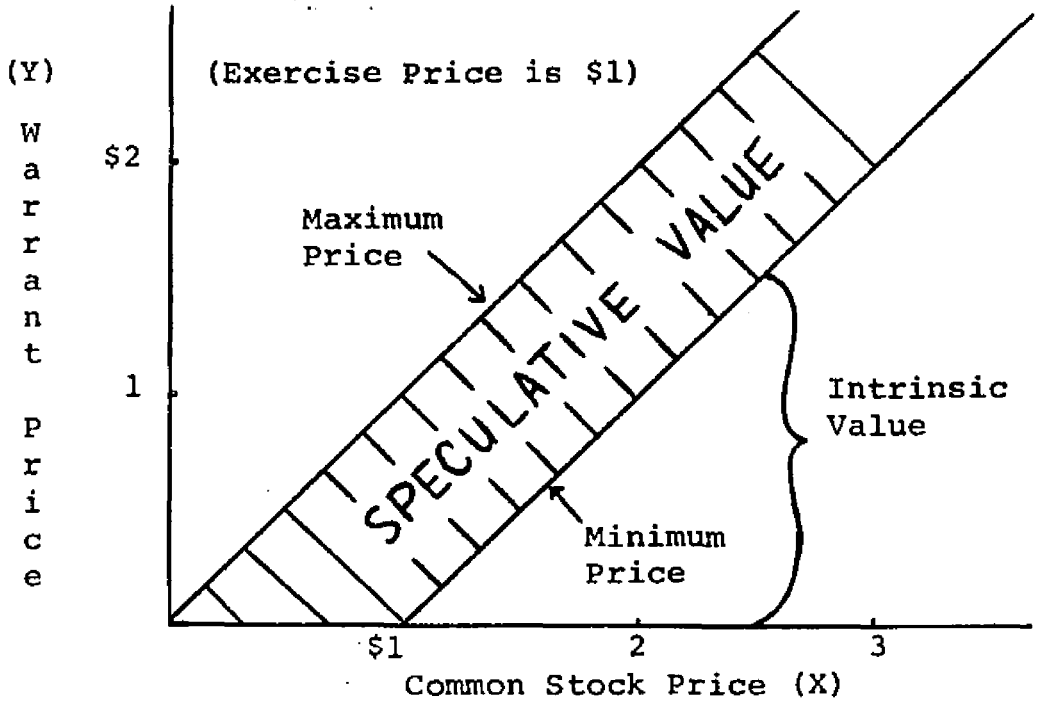
The Gulf & Western warrant did not have an intrinsic value in January, 1970, because its common traded at $20. The warrant, however, traded at $6. This value may be referred to as the speculative value, and its level depends on the anticipated performance of the common. The speculative value of a warrant to buy one share of common stock will have as its upper limit the price of the common stock itself. Knowledgeable investors would never pay more

for an option on the common than they would pay for the share itself. The only exception to this rule would be an investor who has become caught in a short squeeze after selling warrants short.[2]

The maximum speculative value and the minimum intrinsic value may be described graphically. Chart 1 depicts these relationships.

CHART 1

SPECULATIVE AND INTRINSIC VALUES OF WARRANTS



[2]William Schwartz and Julius Spellman, Guide To Convertible Securities (New York: Convertible Securities Press, 1968), p. 28.

Chart 1 indicates that the price of the warrant is confined to speculative value when the price of the common is less than $1, and consists of intrinsic value and possibly speculative value at all levels greater than $1. It is logical that a warrant with a speculative value of $.35 when the common stock is at $.99 will not totally lose its speculative value when the common is at $1.01. Moreover, it does not appear theoretically feasible that the speculative value that existed at $.99 for the common will continue to exist when the common trades at higher levels. Therefore, one may conclude that the speculative value should be at the highest for a given warrant when the common stock price equals the option price. Also, the speculative value should be at its minimum when the common stock price approaches zero and when it approaches infinity.

Dr. Paul Samuelson has written an excellent article on rational pricing of the warrant.[3] His contention is that warrants may have numerous ranges of rational prices--with the ranges dependent on the underlying nature of individual warrants and the expectations of the warrant buyers. Samuelson's Figures 1a and 1b[4] are approximately reproduced

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[3]Paul A. Samuelson and Henry McKean, Jr., "Rational Theory Of Warrant Pricing," Industrial Management Review, Vol. 6, No. 2 (Spring, 1965), pp. 13-39.

[4]Ibid., p. 20.

in Chart 2.

Samuelson's charts are based on the expectations of warrant buyers in relation to common stock buyers. He defines α to be the mean expected growth rate of the common stock per unit in time and β to be the mean expected growth rate of the warrant.[5] Samuelson's Figure 1a then assumes that $\alpha=\beta$. Moreover, he shows with line A that a perpetual warrant will be equal to the price of the common because they are priced to have identical expected growth rates.

Line B is priced less than the common because it is limited by a 25 year maturity. Lines C and D describe warrants with maturities of 4 and 1 years, respectively. Samuelson's Figure 1a is pertinent to the present study due to his identification of time as a major variable affecting a warrant's price. He also contributes an important idea concerning the value of warrants with his graphic and mathematical presentation--namely, that warrants with expected mean growth rates equal to those of their corresponding common will never be converted (or rest on their intrinsic value) as long as a positive period of time exists before maturity.

Samuelson's Figure 1b[6] also provides excellent discussion for warrant valuation. Two reasons are introduced

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[5] Ibid., p. 19.

[6] Ibid., p. 20.

CHART 2

SAMUELSON'S THEORETICAL VALUES OF WARRANTS

Figure 1a

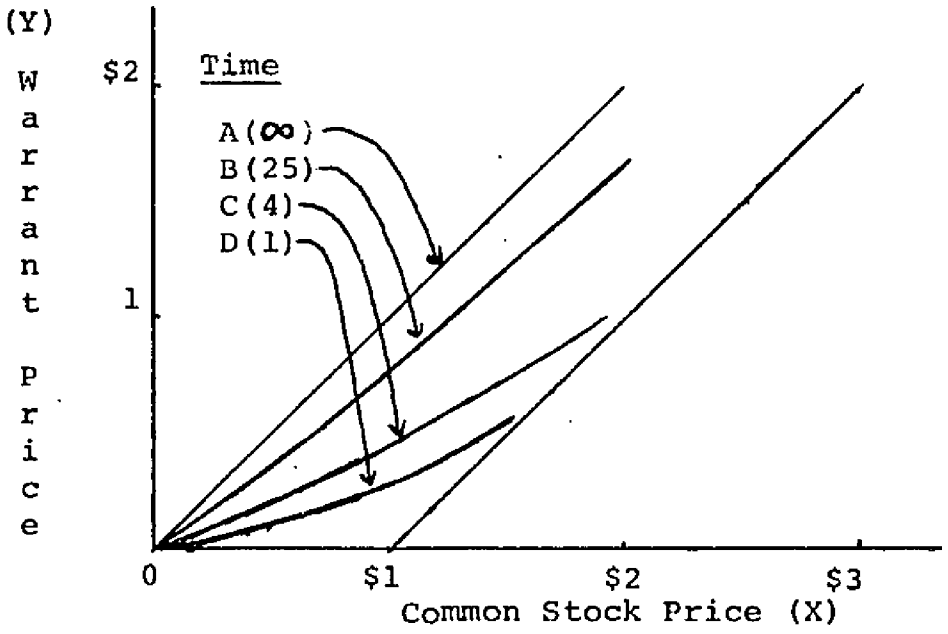
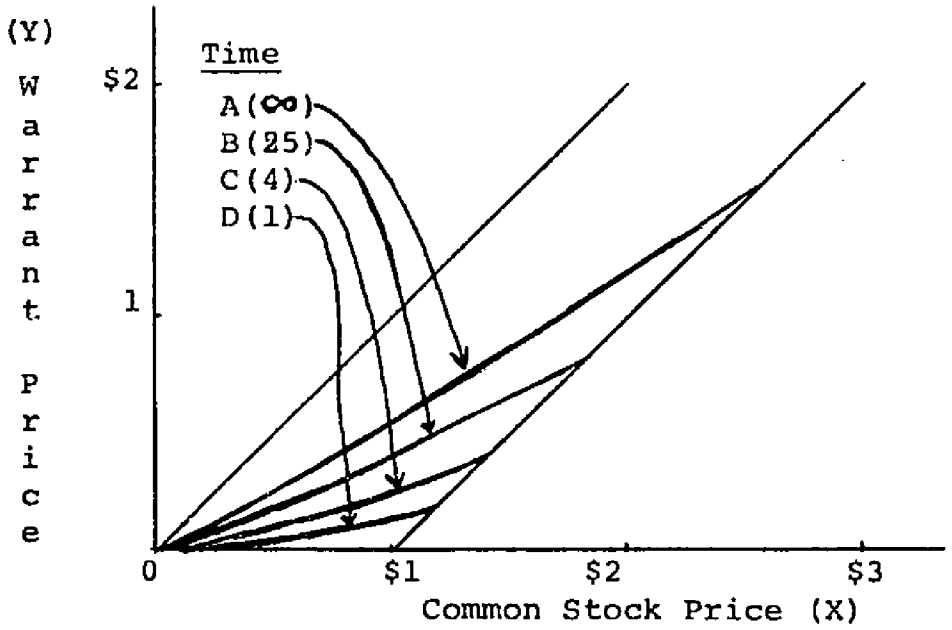


Figure 1b



to show why a warrant will have a $\beta > \alpha$ to represent as attractive a purchase commitment as the common stock. According to Samuelson, investors may require a higher mean return to hold the warrants because warrants are not protected against dividend declarations on the common, and, warrants represent more volatile instruments than common.[7]

Samuelson's Figure 1b shows that a warrant will be converted at some range of x (or rest on its intrinsic value) regardless of its time horizon. This is due to the fact that investors expect a higher yield from the warrant than from the stock, and it must be priced accordingly. At some price of the common the warrant will be at its intrinsic value. At that point it should be converted into common as it will no longer satisfy the return differential. Theoretically, a small return differential and a large exercise price may create an extremely high value of x causing the warrant to be priced at its intrinsic value.

The empirical model that appears in Chapter IV closely follows Samuelson's Figure 1b. There may be other reasons why a warrant will require a high mean return other than dividends and volatility. The warrant may not be fully protected from dilution via small stock splits, additional options being issued, and benefits (both psychic and material) accorded stockholders. Samuelson's presentation

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[7] Ibid., pp. 19-22.

must be described as an innovation in thinking about warrant price behavior.

An existing perpetual warrant may be applied to his two models. For example, the Atlas Corporation perpetual warrant has an exercise price of $6.25. If one assumes an investor purchases the warrant with the intent of holding for 10 years, one finds that the price can approximate either Samuelson's Figure 1a or 1b. Under the assumption that the market place has an equal required rate of return of 10% annually in both the common and the warrant, the common stock will be priced in anticipation of its future performance. If the common were expected to appreciate to $20 in 10 years, it should be priced at $7.72 to provide the desired rate of return (assuming no dividends).[8] The warrant could also be priced at $7.72 if investors at all periods of time had the same desired rate of return. This can be reconciled somewhat by considering the present value of the exercise price discounted at 10% over an infinite period of time. The present value of $6.25 discounted at 10% for 50 years is only $.056. Investors, therefore, who consider a perpetual warrant to remain unconverted forever have little reason for penalizing the warrant's value

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[8]The $7.72 price is determined by multiplying the present value of $1 received 10 years hence and discounted at 10% by $20. Thus .386 times $20 equals $7.72.

relative to the stock.

Consideration can now be focussed on the instance where the market place has different required rates of return for these securities. When the desired rate of return on the common remains at 10%, but the warrant's rate is raised to 15%, investors still view the present value of the exercise price to be nil. The common will still be priced at $7.72, but the warrant will now be priced at $4.94 ($20 discounted @ 15% for 10 years). The warrant will be at its intrinsic value in 10 years because the $20 price assumes the market price will grow to $51.82 at the end of the next ten years, and $51.82 discounted at 15% for 10 years is only $12.80 (which is less than the intrinsic value).[9] It is possible that the desired rates of return will not maintain an equal distance as the stock grows in value, since stock and warrant movements will become highly correlated. Therefore, a higher price may be required to bring the warrants's price to its intrinsic value.

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[9]The intrinsic value at the end of 10 years is $13.75 ($20.00 market price minus 6.25 exercise price), and the estimated price assuming a 15% desired rate of raturn is $12.80 ($51.82 multiplied by .247, the discount factor at 15% for 10 years.

Leverage. The speculative value of a warrant is primarily related to the leverage offered by the individual security. A high speculative value in relation to the price of the common actually retards the degree of leverage for a given warrant. The actual leverage in a warrant may be closely associated to the mechanical leverage observed by Archimedes, a third century scientist.[10] He observed that less force was required to move a given weight as one moved further away from the fulcrum (the base which supported the lever). The leverage afforded by a warrant may be analogous to mechanical leverage in this way: the relationship of the common stock price to the warrant value can be considered the lever, with equality between the two resting at the exercise price, viewed as the fulcrum. An increase in the price of the common stock will generate a higher relative move in the warrant when the price of the common is high relative to the price of the warrant and greater than or equal to the exercise price. An illustration may be helpful at this point.

Assume that the XYZ common trades at $20 and its warrant, which is exercisable at $10, trades at $10. In

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[10]David Jenkins, The Power of Leverage (Larchmont, New York: Investors Intelligence, 1966), p. 1.

this case a given percentage increase in the common will result in twice the percentage increase for the warrant. If the common appreciates 10 per cent to $22, the warrant will increase 20 per cent to $12. The leverage for the warrant in the first example is calculated to be 2, since the warrant rises twice as fast as the common. If the stock were trading at $30 with the warrant at $20 and still exercisable at $10, the degree of leverage would have dropped to 1.5 which can be proved by hypothesizing percentage increases for these securities.

The proceeding example definitely shows leverage to be a function of the stock's price relative to the warrant's price, but it ignores speculative value altogether. What happen's when the price of the common is equal to or less than the exercise price? Will a given increase in the common have a direct impact on the price of the warrant? These are some of the questions that the empirical model in Chapter IV attempts to answer.

Leverage indicator. William Schwartz recognized that the degree of leverage would be a valuable tool in determining the attractiveness of individual warrants.[11] Schwartz

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[11]William Schwartz, Using Warrants For Leverage (New York: Investing and Management Compass, Inc., 1967), pp. 14-20.

also recognized that exact degrees of financial leverage were impossible to calculate when warrant prices included speculative values or premiums. Therefore, Schwartz developed an indication of leverage which he expressed with the following formula.[12]

$$\text{Leverage Indicator} = \frac{\text{Price of optioned stock}}{\text{Price of warrant on one share}}$$

Schwartz contends that the leverage indicator can be effectively used to locate warrants with attractive upside potential. He is of the opinion that the more attractive warrants are those with no intrinsic value and leverage indicators of 4 or more. He further feels that warrants with leverage indicators of less than 2 should be avoided because of the great downside risks relative to small upside potentials.[13]

Warrants also have downside leverage which results in a greater percentage decline in individual warrants than in their associated common stock. Sidney Fried describes the danger of downside leverage with an example of the Universal Pictures warrants which declined from $39 in 1945 to $1.50 in 1947.[14] The percentage decline in the common was

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[12] Ibid., p. 19.

[13] Ibid., p. 19.

[14] Sidney Fried, The Speculative Merits of Common Stock Warrants (New York: R.H.M. Associates, 1961), p. 2.

significantly less.

Two recent examples of unfavorable leverage occurred during the market decline of 1966. McCrory's warrant declined 46.8% from $5.875 to $3.125 while its common stock declined 23.1% from $22.75 to $17.50. Thus, McCrory's warrants experienced negative leverage of greater than 2 from the beginning to the end of 1966. The 1966 performance of the Universal American warrant provides a different example of downside risk. The warrant had 15 months left before expiration at the beginning of 1966. It had an exercise price of $13.75 and the common was trading at $10.875. The warrant was traded at $4. At the end of 1966, the common traded at $11.125 for a percentage gain of 2.3%, but the warrant traded at only $1.25 for a percentage decline of 68.7%.

The preceeding examples suggest the following conclusions: (1) speculative values on warrants may magnify the percentage decline of a warrant when the common declines; and (2) speculative values on soon-to-expire warrants may result in sharp market losses from warrants, even when the common stock appreciates.

One can see that leverage in warrants offers an advantage on the upside but also generates a high degree of downside risk. The volatile nature of these instruments,

especially when speculative values are temporarily out of line, is justification for establishing valuation models that will assist in determining the degrees of positive and negative leverage. Trading tactics aimed toward taking advantage of these discrepancies are explored in Chapter VI.

The Origin of Warrants

Warrants enter the market place in numerous ways. As seen in Chapter I, many warrant issues accompany bonds. The desire of the issuing company to lower the coupon rate on the bonds, while creating an avenue for acquiring equity capital upon exercise of the warrants, accounts for this occurrence. Twenty-six warrants listed on the American Exchange at the end of 1968 had six different ways of coming into existence. Half of them were issued with debentures to acquire capital for various purposes; 7 were used in a package of securities to effect an acquisition; 3 of the warrant issues were tendered for the issuing company's outstanding shares of common stock; and one was used in each of the following ways: (a) as part of the reorganization package, (b) to pay a dividend in warrants to common stockholders, and (c) to be used as a package in selling common stock and bonds to the public. Hayes and Reiling discussed the flexibility of the warrant in a Harvard Business Review

article.[15] They showed how small growth companies may avoid underpricing a common stock issue by attaching warrants. The attached warrant may enable the company to completely avoid underpricing and eventually issue additional stock upon exercise of the warrants.

Warrants may be paid to investment bankers for floating an issue of securities or for locating a merger candidate. The taxation rules pertaining to warrants received as compensation for underwriting services led the Investment Bankers Association to conduct an empirical study of these instruments.[16] The investment bankers concluded that fair values could be determined for taxation purposes. The approach used in their empirical study is pertinent to the present study.

All of the enumerated methods of using warrants seem appropriate except for the tender offers to retire existing shares of common stock. The LTV companies have used this method of recapitalization in at least three separate offerings. The primary objective was to increase reported earnings

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[15]Samuel Hayes and Henry Reiling, "Sophisticated Financing Tool: The Warrant," Harvard Business Review, Vol. 47, No. 1 (January-February, 1969), pp. 137-150.

[16]Federal Income Taxation of Compensatory Options (Including Warrants) Granted To Underwriters and Other Independent Contractors (Washington, D.C.: Investment Bankers Association, 1963), pp. 1-85.

per share through lowering the shares outstanding by more than the decline in net income. The decline was produced by the interest expense of the bonds accompanying the warrants.

Lerner Stores also used warrants and debentures as a package in offering to buy its own stock. As a matter of fact, Lerner Stores pre-dated LTV's use of this approach. The Wall Street Journal of May 19, 1967, carried an announcement stating that Lerner intended to repurchase all of its shares of common not owned by McCrory Corporation through issuing $10 par of 6-1/2% debentures and 1 warrant to purchase common at $15 for each share of Lerner Stores. The market price of Lerner common on that date was $10.625. On October 2, 1967, Lerner announced the completion of the offer and stated that 985,000 shares had been acquired by issuing $9,850,000 in 6-1/2% debentures and 985,000 warrants exercisable at $15 per share. The following statistical presentation (Table III) illustrates the impact of the tender offer.

Table III was prepared by adding the after-tax cost of interest on the debentures to the reported net income for 1968 and 1969. The shares that were acquired through the tender offer were also added back. These data show earnings per share to have been substantially stimulated by the tender offer. The actual data reported show earnings per share

increased more than 80.3% over 1967-1969 while the adjusted data, assuming the tender offer did not occur, showed a growth of only 50.8%. The tender offer to reacquire Lerner's common through a combination of debt and warrants substantially increased earnings growth as reported by the company. This ability to create earnings per share growth through acquiring ones own stock was probably one motivating force in bringing about the Accounting Principles Board Opinion No. 15 on earnings per share.[17]

TABLE III

EARNINGS PER SHARE STIMULUS THROUGH TENDER OFFER

ACTUAL DATA REPORTED	1967	1968	1969
After tax income	$6,734,516	$8,549,958	$10,054,097
Shares outstanding	5,110,912	4,150,475	4,222,055
Earnings per share	1.32	2.06	2.38
WITHOUT TENDER OFFER			
After tax income	$6,734,516	$8.882,888	$10,387,027
Shares outstanding	5,110,912	5,135,475	5,207,055
Earnings per share	1.32	1.73	1.99

Source: Moody's Industrial Manual (New York: Moody's Investors Service, 1969), p. 2670.

[17]APB Opinion No. 15 focuses on the proper accounting determination and presentation of earnings per share data. It includes provisions to adjust these earnings in lieu of potential dilution through various forms of convertible securities. The last section of this chapter discusses accounting for these instruments.

Basic Elements of Convertible Bonds

The convertible bond is an investment package offering two distinct claims: (1) a claim to interest via owning the bond and (2) a claim on the common stock through the conversion feature. The bond provides normal bond features of periodic interest and terminal repayment of the principal at some prespecified date. The conversion privilege enables the bondholder to participate in the appreciation of the common stock. The conversion privilege is similar to a stock purchase warrant with the exception that the bond must be surrendered in order to effect a change into common stock. Thus, the bond and the conversion privilege are inseparable until conversion or expiration of the claim on the common.

Premium on Convertible Bonds

This "inseparable" nature of straight debt value and option value presents a dilemma when determining fair values for this unique investment package. Investors must decide on a fair price by simultaneously considering the straight debt value and the conversion value.[18] It is frequently difficult to decide whether one is basically buying straight

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[18]Conversion value refers to the immediate market value of the shares of stock that may be acquired by surrendering the bond.

debt value with an equity "kicker" (conversion value potential), or whether conversion value is more important with the debt value only providing a degree of downside protection. Either way, the convertible bond purchaser will usually be faced with paying a premium over one of these values in order to acquire the instrument. The determination of a proper price for a given premium is a primary element in evaluating convertible bonds. Chart 3 is a pictorial illustration of the nature of these premiums.

As can be seen from Chart 3, three separate possibilities may arise when considering convertible bonds. The debt value may be greater than the conversion value; the debt value may equal the conversion value; and, the conversion value may exceed debt value. Situations A, B, and C provide respective examples of these three possibilities. It is proposed that situations A and C force an investor to decide on the proper approach to valuation. The investor must decide if A's conversion value is worth $140 and if C's debt value is worth $140.[19] How should one analyze Situation B, since the premium is the same for debt value as it is for conversion value? This question will be further explored in the following section, and in Chapter

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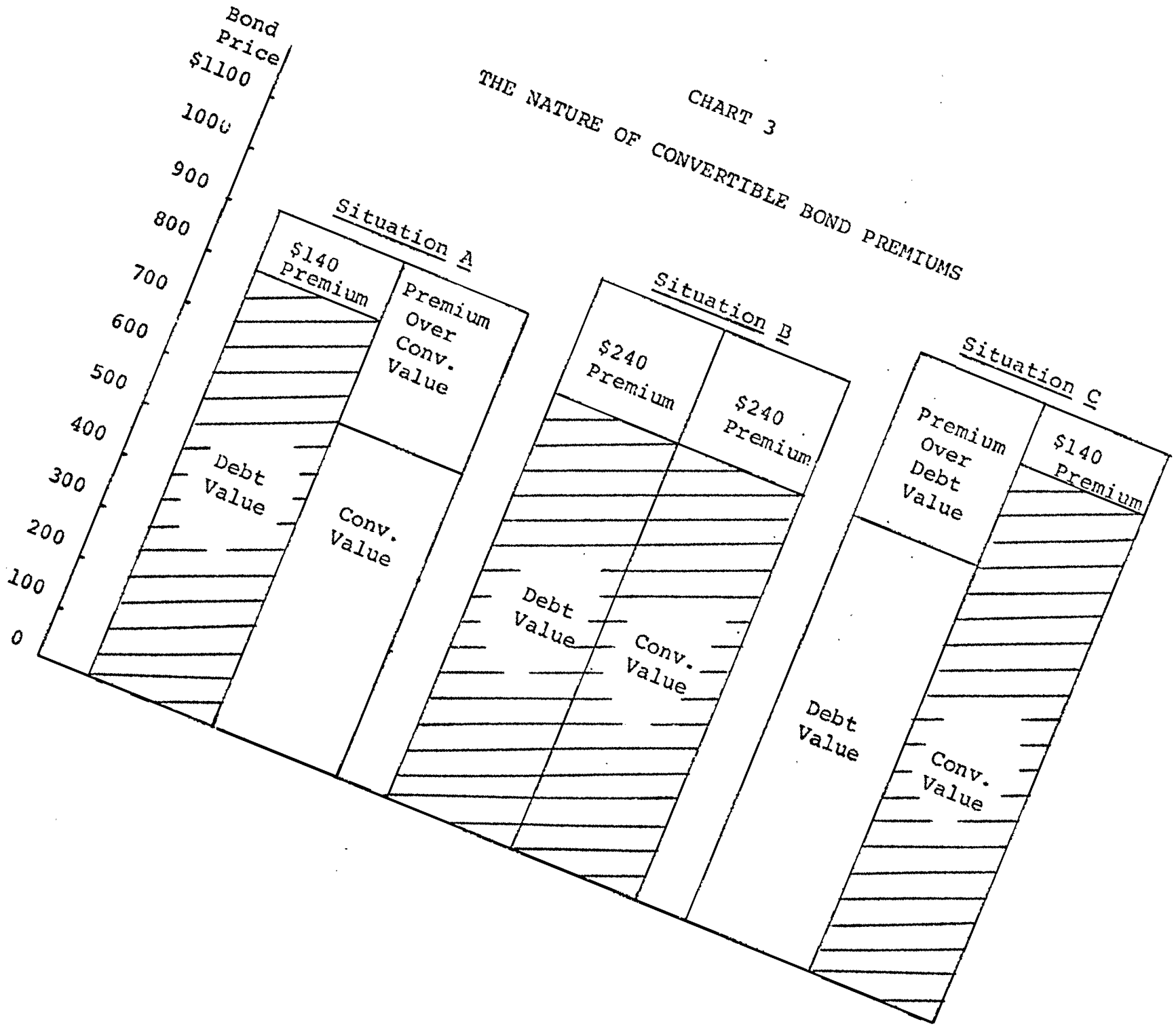
[19]It is common practice to refer to the premium of a convertible bond as being the market price of the bond less the highest of either straight debt value or conversion value.

CHART 3

THE NATURE OF CONVERTIBLE BOND PREMIUMS



V of this paper.

Theoretical Valuation of Convertible Bonds

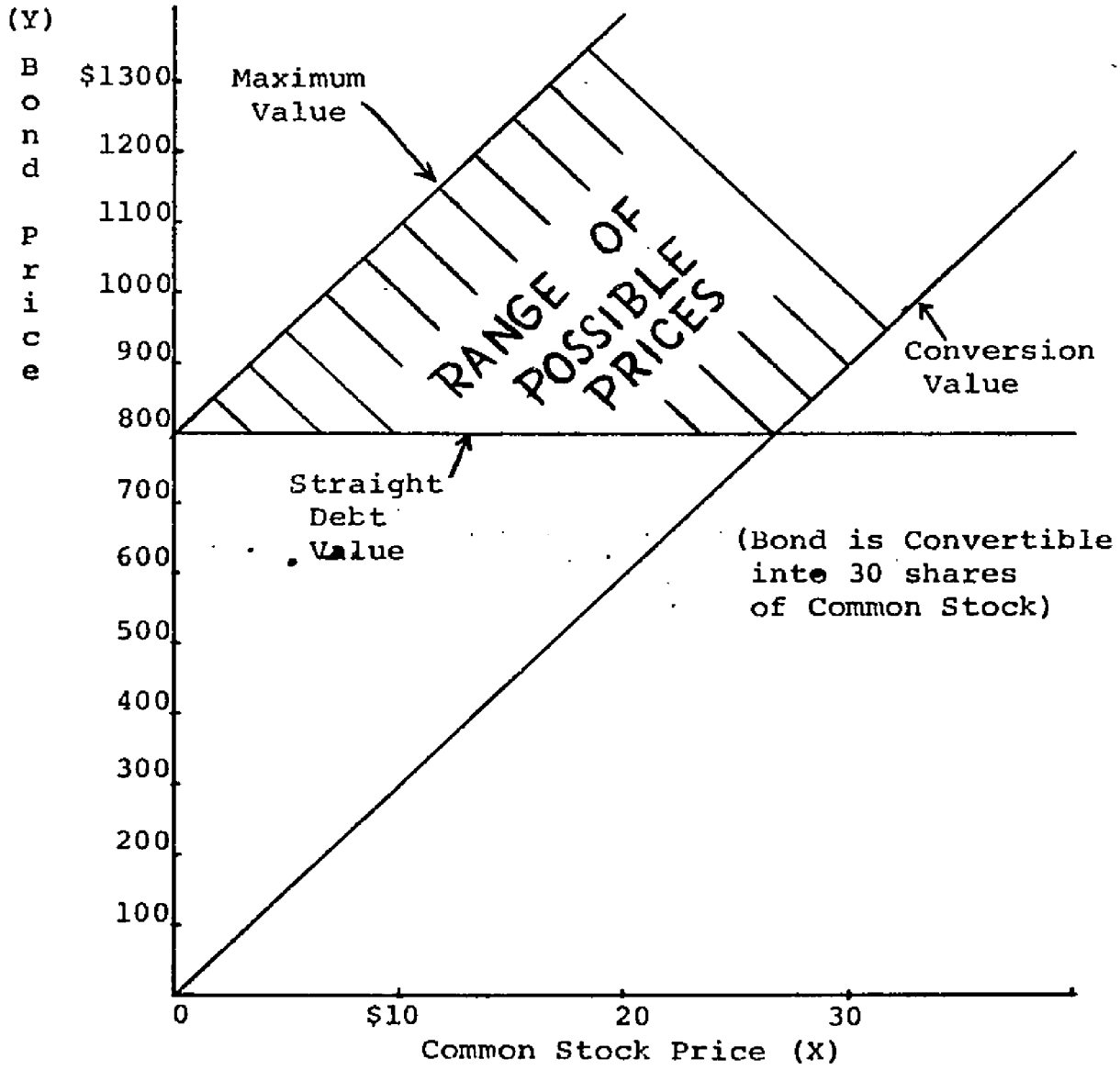
Potential purchasers of convertible bonds desire to obtain both the features of debt and the features of conversion value, or they would surely put their money into common stock or bonds directly. However, a given investor may be primarily interested in acquiring a claim to interest through the bond and would be secondarily interested in the conversion opportunities. His order of priority would require him to first determine the straight debt value of the security package and then add the value of the conversion feature. An investor primarily interested in conversion opportunities would approach the investment package by determining the value of the conversion feature and then including the hypothesized value of the straight debt value.

The two separate approaches to convertible bond valuation lead to confusion as to the fair value of a given security package. Perhaps a graphic examination of a convertible bond's range of possible values will facilitate a decision as to the proper approach to utilize in determining fair value. Chart 4 provides that graphic presentation by depicting the possible range of convertible bond prices.

Chart 4 shows a convertible bond to be priced at its debt value only when the common stock is priced at zero. If

CHART 4

THEORETICAL PRICE RANGE FOR CONVERTIBLE BONDS



the common price increased to $15, the conversion value of $450 (30 shares X $15) would be worth something more than the bond's straight debt value. The convertible bond price could vary from a maximum of $1,250 ($800 DV plus $450) to a minimum of $800.

The maximum value of a convertible bond can be conceptualized in the following manner. Assume that a particular convertible bond is perpetual and the corresponding common stock does not pay dividends. The package can then be hypothesized to provide fixed interest receipts, a bond value, and equal claims on the common through the convertible feature. Moreover, the present value of the debt surrendered at some distant time horizon must be zero in order for the price of the bond to approach the maximum value. Few convertibles come close to these conditions, but the maximum value of a convertible bond is illustrated through adding debt value and conversion value. The minimum value of a bond is described by the debt value until conversion value becomes greater than $800. The minimum value is then reflected by conversion value alone.

The dilemma of evaluating the proper approach to convertible bond valuation remains. However, the possible range of bond prices provides the basic model for evaluating convertible bonds. This model is developed in Chapter V.

Analysis of debt value. Chart 4 shows a convertible bond price to be based on two elements, the potential conversion value as a function of the price performance of the common stock, and the debt value as a function of interest rates and time to maturity. The expected performance of the common stock may be analyzed through both fundamental and technical approaches. Although the methodology of these approachs is beyond the scope of the present study, a brief outline of how to evaluate the downside risks of straight debt values is considered pertinent to this study.

Eugene Brigham's graphic model of a convertible bond assumes interest rates to be constant and describes straight debt value as constantly moving toward par over the bond's maturity.[20] Actually, the ultimate value of the straight debt value depends on the level of interest rates over the maturity of the bond. The market direction of straight debt value will be an inverse function of changes in interest rates. The degree of fluctuation will furthermore diminish with a given interest rate change as the bond approaches maturity.

Yield curve concept. Straight debt value and interest rate relationships can be appropriately described

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[20]Eugene F. Brigham, "An Analysis of Convertible Debentures: Theory and Some Empirical Evidence," Journal of Finance, Vol. 21, No. 2 (March, 1966), p. 37.

through the use of a yield curve. This can be illustrated by assuming two convertible bonds, X and Y, each have coupon rates of 6%. Bond X has 3 years to maturity and Y has 20 years to maturity. The market yield curve for bonds with similar ratings is also 6% for all maturities.[21] Bonds X and Y will each have a straight debt value equal to par or $1,000, since their coupon rates equal the market's rate of interest. If we assume that one year later interest rates have increased to 8% on bonds with 2 year maturities and to 7% on 19 years bonds, the decrease in straight debt values will be greater for the 19 year maturities than for the 2 year maturities. These yield curves are presented in Chart 5.

Chart 5 reveals a smaller yield change for the longer term maturity, but the 1% change must be spread over the 19 years that remain to maturity. Present value tables indicate the long-term bond to have lost approximately $104.20 in straight debt value while the 2 year bond suffered only a $36.30 loss due to the 2% change in effective interest

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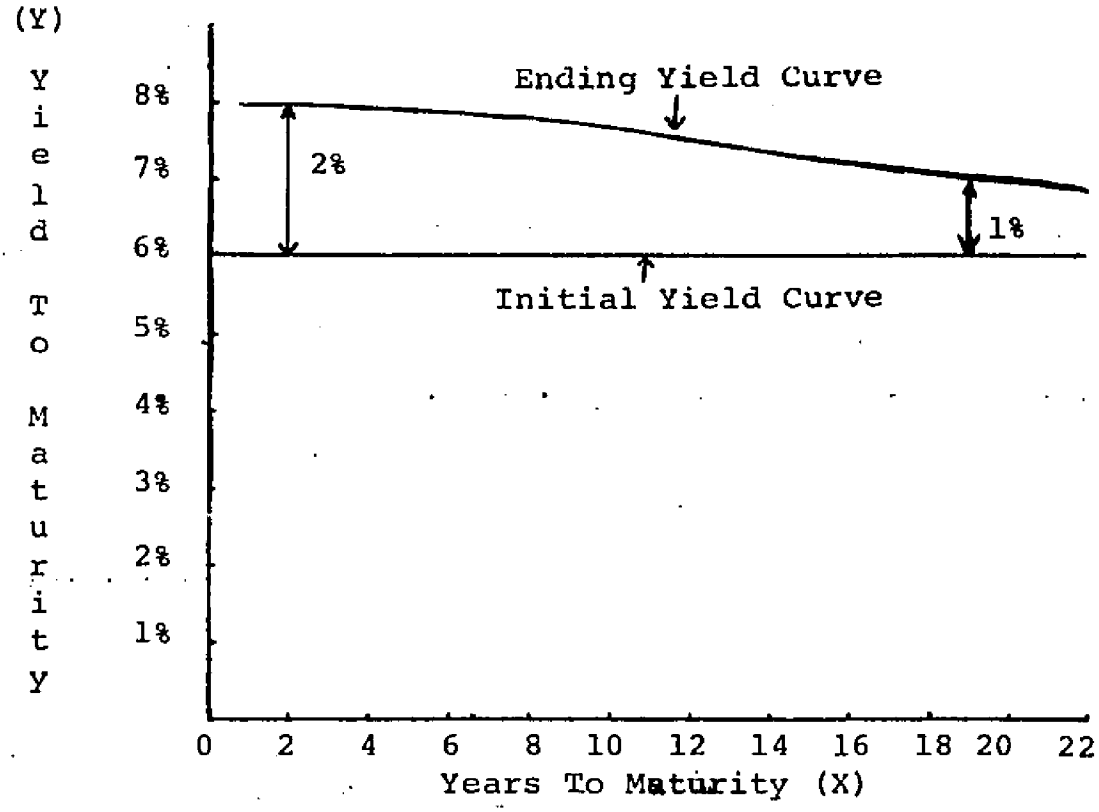
[21] A yield curve where all maturities have equal effective yields is said to be "flat". The flat yield curve is presented here due to its simplicity for explaining price changes.

CHART 5

FLAT AND DOWN-SLOPING YIELD CURVES



rates.[22] Conversely, a decline in interest rates would have

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[22]The approximate decline in value for the long-term bond may be determined in the following manner.

Effective semi-annual yield	3.5%
Semi-annual coupon rate	3.0%
Interest deficiency	.5%
P.V. of periodic $1 @ 3.5% for 38 periods	X20.841
	10.4205%
($1,000 par)(10.4205%) =	$104.20

For the conventional approach in determining bond prices, see Jules J. Bogen, Financial Handbook (4th ed.; New York: The Ronald Press Company, 1968), pp. 27.48.

caused a greater price appreciation in the 19 year bond than in the 2 year bond. The direction of interest rates is then a very important aspect in the analysis of convertible bonds. Many financial experts have concluded that interest rate risks are more important than credit risks when analyzing corporate bonds.[23]

The shape of the yield curve offers indication of the direction interest rates will take in the future. The flat yield curve usually occurs in initial stages of tight money periods and again near the end of restrictive monetary policies. Typically, the yield curve is upsloping with short-term maturities yielding less than longer term securities. This type of yield curve is consistent with ideas concerning liquidity preference, since short-term instruments can be held with little risk of principal loss and a high degree of marketability. As a result, investors are willing to accept smaller yields on short-term investments and require higher yields on longer term commitments because interest rate risks are magnified. The upsloping (normal) yield curve offers a premium for assuming higher interest rate risks.

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[23]See Douglas H. Bellemore, Investments, Principles, and Analysis (Dallas: South-Western Publishing Company, 1966), p. 54 and Harry Sauvain, "Changing Interest Rates and Investment Portfolio," Journal of Finance, Vol. 14, No. 2 (May, 1959), pp. 230-244.

The yield curve where short-term maturities afford higher yields than longer term securities is referred to as a downsloping or hump-backed yield curve. It occurs during tight money periods and is a product of the struggle for reserves by the commercial banking industry and the desire to extend maturities during high interest rate periods. The Federal Reserve System absorbs much of the supply of short-term money through selling government securities. Lenders, anticipating a decline in interest rates at the end of the tight money period, work toward increasing the average length of maturity of their portfolio during these high interest rates. Thus, short-term rates expand at a faster pace than long-term rates, and a hump-backed yield curve is created.

Historically, hump-backed yield curves have been followed by lower short and long-term interest rates as the yields settled back to a normal liquidity preference structure. From the standpoint of straight debt value, long-term convertible bonds are more attractive commitments when a hump-backed yield curve exists because of the appreciation potential of their straight debt values.

Anti-Dilution Clauses for Warrants and Convertible Bonds

Stock purchase warrants and convertible bonds are closely related in that both provide contractual options for

the purchase of common stock. Legal protection of these contractual rights necessitates similar draftmanship. Therefore, the following discussion will be uniformly applicable to each security form.

The anti-dilution clause is an important element in each security package offering a call on the common stock. This clause contains the rights of the claimant on the common stock in respect to various developments. Provisions of anti-dilution clauses may be thought of as determining the relative positions of the option holders and the residual owners themselves, since a concession granted to the option holder dilutes the position of basic owners. Due to the important ramifications of these clauses, their explication is pertinent to this research project.

Stanley Kaplan[24] feels that the anti-dilution clause is often viewed by attorneys and investment bankers as boiler plate-a standard provision that is not worthy of negotiation. He contends that he has seen attorneys prepare convertible securities by cutting anti-dilution clauses from other prospectuses and glueing them to the data of a proposed security issue.

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[24]Stanley Kaplan, "Piercing the Corporate Boilerplate: Anti-Dilution Clauses In Convertible Securities," The University of Chicago Law Review, Vol. 33 (1965-1966), p. 3.

Current prospectuses of convertible bond offerings tend to confirm Kaplan's contention. The following is a statement taken from a preliminary prospectus dated December 31, 1969. It approximates some anti-dilution clauses, and is identical to others.

> The conversion price is subject to adjustment under a formula in certain events, including the issuance of Shares or rights or options to purchase Shares or securities convertible into Shares at less than the current conversion price (with credit under the formula for the issuance or sale of Shares above the current conversion price). The conversion price is subject to automatic proportionate adjustment upon the subdivision or combination of Shares or the payment of Share dividends. No reduction under the formula will be required until cumulative adjustments amount to 1% of the conversion price then in effect. No adjustment under the formula or otherwise will be made upon conversion of the Debentures or of the Trust's 5% Subordinated Convertible Notes, or upon the issuance of Shares in connection with the acquisition of assets, or upon the issuance of Shares pursuant to any present or future stock option plan.[25]

The above statement was selected because it covered the typical developments requiring adjustment for dilution. Chronologically it covers (1) the sale of common stock below a specified price, (2) the proper handling of stock splits or stock dividends, (3) the problem concerning adjustment upon acquisition of assets, and (4) the adjustment due to the issuance of stock options. Possible developments that

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[25]Taken from the preliminary prospectus of Continental Mortgage Investors.

may undermine the conversion or exercise value are infinite. Only the most frequently encountered developments are presented in typical anti-dilution clauses.

Similar provisions are presented when warrants are issued to the public. The prospectus will usually refer to a Warrant Agreement which is defined as an instrument executed by the corporation setting forth the rights of the warrant holders and the corporation.[26] Warrant Agreements can be obtained from warrant agents, and they represent the formal contract between the issuing corporation, warrant agent, and the warrant holders.

Issue of Additional Common Stock

The first provision listed in the anti-dilution clause is perhaps the most difficult to reconcile. The proviso requires that an adjustment be made when common stock is sold for less than the exercise price. The use of exercise price guarantees an adjustment to an option holder whenever common stock is issued for consideration below the conversion price. Sales of common stock at prices higher than the exercise price are considered not to have diluting effects on the option holders.

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[26]Clifford Charles Keith, "Convertible Securities and Stock Purchase Warrants," Rocky Mountain Law Review, Vol. 2, (1929-1930), p. 25.

Some anti-dilution clauses call for adjustments to the conversion price to be based on the market price of the common stock. Sales of common below the market price[27] require adjustments in the conversion or exercise price.[28] The market price formula is fundamentally based upon the concept that the option holder is in the same position as the common stockholders. In other words, if common stockholders are given a benefit, it should be reflected in the conversion formula. This concept, in effect, implies that pre-emptive rights given to common stockholders should also be granted to the option holders. Adjustments in the exercise price are based on the dilutive value of a pre-emptive rights issue.

The market value approach may be compared with the conversion value approach by examining its impact on the following situation:

Company X

Initial shares outstanding	1,000,000
Warrants to purchase shares	200,000
New Shares to be issued	500,000
Average per share market price prior to issue (MP)	$30
Subscription price of new shares (SP)	20
Exercise price of each warrant (EP)	10

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[27]Usually defined as average closing price for a period of time prior to the actual issuance of the securities.

[28]See the preliminary prospectus of Guardian Mortgage Investors dated November 21, 1969.

If the anti-dilution clause specified the exercise price in determining the degree of dilution, there would be no adjustment. However, the use of the market price approach significantly lowers the exercise price.

The adjustment required under the market price approach can be determined by calculating the intrinsic value of each warrant before and after the issue. The intrinsic value prior to the issue is $20 ($30 market price—$10 exercise price).[29] The adjusted number of shares that each warrant is now entitled to can be determined by dividing the original intrinsic value of $20 by the diluted value of $16.67 which yields 1.2 shares for each warrant. The adjusted exercise price per share then become $8.33 ($10 old exercise price ÷ 1.2 shares). By adjusting the exercise price when new stock is issued for less than the market price, the option holder has maintained his claims on the common stock to the same degree as common stockholders. On the other hand, the use of the exercise price approach would have diluted the holder's intrinsic value by $3.33 or 16.65%, since the market price for the newly issued shares exceeded

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[29]The new market price may be determined by the following formula:

$$\text{New MP} = \frac{(MP)(N)+(SP)}{N+1} = \frac{(30)(2)+(20)}{2+1} = 26.67$$

Where: N=number of shares necessary to purchase one new share.

the exercise price.

The exercise price approach has been described as adversely affecting the warrant holder when new issues of common are sold with SP>EP. The approach may also be shown to benefit the warrant holder when SP<EP. The following exemplifies those conditions.

Company Y

Initial shares outstanding	1,000,000
Warrants to purchase shares	200,000
New shares to be issued	500,000
Average per share market price prior to issue (MP)	$8
Subscription price of new shares (SP)	6
Exercise price of each warrant (EP)	10

The adjusted exercise price can now be calculated with a specific formula, since the exercise and the market price are greater than the subscription price. Such a formula is:[30]

$$\text{Adjusted EP} = \frac{PX+NY}{X+N}$$

Where: P = exercise price (or average market price)
X = number of shares outstanding prior to the new price
N = number of shares to be issued
Y = consideration for newly issued shares.

Substituting into the formula to determine the adjusted exercise price under the two alternatives yields:

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[30]Kaplan, op. cit., p. 8.

AVERAGE MARKET PRICE APPROACH	EXERCISE PRICE APPROACH
$\frac{(8)(1,000,000)+(500,000)(6)}{500,000+1,000,000}$	$\frac{10(1,000,000)+(500,000)(6)}{500,000+1,000,000}$
$= \frac{11,000,000}{1,500,000}$	$= \frac{13,000,000}{1,500,000}$
= 7.33 + (EP-MP)	= $8.67
= 7.33 + (10-8) = 7.33 + 2	
= $9.33	

The exercise price approach significantly benefits the option holder, since he can now purchase 1.154 shares ($10 initial exercise price ÷$8.67 adjusted exercise price per share) when only 1.072 shares could be acquired under the market value approach. The market value approach enables the warrant holder to obtain a benefit identical to that of the common stockholder, $.67 (i.e., $10 initial exercise price--$9.33 adjusted exercise price). This can be proven by examining the theoretical value of one right by using the formula:[31]

$$\frac{M - S}{N + 1}$$

Where: M = initial market price of stock
S = subscription price of the stock
N = the number of rights necessary to purchase one new share.

Substituting:

$$\frac{\$8 - 6}{2 + 1} = \$.67$$

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[31]Robert Johnson, Financial Management (Boston: Allyn & Bacon, Inc., 1966), p. 472.

The adjustment to the exercise price is shown to equal the benefit received by each common stockholder through the preemptive rights offering. The option holder has maintained his proportionate share in the firm through the market value approach.

Use of the exercise price approach provides the option holder with twice the benefit received by the common stockholders. Such an approach actually undermines the rights of common stockholders by increasing the claims on the common through a significant lowering of the exercise price on the warrants.

Potential option buyers should be cognizant of the fact that the exercise price approach will protect them from dilution via new stock sales only when the subscription price of the new stock is less than the exercise price. On the other hand, the market price approach will afford protection anytime new shares are sold for less than the average price.

Stock Splits and Stock Dividends

Modern anti-dilution clauses appear to offer adequate protection for stock splits, although there is some variation in the handling of stock dividends. A few clauses enable payment of minor stock dividends without adjusting the exercise price. The clause of the Armour warrants failed

to offer adequate protection for stock dividends. As a result these option holders lost approximately 30% of their value due to the declaration of two 10% stock dividends.[32]

The courts have looked with disfavor on companies that have taken advantage of weak anti-dilution provisions. A decision handed down in 1950 required Merritt-Chapman and Scott to allow 1.4 shares of stock to perpetual warrant holders at $30, due to a 40% stock dividend, even though the company did not have an anti-dilution clause.[33] Court action has not impinged on the right of firms to exempt small stock dividends from anti-dilution adjustments, and there appears to be a trend toward including specific provisions to exempt stock dividends as cash substitutes from anti-dilution provisions.[34]

Merger and Acquisition of Assets Though Issuance of Common Stock

Option holders are fundamentally without recourse when additional shares of common stock are issued in acquiring specific assets. Most anti-dilution clauses provide the right to acquire assets without requiring an adjustment

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[32]Schwartz, op. cit., p. 14.

[33]Benjamin Graham, David L. Dodd, and Sidney Cottle, Security Analysis (New York: McGraw-Hill, 1962), p. 621.

[34]Kaplan, op. cit., pp. 11-12.

to the exercise price. Consequently, option holders must depend on the good faith of the directors in preserving the value of the company's common stock.

Mergers and consolidations present special problems in the proper drafting of anti-dilution clauses. Numerous authorities, including Dewing, have recognized the need to protect an option holder in the event of a merger by allowing a substitute claim on an analogous security.[35] Most option holders are adequately protected upon a simple merger where stock is directly exchanged--because an analogous security is created. The courts have also affirmed the claims of option holders on share-for-share merger exchanges. Berle cites a case where Justice Holmes enforced the conversion privilege by requiring delivery of the new corporation's stock.[36]

Simple exchanges of one share of common for another in a merger is rare today. Shares are now being exchanged for packages of bonds, warrants, and other convertible vehicles that result in dissimilar investment claims. Kaplan states,

> In the more involved forms of corporate combination, such as the sale of 40% of the assets of

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[35]Arthur Stone Dewing, A Study of Corporation Securities (New York: The Ronald Press, 1934), p. 411.

[36]Adolf A. Berle, Jr., Studies In The Law Of Corporation Finance, (Chicago: Callaghan and Company, 1928), p. 138.

> Corporation A to Corporation B for common stock of B followed by the merger of Corporation A into Corporation C, the ordinary "boilerplate" merger provisions of an anti-dilution clause amount to little more than the convertible's prayer for fair play.[37]

A pioneering article by Hills also recognized the weakness of an option holder's claim upon the merger of the obligor company. According to Hills:

> Privilege holders have practically no rights upon a termination of the corporate existence of the company which granted the privilege, whether it terminates by reason of a consolidation, merger, dissolution or otherwise. Such rights as they do have are granted by statute or by an agreement of reorgainzation under which the successor company undertakes to protect the privilege.[38]

Perhaps the only way to properly handle complex combinations of firms with outstanding options is to specify some trustee to be responsible for deciding the rights of the option holders during the merger process. The trustee should be a totally disinterested party. This provision could also be extended to other corporate developments.

Issuance of Additional Options

Quoted conversion clauses do not provide adjustments for options granted to employees, but they do offer adjustments for additional issues of convertible bonds, warrants,

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[37]Kaplan, op. cit., p. 16.

[38]George S. Hills, "Convertible Securities--Legal Aspects of Draftsmanship," California Law Review, Vol. 19, No. 1, (Nov., 1930), p. 31.

or other marketable claims on the common. These options can either be dilutive from an exercise price standpoint or from a market price standpoint. These two approaches have already been adequately treated.

The relevant question is whether adjustments are made upon actual conversion of the instruments or at the time of their issuance. The first approach may nullify any gains from such an issue because the first securities may be exercised ahead of the newly created securities. At the same time, forced conversion through a call of the issue may eliminate the benefits of conversion.

The second approach, adjusting the exercise price immediately, may benefit the convertible holders at the expense of the common holders. Newly issued securities may never be exercised--resulting in greater conversion benefits to those who convert early.

Other Methods of Dilution

Anti-dilution clauses can not provide for all possible developments that may affect the conversion privilege. However, known developments that may prostitute option value should be provided for. Two such occurrences are the creation of an intermediate class of stock and the payment of large cash or property dividends.

New issues of preferred stock may be offered to

existing stockholders. The dividend rate could be designed to absorb normal earnings accruing to the common stockholders, and the options on the common would be rendered nugatory. Berle was probably the first to recognize the danger of newly issued intermediate shares in undermining the value of the call on the common. His discussion of this danger in 1927 failed to generate preventive anti-dilution clauses.[39]

Recognition of intermediate issues may not be satisfactory for modern security forms. For example, LTV's accumulating convertible stock, issued June 28, 1968, may challenge the drafters of anti-dilution clauses. This hybrid security is exempt from cash dividends and offers a 3% annual stock dividend. It is convertible into common on an increasing basis, with each share convertible into .85 common shares through December 30, 1969. This increases annually to a peak of 1.5 shares by 1979.[40] LTV's new security offers tremendous potential dilution and option holders deserve protection.

Option holders may also find their position weakened through payment of unusually high cash or property dividends.

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[39]A. A. Berle, Jr., "Convertible Bonds and Stock Purchase Warrants," Yale Law Journal, Vol. 36, No. 5 (March, 1927), pp. 655-656.

[40]Moody's Industrial Manual (New York: Moody's Investors Service, 1969), p. 1973.

Berle describes a corporation's right to pay dividends:

> The corporation may, if it chooses, strip its surplus account bare, distributing it fully to the stockholders without considering the effect on the value of the shares in respect of which the privilege might operate. If the privilege holder wishes to participate in the distribution he must elect to become a stockholder and convert his obligation, or pay for his stock.[41]

There appear to be three ways in which anti-dilution clauses may be drafted to protect the option value where dividends are concerned. The clause may (1) require a notice prior to the record date to ensure that the options may be exchanged to common stock prior to distribution; (2) require an adjustment in the exercise price when dividends exceed a specific amount, or (3) contain restrictive covenants that place a ceiling on annual dividend distributions.

The first method would not assist an option holder when the market price of the stock is less than the exercise price; the third approach may unduly restrict the firm's dividend policy. The preferred clause would enable adjustments in exercise price when "in kind" or cash dividends exceeds a specific amount. Potential buyers of warrants or convertible bonds should review the specific protection afforded by anti-dilution clauses before making a commitment.

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[41] Berle, op. cit., pp. 140-141.

Accounting For Warrants and Convertible Bonds

The Accounting Principles Board has experienced difficulty in deciding how stock purchase warrants and convertible bonds should be reflected in financial statements. Prior to the issuance of APB Opinion No. 10,[42] warrants and convertible bonds were relegated to footnotes in explaining their importance to a given business firm. The accounting profession's method of reporting for these securities traditionally has followed Internal Revenue Service Regulations.

Straight Debt Value and Option Value

The Accounting Principles Board of the American Institute of Certified Public Accountants rendered Opinion No. 10 in December, 1966. The advent of this opinion altered traditional methodology. It was recommended that the value of the call on the common be separated from the straight debt value by crediting paid in capital for its value. Correspondingly, an offsetting charge was suggested that would be made by debiting the debt discount (or reduction in premium).[43] The discount was then to be amortized over

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[42] Accounting Principles Board, Opinion No. 10, "Omnibus Opinion-1966," (New York: American Institute of Certified Public Accountants, December, 1966), pp. 141-151.

[43] Ibid., pp. 147-148.

the life of the bond issue. The net effect would be a reduction in reported income through an effective increase in the annual interest levy.

APB Opinion No. 10 generated a furor among corporations who had found these instruments helpful in acquiring huge amounts of capital. The airline industry, faced with rising costs, strongly protested. Investment bankers, fearful of losing lucrative convertible bond business, exerted their power in an attempt to have the ruling changed.[44] They argued that the conversion privilege in a bond was simply one of the provisions contained in an indenture, and that the low coupon rate was a function of numerous features of the security. The argument was further presented that convertible bonds were either equity or debt, but not both. Prior to conversion they are debt; after conversion they are equity.[45]

The Accounting Principles Board yielded to the pressure of the business community and their fellow colleagues. Their Opinion No. 12, dated December, 1967, requested that the section of Opinion No. 10 concerning convertible bonds and bonds with warrants be suspended.[46]

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[44]Arthur M. Louis, "The Accountants Are Changing the Rules," Fortune, Vol. 77, No. 7 (June 15, 1968), p. 179.

[45]"Cloud Over Convertible Debentures," Business Week, No. 1988 (Oct. 7, 1967), p. 148.

[46]Accounting Principles Board, Opinion No. 12,

Straight Debt or Stock

It was not until March 1969 that the Accounting Principles Board took a definitive step on these securities. APB Opinion No. 14 directed companies issuing detachable stock purchase warrants to credit paid-in capital for their fair value at date of issuance. The decision was based on the fact that these warrants become separable from the bond issue--thus commanding their own value.

Bonds issued with non-detachable warrants and convertible bonds, however, were to be accounted for as if the entire issue were debt. The underlying reason for this decision was based on the inseparable nature of the debt and the conversion option. The Board evinced concern about the difficulty in properly assessing the value of the conversion option. To quote Opinion No. 14, "In the absence of separate transferability, values are not established in the marketplace, and accordingly, the value assigned to each feature is necessarily subjective.[47]

"Omnibus Opinion-1967," (New York: American Institute of Certified Public Accountants, December, 1967), p. 91

[47]Accounting Principles Board, Opinion No. 14, "Accounting for Convertible Debt and Debt Issued With Stock Purchase Warrants," (New York: American Institute of Certified Public Accountants, (March, 1969), p. 205.

Importance of Opinion No. 15

The Accounting Principles Board rendered its most significant opinion relative to warrants and convertible bonds in May, 1969, with the issuance of Opinion No. 15. This opinion was primarily directed towards earnings-per-share calculations, although it included warrants and convertible bonds by considering their dilutive effect on earnings-per-share data. In brief, Opinion No. 15 states that:

> The Board believes that corporations with complex capital structures should present two types of earnings per share data (dual presentation) with equal prominence on the face of the income statement. The first presentation is based on the outstanding common shares and those securities that are in substance equivalent to common shares and have a dilutive effect. The second is a pro-forma presentation which reflects the dilution of earnings per share that would have occurred if all contingent issuances of common stock that would individually reduce earnings per share had taken place at the beginning of the period (or time of issuance of the convertible security).
>
> The Board has concluded that determination of whether a convertible security is a common equivalent should be made only at the time of issuance. A convertible bond will be considered as a common equivalent at the time of issuance if, based on its market price, it has a cash yield of less than 66-2/3% of the then current bank prime interest rate. Options, warrants and similar arrangements usually have no cash yield and derive their value from their right to obtain common stock at specified prices for an extended period. Therefore, these securities shall be regarded as common stock equivalents at all times.
>
> The Board recommended that the amount of dilution to be reflected in earnings per share data should be computed by application of the "treasury stock" method. Under this method, earnings per

> share data are computed as if the options and warrants were exercised at the beginning of the period and as if the funds obtained thereby were used to purchase common stock at the average market price during the period.[48]

Opinion No. 15 is relevant to the study of warrant and convertible bond valuations because these instruments can now affect the amount of primary earnings-per-share and/or fully diluted earnings-per-share. In addition, the price of the common stock relative to the option price has a direct relation on the reported per-share-earnings. The influence of the market price is demonstrated in Table IV.

TABLE IV

IMPACT OF COMMON STOCK PRICE ON EARNINGS PER SHARE

	19X1	19X2
After tax earnings	$2,000,000	$2,000,000
Average shares outstanding	1,000,000	1,000,000
Warrants	200,000	200,000
Exercise price per share	$20	$20
Average market price of common	$20	$40
Adjusted common shares	1,000,000	1,100,000
Primary earnings per share	$2.00	$1.82

An examination of Table IV illustrates the relationship between the market price of the common and the option

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[48] Accounting Principles Board, Opinion No. 15, "Earnings Per Share," (New York: American Institute of Certified Public Accountants, May, 1969), selected pages.

price. The decline in per-share-earnings for 19X2 is a result of the average market price appreciating to $40--it is unrelated to the earnings production of the firm. Thus, Opinion No. 15 has delivered a new accounting concept; that the market capitalization rate should be reflected in earnings-per-share.

Earnings-per-share determination, under the present approach, has an important message for the market place. It conveys the image that options have no dilutive effect until the market price of the common approaches the exercise price. This suggests that options with exercise prices high relative to the conversion prices should be favored because dilutive effects are not recorded. In contradiction, options with exercise prices less than or equal to common stock prices should be avoided because each dollar the common appreciates will result in lower earnings per share. It is conceivable that a rapidly growing company may report declining earnings per share due to a growth in the price of its common stock. The question that presents itself is: How meaningful are earnings per share data under these circumstances? One obvious answer is that the APB may have eliminated the traditional concept of earnings per share data with Opinion No. 15.

The negative correlation between market prices of common stock and earnings per share (higher prices = lower

reported earnings per share) should produce a leveling effect. Earnings per share should be cushioned during recessions by falling stock prices and curtailed during boom periods by higher prices. Such a development may be a desirable overall goal, but the impact on individual companies could be crippling.

Before concluding the discussion of accounting for these instruments, a brief address to the classification of these instruments as common equivalents is in order. The APB decided that the coupon rate relative to the prime lending rate should be used in classifying the convertible bond. It is interesting to note that stellar firms would come closer to having their securities classified as common equivalents due to their ability to float debt securities at lower rates. These firms would probably offer less dilution potential than highly leveraged firms. Under present conditions then, Opinion No. 15 favors the highly leveraged firms. Whether primary earnings per share will be more significant than fully diluted earnings to the investors of capital is still to be tested.

It may be surmised that earnings per share data, as drafted in Opinion No. 15, no longer reflect operating performance but are market oriented. Consequently, growth rates based on per share data must be adjusted for the impact of average market prices over a given period of time.

CHAPTER III

REVIEW OF SELECTED RESEARCH PROJECTS

Introduction To Stock Purchase Warrant Models

This chapter entails a discussion of selected research projects that have made significant contributions toward proper valuation procedures for warrants and convertibles. The projects are reviewed on a chronological basis, and the overview-approach does not permit exhaustive treatment of any one project. The organization of this chapter is well served by following the natural dichotomy inherent in the subject matter. Warrant models will be reviewed first, followed by a consideration of the convertible models.

The first few warrant models are based on simple warrant-common stock relationships. The approaches utilized in the models may be described as pioneering efforts that are still useful today. These simple models are followed by two mathematical models, justified by comparing actual prices with predicted data. A massive research project, in which the warrant-stock price relationship is measured through an empirical study, is the fifth project to receive

attention. Finally, a study which involves a multiple regression approach, where independent variables are used to predict the price of warrants, brings this section to an end.

Paul Hallingby's Approach To Warrant Valuation

Literature pertaining to the nature of stock purchase warrants were ubiguitous in the 1920's and 1930's. Valuation techniques or models were not described until empirical data on warrant performance became available. In a 1947 article, Hallingby, discussed a method of determining a "correct price" for a warrant.[1] Hallingby concluded that historical price action of the warrant in relation to its corresponding common stock could provide a basis for determining a fair price for a warrant. An editor's annotation states, "If the stock is selling at one quarter of the distance between its high and low, the warrant should also sell at one quarter of the distance between the warrant's high and low--and so for any other price of the stock."[2]

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[1]Paul Hallingby, Jr., "Speculative Opportunities In Stock Purchase Warrants," The Analysts Journal, Vol. 3, No. 3 (Third Quarter, 1947), pp. 41-49.

[2]Ibid., p. 49.

The "correct price", according to Hallingby's approach, is determined by comparing the current market price of the stock to its all-time high price. If the stock currently trades at one quarter of its high, the "correct price" for the warrant would be one quarter of its all-time high price. Warrants trading at less than the calculated price would be deemed attractive purchases. Hallingby also alluded to a desirable stock-warrant price relationship by stating, "The lower the price of a long-term warrant is in relation to that of its common, the greater the degree by which price movements of the warrant tend to surpass those of the common."[3] The stock-warrant relationship could have provided a mathematical indication of leverage by dividing the price of the warrant into the price of the common stock.

Hallingby's article seemed to be directed toward the upside leverage of warrants during bull markets. Therefore, his comments on proper valuation were not intended to be comprehensive. The major weaknesses in his approach are: (1) historical highs on the common stock do not serve as true limits as they are frequently penetrated; (2) past stock-warrant relationships are not necessarily indicative of fair values; (3) the approach tended to overstate the correct price for warrants when the common was depressed; and (4) a linear relationship between common and warrant

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[3] Ibid., p. 45.

prices was assumed to exist when a curvilinear relationship appears more accurate. (The curvilinear relationship will be discussed in Chapter IV.)

Russel Morrison's Basic Formulas

Morrison contributed a succinct article on warrant price assessment in 1957.[4] Morrison presented two ways for examining existing warrants. The first utilized the imputed return required on alternative investment. The second used the break-even price of a common when buying warrants.

Imputed return. The imputed return and the break-even price are both determined by recognizing two possible costs when buying warrants versus common stock. The warrant usually sells at a premium over intrinsic value, and this premium disappears at expiration. The disappearance of the premium on warrants with given expiration dates is one cost. The other cost is the dividend, if any, that is lost by holding warrants instead of stock.

The formula for calculating the imputed return may be described by:

$$\frac{P + D}{(N)(Ps-Pw)}$$

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[4]Russel J. Morrison, "The Warrant Or The Stock," The Analysts Journal, Vol. 13, No. 5 (November, 1957), pp. 51-52.

Where P = premium on the warrant

D = anticipated dividends per share for N years

N = number of years to expiration

Ps = market price of common stock

Pw = market price of warrant to buy one share

The formula relates the smaller commitment in the warrant over the holding period to the costs of buying the warrant, rather than the common stock. The resultant percentage can be conceptualized as the minimum rate of return necessary on other investments before a given warrant is preferred over the common stock.

The imputed return approach is best suited for warrants with relatively short maturity horizons. National General's old warrant provides an example of such an instrument. It is exercisable into common stock at $15.00 per share and expires May 14, 1974. On April 3, 1970, the common stock traded at $18.00 and the warrant closed at $9.25. National General also pays an annual cash dividend of $.20 per share. By substituting these data into Morrison's formula, one obtains an imputed return of 19.65%. The return is calculated as follows:

$$\frac{P + D}{(N)(P_S - P_W)} = \frac{6.25 + .80}{(4.1)(18-9.25)} = \frac{7.05}{35.875}$$

$$= .1965$$

Break-even price. Morrison's second method in appraising warrants deserves special attention, because it enables an investor to quantify the price appreciation of the stock necessary for the warrant to be superior to the common as an investment. For example, if a stock trades at $6 and the "break-even price" is calculated to be $12, the investor can assess the probability of the stock reaching $12 over a given period of time. High break-even prices in relation to existing prices would expose specific warrants as being overpriced.

Morrison provided the following formula for calculating the "break-even price:"

$$A = \frac{W}{1 - X/Y} + Z$$

Where: A = "break-even price"

W = exercising price of warrant

X = current market price of warrant

Y = current market price of a share

Z = total dividends anticipated on a share during the future life of a warrant.

Morrison's formula can be applied to National General's warrant in the following manner.

$$A = \frac{15}{1-(9.25/18)} + .80$$

$$A = \frac{15}{.514} + .80$$

$$A = 29.18 + .80$$

$$A = \$29.98$$

The calculated price of $29.98 enables one to assess the merits of a commitment in the warrants. A high probability of National General's common stock attaining a price of more than $29.98 in the next four years would indicate the warrants to be more attractive than the common stock.

The primary weaknesses of his schema stem from its inflexibility. Premiums do not evaporate in a step-wise fashion to a given expiration date. Given purchasers may take short-term positions in warrants and not be penalized by an eroding premium. The action of premiums is even more unpredictable on perpetual or long-term warrants. It may be surmised that Morrison's methodology applies only to warrants approaching expiration. Perhaps the major weakness of the break-even formula is its failure to recognize downside risks. This can be illustrated with the following exmaple. Assume that a warrant trades at $21 and expires in 2 years. The exercise price is $10, and the common trades at $30 and pays no cash dividend. The break-even price is calculated to be:

$$A = \frac{10}{1 - 21/30} = \$33.33$$

The break-even price of $33.33 may look attractive, since it is only 11.1% above the price of the common. However, an investment in the warrant could expose a purchaser to unsuspected downside risks. For example, if the common declines by 1/3, the warrant should lose about 1/2 its value.

Morrison's formula can be ruled ineffective when the warrant trades near its intrinsic value due to high prices of the common stock.

Guynemer Giguere's Mathematical Model

Giguere's article[5] contributed a new approach to warrant valuation. He suggested that the relationship between the value of a warrant and its corresponding stock is given by a parabola with its vertex at (0,0) and whose equation is:

$$W = \frac{P^2}{4a}$$

Where: W = Value of the warrant

P = Price of the stock

a = Exercise price

Giguere demonstrated the correlation between his formulated warrant values and the actual warrant prices by using price data for Tri-Continental and Atlas Corporation, both perpetual in nature. These data were taken from market quotes in the early and middle 1950's. The observed relationships led Giguere to conclude that his mathematical formula could prove helpful in locating attractively priced warrants.

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[5]Guynemer Giguere, "Warrants-A Mathematical Method of Evaluation," The Analysts Journal, Vol. 15, No. 5 (November, 1958), pp. 17-25.

Giguere recognized that his model would not apply uniformly to all warrants. Warrant uniqueness in time to expiration, variable exercise price, marketability, and speculative nature of the corresponding common had to be considered before proper pricing could be determined. He made a specific adjustment to his formula by deducting (a÷16) from the calculated warrant price, when it had from one to five years to expiration. The a÷16 adjustment could be changed to a÷8 or a÷20 depending on the experience of the historical relationship, but Giguere failed to elaborate on the specific method of making such a determination. His formula deemed warrants with expiration dates of more than 5 years to be valued identically with perpetual instruments. He did not feel that warrants with less than one year could be properly evaluated with a formula.

Giguere failed to adjust his model for different ranges of marketability, exercise price adjustments, or expectations of the related common stock. He mentioned the incongruencies of these factors but failed to recommend solutions. Giguere also recognized the weakness of his basic formula when the $P > 2a$ by requiring that $W = P-a$ (warrant be priced at its intrinsic value) when $P > 2a$. Thus, Giguere's basic formula as modified reads:

$W = \frac{P^2}{4a} - \frac{a}{16}$ ($\frac{-a}{16}$ is utilized when expiration date is from 1 to 5 years hence.)

Subject to: $W = P-a$

When: $P > 2a$

Giguere's formula reveals a fair value for National General's old warrant to be $4.46. The value is determined by:

$$W = \frac{(18)^2}{(4)(15)} - \frac{15}{16} = \frac{324}{60} - .9375$$

$$= 5.40 - .94$$

$$= \$4.46$$

The actual price of National General's warrant is $9.25 or more than twice as high as Giguere's calculated fair value of $4.46 on April 3, 1970.

The formula presented by Giguere is helpful in describing how a warrant's value is a function of its related stock's price. The simplicity of his model is also commendable. However, some basic inconsistencies are present in his article. For example, his adjustment of minus a÷16 or a÷20 may create a negative value for individual warrants. He discussed the importance of expiration dates and suggested that warrants with long periods to expiration should be valued higher than near term warrants. Mathematically, however, he expressed warrants with expiration dates in excess of 5 years to be valued identically with perpetual warrants.

Giguere also justified the appropriateness of his model based on empirical data existing in the 1950's. He further indicated that his predicted values for warrants were usually low or conservative. Actually, his approach is entirely too conservative to be useful in selecting warrants for purchase today. Chapter IV will explicate this conclusion in a more comprehensive fashion.

Kassouf's Norm Value Model

Another mathematical model, focusing on the relationship between the stock's price and the warrant's value, was developed by Kassouf. Kassouf's model is defined as the hyperbolic equation:[6]

$$Y = \sqrt{a^2 + x^2} - a$$

Where: Y = warrant's value

a = exercise price

x = price of common

Kassouf refers to the resultant value of the warrant as the norm value.

Kassouf's formula estimates National General's warwants to have a norm value of $8.34. The normal value is calculated by:

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[6]S. T. Kassouf, Evaluation of Convertible Securities (New York: Analytical Publishers Co., 1966), p. 26.

$$Y = \sqrt{(15)^2 + (18)^2} - 15$$

$$Y = \sqrt{549} - 15$$

$$Y = 23.43 - 15$$

$$Y = \$8.43$$

The actual price of the warrant is close to Kassouf's normal value.

Kassouf does not claim to have a sound set of assumptions leading to development of the model, but he does claim that empirical evidence lends support to his model. Kassouf, like Giguere, demonstrated the accuracy of his formula by comparing norm (predicted) values to actual prices for individual warrants.

Kassouf also places constraints on the norm value by excluding warrants that do not meet the following normal conditions: (1) a life of 4 or more years; (2) potential dilution of less than 15%; and, (3) a dividend rate on the common of less than 4%.[7] Kassouf's overall valuation of warrants introduces two new variables that influence warrant values. They are dividends on the associated common stock and potential dilution from convertible securities. He failed to discuss the rationale supporting the negative impact of these variables on the common, but he obviously

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[7] Ibid., p. 10.

assumed high dividend payouts and high amounts of dilution have restrictive influences on the value of warrants.

Kassouf felt warrants that were priced at less than norm values and met the specified normal conditions offer attractively priced investment vehicles. His normal conditions and norm value, however, appear too restrictive for the majority of warrants. It is illustrated in Table V that either the norm value or one of the normal conditions eliminates all of the following warrants from purchase commitments. These data are taken from Kassouf's Table I and represent closing prices for September 21, 1962.

Tri-Continental's warrant meets all the conditions except that of the dividend yield which is to be less than 4%. Tri-Continental's yield was exactly 4%. Table V illustrates the restrictiveness of Kassouf's approach to warrant valuation. In their original forms, Kassouf's and Giguere's models are conservative in selecting warrants to purchase. In Chart 6 the relationship of Kassouf's approach to the basic formula of Giguere is portrayed.

The hyperbolic formula of Kassouf provides warrant values almost double those of Giguere when the common price is less than 50% of the exercise price, as can be seen in Chart 6. For example, at .5X Kassouf provides a warrant value of $.118 whereas Giguere calculated the warrant at $.0625. Also illustrated is a major weakness of the Kassouf

TABLE V

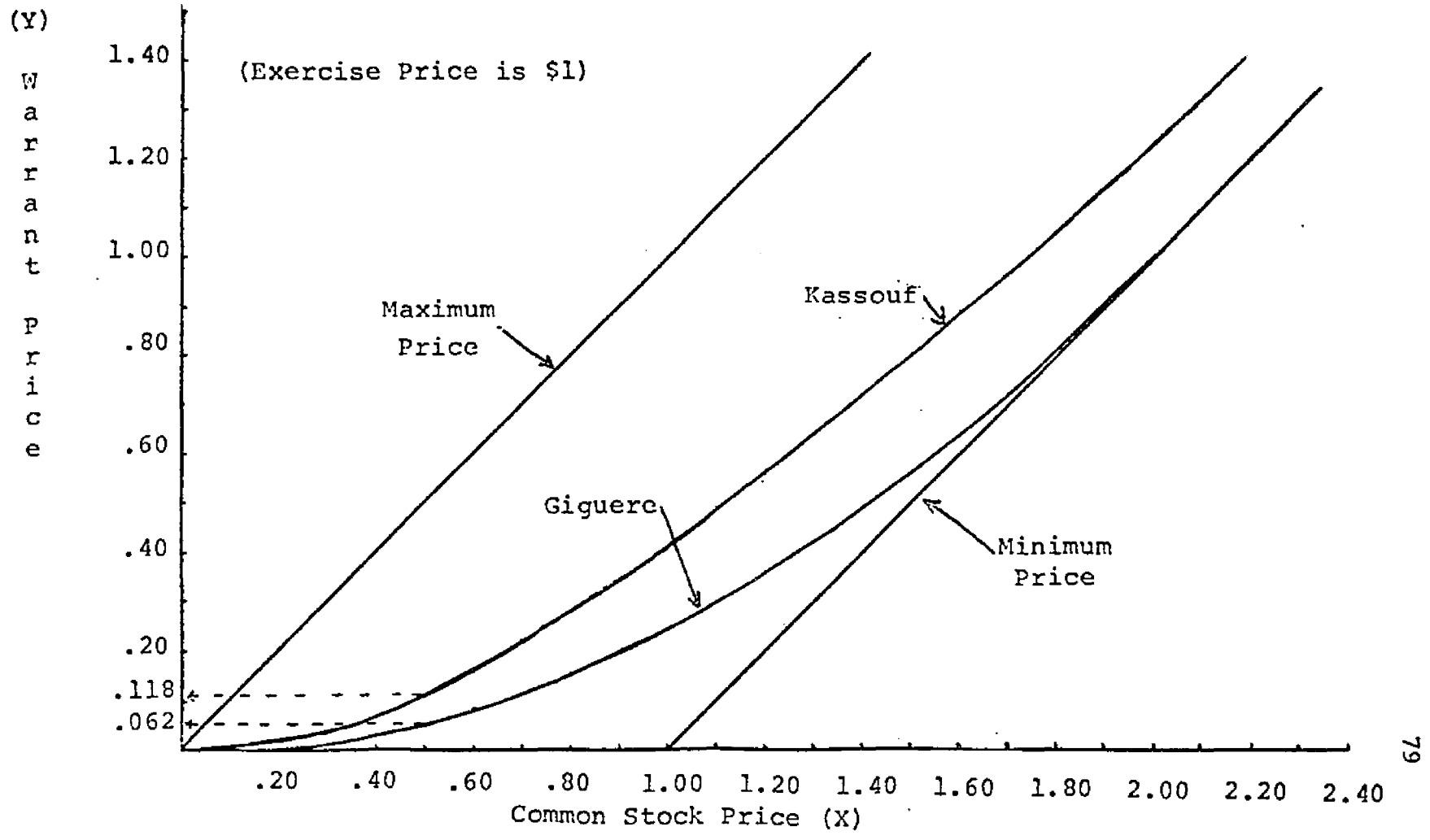
KASSOUF'S NORM VALUE AND NORMAL CONDITIONS

Company	Terms	Com. Price	War Price	Norm Value	% Pot. Dilut.	% Div. Com.
Alleghany	1 sh. @ 3.75; perp.	8.00	4.88	5.00	26	0
Armour	1 sh. @ 20; to 12/64	36.00	22.50	21.10	12	3.9
Atlas	1 sh. @ 6.25; perp.	2.50	.94	.63	33	0
First Nat. Realty	1.05 sh. @ 6.00 to 12/66 6.75 to 12/71	3.25	1.25	.90	30	3.1
Gen. Acceptance	1 @ 20 to 11/69	19.50	6.50	8.00	47	5.1
Hilton Hotels	1 @ 42 to 10/63; 46 to 10/67, etc.	24.00	6.50	6.25	11	6.3
Jeff. Lake Petro.	1 @ 7 to 6/65; plus 1 ea. yr. to 6/71	5.63	2.38	1.96	20	0
Mack Trucks	1.4 @ 45 to 9/63; 47.50 to 9/65; 50 to 9/66	35.13	15.63	21.60	21	5.1
Martin Marietta	2.73 @ 40 to 11/63; 45 to 11/68	22.63	28.50	33.80	2	4.4
McCrory Corp.	1 @ 20 to 3/76	22.25	7.13	10.10	42	4.0
Molybdenum	1.05 @ 30 to 10/63	28.50	14.25	12.90	10	0
Pacific Petro	1 @ 19 to 3/68	12.88	6.38	3.90	6	0
Realty Equities	1 @ 7.50 to 8/63; plus .50 ea. 1.5 yr	7.13	1.75	2.82	44	2.8
Rio Algom	.135 @ 22.23 to 12/66	9.69	.44	.28	4	0
Sperry Rand	1.08 @ 25 to 9/63; 28 to 9/67	12.75	7.12	5.20	7	0
Symington Wayne	1 @ 10 5/63; 15 to 5/68	15.13	6.25	8.12	26	5.3
Teleregister	1 @ 15 to 5/63; 17 to 5/65	5.13	2.50	.85	4	0
Textron	1 @ 25 to 5/64; plus 5 ea. 5 yrs.	25.00	10.00	10.40	23	5.0
Trans World Air	1 @ 20 to 6/65; 22 to 12/73	10.00	4.25	2.40	29	0
Tri-Continental	1.27 @ 22.60; perp.	36.88	27.63	29.30	11	4.0
Univ. American	1 @ 13.75 to 3/67	7.88	2.63	2.15	22	0

Source: S. T. Kassouf, Evaluation of Convertible Securities (New York: Analytical Publishers Co., 1966), p. 26.

CHART 6

GRAPHIC COMPARISON OF KASSOUF AND GIGUERE MODELS



formula; the calculated norm for warrant values always lies above the intrinsic value. Giguere's formula, as previously described, shows the warrant equal its intrinsic value when the common price is 2 times its exercise price. These hueristic models have provided new insight into the dynamics of warrant pricing. Both Kassouf and Giguere, however, failed to test their model's efficiency in selecting warrants for purchase. Chapter IV includes a test of how well these models select warrants for purchase.

Investment Bankers Study of Warrants

Perhaps the most comprehensive study of warrant price behavior was undertaken by the Investment Bankers Association of America. The Association wanted to establish uniform principles of taxation for these securities when they were received as compensation by underwriters and other independent contractors. Their study was concerned with establishing valuation principles for these securities.

The original statistical format called for an analysis of 106 individual warrants from July 1957 through June 1963. However, 19 of these warrants failed to meet specific criteria and were eliminated from the study. The criteria necessary for a warrant's inclusion in the study follow:

(1) the warrant had to have been issued by an American company, defined as one incorporated in the U.S.;

(2) the warrant had to be exercisable only with cash, as opposed to some warrants which provide for exercise through the tender of senior securities of the issuing company;

(3) the warrant had to be exercisable for common stock, which excludes some warrants entitling the holder to purchase other securities of the issuer; and

(4) the warrant had to be actively traded for three months. In this connection, the criterion employed was that quotations for the warrant and optioned stock be available in issues of the R.H.M. Survey for three consecutive months.[8]

The final results of the study were based on an analysis of 87 warrants and included examination of over 3,200 market price quotations. Each warrant was standardized by dividing its exercise price into the price of the common and the price of the warrant. Therefore, the statistical data were in terms of two ratios--one referred to as the relative value of the warrant and the other the relative value of the common stock. The relationship of these relative values was then measured by a regression equation which may be written as $Rw = a(Rc)^b$. Rw and Rc represent the relative values of a warrant and of a common stock while a and b are constants determined by using a least-squares regression program.

The basic study was limited to warrants with lives

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[8] Federal Income Taxation of Compensatory Options (Including Warrants) Granted To Underwriters And Other Independent Contractors (Washington, D.C.: Investment Bankers Association, 1963), p. 45.

in excess of two years, with steady exercise prices for at least 1 year. The findings revealed that average warrants sell at 41% of their exercise prices when market prices and exercise prices are at parity. The average value of the warrant, when the common trades at various levels relative to the exercise price, is revealed in Table VI.

TABLE VI

NORMALIZED WARRANT VALUES

Ratio of Market Value of Optioned Stock To Exercise Price	Ratio of Market Value of Option To Exercise Price
80%	28%
90	34
100	41
110	48
120	55

Source: Federal Income Taxation of Compensatory Options (Including Warrants) Granted To Underwriters And Other Independent Contractors (Washington, D.C.: Investment Bankers Association, 1963), p. 45.

The basic regression equation for the 87 warrants was not revealed in the study, but the empirical approach to warrant valuation seems to have merit. The study also recognized that other factors may influence warrant values, and it discussed the influence of four possible developments, namely;

(1) approaching expiration, (2) step-up in exercise price, (3) deferred transferability, and (4) deferred exercisability.[9]

The study tested the significance of approaching expiration by calculating a standard premium for all warrants. This was accomplished by determining the difference between the standard valuation and the intrinsic value. The ratio of actual premiums to standard premiums was then determined for various intervals prior to expiration. Results indicated that approaching expiration is unimportant until a warrant has 2 years or less to maturity.[10]

An interesting finding was one that pertained to warrants containing step-up prices. Nearly 1,500 price quotations were collected for warrants with increasing exercise prices. The findings indicated that step-ups significantly affect warrant prices within a period of 9 months or less before the step-up. The results also show that the warrants with step-ups 3 years or more in the future actually sell at higher premiums than those that exist on average warrants.[11]

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[9] Ibid., p. 52.

[10] Ibid., pp. 53-55.

[11] This unusual phenomenon may be observed in their Chart 4. It is not discussed in the text of their paper, however.

Also measured in this study was the influence of warrants with a period of time before they may be detached from another security and trade alone. An insignificant effect on warrant value with transferability restricted for one year or less was evidenced. A similar conclusion was rendered for warrants with deferred exercisability of 1 year of less.[12]

The IBA study must be regarded as a comprehensive compilation of statistical data concerning warrant values; its conclusions appear logical in most instances. However, one must question the conclusion that warrants with two years to maturity should be given identical values as warrants with perpetual lives--especially at all ranges of the common stock's price. Other differences will be explored in Chapter IV.

John Shelton's Multiple Regression Study

Shelton introduced numerous innovative approaches to warrant valuation in a 1967 article in the Financial Analysts

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[12]Federal Income Taxation of Compensatory Options (Including Warrants) Granted To Underwriters And Other Independent Contractors (Washington, D.C.: Investment Bankers Association, 1963), p. 45.

Journal.[13] He established a zone of plausible warrant prices by recognizing the minimum price of a warrant to be its intrinsic value. He deduced, from empirical results and theoretical concepts, that warrants will trade at their intrinsic value when the common stock is priced at four times its option price. For example, if a common stock trades at $40 and the warrant provides the right to buy one share at $10, the warrant will sell at $30. Shelton concludes that the speculative value of a warrant will disappear when the common is priced 4 times the warrant's exercise price.

Shelton carried the logic of 4 times the exercise price to lower levels of stock-option price relationships. He concluded that an appropriate upper limit on the warrant price is three-fourths of the common stock price. He found support to his zone of plausible prices by plotting 157 observations of warrant-stock relationships. Only one observation exceeded the upper limit. Shelton's zone of plausible warrant prices can be described symbolically as:

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[13]John P. Shelton, The Relation of the Price of a Warrant to the Price of Its Associated Stock," Part 1," Financial Analysts Journal, Vol. 23, No. 3 (May-June, 1967), pp. 143-151. For a similar approach, see Sheen Kassouf, "A Theory And An Econometric Model For Common Stock Purchase Warrants," (unpublished Doctor's dissertation, Columbia University, 1965), pp. 1-88.

$$(Ps-Po) < Pw < 3/4Ps \quad \text{if } Ps < 4(Po)$$
$$Pw = (Ps-Po) \quad \text{if } Ps \geq 4(Po)$$

Where: Ps = Price of the stock

Pw = Price of the warrant

Po = Option price

The variation of warrant prices within the newly defined zone concerned Shelton. He decided to analyze specific variables that could lead to positioning of these prices and selected the following independent variables for trial:

(1) the longevity of the warrant (measured in months with an arbitrary truncation of 120 months for perpetual warrants);

(2) the current yield on the common stock

(3) whether the warrant was listed on the American Stock Exchange or traded over the counter;

(4) whether the warrant sold for more or less than $500;

(5) the past volatility of the common stock (measured by averaging the ratios of annual high dividend by annual low for each of the three preceding years); and

(6) the recent trend of the stock price (measured by the percentage change of the stock over the past year).[14]

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[14]John P. Shelton, "The Relation Of The Price Of A Warrant To The Price Of Its Associated Stock, Part II," Financial Analysts Journal, Vol. 23, No. 4 (July-August, 1967), pp. 91-92.

These variables were tested for colinearity by using a simple correlation approach. They were all deemed acceptable for a multiple regression program, since none were highly correlated. These independent variables were then tested to see how well they predicted the variation of warrant prices in the aforementioned zone of plausible prices. The most highly affective variable in predicting warrant pricing was found to be the dividend yield of the stock--with the higher yield indicating a lower price for the warrant. Thus, the coefficient was negative. Next in importance was whether the warrant was listed on the American Stock Exchange or traded over-the-counter. Warrants listed on the A.S.E. had positive coefficients. The next important variable was the longevity of the warrant. The three other variables did not prove to be significant to Shelton in predicting warrant price behavior. Shelton's significant variables provided a multiple correlation value of .61 indicating that unexplained variation remains high.

Shelton reasoned that the longevity calculation was not a linear function, therefore, he substituted a "fourth-root" adjustment. He employed the findings of the previously mentioned Investment Bankers Study to justify use of the adjustment factor. The resultant equation for locating a

$$\text{warranY} = \left(\sqrt[4]{\frac{\text{longevity}}{72}}\right) \quad [.47-4.25(\text{yield})+.17 \text{ if listed}]$$

$$Y = \left(\sqrt[4]{\frac{\text{longevity}}{72}}\right) \quad [.47-4.25(\text{yield})+.17 \text{ if listed}]$$

This formula can be applied to individual warrants in the following manner:

(1) Multiply the price of the stock by .75 to determine the upper limit.

(2) Deduct the option price from the price of the stock to determine the lower limit or intrinsic value (if any).[15]

(3) Subtract the lower limit from the upper limit to determine the zone of plausible prices.

(4) Use the above formula to determine the positioning of each warrant in the zone and then add the minimum value if any. For example, data for National General's old warrant were as follows:

TABLE VII

SHELTON'S APPROACH IN DETERMINING WARRANT VALUES

Price of common stock	18.00
Exercise price	15.00
Dividend yield	1.1%
Longevity (months)	49
Where traded	ASE

(1) 18.00 (3/4) = 13.50 (upper limit)
(2) 18.00 - 15.00= 3.00 (lower limit)
(3) 13.50 - 3.00= 10.50 (zone of P.P.)

$$= \left(\sqrt[4]{\frac{49}{72}}\right)(.47-4.25(.011)+.17)$$

$$= \left(\sqrt{.8248}\right)(.423+.17)$$

$$= (.9082)(.593) = .539$$

$$= (.539)(10.50) = \$5.66 \text{ plus } \$3.00$$

$$= \$8.66$$

[15]If the option price of the stock is greater than the

Data for the National General warrant were not complicated by uneven number of shares or step-up exercise prices. The complexity of the calculation is grossly understated. These prices are calculated for over 172 observations in Chapter IV to test the efficiency of the model.

Shelton should be complimented for presenting a new approach to warrant valuation as his zone of plausible prices is totally new. However, his relatively small sample in conjunction with his low coefficient of determination leave the statistical inference suspect.

Introduction To Convertible Bond Models

The development of convertible bond models has occurred in a short period of time. Most of the research has taken place in the 1960's with important projects contributed in the last 5 years.

Eugene Brigham's Convertible Bond Model

Perhaps the most penetrating discussion of convertible bonds was presented in Brigham's 1966 article.[16] In it he constructed a graphic model and described its important

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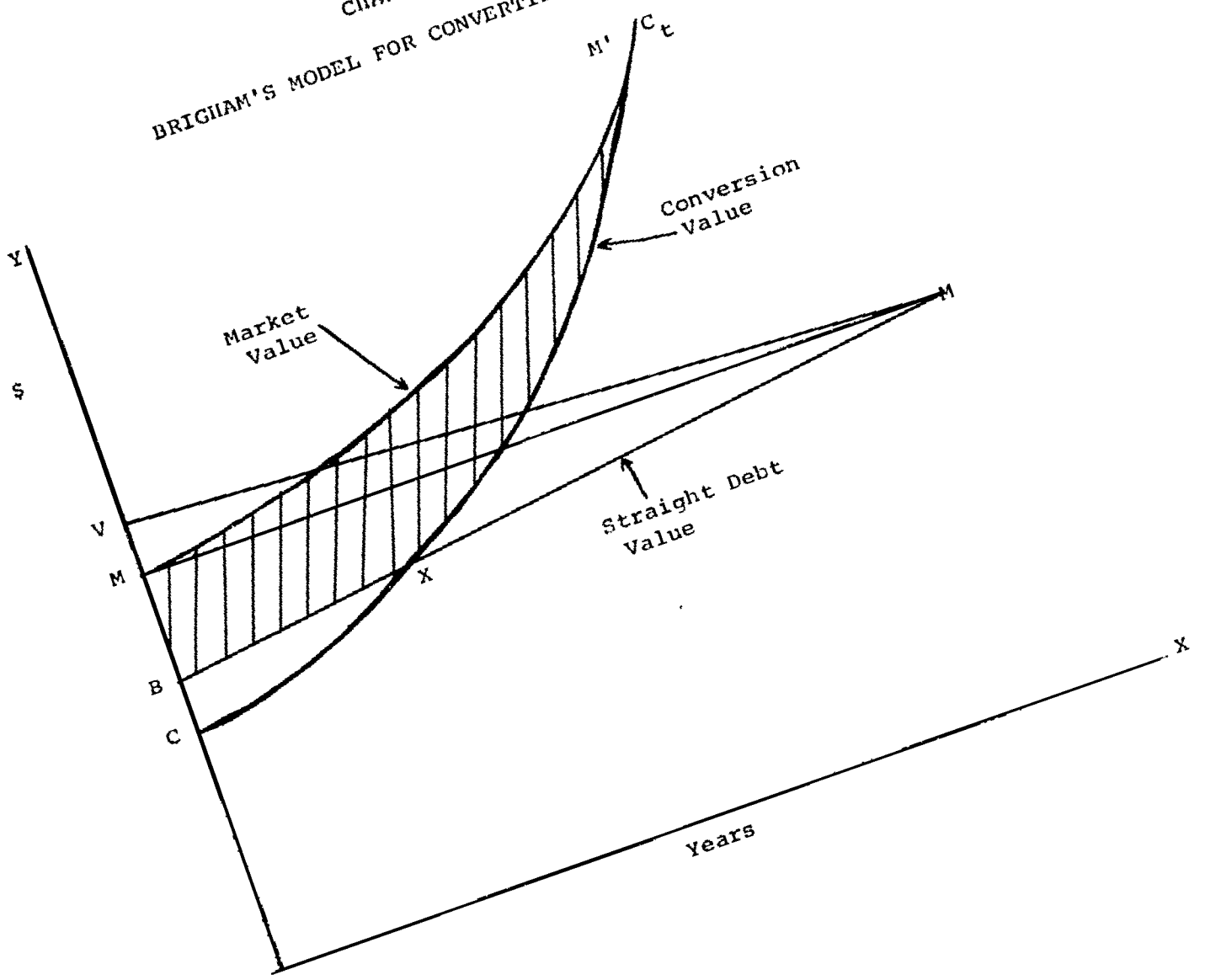
stock price, the lower limit will be zero. The warrant's price in this case is based solely on speculative value.

[16]Eugene F. Brigham, "An Analysis of Convertible Debentures: Theory And Some Empirical Evidence," The Journal of Finance, Vol. 21, No. 1 (March, 1966), pp. 35-54.

CHART 7

BRIGHAM'S MODEL FOR CONVERTIBLE BONDS



features. Interactions of the key variables were analyzed, as was the rationale behind convertible bond issuance.

Brigham's hypothetical convertible bond model, reproduced as Chart 7, focuses on straight debt value and conversion value. The conversion value (C) on the "y" axis was determined by multiplying the number of shares the bond may be exchanged for by the market price per share. The line (CC_t) illustrates the growth in market price over a period of years. The entire curve (CC_t) may be expressed by the following equation:

$$C_t = Po(1+g)^t R$$

Where: C_t = the conversion value at time t;

Po = the initial market price of the stock;

g = rate of growth of the stock's price;

R = the number of shares into which the bond may be converted

The straight debt value (B) may be described as a function of the bonds' coupon rate, compared with existing yields on similar rated bonds, over a definite time horizon. B is shown to rise toward par (M) over its life; suggested that initial coupon rates on convertible bonds are typically less than the going rate of interest. The call price (V) is shown to decrease to par over the life of the security. The line (MM') represents the price of the bond. The premium on the bond is designated by the slash marks and is shown to be

confined by either BX or XC_t. BXC_t is described by Brigham as a price floor. The spread between MM' and the floor represents the premium marginal investors are willing to pay for the conversion option. It is interesting to note that Brigham shows the greatest premium at point X, where C = B, which confirms the premiums described in Chart 3 in Chapter II. The amount of the premium grows as C approaches B, then diminishes steadily until MM' rests on CC_t.

Possible reasons for the elimination of the premium are numerous. The domination of the straight debt value by the conversion value, at some point, is of primary importance. At this point, the market place considers the security to be identical with common stock. Obversely, straight debt value may dominate conversion value, at some point, and the market price will equal B. For example, what price would a $900 straight debt value convertible bond command with a conversion value of $10? The market place would probably ignore the conversion value in pricing the security as it ignores straight debt value, at some point, for the common stock.

Brigham's graphic model has provided a framework that has led to productive research on convertible bond financing. It indirectly influenced the creation of the valuation model introduced in Chapter V.

Probability Distribution Model for Convertible Bonds

In 1966 a pertinent article on convertible bonds was presented by William Baumol, Burton Malkiel, and Richard Quandt.[17] They recognized that convertible bonds generally sell at a premium over the higher straight debt value or conversion value. They referred to the convertible feature and/or the straight debt value as providing an insurance value which cushions price declines. Prior to introducing their valuation model, they suggested that proper evaluation required an assessment of the present value of interest receipts and an assessment of principal value along with the expected value of the convertible feature.

The valuation models developed by these writers assume that the determination of convertible bond premiums are revealed in two specific ways. First, one may determine the conversion value and then add the value of the insurance afforded by the straight debt value. Such an approach was described by the following model:

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[17] William J. Baumol, Burton G. Malkiel, and Richard E. Quandt, "The Valuation of Convertible Securities," Quarterly Journal of Economics, Vol. 80, No. 1 (Feb., 1966), pp. 48-59.

$$C \geq C_s = PS + \int_0^{\bar{B}/PS} f(i,to)\ [\bar{B} - i(t)PS]di(t)$$

Where:

C = value of convertible

$\bar{B}$ = bond value

S = number of shares of stock into which the convertible may be exchanged

P(t) = the price per common share at date t

PS = initial conversion value

i(t) = a price-relative of a share at date t

to = date on which the convertible is being evaluated

This equation shows the insurance value of the straight debt instrument to be positive only when the conversion value is less than $\bar{B}$. This is shown by the limits of integration with i(t) = 0 and i(t) = $\bar{B}$/PS. Therefore, a probability distribution for P(t) which assumes all values of P(t)S to be greater than $\bar{B}$ will not enable a premium to be based on the insurance value of debt. Rather, the entire value of the security will be based on PS in such a situation. Conversely, a probability distribution that contained P(t)S values lower than $\bar{B}$ would add to the initial conversion value of the bond. This model approaches convertible bond valuation by first examining conversion value and then adding an appropriate premium for the straight debt value cushion. The cushion would be nugatory if there were a 100% probability that the conversion value would never be lower than the

debt value.

The second approach to convertible bond valuation is to determine the debt value, then add the value of the convertible feature. Their model for such an approach is:

$$C \geq C_b = \bar{B} + \int_{\bar{B}/PS}^{\infty} f(i,to)[i(t)PS-\bar{B}]di(t)$$

This equation assumes $\bar{B}$ to remain constant over the time horizon in which the probability distribution of the common stock is determined. The formula shows the premium over straight value to be governed by the probability of the conversion value exceeding the straight debt value. Baumol, Malkiel, and Quandt reverse their limits of integration somewhat for this formula. The lower limit should show $PS/\bar{B}$ because only at $PS>\bar{B}$ will a premium be contributed. The maximum value of i(t) is∞, since infinitely high values of the common contribute a premium over debt value.

The authors properly show that the maximum value of the two equations should determine the value of the convertible bond. This is noted by:

$$C = \max (C_s, C_b)$$

Their two models are consistent with the graphic description of the convertible bond as described by Brigham. For example, the first equation would provide a low or non-existent premium if the initial price of the stock were high, since the probability of P(t)S being less than $\bar{B}$ would be small.

The price of the convertible would then rest on its conversion value. The second equation would also reveal a low or zero premium over $\bar{B}$ when the probability of the conversion value exceeding $\bar{B}$ over a definite time horizon was low. The highest premiums would be calculated when $\bar{B} \cong PS$. This is consistent with Chart 3 in Chapter II.

The writers tested the efficiency of their model by assuming two time horizons--one of 24 months and the other of 36 months. They determined the subjective probability distributions of the common stock by extrapolating past distributions into the future. These magic forecasts were made for 7 convertible bonds, and their resultant values were compared to actual market prices. The spread between predicted values and actual values was extremely high, but the significance of the models was not undermined. The poor estimates arose from the improper assessment of the common stock's future performance.

A Regression Model for Convertible Bond Premiums

The determination of a fair price for a convertible bond leads directly to an assessment of the premium. Weil, Segall, and Green contributed an interesting approach to determining convertible bond premiums in a recently published

article.[18] They surveyed current literature and identified seven independent variables that have been ascribed as influential on premiums. These variables were: (1) transactions costs, (2) income differences, (3) financing costs, (4) anti-dilution clauses, (5) price floors, (6) volatility of price, and (7) duration.[19] It was reasoned that some of these variables should not be tested because they were not quantifiable. For example, how could one properly quantify anti-dilution clauses? The authors recognized the divergence of different clauses but reasoned their significance to be of little importance.

The writers also recognized the potential importance of the price volatility of the common stock. However, they decided to test only bonds whose conversion values exceeded par value. They felt that bond prices would already reflect a risk measurement and therefore excluded price volatility as an independent variable.

The duration of the convertible privilege and the financing costs were also eliminated. Even though the importance of the life of the convertible feature was discussed,

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[18]Roman L. Weil, Jr., Joel E. Segall, and David Green, Jr., "Premiums on Convertible Bonds," The Journal of Finance, Vol. 23, No. 3(June, 1968), pp. 445-463. For similar and more detailed study see Gary Stone, "An Analysis of The Investment Nature of Convertible Bonds," (unpublished Doctor's dissertation, Stanford University, 1967), pp. 128-205.

[19]Ibid., pp. 445-447.

it was not included as an independent variable due to the lack of an operational measure. Financing costs were deemed to be quantitatively unimportant on the basis of substitute credit sources and small differences in cost.

Three independent variables then remained for use in predicting premiums on convertibles. They were, the price floor, income differences, and differences in transaction costs. The general form of the equation was:

$$\frac{P}{B} = f(F) + \beta\frac{Yd}{B} + \gamma\frac{T}{B} + \delta\frac{1}{B}$$

Where: P = calculated premium

B = bond price

F = floor variable; the difference between the bond price and the straight debt value of the bond as reported in Moody's Bond Survey.

Yd = difference in current income streams; bond income (coupons) less stock income (dividends per share multiplied by number of shares into which the bond is convertible).

T = transaction cost difference; cost to buy the stock into which the bond is convertible less cost to buy the bond including all transfer taxes.

The general equation was modified for the non-linear relationship of the premium to the straight debt value of the bond. The premium was claimed to be a monotonically decreasing function of the floor variable and therefore divided into ten ranges: (1-10), (10-20), (20-30), (30-40), (40-50), (50-75), (75-100), (100-150), (150-200), and (above

200).[20]

The authors utilized 452 observations of convertible bond values gathered for 18 different dates to evaluate their model. These observations were separated into two sets through the use of a random-number series. One half of the observations were used to calculate the coefficients of the specific equation, the other half of the observations were tested using the estimated coefficients.

The results of the regression on the first half of the data revealed an R^2 of .57. When the estimated coefficients were used to predict the premiums in the other half of the sample, the coefficients failed to perform as well as on the primary set. The quasi--R^2 dropped to .36.[21] The t values all proved significantly different from zero for the floor variable--thus, the floor variable was omitted and a similar regression program was fitted. Again the results were not powerful.

The major finding of this research was claimed to be the negligible importance of the floor in explaining the premium. Historically, it was claimed, too much emphasis has been placed on the significance of the floor. The

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[20] Ibid., p. 454.

[21] Ibid., p. 456. Quasi-R^2 defined as $1-{}^{b}/a$ where a is variance of P/B and b is residual variance.

studies previously reviewed in this paper were cited as examples. It was also claimed that either stop-loss orders to sell at a set price or the use of a put contract may establish a floor with greater significance. In effect, it was proposed that straight debt value has little or no influence on the premium of a convertible bond with the conversion value greater than par.

Weil, Segall, and Green seem to have overlooked the cost of their hypothetical floor. The use of stop-loss orders may cause a whipsawing effect which could result in excessive commissions of selling for protection and then buying back for appreciation. The cost of put contracts may consume the entire value of convertible bonds if they are used over a long time horizon.

Concluding Remarks

All of the research projects reviewed herein provide insight into the proper evaluation of the securities under discussion. These research efforts, however, are circumscribed in the following ways:

(1) Static models have been designed for dynamic relationships. A mathematical model verified by empirical data over a limited time horizon can not be extrapolated into the future. The models should offer adjustments in lieu of changing conditions.

(2) Recent research projects have placed too much emphasis on the power of a particular statistical technique and have failed to fully evaluate the parameters of the problem in question. For example, multiple regression approaches used in identifying significant independent variables have provided contradictory findings.[22] Most researchers using this technique have made allowances for the non-linear relationships that exist for most of these variables, but they have still failed to fully treat the variety of influences of these variables. Longevity has been accorded varying degrees of importance. Obviously, longevity may contribute nothing to value when the common exceeds the exercise or conversion price by a sizeable amount, but it may prove highly significant at low prices of the common relative to these critical values.

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[22]The Shelton regression model described the foregone dividend yield as second only to the stock price in predicting warrant value. Shelton also concluded that price volatility had an insignificant influence on warrant value. A recent article by James C. Van Horne yielded an entirely different set of relationships. Van Horne found a regression coefficient for volatility of .184 and a "t" value of 2.95 while dividend yields had a small regression coefficient of -.011 and a "t" value of only 1.16. See James C. Van Horne, "Warrant Valuation In Relation to Volatility and Opportunity Costs," The Industrial Management Review, Vol. 10, No. 3 (Spring, 1969), pp. 19-32.

(3) Recent research projects have attempted to analyze too many variables in testing the predictability of independent variables. This has led to inconclusive and contradictory findings.

Chapter IV attempts to identify the warrant-stock relationship as it is influenced by the longevity of the option. Convertible bond values are evaluated in a similar fashion in Chapter V.

CHAPTER IV

AN EVALUATION MODEL FOR STOCK PURCHASE WARRANTS

Introduction

This chapter is comprised of two parts, the first of which deals with the development of a warrant model. The model is created by normalizing the common stock price and the warrant price by dividing the exercise price into these values. Observations are separated into six time-to-expiration categories. A power function is utilized to convert these observations to predictive equations that are used in determining fair values for warrants.

The second part of this chapter, which focuses on testing the model, involves a study of all of the common stock warrants listed on the American Stock Exchange from 1960 through 1969. Performance data are gathered for the four separate models used in the study. Through this process conclusions concerning the utility of the empirical models are drawn.

Development of the Model

Influence of Prior Research Projects

Numerous researchers have concluded that the most

important variable influencing warrant pricing is the corresponding price of the common stock. As previously mentioned, other independent variables such as dividend yields on the common, volatility of the common, and longevity of the warrant have been accorded different levels of significance in separate research projects.

At first glance the impact of dividends on the value of a warrant appears to be negative. Cash dividends are paid directly to common stockholders and warrant owners receive no direct benefits. Moreover, the warrant owner finds his potential equity in the firm reduced by the amount of the dividend on each payment date. Before concluding that warrants on dividend paying stocks should be valued lower than warrants on low or non-dividend stocks, it must be recognized that dividend rates may influence the value of the stock and consequently the value of the warrant. Empirical studies have generally found regression coefficients for dividends to be greater and more significant than those for retained earnings.[1] The higher positive coefficients indicate that dividends influence the value of the common stock to a greater extent than do retained earnings.

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[1]James C. Van Horne, Financial Management and Policy (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1968), pp. 188-189.

Therefore, the higher price for the common stock would produce higher market values for warrants.

The past volatility of the common stock also seems to have a role in determining market value of warrants. Theoretically, the warrants of a company whose common stock has ranged in price from $10 to $30 would be more attractive to a potential warrant buyer than one with a range of $18 to $22. However, empirical studies using the past trading range of the common stock have failed to produce conclusive evidence as to its influence on warrant values.[2] General Acceptance Corporation (Now GAC Corporation) offers a classic example of how a stock may trade in a narrow range over a long period of time and then realize a substantial appreciation. GAC's stock never rose above 26.75 nor fell below 17.00 from 1960 through 1966. By the end of 1968 the common had risen to 64.125 and the warrant gained 864% to a price of 43.375. Numerous examples may be used to confirm the inefficiency of past trading ranges to predict future price

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[2]See James C. Van Horne, "Warrant Valuation in Relation to Volatility and Opportunity Costs," The Industrial Management Review, Vol. 10, No. 3 (Spring, 1969), pp. 24-26. See also John P. Shelton, "The Relation of the Price of a Warrant to the Price of Its Associated Stock," Financial Analysts Journal, Vol. 23, No. 4 (July-August, 1967), pp. 94-97. The researcher also experimented with a crude measure of past volatility of the common stock in predicting market moves in warrants. The findings revealed an insignificant and negative coefficient. It was concluded that past volatility fails to predict future price levels or trading ranges for individual warrants.

movements of warrants.[3]

The influence of longevity receives special treatment in this paper. Prior researchers indicate this variable to be highly significant only for warrants approaching expiration. The reviewed research seemed to have produced conclusive evidence that a time horizon beyond a specific number of years has little or no influence on warrant values. The Investment Banking Study used 2 years, Kassouf 4 years, Giguere 5 years, and Shelton 10 years. The researchers all concluded that warrants beyond some specific number of years to maturity should be valued identically regardless of their actual longevity.

The arbitrary truncation of all warrants beyond some specific number or years arose from empirical investigations--because theory can not support such a decision. Warrants often expire with the price of the common at less than the exercise price and thus become worthless, whereas a longer maturity would have given rise to a tangible value with the common priced above the exercise price. Theoretically, the

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[3]Jefferson Lake Petroleum common ranged from 7.00 to 10.00 in 1964. One year later the common closed at 22.50 and the warrant appreciated 339% to 15.375. The common of National General also traded from a high of 12.50 in 1965 to a low of 7.50 in 1966. National General's warrants appreciated from 4.25 to 13.875 in 1967 for a 226.5% increase. These examples support the lack of correlation found between past volatility and future price performances.

time horizon of the warrant should become increasingly relevant as the price of the common is lower in relation to the exercise price. Time to expiration would be more important for a warrant with an exercise price of $50 and a common price of $10 than with an exercise price of $50 and a common price of $40. When the latter situation exists, investors may well reason that a 2 year warrant should be valued approximately equal to a 4 year warrant because an appreciation of only 25% or $10 would result in parity. The influence of time would be negligible if the probability were high that the stock would reach $50 in less than 2 years. On the other hand, the common must appreciate 400%, or 40 points, to reach parity in the first case. The probability of such a move may be low over the two year horizon and significantly higher over the four year horizon. Longevity can be theoretically viewed as becoming more significant as the ratio of the market price to the option price declines and less significant as the ratio rises.

Prior researchers failed to recognize that time to expiration may well prove significant when common prices are low in relation to option prices and insignificant when common prices are well above option prices. Typical multiple regression studies have included time to expiration (in months) with such other variables as the price of the common stock, dividend yields, volatility of the common, and

potential dilution of the convertible securities. The dependent variable, price of the warrant, is then regressed on these independent variables.

Studies have generally shown the price of the common to be the most significant predictor of warrant values. Heretofore, time to maturity has not been shown to exert much influence on warrant value, except when two years or less remain to expiration. It is likely that these findings have been biased due to the failure to structure the time variable as a function of the ratio of the common stock price to the exercise price. In other words, empirical studies conducted when common prices were high, in relation to option prices, have found longevity to be mildly significant. Empirical observations analyzed when low ratios existed have found the time variable to be a better predictor of warrant value. This weakness may be overcome in three possible ways, namely: (1) adjusting the time variable to include the ratio of the common to the exercise price, (2) separating the observations according to the ratio of the common to the exercise price, or (3) separating the observations according to their time to maturity. Prior researchers have failed to properly consider these alternatives in their regression models, and the investment community still lacks a sophisticated technique for appraising these securities.

Structuring the Time to Expiration

The supply of actively traded stock purchase warrants is definitely limited. Most of these are listed on the American Stock Exchange, as the New York Stock Exchange does not allow trading in these securities. Warrants are traded over-the-counter, but the inaccuracy of the price structure may negate the efficiency of an empirical model.

Accepting these limitations, it was decided to analyze the common stock--warrant price relationships by using a sample of the existing warrants on the American Stock Exchange over the 1960-1969 period. The warrants were structured according to their time to expiration and were segregated into the following classifications: Category 1 (6 months to 1 year); Category 2 (1 to 2 years); Category 3 (2 to 4 years); Category 4 (4 to 7 years); Category 5 (over 7 years); and Category 6 (perpetual warrants). These classifications arose from the limited number of price quotes that would accomodate a greater number of time intervals.

Initially, all of the warrants that fitted these categories were selected for study. Immediate problems arose such as a accurate measurement of the life of a warrant with step-ups in the exercise price at predetermined intervals, and the need for a method of accounting for exercise prices that could be settled by either cash or the principal value of a fixed income instrument that may trade at less than par. These

differences were important, and warrants with either step-ups in exercise price or the exercise price payable in principal value of some security were eliminated from the study. Some of the warrants that had been recognized as selling at excessively high or low premiums were also eliminated from the study. For example, Molybdenum was deemed to trade at an excessive premium and was not used in the study.

Companies were selected for the 6 months to 1 year and 1 to 2 year classifications on the basis of available data for each of these categories. In other words, the same companies were used for both of these classifications in order that only time to expiration varied. The companies used in the first two categories were also used in subsequent categories where possible. Table VIII lists the warrants used in developing the model.

For comparative empirical trading patterns, Table VIII presents a less-than-perfect selection of warrants. In Category 3 (2 to 4 year maturities) United Industrial is substituted for Armour, since the Armour warrants contained a step-up in exercise price 3 years before expiration. The other categories contain all of the warrants that met these time intervals during the 1960-1969 period. General Acceptance is the only warrant listed in all categories except Category 6 (perpetual maturities).

TABLE VIII

LIST OF WARRANTS USED IN EACH CLASSIFICATION

Category 1 (6 months to 1 year)	Category 4 (4 to 7 years)
Armour	First National Realty
General Acceptance	General Acceptance
Martin Marietta	Martin Marietta
Rio Algom	National General
Sperry Rand	Rio Algom
Universal American[62]	United Industrial
	Universal American[62]
	Uris Building
Category 2 (1 to 2 years)	**Category 5 (Over 7 years)**
Armour	General Acceptance
General Acceptance	Gulf & Western
Martin Marietta	Lerner Stores
Rio Algom	McCrory Corp.
Sperry Rand	National General
Universal American[62]	National Industries
	Uris Building
Category 3 (2 to 4 years)	**Category 6 (Perpetual)**
General Acceptance	Alleghany
Martin Marietta	Atlas Corp.
Rio Algom	Tri-Continental
Sperry Rand	
United Industrial	
Universal American[62]	

Price data for these securities were gathered on an interval basis. For the first three categories, monthly data were used. For example, 24 price quotations were extracted for each warrant listed in Category 3. The quotations were taken from Standard & Poors ISL Daily Stock Price Indexes starting when each warrant had four years to maturity and taking monthly closing prices until only two years remained to maturity. The total number of observations for Category 3 was 144 (24 months X 6 warrants). Quarterly data were usually extracted for the remaining classifications, except where a warrant had a expiration of less than 7 years in Category 4 (4 to 7 year maturities). The only warrant to perfectly fit the category was General Acceptance. Therefore, improvisations had to be made to insure a sizeable sample. Monthly price data were acquired for warrants with 5 year expirations and bimonthly data were acquired for 6 year warrants. It was felt that each warrant should contribute an equal number of observations, necessitating a different time interval (i.e., monthly, bimonthly, or quarterly) for individual warrants.

The number of observations for each classification varied due to the number of warrants available for analysis and the time span in each grouping. Category 1 (6 months to 1 year maturities) consisted of only 42 observations (7 for each warrant ranging from 12 to 6 months to expiration). The

highest number of observations was 144; these were gathered for Category 3. The number of observations totaled 540 for the six maturity categories.

The validity of these data may be questioned because different companies are compared in the individual categories. The limited population of warrants dictated this comparative technique--a deficiency future researchers may circumvent since the supply of actively traded warrants has grown sharply in 1969 and 1970.

Normalization of the Dependent and Independent Variables

Comparisons of the actual prices of the warrants to the prices of the common stock would not provide meaningful data since the warrant prices are influenced by the exercise price on each security. For exmaple, National General's old warrants have an exercise price of $15 while Gulf & Western's warrants carry a $55 exercise price. On March 6, 1970, National General's common closed at 17.125 while Gulf & Western's common ended up at 17.625. The corresponding prices for the warrants were 9.50 and 5.625 on the same date. These data are not meaningful without some common denominator. The method selected for comparing these values for this study is a normalization of the warrant and the common by dividing the exercise price into each value.

Normalized values can be thought of as ratios of the market price to the current exercise price of the warrant. This normalization procedure involved two steps. First, the relationship between the ratio of the market price and exercise price is measured for each warrant. Then, the ratio of the market price of the common stock to the exercise price is calculated. These normalized values become the two basic variables of the study. The normalized common stock ratio is the independent variable and is used to predict the dependent variable, or the warrant ratio.

The normalized values for the National General and Gulf & Western warrants may be determined as follows:

National General	Gulf & Western
Common stock ratio: $\frac{\text{Market price}}{\text{Exercise price}} = \frac{17.125}{15} = 1.142$	Common stock ratio: $\frac{\text{Market price}}{\text{Exercise price}} = \frac{17.625}{55} = .320$
Warrant ratio: $\frac{\text{Market price}}{\text{Exercise price}} = \frac{9.50}{15} = .633$	Warrant ratio: $\frac{\text{Market price}}{\text{Exercise price}} = \frac{5.625}{55} = .102$

These data show the normalized values to be substantially different even though the market prices of the two common stocks differs by only $.50. These normalized ratios are extracted for numerous trading dates and are used in determining the empirical relationship between the two ratios. These empirical data are then used to predict the normalized

or expected value of a warrant when a common stock has a given relationship to its exercise price.

Selecting the Proper Statistical Approach

The normalized ratios are plotted on a graph by letting the X axis represent the common stock ratio and the Y axis represent the warrant ratio. The empirical points reveal a curvilinear relationship as a given change in the common stock ratio fails to generate an equal magnitude of change on the Y axis. A free-hand curve through these observed points indicates a simple parabola. Free-hand curves are drawn for data in the aforementioned classifications, and the time horizon appears to prove important in influencing the values of the warrants. Each curve appears higher as the time to expiration is increased.

Free-hand curves are too inaccurate for decision making purposes; therefore, mathematical relationships of these curves are evaluated. The first equation used to calculate a line of best fit through the observed points is a second-degree polynomial expressed by:

$$Y_c = a + bX + cX^2$$

The second-degree polynomial produces the following predictive equations:

Category 1
(6 months to 1 year maturities)
$Y_c = -.08742 + .2148(X) + .1666(X)^2$

Category 2
(1 to 2 year maturities)
$Y_c = -.1146 + .3625(X) + .1234(X)^2$

Category 3
(2 to 4 year maturities)
$Y_c = .0084 + .2086(X) + .1546(X)^2$

Category 4
(4 to 7 year maturities)
$Y_c = -.0856 + .4063(X) + .0891(X)^2$

Category 5
(over 7 year maturities)
$Y_c = .1316 + .0251(X) + .2501(X)^2$

Category 6
(Perpetual maturities)
$Y_c = -.05834 + .5693(X) + .0484(X)^2$

These data generally support the contention that the time horizon of each warrant provides a positive influence on warrant value. The second degree polynomial, however, produces some incongruous results. Category 3 equations provide higher values than those for Category 4 at common stock ratios of less than .6. Category 3 equations also provide lower values for warrants than those for Category 2 at common stock ratios greater than 1.2. These empirical results do not fit the prior theory developed for these instruments.

The equations reveal four of the intercepts to be

negative and only two positive. The negative intercepts are ruled theoretically impossible, since the price of a warrant can not be less than zero even when the common stock is zero. The calculated intercept for Category 3 of .0084 is insignificantly different from zero. Therefore, the high positive intercept for the Category 5 is viewed as a chance occurrance.

Various mathematical equations are applied to the observed data in an attempt to obtain a superior fit. A third degree polynomial is tried, a second degree polynomial with a zero intercept is attempted, and transformations of the independent and dependent variables are utilized. These predictive equations are graphed with the observed data in arriving at the best fit. The coefficient of determination, R^2, is also used to assist in selecting the superior approach.

Most of the statistical procedures applied to these data have some strong points. The cubic parabola provides a better fit for all classifications, but the problem of having Category 3 overstated at low common stock ratios exists as with the second-degree polynomial. Exponential trends and the Gompertz curve are found to fit Category 1, but they fail to uniformly apply to each series of data. The superior method of fitting these data is found by converting both the common stock ratio and the warrant ratio to natural logarithms and fitting a linear equation for

log X and log Y. Predicted regression lines are drawn through observed points, and the closeness of fit is remarkably high. The calculated R^2 is in excess of .94 for 4 of the classifications.

The Final Model for Warrants

The use of logarithms in transforming both the dependent and independent variables is referred to as a power function.[4] It may be represented by:

$$Y_c = a(X)^b$$

The exponent, b, provides a measure of elasticity which is an important element in a warrant's price, since leverage plays an important role in warrant pricing. The simple conversion of these data to logarithms provides a good curvilinear approximation of a warrant's price relative to its corresponding common stock.

Final equations using the power function are as follows:

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[4]John E. Freund and Frank J. Williams, Modern Business Statistics (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1969), p. 419.

Category 1
(6 months to 1 year maturities)

$Y_c = .2753(X)^{1.8585}$ $R^2 = .9622$

Category 2
(1 to 2 year maturities)

$Y_c = .3547(X)^{1.6124}$ $R^2 = .9421$

Category 3
(2 to 4 year maturities)

$Y_c = .3703(X)^{1.4317}$ $R^2 = .8039$

Category 4
(4 to 7 year maturities)

$Y_c = .4062(X)^{1.3095}$ $R^2 = .9582$

Category 5
(Over 7 year maturities)

$Y_c = .4263(X)^{1.1973}$ $R^2 = .8141$

Category 6
(Perpetual maturities)

$Y_c = .5509(X)^{1.2155}$ $R^2 = .9926$

These equations convey the importance of time in warrant pricing. The calculated constant for "a" is shown to increase as time to expiration is lengthened. When the price of the common is equal to the exercise price, the speculative value of the warrant increases from .2753 of exercise price for 6 months to 1 year maturities to .3547 for 1 to 2 year maturities. These speculative values increase gradually as longevity is stretched out. The large speculative value of .2753 for warrants with average lives of nine months indicates the critical nature of time when the expiration horizon is short, as

warrant premiums will trend toward zero when maturity approaches. This phenomenon is incorporated into trading tactics in Chapter VI.

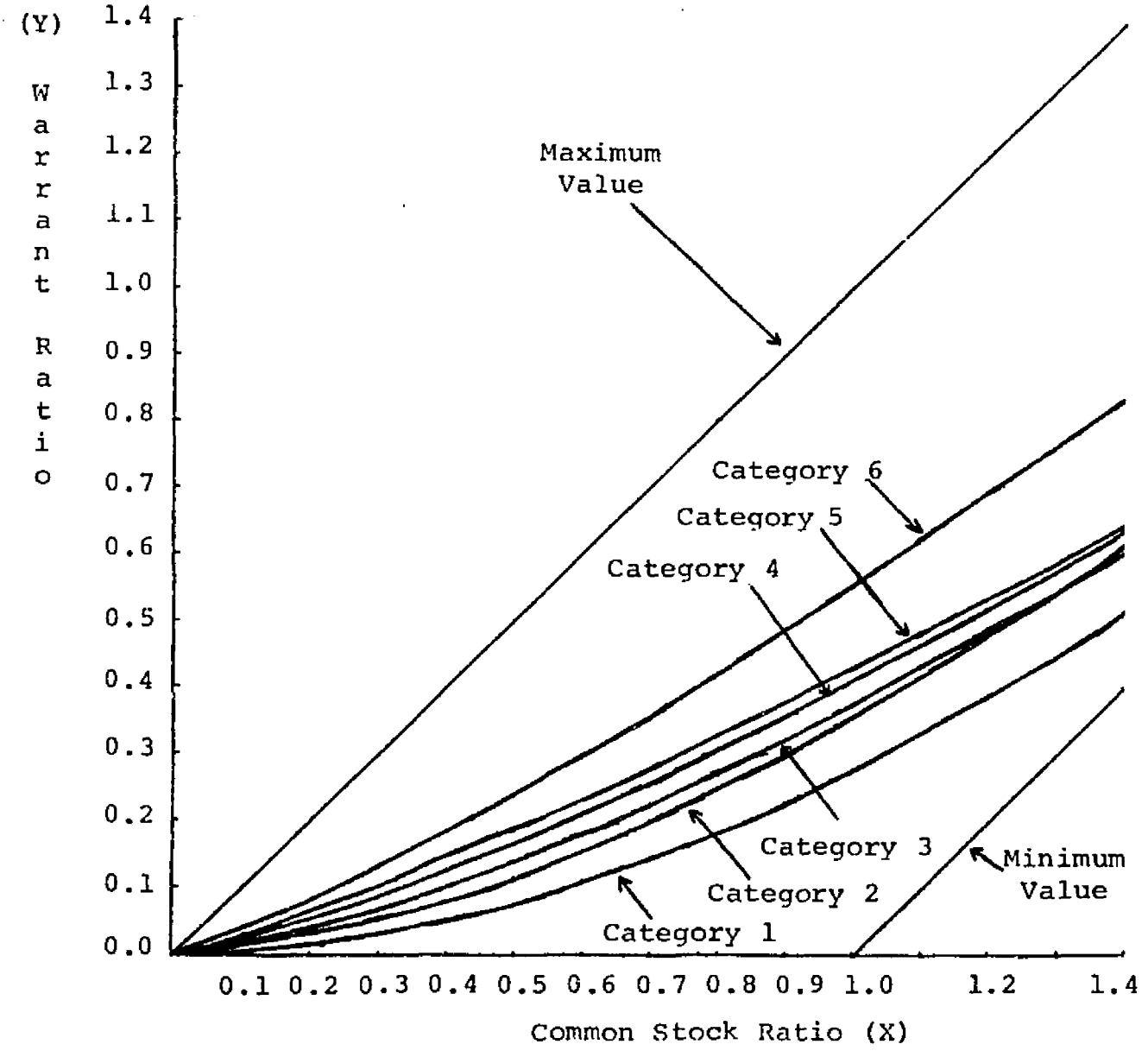
The calculated exponents also support previous theory concerning warrant pricing. Exponents decline as a function of time except for perpetual warrants, and the importance of time is minimized as the common stock ratio increases above 1.0. In other words, warrant prices appreciate and depreciate faster with the higher exponents. Time is less important at higher common stock ratios and more important at lower ratios. Chart 8 provides a graphic representation of the empirical price of the warrant, compared to the common stock, by utilizing these ratios.

As can be seen in Chart 8 the warrant is valued relatively higher as the common stock ratio and time to expiration increase. At the common stock ratio of approximately 1.2, the categories show a tendency to change relative positions--especially for Category 2. These interchanges can primarily be explained by the relevant ranges of the empirical data. Observations for Category 4 consist of common stock ratios ranging from .30 to 1.20. Therefore, data beyond this relevant range are not comparable. Their sole function is to facilitate an understanding of the mathematical model.

The empirical model is designed to provide a method of evaluating fair prices for warrants based on past

CHART 8

GRAPHIC REPRESENTATION OF WARRANT MODEL



relationships. The model is employed in the following manner:

(1) Determine the proper category for an existing warrant according to its time to expiration.

(2) Divide the exercise price of the warrant into the price of the common stock to determine the value of X.

(3) Calculate the normalized value of the warrant by taking the logarithm of X and multiplying it times the exponent. Take the antilog of the product and multiply it by the constant "a" value.

(4) Multiply the normalized value by the exercise price per share to determine the fair value of the warrant.

(5) Compare the calculated value with the existing market price to determine if an attractive buying opportunity exists.[4]

The complete model for warrant valuation has potential uses other than determining fair values for purchase opportunities. It can be used in determining the taxable benefit when warrants are received as compensation (i.e.,

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[4]A potential investor should analyze factors other than the empirical price compared to the actual price. The expectations of the individual company should be integrated with the overall economic outlook. The calculated price should be used only in judging relative prices and not as the only factor in deciding what to pay.

when inactively traded warrants are paid to underwriters as compensation for distributing a security issue). The model may also be applied to potential new issues of warrants in determining the terms, especially longevity, of the Warrant Agreement. The terms of the warrants can be fitted to management's utility preferences that will interact in the tradeoff of a higher price for the longer life of the warrant. This model enables the elements of this tradeoff to be quantified on an empirical basis.

Testing the Empirical Warrant Model

Purpose of the Test

Historically, warrant models have been tested by comparing predicted prices to actual warrant prices. Kassouf, Giguere, and Shelton[5] utilized such an approach in confirming the validity of their models. These three models were aimed at determining fair or normalized values for warrants. Each researcher provided an approach designed to assist investors in the selection of warrants for purchase.

The empirical model discussed in the first part of this chapter has already been described as providing a close approximation of warrant prices. A further test accomplished by comparing predicted prices with actual prices would not support the utility of the model in selecting warrants for

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[5] Op. cit.

purchase. The logical test for a predictive model is to determine if normalized or fair values assist investors in selecting securities for purchase. Theoretically, models aimed at determining fair values for warrants should provide superior performance than average warrants, since capital will not be committed to overpriced warrants.

Structure of the Test

A basic tenet of the empirical model is that its coefficients will change as long-term market conditions change. Empirical data gathered for its construction are taken from the most recent ten year period to enhance its applicability in today's markets. Extrapolation of this model into the distant future or past would probably rule it as ineffective in determining appropriate values.

The period from the beginning of 1960 to the end of 1969 is chosen to test the efficiency of the model. First-of-the-year price data are gathered for each warrant on the American Stock Exchange. Closing prices at the end of the year are used in determining the performance of individual warrants. Performance is measured in terms of percentage gained or lost for each year. These net percentage changes are then averaged to determine the overall performance of the listed warrants for each year.

Average percentage changes provide a method of simulating purchase commitments in these warrants by assuming equal

dollar investments in each warrant at the beginning of the year and closed out at the year's end. The average percentage change of all warrants determines the net effect on the initial investment. The original investment at the beginning of 1960 is assumed to be $100, and it fluctuates according to the annual cumulative percentage changes of the assumed investments.

Valuation models of Giguere, Kassouf, and Shelton are also included in this study. These models are added to the empirical test to provide further comparisons. The Giguere model provides conservative normalized values while the other two models offer higher predicted prices.

Decision Rules for the Models

All of the models included in the test provide normalized values for the warrants, but the prior researchers failed to specify rules for determining when a warrant should be purchased. A uniform decision rule is therefore deemed appropriate and is applied to each normalized value.

There are various approaches to establishing an automatic decision making process. One may reason that a 50% confidence limit placed on the model value would include attractively priced securities. Those in the upper quartile would be deemed overvalued; those in the lower quartile would be ruled unattractive due to the market's extremely cautious appraisal of them. Confidence limits could be increased or

narrowed and multiple purchase decisions could be simulated.

Purchase decisions could also be based on the percentage differences in the normalized values and actual prices. For example, normalized values that exceed actual prices by more than 3% might be deemed as desirable commitments.

The decision to use a 3% filter for purchase decisions does not have greater theoretical support than a 2%, 1%, or 5% filter. One could logically argue that any price of a warrant less than a model's appraisal value would offer an attractive buying opportunity. The selected decision rule--to simulate a purchase when the normalized value is 3% greater than the price of the warrant is based on objectivity and selectivity. It is objective in that technical buy or sell signals are usually based on a 3% penetration of a resistance or support level. It is selective in that fewer than 50% of the warrants will be selected for purchase with the least squares technique used in the model development.

The 3% buy signal is used for each of the four models. Each time the model value exceeds the initial price of the warrant by more than 3% a buy signal is given. Each model's performance is measured by averaging the percentage gains and/or losses of the simulated purchases.

Unique Characteristics of Each Model

The empirical model is limited by a relevant range of sample calculations used in its construction. Category 2

provides higher normalized values than Category 3 when the common stock ratio is greater than 1.2; therefore, normalized values are not determined for common stock ratios greater than 1.2. The exclusion of warrants with common stock ratios in excess of 1.2 also eliminates low leverage warrants. The potential leverage of any particular warrant is an inverse function of the price of its corresponding common stock. Higher prices of the common stock mean less leverage in the warrant and vice versa.

The empirical model is also limited in accurately determining warrant values at low common stock ratios, since sample data were unavailable for these low values. Normalized values are not calculated for warrants with common stock ratios of .5 or less. The exclusion of these low common stock ratio warrants tends to lower the potential leverage of the empirical model selections.

Kassouf's normal conditions are deemed too restrictive to be useful in studying the benefit of his model. Calculations of normalized values for his model are based solely on his formula except for warrants with step-ups in exercise price. In these instances the exercise price is raised to the higher level when less than one year to the step-up exists. Step-ups for the Shelton model are treated according to his format with adjustments of 2.5 times the percentage increase in step-up price used to lower the time to expiration.

Step-ups for the empirical model and Giguere's model are treated by calculating a normalized value for each possible exercise price. The immediate exercise price is applied to the appropriate equation according to time to step-up. Higher exercise prices are applied to longer time to expiration categories, and the highest normalized value is compared with the actual price of the warrant for decision purposes.

Return on Investment

Stock purchase warrants do not provide a cash income. Capital appreciation is the only source of income from these instruments. The cumulative impact of the annual commitments is compared with the initial investment in determining the overall performance of each model and the overall performance of average warrants.

The individual models call for purchasing different numbers of warrants in each year. The initial investment is assumed to be equally divided among all warrants purchased. Where the model does not select any of the given warrants for purchase, the money is assumed to be invested in 91 day Treasury bills--at the annual rate in existence during that year.

The cumulative amount of the investment at the end of 10 years is compared for each category. The final balance is converted to an annual compounded rate of return through

use of present value tables. Present value rates of return are used to rank the individual classifications in terms of overall performance.[6]

Results of the Test

The chosen time horizon provides an interesting framework for warrant analysis. Different degrees of bullish and bearish sentiment are portrayed in specific years. Price fluctuation of individual warrants is also shown to be extremely high. Appendix A shows First National Realty warrants to have appreciated 1114.3% in 1968 while Ling-Tempco-Vought warrants declined 80.1% in 1969. Of the 177 warrant observations listed in Appendix A, 80 appreciated, 94 declined, and 3 were unchanged. The contrasting performances provide a good challenge for the valuation models.

Table IX depicts the overall gains and losses for each year, and Appendix A contains individual observations. Examination of Table IX reveals that average warrants appreciated sharply over the 10 years. They actually appreciated more than the warrants selected for purchase by either of the valuation models. This suggests that valuation models should be

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[6]The cumulative funds are based on market changes without consideration of brokerage fees and capital gain taxes. These items are ignored due to the assumption of equal investment in each security. Odd-lot fees on low priced warrants would complicate the procedure and may even bias the findings. The exclusion of these normal costs tends to inflate the present value rate of return.

TABLE IX

SUMMARY OF TEST RESULTS FOR WARRANTS

	Total Warrants	Giguere	Kassouf	Shelton	Empirical Model
1960 Data					
No. Purchased	11	None	3	4	1
No. Increasing	2	0	1	1	0
No. Decreasing	9	0	2	3	1
Average Change (%)	-18.850	+4.100*	-14.460	-23.920	-52.300
Initial Investment	100.000	100.000	100.000	100.000	100.000
Ending Balance	81.150	104.100	85.540	76.080	47.700
1961 Data					
No. Purchased	14	None	9	8	3
No. Increasing	12	0	9	7	3
No. Decreasing	2	0	0	1	0
Average Change (%)	+39.290	+2.200*	+50.530	+40.030	+79.270
Ending Balance	113.034	106.390	128.763	106.535	85.512
1962 Data					
No. Purchased	16	None	5	8	None
No. Increasing	1	0	0	1	0
No. Decreasing	15	0	5	7	0
Average Change (%)	-25.070	+2.700*	-21.420	-21.200	+2.700*
Ending Balance	84.696	109.263	101.182	83.950	87.821
1963 Data					
No. Purchased	19	1	8	12	4
No. Increasing	8	0	2	3	0
No. Decreasing	11	1	6	9	4
Average Change (%)	+5.840	-21.400	-14.550	+3.020	-29.620
Ending Balance	89.642	85.881	86.460	86.485	61.804

*Indicates estimated treasury bill rate for the year.

TABLE IX (CONTINUED)

		Total Warrants	Giguere	Kassouf	Shelton	Empirical Model
1964 Data						
No. Purchased		18	1	11	14	7
No. Increasing		9	0	6	7	3
No. Decreasing		7	0	4	6	3
Average Change	(%)	+5.570	0.000	+15.240	+8.540	+5.260
Ending Balance		94.635	85.881	99.637	93.871	65.055
1965 Data						
No. Purchased		19	None	11	14	6
No. Increasing		15	0	9	11	5
No. Decreasing		3	0	1	2	0
Average Change	(%)	+82.710	+3.800*	+88.720	+96.190	+130.080
Ending Balance		172.908	89.144	188.035	184.166	150.980
1966 Data						
No. Purchased		18	1	7	11	2
No. Increasing		5	0	1	5	0
No. Decreasing		13	1	6	6	2
Average Change	(%)	-18.070	-46.800	-19.230	-9.580	-32.500
Ending Balance		141.664	47.425	151.876	166.523	101.912
1967 Data						
No. Purchased		16	1	8	9	3
No. Increasing		14	1	6	8	3
No. Decreasing		2	0	2	1	0
Average Change	(%)	+135.240	+240.000	+134.250	+157.890	+257.370
Ending Balance		333.250	161.245	355.770	429.446	364.203

*Indicates estimated treasury bill rate for the year.

TABLE IX (CONTINUED)

	Total Warrants	Giguere	Kassouf	Shelton	Empirical Model
1968 Data					
No. Purchased	18	None	7	9	1
No. Increasing	13	0	7	7	1
No. Decreasing	5	0	0	2	0
Average Change (%)	+127.450	+4.900*	+75.930	+69.250	+114.500
Ending Balance	757.977	169.146	657.321	705.660	781.215
1969 Data					
No. Purchased	28	None	9	10	3
No. Increasing	1	0	0	1	0
No. Decreasing	27	0	9	9	3
Average Change (%)	-53.260	+6.300*	-52.940	-53.410	-68.800
Ending Balance	362.863	179.802	309.335	328.770	243.739
Total Data					
No. Purchased	177	4	77	99	30
No. Increasing	80	1	41	51	15
No. Decreasing	94	2	35	46	13
Annual P.V. ROI	13.8%	6.0%	12.0%	12.7%	9.4%

*Indicates estimated treasury bill rate for the year.

ignored in warrant selection because average warrants perform better than any of the sophisticated models. This conclusion is decidedly premature as the performance of average warrants is biased upward by one observation, namely, First National Realty's 1114.3% increase in 1969. At the end of 1967, three of the models held a slight advantage over the average warrant category. If this observation were deleted, average warrants would show a cumulative value of 564.525 instead of 757.977 at the end of 1968 and would exceed only Giguere's cumulative value.

Arbitrary selection of beginning and ending years could also change the results of this study. If the initial year were 1961 and the ending year 1968, $100 invested via the empirical model would have grown to $1637.74 for a present value rate of return in excess of 40% annually. A selection of those years could have shown the empirical model to be totally superior, but the purpose of this research project would not be satisfied.

The decision to restrict warrant selection to warrants with common stock prices less than 20% above the exercise price (common stock ratio of less than 1.2) naturally provides more leverage for the warrants selected by the empirical model. The danger of this downside leverage is properly shown by having a bear market exist in the first and

last years of the study. Such a test forces the empirical model to demonstrate superior percentage gains in bull market periods in order to surpass the other models. Table IX shows the empirical model's selections to have gained 85.51%, 130.08%, 257.37%, and 114.50% in the bull markets of 1961, 1965, 1967, and 1968 respectively. These remarkable gains place its performance above all of the other classifications by the end of 1968. The downside leverage of the model results in huge percentage losses during the 1969 bear market to erase its overall advantage.

The most interesting fact about the empirical model is its tendency to recognize generally inflated markets. Table IX shows average warrants declined sharply in four of the ten years. Specifically, average warrants declined 18.85% in 1960, 25.07% in 1962, 18.07% in 1966, and 53.26% in 1969. Table IX also shows the empirical model to have given only 6 buy signals in these four years, even though it provided 30 buy signals over the ten year period. If potentially dangerous markets can be identified by few buy signals and/or by generally inflated warrants, the empirical model could be utilized to generate tremendous gains.

A modification in the decision rule to buy warrants only when they are favorably priced in general would significantly change the results. The decision rule could dictate buying only when 30% or more of relevant warrants are

favorably priced by the empirical model. This decision rule would avoid all of the major bear markets and eliminate only 1 bull market. The initial investment in warrants would grow from $100 to $1,102 by the end of 1967, and warrants would not be purchased in 1968 and 1969. The timing device is not sophisticated, but it illustrates the importance of avoiding overpriced markets. These data can be verified in Appendix A.

Giguere's model makes only four purchases over the ten year period. The conservatism of the model basically keeps it out of the market--when average warrants experience sizeable gains. Giguere's model is not applicable to warrant prices in the 1960's, but it is not ruled ineffective for warrant markets in the future.

The performances of the Kassouf and Shelton models are viewed as applicable in today's markets. Shelton's model suggests that more than half of the available warrants should be purchased, as it specifies 99 buy signals from the 177 observations. Shelton's model appears consistent in selecting warrants, but it seems to place excessive negative influence on dividends per share. Kassouf's model would probably prove more efficient if some upper limit were placed on the ratio of the common stock price to the exercise price. The overall results for the models show Shelton's model to be superior with a 12.7% compound rate of return.

Kassouf's model provides a 12.0% return while the empirical model and Giguere's model produce returns of 9.4% and 6% respectively.

Conclusion

It is probably desirable that average warrants performed better than warrants selected by either of the models. These models are not designed to answer all questions pertaining to warrant behavior. Future warrant performance will depend on the performance of corresponding common shares, and these models do not evaluate fundamental and technical strengths of each common stock. The empirical model provides only an approximation of average prices based on historical relationships.

The empirically designed model can be best described as a guide to thinking and not an end in itself. Prospective purchasers of warrants may find the empirical relationships helpful in arriving at fair prices. Users should be especially careful in selecting warrants on the basis of the model values alone. A warrant may be priced at lower than model values due to the market's knowledge of the weaknesses of the common stock. Users of these models should engage in a thorough analysis of the common stock before making a commitment in the warrant.

The normalizing procedure tends to understate the

price for warrants with low exercise prices and overstate warrant values with high exercise prices. For example, an exercise price of $1 for one warrant and $100 for another will result in identical relative prices for the warrants when their common stock price equals their respective exercise prices. The market price will probably favor the lower exercise price, however, as investors would prefer to pay $.30 for the low priced warrant rather than $30 for the higher priced security. Users of the empirical model should recognize this limitation in the normalizing procedure. Prospective issuers of warrants may also utilize this observation by enabling warrants to be exercisable into fractions of shares, rather than whole shares, in order to effectively increase the initial price of the warrant's speculative value.

Those who employ the empirical model should also recognize the extreme leverage they are subjected to when using the approach presented in this chapter. Warrants with low common stock ratios possess more leverage, upside as well as downside, than do average warrants. Purchasers of warrants with low common stock ratios should invest a smaller fraction of their overall portfolio in these securities, since leverage will be magnified.

CHAPTER V

AN EVALUATION MODEL FOR CONVERTIBLE BONDS

Introduction

The empirical warrant model shows the price of warrants to be a function of time to expiration and the common stock ratio. Warrant prices are based solely on their claim to common stock which is similar to the claim offered by a conversion feature on convertible bonds. A proper method of valuing the conversion feature provides a method of analyzing convertible bonds in a similar format to that used for warrants in Chapter IV.

This chapter consists of two distinct sections. First, an evaluation model for convertible bonds is developed. The second section concentrates on the testing of that model.

The model is developed by normalizing the conversion value and the convertible feature in terms of straight debt value. Observations are gathered for these securities and are used in determining final equations for the model.

A sample of 100 convertible bonds is selected to accomplish the testing of the model. Tabular data are provided to illustrate how well the model selects convertible bonds for

purchase. The performance of the convertible bond model is measured in terms of average convertibles used in the study with conclusions based on those comparisons.

Development of the Model

Value of the Conversion Feature

Convertible bonds offer fixed coupon payments and set maturity values similar to straight debt instruments. When convertible bonds offer a greater than zero probability of conversion value (common stock price multiplied by the number of shares the bond may be converted into) exceeding straight debt value during the life of the feature, the right to convert is reflected in their price. Market participants evaluate the worth of straight debt value concomitant with the conversion feature in determining the price they will pay for the convertible bond. In order to develop a convertible bond model similar to the warrant model, the price of the convertible bond must be segregated into a value for the conversion feature.

Charts 3 and 4 in Chapter II illustrate a convertible bond's price to always be equal to or greater than its straight debt value. Therefore, the difference in the bond's price and its straight debt value is identical with the premium on a convertible bond when straight debt value is greater than conversion value, and it is always equal to the premium over straight debt value. The conversion feature's value is,

however, improperly determined by deducting conversion value from the price of the bond, since a measurement of straight debt value's influence is provided with such an approach.

Normalization of the Variables

The empirical model for warrants, illustrated in Chapter IV, uses the exercise price as the common denominator in determining common stock and warrant ratios. The exercise price component for a convertible bond is the bond itself, since it is given in exchange for common stock. Bonds exchanged for common stock usually consist of redemption value, par value, and straight debt value. These are three distinct values, and it is conceivable that any one of them could represent the most important factor in a bond's price at different times.

Redemption value governs the price of a convertible bond when it is being called by the issuing company. It also restricts the upward movement of a bond as interest rates drop due to the possibility of a call. Par value is dominant when maturity is near. Straight debt value, or the present value of future coupon payments and principal discounted at existing interest rates, is a pervasive element that seems to always have a role in a bond's price. These coupon payments are usually considered when bonds are called for redemption. As bonds near maturity, straight debt value approaches par

value. Thus, straight debt value is the proper foundation for developing a convertible bond model.

A division of straight debt value into conversion value normalizes the common stock contribution to a bond's total value. A low ratio suggests a small contribution to the bond's overall price, since the stock must appreciate significantly before conversion value exceeds straight debt value. Ratios of greater than 1.0 contribute tangible values above the coupon and principal values.

The normalization of straight debt values and conversion values is hereafter referred to as the conversion value ratio. It becomes the independent variable that is used to predict the dependent variable, or the value of the conversion feature. An empirical determination of how conversion value ratios influence the value of the conversion feature enables proper valuation of convertible bonds.

Dividing the straight debt value into the difference between the actual price of the bond less the straight debt value normalizes the conversion feature and becomes the dependent variable referred to as the conversion feature ratio.

Selection of the Observations

Data for the warrant model are based on the most recent ten years (1960-69), and consistency with the warrant model dictates a similar time horizon for the convertible bond

model. The supply of convertible bonds is, however, much greater than the supply of warrants. This circumstance permits a more objective method of gathering data than was possible for the warrant model.

Test cases are determined from issues of Barron's and The Commercial and Financial Chronicle for the end of each year, 1960-1969, and data are obtained for industrial convertible bonds with ratings of Baa or higher. Each convertible issue is required to be listed at least two years prior to the respective cutoff dates. Each individual issue is limited to five observations.

Moody's Industrial Manual provides ratings no lower than Baa due to the erratic nature of lower rated bonds.[1] In addition, many institutional investors are limited to buying the top four grades. Therefore, only high quality bonds are included in the observations. The universe is limited to industrial bonds in an attempt to obtain consistent interaction between conversion value ratios and conversion feature ratios. Single observations of one company are eliminated due

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[1]In a personal correspondence Miles H. Riggs of Moody's Investors Service, Inc. explained that bonds rated below Baa are too unpredictable for average yields on these categories to be meaningful. Mr. Riggs' letter of November 11, 1969 stated that bonds with Ba ratings yield from 7.40 to 11.00%, B bonds, about 8.00 to 12.60%, and Caa issues, anywhere from 9.00 to 15.00% in the current market.

to some underwritings that are nothing more than disguised equity issues. These securities are deemed to misrepresent the nature of average convertible bonds. At the same time, convertible bonds that have remained outstanding over a long period of years are ruled atypical, and their observations are limited to five years.

Inactively traded convertible bonds are excluded from consideration because of the possibility of a lengthy time lapse between recorded closing prices for the common stock and convertible bond--a situation that could result in sizeable error if the normalizing process were to be attempted. The closing price of each convertible bond selected is extracted concurrently with the closing price of the stock. The terms of each convertible feature are determined from Moody's Industrial Manuals and adjustments are made for stock splits, dividends, and other capitalization modifications.

Conversion values are determined by dividing the conversion price into the price of the common stock and multiplying the product by the par value of the bond. Straight debt values are determined by deducting the present value of the coupon deficiency from par. Redemption value provides the upper limit on straight debt values when coupon rates exceed existing interest rates. In such situations, the present value of the coupon advantage is added to par.

Structure of the Model

Few convertible bond observations provided short periods to expiration of the convertible feature. Most of the observed issues contained significantly longer maturities than did the warrants. Time categories for convertible bonds differed significantly than those for warrants.

A total of 181 observations are taken from the end of 1959 to the end of 1969.[2] These observations are subdivided into six categories. Two categories are created for convertibles with step-ups in exercise price. Convertibles with step-ups of less than five years hence comprise one category. Actual observations for this category amounted to thirty-seven, and the predictive equation was determined to be:

$$Y_c = .1593(X)^{2.6820}$$

a second category consisting of twenty-six observations with step-ups of five to ten years hence provided the following equation:

$$Y_c = .1574(X)^{3.1948}$$

These results are in contrast with theory, since the longer period to step-up provides lower values for the conversion

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[2]These observations are shown in Appendix B. These data reveal the supply of highly rated convertibles to have declined sharply in the middle 1960's and then increased in 1968 and 1969.

feature at conversion value ratios less than 1.0. These data are deemed inappropriate in valuing convertibles with step-ups, and they are eliminated from the study.

The remaining observations (all free of step-ups in exercise price) are separated into four categories. Twenty-one observations with conversion features less than ten years to expiration are included in Category A. Category B is comprised of thirty-two observations with convertible features expiring from ten to twenty years. Category C consists of thirty-seven observations with over twenty years to expiration and straight debt values in excess of $700. Category D contains twenty-eight observations with over twenty years to expiration and straight debt values less than $700.

The separation of long-term convertible feature instruments into two distinct categories is based on the conversion value ratios' relationship to a convertible bond's call price. For example, a convertible bond with a straight debt value of $500 provides a conversion value ratio of 2.0 when conversion value is $1,000. At a ratio of 2.0, the convertible still provides upside potential, since the redemption price is yet to be reached. On the other hand, a straight debt value of $1,000 produces a conversion value ratio of 1.0 when conversion value is $1,000, and upside potential is similar to a $500 straight debt value with a conversion value ratio of 2.0. The convertibles are

segregated into these categories on the contention that low straight debt values should produce higher values for conversion features at given conversion price ratios.

The Final Model for Convertible Bonds

These data are regressed in a fashion similar to those for warrants. The independent and dependent variables are converted to natural logarithms and linear least squares fit is mathematically calculated. The final model is given by the following equations.

Category A
(less than 10 years to expiration)

$$Y_c = .1871(X)^{2.9190} \qquad R^2 = .6812$$

Category B
(10 to 20 years to expiration)

$$Y_c = .2679(X)^{1.9112} \qquad R^2 = .8852$$

Category C
(Over 20 years to expiration--SDV>$700)

$$Y_c = .2772(X)^{1.7521} \qquad R^2 = .8827$$

Category D
(Over 20 years to expiration--SDV<$700)

$$Y_c = .3173(X)^{1.6809} \qquad R^2 = .9135$$

These mathematical equations provide a simple method for determining a fair price for a convertible bond when straight debt values and conversion values are available. The procedure follows:

(1) Divide straight debt value into conversion value to determine X, or the conversion value ratio.

(2) Select the proper equation by determining the years to expiration.[3]

(3) Take the log of X and multiply it by the exponent. Take the antilog of the product and multiply it by the constant "a".

(4) Multiply the resulting decimal by the straight debt value to determine a proper value for the conversion feature.

(5) Add the calculated conversion feature to the straight debt value in determining a fair price for the convertible bond.

The final model reveals that time plays a significant role in convertible bond pricing. The constant "a" value increases as a function of time. The "a" value also rises for convertibles with low straight debt values for the observations in Category D. The exponents decline in an expected fashion to show time less important at conversion value ratios above 1.0. In other words, the convertible feature increases 2.919

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[3]Convertible bonds with step-ups are treated in an identical fashion as warrants. The initial conversion price is applied to the appropriate category. Higher conversion prices are then applied to longer term categories as they exist. The higher of the calculated values is deemed to be the fair price. The use of a Wang calculator enables these calculations to be made with great speed.

times as fast as conversion value when the conversion value is equal to 1.0 in Category A. This upside leverage rules the equation worthless at conversion value ratios of greater than 1.2 as can be seen in Chart 9. The descending exponents reveal the model to be consistent with traditional convertible bond theory which suggests that all convertible bond prices should rest on conversion value at some percentage above par.

This graphic presentation shows the critical nature of time to expiration. Category A consists of observations with an average of five years to expiration. The observed points do not fit the regression line too closely as some of these convertibles have zero values for the conversion feature. Regression lines for the three other categories fit their observed points well, with the lowest R^2 = .8827 for Category C.

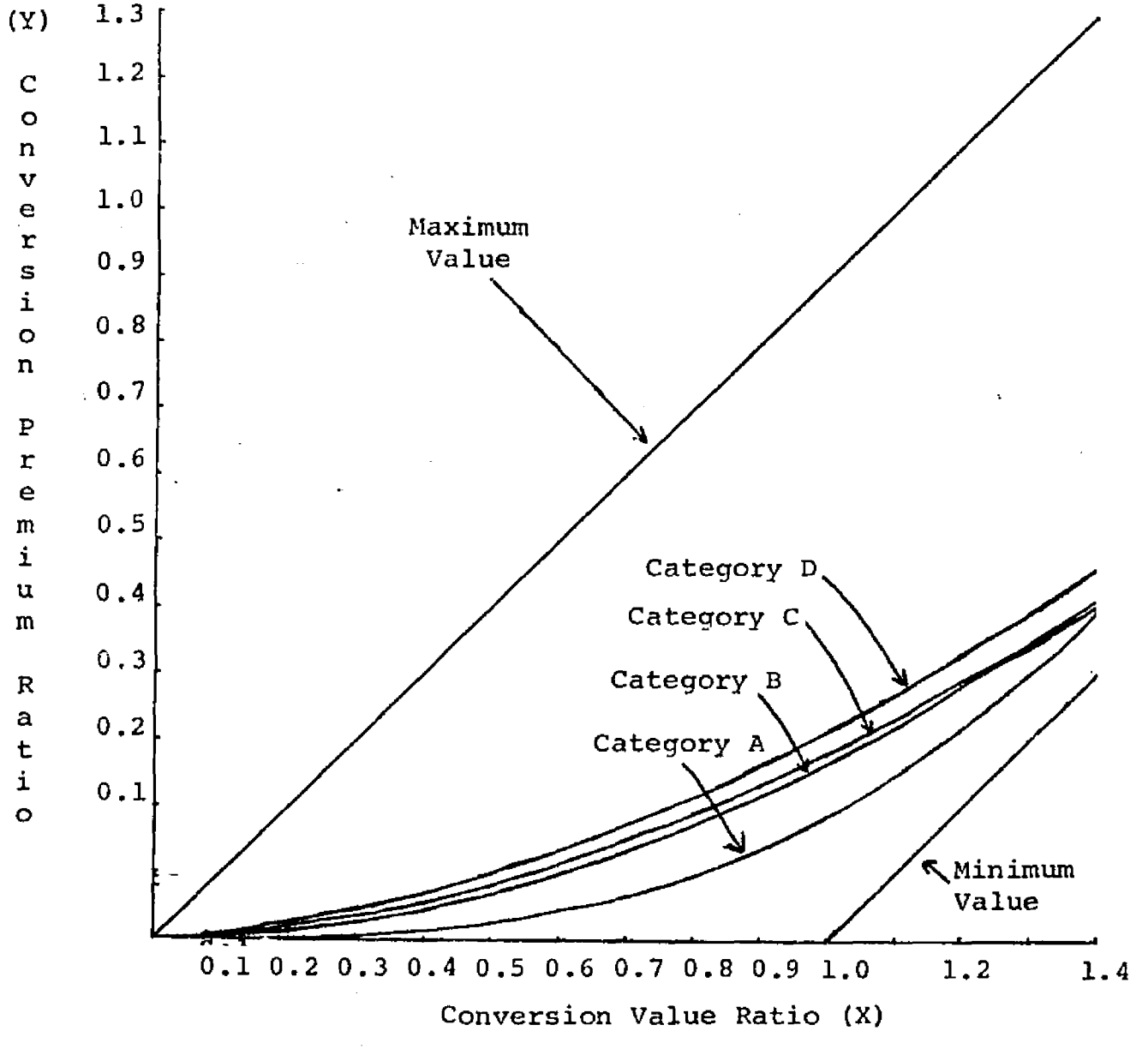
Chart 9 can be used to approximate fair values for individual convertibles by calculating the conversion value ratio and reading off the value of the conversion feature. The conversion feature is multiplied by the straight debt value to determine the actual dollar worth of this contractual right. The conversion feature plus the straight debt value yields the total value of the convertible bond.

The observations could conceivably be carried to infinitely high conversion value ratios. For example, if

CHART 9

GRAPHIC REPRESENTATION OF THE CONVERTIBLE

BOND MODEL



the conversion value is $5,000 and straight debt value is $500, the conversion value ratio is 10. The conversion feature should have ratio of 9 due to arbitrage conditions, described in Chapter II, and the resultant value of the convertible bond is $5,000 [(9 x $500) + 500].

The empirical model for convertible bonds enables these instruments to be appraised regardless of whether conversion value is higher or lower than straight debt value. The empirical model is, however, bound by the relevant range of the observations. A conversion value ratio of .3 is deemed the appropriate lower boundry and 1.2 is the upper limit.

Testing the Empirical Model

Selecting the Proper Test

The logical method of determining the appropriateness of the empirical convertible model is to determine how well it selects convertible bonds for investment since its construction is aimed at this goal. The model's selection of attractively priced combinations of straight debt value and conversion value should produce results superior to those achieved by investments in average convertibles. Theoretically, overpriced convertibles are avoided with such a model.

The test procedure for the warrant model includes too many observations outside the model's relevant range. The test procedure for the convertible model attempts to overcome these prior deficiencies, as the abundant supply of

convertibles enables a superior test for the convertibles model.

The relevant range for the convertible model is a conversion value ratio of .3 to 1.2. Actually, model values appear close to empirical observations at extremely low conversion value ratios. Therefore, convertible issues with conversion value ratios of less than .3 are deemed acceptable for this test. However, selection of only low conversion value convertibles would severely restrict the upward mobility of the sample, since the common stock must appreciate significantly before parity would be attained.

The procedure necessary to objective selection of convertibles with comparable conversion and straight debt values is accomplished by selecting those convertible bonds selling near par. These bonds should have some of their value occurring from conversion value, since coupon rates on convertibles are typically lower than existing interest rates on similarily rated companies.

The first issues of The Commercial and Financial Chronical and Barron's of each year from 1960 to 1969 are used to locate convertible bonds selling near par. The number of convertibles trading within 6 to 8 points of par are listed. Closing prices are used to locate the ten convertibles nearest to par and these securities are included in the final sample of securities. The sample is structured to exclude the following convertibles: (1) those rated below B by Moody's

Corporation, (2) those not listed in Moody's Industrial Manual, (3) those convertible into instruments other than common stock, and (4) those requiring additional monetary payments upon conversion.

A minor improvisation is required in order to obtain sample convertibles trading near par. Bonds rated Ba and B are included in the test observations even though the model is based on Baa or higher instruments. It is reasoned that the lower rating only influences straight debt values and does not change conversion feature values. As long as straight debt values are properly estimated, the model should properly extend to all classes of convertibles.

The inclusion of Ba and B rated bonds requires their yields to be estimated, since Moody's does not list these yields. The estimation procedure includes an estimation of the percentage difference in coupon rates on newly issued Ba and B bonds when compared with yields on Baa bonds. The method of comparison shows Ba bonds to usually carry an interest rate of 5 to 6 per cent higher than Baa bonds. Therefore, the yield on Baa provides the basis for the adjustment. B rated bonds are found to have from 6 to 7 per cent higher yields than Ba bonds.[4]

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[4]Estimated yields for all of these securities appear in Appendix C.

Fair values are determined for the 100 observations. Buy decisions are simulated using the 3 per cent filter in a fashion identical to that used in testing the warrant model.

Results of the Test

A simulated test produces findings that must be treated with caution. Failure to include the entire universe of convertibles limits reliability of cumulative changes as measured in this test. Thus, the test results obviously contain bias.

Table X shows average convertible bonds used in the study to have slightly declined in market value during the ten year period. Coupon payments are not included in the cumulative balances since capital appreciation appears to best test a model's efficiency. These data also reveal the average convertible to have appreciated in five of the years and declined in the other five years. Large percentage gains were registered in 1965 while severe capital losses were suffered in 1969. The market performance of the convertible bonds was similar in direction to the performance of the warrants. The major capital losses for each of these securities occurred in 1962, 1966, and 1969.

The performance of the convertible bond model is outstanding. Table X shows it to have performed better than average convertibles in each year excepting 1966--a

TABLE X

SUMMARY OF TEST RESULTS FOR CONVERTIBLE BONDS

	Total Conv. Bonds	Empirical Bond Model		Total Conv. Bonds	Empirical Bond Model
1960			1965		
No. Purchased	10	7	No. Purchased	10	6
No. Increasing	4	3	No. Increasing	9	6
No. Decreasing	6	4	No. Decreasing	1	0
Aver. Chg. (%)	-1.600	+0.200	Aver. Chg. (%)	+31.250	+39.700
Init. Invest.	100.000	100.000	ENDING BAL.	150.847	205.258
ENDING BAL.	98.400	100.200			
1961			1966		
No. Purchased	10	7	No. Purchased	10	1
No. Increasing	10	7	No. Increasing	0	0
No. Decreasing	0	0	No. Decreasing	10	1
Aver. Chg. (%)	+17.610	+18.100	Aver. Chg. (%)	-17.420	-23.200
ENDING BAL.	115.718	118.336	ENDING BAL.	124.600	157.638
1962			1967		
No. Purchased	10	1	No. Purchased	10	4
No. Increasing	0	0	No. Increasing	6	4
No. Decreasing	10	1	No. Decreasing	3	0
Aver. Chg. (%)	-9.210	-2.200	Aver. Chg. (%)	+9.440	+24.200
ENDING BAL.	105.072	115.733	ENDING BAL.	136.362	195.786
1963			1968		
No. Purchased	10	3	No. Purchased	10	2
No. Increasing	6	3	No. Increasing	6	2
No. Decreasing	4	0	No. Decreasing	4	0
Aver. Chg. (%)	-0.200	+2.300	Aver. Chg. (%)	+5.560	+7.750
ENDING BAL.	104.862	118.395	ENDING BAL.	143.944	210.959
1964			1969		
No. Purchased	10	3	No. Purchased	10	0
No. Increasing	8	3	No. Increasing	0	0
No. Decreasing	2	0	No. Decreasing	10	0
Aver. Chg. (%)	+9.560	+24.100	Aver. Chg. (%)	-32.500	0.000
ENDING BAL.	114.887	146.928	ENDING BAL.	97.162	210.959

in which the model selected only one of the ten declining convertibles. In other words, it reveals nine of the convertibles to be overvalued and only one favorably priced. An astute investor could have used these data as indicative of convertibles in general being overpriced and would have failed to commit any capital in convertibles in 1966.

The convertible bond model shows a remarkable ability to anticipate overpriced markets, as indicated by its selection of one purchase in 1962, one in 1966, and none in 1969. Further testing of the model is needed, however, before it proves itself effective in determining the direction of the stock market.

The convertible bond model is found to be highly accurate in its selection of individual securities. The following summary data from Table X confirm its accuracy:

	Total Conv. Bonds	Conv. Bond Model
Total No. Purchased	100	34
No. Increasing	49	28
No. Decreasing	50	6
Compound Capital Appre.	Neg.	7.8%

Few people in the investment community would be dissatisfied with a model providing profits on more than 80 per cent of its selections, especially when 50 per cent of the observations in the universe declined.

Appendix C reveals that four of the six declining convertibles selected by the empirical model experienced only marginal declines. For example, in 1960 W. R. Grace lost 2.3%, J. R. McDermott 2.2%, and SCM 1.2%. Royal McBee also had a 2.2% loss in 1962. When these minor losses are considered, the convertible bond models' selection record is marred only by Seiberling Rubber's 12.0% decline in 1960 and by Crescent Corporations's 23.2% decline in 1966. It must, however, be recognized that many of the model's selections experience minor capital appreciations.

The compound rate of return of 7.8% considers only the capital appreciation of these instruments. Coupon rates on the purchased convertibles average approximately 5% and the Treasury bill rate for 1969 would be in excess of 6%. One can estimate the overall benefit from investing in these securities at approximately 12.8% per year.

The calculated rate of return may well be downward biased for two reasons. First, the sample included only securities initially priced close to par. These securities probably provide less upside potential than higher priced convertibles since conversion values tend to be more operative in higher priced convertibles. Second, the ten year period contains bear markets at the beginning and end of the period.

Conclusion

The convertible bond model developed in this chapter can be described as innovative. It yields objective prices for existing convertible bonds and can be tested by astute market students. It is simple, and it removes the mystique surrounding the convertible bond premium since straight debt value is the fulcrum on which convertible bond analysis should be based.

The convertible bond model is limited in that it fails to include an analysis of the common stock. A fair valued combination of straight debt value and conversion value will never substitute for a sound fundamental analysis of a common stock because common stock performance will determine eventual performance of convertible bonds.

The convertible bond model is not a proven device for timing critical market junctures. It must be adequately tested before being used for such purposes. However, there appears to be theoretical support behind speculative activity preceeding bear markets. Investors may well shift to convertible bonds as stocks become inflated, and the convertible bond model may be useful in predicting these turning points.

The convertible bond model is not intended to be a permanent tool in appraising convertible bond prices. It should be up-dated periodically with new bond data. For

example, statistics for 1960 should be replaced with 1970 data when they become available. The number of observations should also be increased in order that more categories may be included. Data for convertibles with short-term expiration dates are especially needed for a more complete set of equations.

CHAPTER VI

SPECIAL TRADING TACTICS FOR WARRANTS AND CONVERTIBLE BONDS

Introduction

This chapter begins with a discussion of terminology used in the literature, followed by a brief description of short selling. Potential trading tactics are discussed and are then applied to the warrant and convertible bond models. Simulated commitments are employed to demonstrate the profitability of the recommended specialized tactics. The chapter is brought to a close with a presentation of the conclusions drawn from the analysis of the data.

Arbitrage Verses Hedging

Chapter II identifies the minimum price of a warrant to be its intrinsic value due to available arbitrage profits at lower prices. Arbitrage, in this instance, involves selling the common stock short and buying warrants exercisable into the number of shares sold short. The warrants are exercised and the acquired common is used to cover the short position in the stock. Arbitrage profits are the difference in the selling price less the buying price (price

of the warrants and exercise costs), minus commissions and other trading expenses.

A strict definition of arbitrage is "the simultaneous purchase and sale of the same or equivalent things to take advantage of a price discrepancy."[1] This definition fits the purchase of warrants and the short sale of the corresponding common, but it fails to uniformly apply to numerous security transactions. For example, the warrant may be sold short and common purchased. The common is then used to protect against declines in the warrant and to rise faster than the warrant in bull markets. In other words, possible profit exists on the upside while the downside is semi-protected with the common stock. The position has elements of both an arbitrage and a hedge, since a hedge is defined as "a sale or purchase of a contract for future delivery against a previous purchase or sale of an equal quantity of the same commodity or an equivalent quantity of another commodity."[2] Hedge positions have generally been associated with commodity trading where a hedger creates a neutral position. However, numerous financial writers refer to transactions in convertible bonds

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[1]Jules I. Bogen, Financial Handbook (4th ed.; New York: The Ronald Press Company, 1968), p. 10.58.

[2]Ibid., p. 23.23.

and warrants as hedge positions.[3]

Arbitraging and hedging can involve a simultaneous purchase and sale of securities, but the former seems to convey a concept of profit whereas the latter creates a neutral position to avoid losses. The trading tactics described in this chapter seem to convey profit opportunities with limited risk exposure. This dual nature calls for new terminology which is designated as arbitrage-hedge. The positions are arbitraged in that profits exist in one direction and hedged to the extent that losses are minimized in the other direction.

Importance of Short Selling

The arbitrage-hedge position requires an investor to be long and short in securities at the same time. The short position is often misunderstood. Therefore, it is discussed prior to presenting complicated trading tactics.

Short selling simply involves a sale of securities where the seller delivers the stock by borrowing it from a third party. The act of borrowing creates short interest in that the seller must later repay his short position by buying stock (hopefully at a lower price than he sold it

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[3]Financial writers often refer to identical transactions as either arbitrages or hedges. See, Wilford J. Eiteman, Charles A. Dice, and David K. Eiteman, The Stock Market (New York: McGraw-Hill, 1966), p. 359; William Schwartz and Julius Spellman, Guide To Convertible Securities (New York: Convertible Securities Press, 1968), p. 55; and Reynolds Griffith, "Arbitrage for the Amateur," North Texas State University Business Studies, (Fall, 1967), pp. 6-7.

for) and returning it to the lender.[4]

A short seller's account executive is responsible for obtaining the stock to be delivered in a short sale. He may have it in inventory, obtain it from customers who frequently lend stock, or obtain it from other brokerage firms.[5] Typical borrowing arrangements call for the proceeds of the short sale to be turned over to the lender. The usefulness of the cash proceeds is ample compensation to the lender, and most loans are made on a "flat" basis with no further compensation passing from either party. Mutual benefits are accorded each. The lender has a deposit of cash and the borrower has the shares to deliver. The short seller must, however, pay dividends paid on the borrowed stock if any are declared.

Short sales are unique in that each is made on margin. The short seller is required to pay current margin requirements when selling short and is required to maintain

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[4]Some short sales are made by individuals who own the security being sold short but elect not to deliver their shares. Such a transaction is referred to as an "against the box" short sale. It is primarily used to lock in a profit to be carried to a different tax year or as a hedging device in a bear market.

[5]George L. Leffler and Loring C. Farwell, The Stock Market (New York: The Ronald Press Company, 1963), p. 223.

a minimum of 30 per cent margin in his account.[6] However, Regulation T of the Federal Reserve Board specifies that the margin on short sales is ignored when securities exchangeable into the securities sold short are in the seller's account. Regulation T enables a small investment in most arbitrage-hedges, since the price of the exchangeable security is often less than the one sold short.

One major problem in executing favorable arbitrage-hedges is that short sales can be executed only after an up-tick. The rule requires that the direction of the last change in price of the security must be up before a short sale can be executed. For this reason, most arbitrage-hedges should be engineered by establishing the short position and then executing the long purchase. Instantaneous purchases and short sales are usually superior when possible.

Various Types of Arbitrage-Hedges

Convertible bonds and warrants offer avenues for profits other than their appreciation potential. Charts 9 and 10 showed the price of the common stock to have significantly higher amounts of change than either warrants or convertible bonds at less than parity. These charts are

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[6]For a discussion of margin requirements and margin maintenance, see, Donald E. Vaughn, Survey of Investments (New York: Holt, Rinehart and Winston, Inc., 1967), pp. 140-142.

based on empirical data and represent fair approximations of reality. If all convertibles and/or warrants performed identically to the models, arbitrage-hedges would be impractical. Attractive arbitrage-hedges exist when relatively small amounts must be paid for the call on the common, or when the call is grossly inflated.

A frequently used investment combination[7] is that of buying a convertible bond and shorting the corresponding common stock. Convertible bonds are sought where conversion price, straight debt value, and the bond's price are all close together. Small premiums over straight debt value and conversion value enable the stock to be sold short. Conversion value hedges against price appreciation in the common; straight debt value enables an arbitrage profit when the common stock declines. In other words, straight debt value provides a floor price for the bond even though the common stock declines sharply.

Tactics where the number of shares on the long convertible are equal to the shares of the common sold short and referred to as full arbitrage-hedges. Profits accrue when the stock declines and the position is hedged at higher common stock prices. A partial hedge, on the other hand,

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[7]Graham, Dodd, and Cottle consider these commitments to properly be classified as investment operations. Op. cit., p. 49.

permits one to earn profits from the transaction regardless of the direction of the common stock. If less than the full number of shares is sold short against the convertible bond (i.e., 100 shares of common are sold short against a bond convertible into 200 shares), conversion value rises faster than the common on the upside and straight debt value minimizes the loss on the downside. Profits are dependent on the common stock fluctuation and the direction is not important.[8]

Warrants permit various tactics aimed at creating limited risk profits through arbitrage-hedge arrangements. A normal tactic is to locate a warrant with a minimum speculative price and trading near parity. The warrant is then purchased and the corresponding common is sold short. The position is hedged on the upside through the exercise feature on the warrant, and the downside produces profits as the common stock declines faster than the warrant.[9]

Warrants containing excessive speculative prices can be sold short against a long position in the common stock. The theory behind such a tactic is that the common will

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[8]If the common stock price remains stable, the bond price will be unaffected, and profits are non-existent.

[9]The common declines faster in absolute dollar amounts. The warrant often declines from $.10 to $.50 while the common declines a full $1.00 per share. The percentage decline is usually higher for the warrant, since it declines from a smaller base. See the discussion of warrant leverage in Chapter II.

appreciate faster than the warrant on the upside--as the warrant premium or speculative value declines. The position is somewhat hedged on the downside as the warrant declines in a fashion similar to the common stock. Such a transaction fails to provide a perfect hedge, since the common may decline much faster than the warrant. This is especially true if a sharp market set-back is experienced.

An alternative approach is to short the inflated warrant and buy a call[10] on the common stock. An increase in the price of the common results in a profit as the call appreciates faster than the warrant. A decline in the price of the common forces the warrant to decline, and a profit is realized on the short sale of the inflated warrant. The only weakness with this tactic is the time limitation and the cost on the call. A profit is realized only if a sizeable price fluctuation occurs in the common during the contract period.

A special tactic, aimed at reaping the benefits from an inflated warrant, is to short a soon-to-expire warrant and hedge the position with a call. Such a transaction is a near "foolproof" arbitrage. The price of the call is

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[10] A call is defined as a contract which gives the holder the right to purchase a specified security within a certain time period at a specified price. For a further discussion of these instruments see, Herbert E. Dougall, Investments (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968), pp. 303-304.

compared with the premium on the warrant to determine if a profitable transaction can be designed. The tactic resembles an arbitrage in that the speculative premium will gradually decline as expiration of the warrant nears. The empirical warrant model reveals a speculative parity premium of .2753 is expected for warrants with an average of nine months to expiration. Such a huge premium can often result in significant profits by selling the warrant short and using a call for upside protection.

Warrants also offer different combinations of shares to be sold short or acquired long in relation to the number of warrants long or short. A logical trading tactic would be to locate warrants selling well below the empirical warrant model's value, buy two warrants, and sell one share of common short. The upside results in profits as the two warrants rise faster than the common; the downside is cushioned by the warrants being fairly priced. This trading tactic is referred to hereafter as a double arbitrage-hedge in warrants.

Warrants found to be significantly inflated according to the empirical model's value could be handled in the reverse fashion. Two inflated warrants could be sold short and one common share purchased long. The downside offers profit potential as the two inflated warrants should depreciate faster than one common share. The common should cushion

the gain of the two inflated warrants on the upside. This tactic is referred to hereafter as a reverse double arbitrage-hedge in warrants.

Use of the Empirical Warrant Model in Locating Arbitrage-Hedges

Empirical model values can be compared to actual warrant prices in locating actual values that are significantly higher or lower than predicted values. For example, model values that are fifty per cent higher than actual prices may be deemed attractive for a double arbitrage-hedge. On the other hand, actual prices fifty per cent above model values may be used in establishing reverse double arbitrage-hedge positions.

The warrants of McCrory Corporation, General Acceptance, and Realty Equity generally fit the requirement of being priced fifty per cent above model values.[11] Price data for the beginning of 1967 indicate both McCrory and General Acceptance have model values more than fifty per cent above their actual prices. A simulated double arbitrage-hedge is depicted in Table XI.

Table XI illustrates the high profit opportunities from double arbitrage-hedge positions when the common stock

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[11]These model value--warrant price relationships may be analyzed in Appendix A.

TABLE XI

SIMULATED DOUBLE ARBITRAGE-HEDGE

	Cost	Selling Price	Profit (Loss)
General Acceptance			
Long:			
200 warrants	$900	$3650	$2750
Short:			
100 shares of common	3500	1962	(1538)
Gross Profit			$1212
McCrory			
Long:			
200 warrants	625	2125	1500
Short:			
100 shares of common	2650	1750	(900)
Gross Profit			$ 600

Price Data	Common Stock	Warrant
General Acceptance:		
12-30-66	$19.62	$ 4.50
12-29-67	35.00	18.25
McCrory:		
12-30-66	17.50	3.12
12-29-67	26.50	10.62

appreciates. The initial investment is not as high as typical long positions, since the warrants serve to offset potential losses from the short position. Margin requirements on the short sale are considered to be covered by the warrants in the account, and only the cost of the warrants is committed. The gross profit is lowered by commissions and dividends paid on the short interest.

Data for 1966 illustrate how the double arbitrage-hedge and the reverse double arbitrage-hedge protect an investor in a bear market. Appendix A shows McCrory and Pacific Petroleum to fit the fifty per cent requirements. McCrory is favorably priced and Pacific Petroleum is overpriced. The decision calls for purchasing McCrory warrants and shorting Pacific Petroleum warrants. Each Pacific Petroleum warrant is exercisable into 1.1 shares at $19 per share. The increased number of shares obtainable with each warrant requires a purchase of 110 shares of common stock. Table XII illustrates the unique elements of these simulated commitments.

Table XII illustrates the importance of favorably priced and overpriced warrants. McCrory warrants declined much less than Pacific Petroleum warrants when the common stock of each company dropped. The weakness of Pacific Petroleum's warrants created a significant gain from the short position while the McCrory warrants held up well in lieu of the common's decline. Dividends on McCrory's common

TABLE XII

SIMULATED DOUBLE AND REVERSE DOUBLE

ARBITRAGE-HEDGE

	Cost	Selling Price	Profit (Loss)
McCrory			
Long:			
200 warrants	$1075	$ 625	$(450)
Short:			
100 shares of common	1750	2275	525
Gross Profit			$75
Pacific Petroleum			
Long:			
110 shares of common	1169	1100	(69)
Short:			
200 warrants	650	1125	475
Gross Profit			$406

Price Data	Common Stock	Warrant
McCrory:		
12-31-65	22.75	5.87
12-30-66	17.50	3.12
Pacific Petroleum:		
12-31-65	10.62	5.62
12-30-66	10.00	3.25

stock and commissions completely eliminate the $75 gain. Superior price combinations can probably be obtained, since the chosen price data are arbitrarily taken from closing prices at the year's end.

The warrant model also assists in locating inflated warrants prior to maturity. Molybdenum warrants are a case in point. These warrants had an expiration date of October 18, 1963. On January 4, 1963 they closed at $10.875 and were exercisable into 1.041 shares of common at $30. On the same date the common stock closed at $25 per share. The calculated model value was $6.34 and this compared with an actual price of $10.875. An arbitrage-hedge was in order. The warrant did not provide an intrinsic value, and the full $10.875 premium would be eliminated by expiration--only nine months and fourteen days away. The reverse double arbitrage-hedge called for 200 warrants to be sold short at $10.875 and 104 shares (each warrant is exercisable into 1.041 shares) of common acquired long at $25 per share. This transaction would have resulted in profits from each side as seen in Table XIII.

The price of the common appreciated to $29.875 on October 11, 1963, and the warrant closed at $1.125 on the same date. The cost of covering the warrants and the selling price of the common is shown in Table XIII. Profits in both directions resulted due to a totally overpriced warrant only

nine months prior to exercise.

TABLE XIII

ILLUSTRATION OF ARBITRAGE-HEDGE ON SOON TO EXPIRE WARRANTS

	Cost	Selling Price	Profit (Loss)
Molybdenum			
Long:			
104 shares of common	$2600	$3107	$ 507
Short:			
200 warrants	225	2175	1950
Gross Profit			$2457

The overpriced warrant could also have provided profits via purchasing a call on the common to protect a short position in the warrant. A nine month call on Molybdenum may have been priced at approximately $600 per 100 shares. The cost of the call would have resulted in a lower overall profit, since the price of the common appreciated during the period. On the other hand, if Molybdenum had closed significantly below its purchase price, a call would have provided larger profits since downside exposure is limited to the price of the call.

The use of a "call" in protecting against a short position in warrants can be illustrated by examining Sperry

Rand's warrants that expired September 15, 1967. These warrants were exercisable into 1.08 shares of common stock at $28. Table XIV illustrates the net profits received by simulating an arbitrage-hedge through the use of a call.

Table XIV illustrates the upside as well as downside protection offered by using call to reap the benefits on an inflated warrant. The profit of this combination appears low, but it is realized in only nine months. The use of a call lowers the initial investment base when compared to a short sale of the common stock. Moreover, the downside risk is protected by the call. A short sale on Sperry Rand's common would have resulted in a significant loss if the common declined to $10 during the nine month period.

Use of the Convertible Bond Model in Locating Arbitrage-Hedge Candidates

An arbitrage-hedge in convertible bonds requires small premiums over straight debt value and conversion value. Candidates for arbitrage-hedges can therefore be chosen by examining straight debt and conversion values in an attempt to locate minimum overall premium bonds. Once these securities have been located, the direction of the common stock can be estimated through a fundamental and technical analysis. A full or partial arbitrage-hedge can be established, based on the estimated direction of the common. Full hedges would

TABLE XIV

SIMULATED ARBITRAGE-HEDGE IN SPERRY RAND WARRANTS

	Common Stock	Warrant
Sperry Rand		
11-7-66 (closing price)	24.62	7.12
8-7-67 (closing price)	39.25	14.25

Actual Data	Cost	Selling Price	Profit (Loss)
Long:*			
Purchase 9-month call on 108 shares	$ 540	1579	1039
Short:			
100 warrants	1425	712	(713)
Gross Profit			326
Less commissions			118
Net Profit			208

Assumed Data (Common Stock $10 per share, 8-7-67)

Long:			
Purchase 9-month call on 108 shares	$ 540	0	(540)
Short:			
100 warrants**	$ 50	712	662
Gross Profit			$ 122
Less Commissions			17
Net Profit			105

Required Investment	
Call price	$ 540
Margin on short sale	509
Total Investment	1049

*The price of a call was determined by a price quotation from John Womack, Account Executive with Merrill Lynch in November of 1966. At that time a nine-month call on Sperry Rand common was $500 per 100 shares. The price was confirmed in a telephone conversation.

**Cost to cover warrant position was assumed to be a nominal $50 per 100 warrants. The warrants were to expire in one month, and the exercise price was well above the price of the common stock.

be established under bearish expectations; partial hedges if uncertainty were forecasted.

The convertible bond model can be used in locating arbitrage-hedge candidates. Predicted values can be compared with actual values in a first screening of the convertible bonds allowing straight debt value and conversion value relationships to be analyzed. A review of Appendix C shows only 3 of the 100 convertibles to simultaneously offer downside protection and upside potential. These are Rohr Aircraft and Scott Paper in 1960, and NAFI Corporation in 1961.

The model values for these convertibles are well above the market price. The premiums over each value are also low when compared with average convertibles on the list. The convertibles are subject to increases in interest rates which lower straight debt values.

Table XV illustrates the gross profits received from a half arbitrage-hedge in these three convertibles. The trading tactic employed is to buy one convertible bond and to sell short one-half the shares into which it is convertible. These positions are assumed to be taken simultaneously and closed out one year later.

The gross profit amounts of $11, $41, and $113 indicate that arbitrage-hedges in convertibles were not too attractive

TABLE XV

ARBITRAGE HEDGE IN SELECTED CONVERTIBLE BONDS

	Cost	Selling Price	Profit (Loss)
Rohr Aircraft (1-4-60 to 1-2-61)			
Long:			
1 convertible bond	$1025	$1090	$65
Short:			
27 shares of common	496	442	(54)
Gross Profit			$11
Scott Paper (1-4-60 to 1-2-61)			
Long:			
1 convertible bond	1025	1111	$86
Short:			
6 shares of common	516	471	(45)
Gross Profit			$41
NAFI (1-2-61 to 1-1-62)			
Long:			
1 convertible bond	1029	1180	$151
Short:			
14 shares of common	472	434	(38)
Gross Profit			$113

in the 1960's.[12] Numerous convertible bonds not on the list were evaluated in order to locate a desirable candidate for the designed trading tactic. All were found to have premiums too high to enable profitable short sales against long positions.

It is interesting to note that convertible bonds in recent years offer poorer opportunities for these special tactics than convertible bonds in the early 1960's. The low straight debt values rule arbitrage-hedge techniques less effective, and straight debt values have declined sharply due to higher interest rates.

Future markets may again enable convertible bonds to become instruments for special trading tactics, but today these tactics are more theoretical than practical. If sound dual positions exist, they are certainly obscure.

Conclusion

Warrants are shown to be highly appropriate for various types of arbitrage-hedge positions. The warrant model is also shown to be capable of locating profitable candidates for simultaneous short and long positions.

The most interesting commitment involves warrants just

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[12]An inclusion of commissions and dividends on the common stock sold short would consume most of these profits.

prior to maturity. These securities can often be sold short to obtain the benefits of the high speculative values. Corresponding shares of common can be purchased to protect the upside, or call options can be utilized. The higher cost of the call option tends to lower profit potential, but it provides protection on each side.

Convertible bonds are found to generally offer arbitrage-hedge positions inferior to those of warrants. Low straight debt values tend to reduce the possibility of desirable arbitrage-hedges, and straight debt values of convertible bonds tend to be less than conversion values in current markets.

The examples used in this chapter were generally taken from year end closing prices to correspond with previous data. Profitable arbitrage-hedges require close scrutiny of prices in order to locate opportune commitments. Daily fluctuations should be watched closely in executing these transactions since ultimate profits are based on initial price combinations with the arbitrage-hedge technique.

CHAPTER VII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

A statement by an alleged financial expert on Johnny Carson's Tonight Show provided the initial impetus for this research work. He conveyed to millions of viewers that convertible bonds offer little downside risk with ample upside potential. One of his recommended convertible bonds subsequently declined more than fifty per cent, and his limited downside risk contention was proven incorrect.

At the same time financial services were touting warrants as offering tremendous upside potential with a minimum investment. The exuberance concerning these instruments appeared exaggerated; therefore, a decision was made to analyze these securities from theoretical and empirical vantage points.

A natural starting point was a review of the literature concerning the investment nature of these securities. Theoretically, warrants were found to have potential trading ranges bordered by the price of the common stock as a

maximum value and intrinsic value as a minimum.[1] Convertible bonds were described as having a floor based on the higher of straight debt or conversion values. Prior researchers failed to fully consider the upper limit of a convertible bond's price, but they seemed to acknowledge that premiums gradually dissipate as the bond's price increases.

Existing literature on stock purchase warrants contains numerous valuation models for these securities. Two mathematical models, one by Giguere and another by Kassouf, offered objective methods in determining a normal or fair price for these instruments. Giguere's model provided conservative values while Kassouf's model offered values almost double those of Giguere, especially when the price of the common was low relative to the exercise price. Shelton presented a modified regression model for valuing warrants. All of these models provided insight into the behavior of warrants and influenced the creation of the warrant model described in Chapter IV.

Prior research on model building for convertible bonds failed to contribute one approach providing objective prices for these securities. Convertible bond models were found to

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[1]See Chapter II, p. 18.

be theoretical exercises requiring subjective judgments.to arrive at a proper price for these instruments. These theoretical models, however, influenced the creation of the empirical convertible bond model.

Time to expiration on warrants was given different emphasis by individual theorists. Samuelson viewed time to expiration as a relevant variable in warrant pricing. Warrants with longer expiration dates were described as having generally higher values. Other researchers that had empirically analyzed warrants generally concluded that expirations in excess of some specific number of years (i.e., Giguere 5 years and Shelton 10 years) should be valued on an identical basis. These inconsistencies in addressing the time horizon led to a special structuring of time to expiration dates. The six time categories also required six different equations for determining attractively priced warrants.

The warrant model's predictive efficiency was tested by applying it to all warrants listed on the American Stock Exchange from 1960 to 1969. Warrants were regarded as favorably priced when model values exceeded warrant prices by more than three per cent. Investment commitments were simulated by comparing the performance of average warrants with the performances of warrants selected for various valuation models.

The warrant model was created by normalizing warrant

and common stock prices in terms of exercise prices for individual instruments. The normalization procedure created two ratios: the common stock ratio (defined as the common stock price relative to the exercise price) and the warrant ratio (defined as the warrant price relative to the exercise price). The common stock ratio became the independent variable and was employed to predict the dependent variable, or the warrant ratio. Warrant ratios were converted to actual prices by multiplying the ratios by exercise prices for individual warrants.

The convertible bond model was developed in a fashion similar to that utilized in the development of the warrant model. Straight debt value was substituted for the exercise price component in the warrant model, and it was utilized in normalizing key elements of convertible bonds. Straight debt value was then divided into conversion value and into the conversion feature value. These manipulations produced the conversion value ratio and the conversion feature ratio, respectively. The conversion value ratio became the independent variable and was used to predict the dependent variable, or the conversion feature ratio. The latter ratio was converted to an actual amount by multiplying it by straight debt value, since straight debt value provided the foundation on which normalization was based. The value of the conversion feature was then added to the straight debt value

of each bond to predict an approximate price to pay for each convertible bond.

Four categories were created for these securities. Three of these were based on different time horizons; the fourth division was based on different straight debt values. These categories required four separate equations.

The convertible bond model was tested by simulating purchases on one-hundred investment opportunities. Ten observations were chosen for each year from 1960 through 1969. The model recommended thirty-four purchases. The three per cent decision rule used for the warrant model was again invoked for the convertible bond model.

The versatility of the empirically determined models was tested in a special chapter (Chapter VI) on trading tactics. The warrant model was found to profitably select underpriced and overpriced instruments that qualified for special trading tactics. Overpriced warrants were sold short against one-half as many common shares purcahsed long. Conversely, underpriced warrants were purchased against one-half as many shares of common sold short.

Warrants containing sizeable speculative values prior to expiration were also found to offer exceptional profit opportunities; they were sold short against a long position in the common. An alternative arbitrage-hedge using a "call" to protect the short position in the warrant

was also illustrated.

Simulated arbitrage-hedges in convertible bonds were illustrated. The convertible bond model was used to locate candidates for these special tactics, but the positions proved generally unprofitable.

Conclusions

Prior researchers have failed to properly evaluate the influence of time to expiration on warrants and convertible bonds. The equations for the warrant model revealed that time positively affected warrant prices when exercise prices exceeded prices of the corresponding common stock. However, time became less important as common stock ratios increased. Students of warrant behavior would uncover such a finding only by structuring time in relation to the common stock ratio. The fact that researchers have arrived at different expiration date cut-offs has likely arisen due to stock market levels in existence during their study. Studies conducted during bear markets (low common stock ratios) would probably find time to be a significant variable whereas studies conducted during bull markets would lead to opposite conclusions.

It is felt that equations for the warrant and convertible bond models provide new insight as to how expiration dates influence warrant prices. Expiration dates critically

affect warrant prices when common stock prices are low in relation to exercise prices. As these common stock ratios increase, time to expiration becomes less and less significant.

The use of the equations in the determination of the normal values for warrants provided a close approximation of reality. Predicted graphical points closely approximated observed points, and coefficients of determination were above .80 for all categories. The structure and testing of the warrant model led to these basic conclusions:

(1) Model values approximated average prices of warrants with similar time horizons, but these normal prices did not provide a sound basis for predicting price performance. This conclusion is based on the inferior performance of warrants selected by the model versus the performance of average warrants.

(2) Warrants selected by the empirical model tended to have greater leverage than average warrants. The model's relevant range of a common stock ratio (.5: 1.2) restricted its selections to more volatile warrants.

(3) Some type of timing device should be found to minimize the downside leverage of warrants during bear markets. The warrant model was shown to select fewer warrants prior to bear markets. In subsequent

research it may be used to indicate generally inflated markets for warrants.

(4) The warrant model was found to locate warrants suitable for arbitrage-hedge tactics. Warrants deemed overpriced by the warrant model appeared well suited as candidates for short sales. Long positions were recommended for those obviously underpriced.

(5) The warrant model showed only modest premium increases for warrants with expirations in excess of five years. Prospective issues of warrants may utilize these data in arriving at optimum expiration dates for these instruments.

The convertible bond model provided an innovative approach to valuing these securities. Heretofore, objective prices had not been provided by convertible bond models. Therefore, empirical testing of these models was impractical. Conclusions pertaining to the structure and testing of the empirical convertible bond model are:

(1) Straight debt value is the proper foundation for appraising the worth of convertible bonds. It is similar to the exercise price on warrants since it represents the value given up at conversion. Moreover, it provides a consistent approach to valuing the premium on convertible bonds.

(2) The convertible bond model was shown to efficiently select investment commitments. It selected individual bonds poised for appreciation and also seemed to be useful as a device in timing overall market purchases and sales. Use of the convertible bond model would likely produce increased returns to investors in these securities.

(3) The equations for the convertible bond model revealed time to expiration of the call privilege to have a positive impact on the value of the conversion feature. Companies planning to issue convertible bonds should therefore lengthen the expiration of the call privilege to twenty or more years, since a higher premium can be expected for the longer expiration.

(4) Convertible bonds are not suited for arbitrage-hedge techniques in today's markets. The convertible bond model revealed excessive premiums for these securities.

Equations for the warrant and convertible bond model revealed warrants to command premiums higher than those for convertible bonds. The warrant model estimated warrants in Category 4 (4 to 7 years from expiration) to carry premiums of 40.62 per cent at parity. The convertible bond model estimated a parity premium of only 26.79 per cent for

convertible bonds in Category B (10 to 20 years from expiration). Warrants in Category 1 (6 months to 1 year prior to expiration) were estimated to have higher parity premiums than those for convertible bonds with ten to twenty year conversion expirations.

These two models appear well suited for analyzing the proper benefit of these security forms when new issues of bonds are being considered. For example, management may plan to issue a bond worth $800 on its straight debt value alone and be undecided as to whether to include a conversion feature or warrants with the bonds. If each bond is to provide a claim on eight shares of common stock with a market price of $100 per share, the total price of the bond may be estimated by applying the two models. Assuming a fifteen year expiration on each privilege, the convertible bond would have an estimated market price of $1,014 [(.2679) (800) + 800].[2] The bond containing warrants to purchase eight shares of common for $800 provides an estimated price of $1,141 [(.4263)(800) + (800)].[3]

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[2]See Chapter V, p. 147.

[3]See Chapter IV, p. 122.

The models enable management to evaluate the trade-off between a higher price for the warrants versus the flexibility of the call provision with the convertible bonds. They also provide a technique with which management can establish optimum terms of new issues that will lower overall capital costs and assist in maximizing the value of the firm.

Recommendations

Stock purchase warrants and convertible bonds remain a fruitful area for additional research. The present study will be construed to have rendered a meaningful service if new and better techniques of analysis are developed by others who review these pages.

The models designed in this work are not intended to be everlasting. Constant updating and revision are inherent in these equations. It is also recommended that the time categories be enlarged by including more varieties of observations. Additional tests should also be administered for each model.

The usefulness of these models in explaining the dynamics of new issues is untried. It is therefore recommended that equations be designed for newly issued securities in order that management can properly quantify meaningful components of new issues.

Inferences concerning the models' ability to time

security market turning points are based on crude decision rules. These models should be tested at disparate market junctures to measure their predictive efficiency. Perhaps both models should be compared in selecting turning points for common stock in general. It is conceivable that the convertible bond model could be used in signaling purchase opportunities in common stocks or other securities.

It is finally recommended that the newly created warrant and convertible bond models should be viewed as "guides to thinking"--not as automatic tools enabling instant profits in these securities. It should be remembered that a successful long position in warrants and convertible bonds usually requires the common stock to rise. Therefore, common stocks should always be analyzed before committing money to these securities.

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APPENDIX A

LIST OF OBSERVATIONS USED IN TESTING THE WARRANT MODELS

1960 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	9.50	7.12	-25.1	8.75	9.30	9.20	( )*
Armour	19.50	22.75	+16.7	16.03	B21.29	B21.15	( )
Atlas	2.87	1.37	-52.3	1.56	2.59	B 3.41	B3.44
Mack Trucks	33.62	15.75	-53.2	24.68	B37.97	B37.89	( )
Martin Co.	23.75	31.50	+32.6	10.38	20.52	14.36	19.39
Molybdenum	37.25	23.62	-36.6	22.80	32.13	32.97	( )
Pacific Pet.	9.62	6.00	-37.6	1.97	3.60	6.29	4.79
Sperry Rand	11.25	8.62	-23.4	6.16	11.26	8.62	11.06
Symington Wayne	6.00	5.25	-12.5	2.75	5.33	3.61	4.71
Tri-Continental	27.25	25.37	- 6.9	27.30	B32.14	B32.77	( )
Van Norman	5.50	5.00	- 9.1	1.24	4.05	5.39	( )
Net Total			-207.40	None	-43.40	-95.70	-52.30
Average Percent Change			- 18.85	+4.10	-14.46	-23.92	-52.30
Initial Value			100.000	100.000	100.000	100.000	100.000
Ending Value			81.150	104.100	85.540	76.080	47.700

*Value is not determined as the common stock ratio is outside the model's relevant range.

BIndicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1961 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	7.12	7.87	+10.5	6.75	B 7.40	B 7.57	( )
Armour	22.75	32.00	+40.7	20.64	B25.25	B24.91	( )
Atlas	1.37	1.12	-18.2	.39	.74	B 1.70	( )
General Acceptance	4.75	10.50	+121.1	4.16	B 7.07	B 5.96	B 7.64
Guerdon	.81	.94	+15.4	neg.	B 1.28	B 1.99	( )
Mack Trucks	15.75	24.00	+52.4	10.67	B19.50	15.15	B18.31
Martin Co.	31.50	38.50	+22.2	25.09	B38.20	B36.50	( )
Molybdenum	23.62	16.25	-31.2	10.00	18.21	17.44	( )
Pacific Pet.	6.00	9.37	+56.2	1.28	2.41	4.79	3.70
Rio Algom	.37	.50	+33.3	neg.	.16	.30	( )
Sperry Rand	8.62	13.37	+55.1	4.51	8.62	B10.10	8.54
Symington Wayne	5.25	8.12	+54.8	3.36	B 6.10	5.04	( )
Textron	1.75	2.87	+64.3	.96	B 1.99	1.48	B2.15
Tri-Continental	25.37	44.00	+73.4	25.08	B30.15	B30.76	( )
Net Total			+550.00	none	+454.80	+320.20	+237.80
Average Percent Change			+ 39.29	+2.20	+ 50.53	+ 40.03	+ 79.27
Cumulative Balance			81.150	104.100	85.540	76.080	47.700
Ending Value			113.034	106.390	128.763	106.535	85.512

B Indicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1962 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	7.87	7.75	- 1.6	7.25	7.87	7.98	( )
Armour	32.00	27.25	-14.8	32.00	B35.71	B35.38	( )
Atlas	1.12	.94	-16.7	.28	.53	B 1.43	( )
First National Realty	1.87	1.25	-33.3	.84	1.50	2.01	1.77
General Acceptance	10.50	7.00	-33.3	8.94	B13.40	B13.59	( )
Mack Trucks	24.00	19.87	-17.2	18.03	31.01	28.86	( )
Martin Mar.	38.50	29.62	-23.1	31.14	B43.57	B42.55	( )
McCrory	10.50	4.87	-53.6	5.98	9.64	9.90	9.49
Molybdenum	16.25	10.87	-33.1	5.00	11.53	10.20	9.92
Pacific Pet.	9.37	6.00	-36.0	2.77	4.90	7.03	5.42
Rio Algon	.50	.37	-25.0	neg.	.24	.39	( )
Sperry Rand	13.37	7.75	-42.1	5.72	10.58	11.43	9.96
Teleregister	8.12	2.75	-66.2	3.40	6.05	6.07	4.29
Textron	2.87	3.62	+26.1	1.46	2.86	B 2.27	2.76
TWA	5.00	4.37	-12.5	2.64	4.86	B 6.44	4.54
Tri-Continental	44.00	35.75	-18.7	42.53	B46.33	B46.22	( )
Net Total			-401.10	none	-107.10	-169.60	none
Average Percent Change			- 25.07	+2.70	- 21.42	- 21.20	+2.70
Cumulative Balance			113.034	106.390	128.763	106.535	85.512
Ending Value			84.696	109.263	101.182	83.950	87.821

[B]Indicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1963 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	7.75	7.25	- 6.5	7.87	B 8.46	B 8.45	( )
Armour	27.25	25.00	- 8.3	23.75	B 28.10	B 27.31	( )
Atlas	.94	1.37	+46.7	.20	.39	B 1.23	( )
First National Realty	1.25	1.00	-20.0	.37	.78	1.24	.88
General Acceptance	7.00	4.50	-35.7	5.32	B 8.73	B 7.28	B 8.46
Jeff. Lake Pet.	2.25	3.12	+38.9	.55	1.42	1.94	1.56
Mack Trucks	19.87	17.87	-10.1	13.22	B 24.21	B 20.52	B 20.98
Martin Mar.	29.62	21.00	-29.1	20.28	B 30.33	B 30.64	B ( )
McCrory	4.87	2.37	-51.3	3.61	B 6.25	B 6.37	B 7.02
Pacific Pet.	6.00	6.12	+ 2.1	1.63	3.02	5.14	3.83
Rio Algom	.37	.25	-33.3	neg.	.32	B .47	( )
Realty Equity	1.75	1.37	-21.4	B 2.02	B 3.90	B 3.64	B 3.38
Sperry Rand	7.75	9.37	+21.0	.30	3.89	5.93	5.13
Teleregister	2.75	3.50	+27.3	.09	1.35	1.87	( )
Textron	3.62	4.12	+13.8	2.09	B 3.80	3.35	( )
TWA	4.37	16.37	+274.3	2.04	3.95	B 5.87	4.26
Tri-Continental	35.75	36.87	+ 3.1	33.97	B 38.30	B 38.72	( )
Univ. Amer. 55	2.50	1.12	-55.0	neg.	1.89	B 3.05	( )
Univ. Amer. 62	2.75	1.50	-45.5	.34	2.22	B 3.58	2.80
Net Total			+111.00	-21.40	-116.40	+36.20	-118.50
Average Percent Change			+ 5.84	-21.40	- 14.55	+ 3.02	- 29.62
Cumulative Balance			84.696	109.263	101.182	83.950	87.821
Ending Value			89.642	85.881	86.460	86.485	61.804

[B]Indicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1964 DATA

Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	7.25	7.25	0.0	6.62	7.28	7.36	( )
Atlas	1.37	1.00	-27.3	.42	.79	B 1.77	B 1.56
First National Realty	1.00	.62	-37.5	.43	.90	B 1.39	B 1.35
General Acceptance	4.50	5.50	+22.2	4.45	B 7.50	B 5.51	B 7.53
Jeff. Lake Pet.	3.12	3.50	+12.0	1.37	2.99	2.03	2.55
Mack Trucks	17.87	17.25	- 3.5	13.75	B26.20	B22.25	B21.97
Martin Mar.	21.00	18.00	-14.3	13.76	B25.76	B22.30	( )
McCrory	2.37	3.75	+57.9	1.41	B 2.65	B 2.90	B 4.00
Pacific Pet.	6.12	6.25	+ 2.0	1.63	3.02	4.87	3.83
Realty Equity	1.37	1.37	0.0	B 1.50	B 3.21	B 2.57	B 2.74
Rio Algom	.25	.31	+25.0	.05	B .36	B .48	B .42
Sperry Rand	9.37	6.25	-33.3	2.43	7.70	9.42	7.41
Textron	4.12	7.25	+75.3	3.42	B 5.10	B 5.25	( )
TWA	16.37	29.37	+79.4	12.10	B18.27	B20.72	( )
Tri-Continental	36.87	39.62	+ 7.5	31.19	B40.37	B40.56	( )
Univ. Amer. 55	1.12	.37	-66.7	neg.	B 1.23	B 2.11	( )
Univ. Amer. 62	1.50	1.75	+16.7	neg.	1.46	B 2.68	( )
Uris Bldg.	8.75	7.37	-15.7	7.19	B10.21	B10.23	( )
Net Total			+100.20	0.00	+167.60	+119.50	+36.80
Average Percent Change			+ 5.57	0.00	+ 15.24	+ 8.54	+ 5.26
Cumulative Balance			89.642	85.881	86.460	86.485	61.804
Ending Value			94.635	85.881	99.637	93.871	65.055

BIndicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1965 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	7.25	10.12	+39.7	6.25	6.93	7.04	( )
Atlas	1.00	1.37	+37.5	.23	.44	B 1.30	( )
First National Realty	.62	.44	-30.0	.18	B .65	B 1.35	( )
General Acceptance	5.50	5.50	0.0	4.39	B 9.18	B 7.09	B 8.80
Jeff. Lake Pet.	3.50	15.37	+339.3	2.07	B 4.09	B 3.97	3.47
Mack Trucks	17.25	23.50	+36.2	10.29	B22.00	B19.02	B 18.78
Martin Mar.	18.00	28.25	+56.9	10.59	B21.64	15.95	B 18.90
McCrory	3.37	5.87	+74.1	2.32	B 4.20	3.08	B 5.38
National General	4.75	4.12	-13.2	2.16	3.83	B 5.53	4.59
Pacific Pet.	6.25	5.62	-10.0	.65	3.38	4.87	3.57
Realty Equity	1.37	6.00	+336.4	1.21	B 2.96	B 2.23	B 2.56
Rio Algom	.31	.50	+60.0	neg.	.30	B .37	( )
Sperry Rand	6.25	10.25	+64.0	.25	3.76	5.90	4.24
Textron	7.25	13.75	+89.7	5.49	B 7.83	B 8.11	( )
TWA	29.37	44.25	+50.6	25.75	B30.57	B32.86	( )
Tri-Continental	39.62	45.25	+14.2	39.36	B43.34	B43.25	( )
Unit. Ind. Corp.	1.12	4.37	+288.9	.06	1.12	B 2.05	B 1.50
Univ. Amer. 62	1.75	4.00	+128.6	neg.	1.57	B 2.54	( )
Uris Bldg.	7.37	8.00	+ 8.5	5.85	B 8.69	B 9.11	( )
Net Total			+1571.40	None	+975.90	+1346.70	+792.50
Average Percent Change			+ 82.71	+3.80	+ 88.72	+ 96.19	+132.08
Cumulative Balance			94.635	85.881	99.637	93.871	65.055
Ending Value			172.908	89.144	188.035	184.166	150.980

BIndicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1966 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	10.12	5.37	-46.9	9.75	10.26	10.00	( )
Atlas	1.37	1.62	+18.2	.30	.58	B 1.91	( )
First National Realty	.44	.31	-28.6	.08	.21	B .70	( )
General Acceptance	5.50	4.50	-18.2	4.00	B 8.64	B 5.71	B 7.67
Indian Head	9.37	8.00	-14.7	4.20	8.64	8.71	8.39
Jeff. Lake Pet.	15.37	16.75	+ 8.9	14.40	15.45	B16.40	( )
Martin Mar.	28.25	25.50	- 9.7	17.00	B29.78	25.83	( )
McCrory	5.87	3.12	-46.8	B 6.47	B10.29	B 9.51	B 9.95
National General	4.12	4.25	+ 3.0	1.67	3.02	B 5.15	3.94
Pacific Pet.	5.62	3.25	-42.3	.61	3.31	4.39	3.51
Realty Equity	6.00	3.37	-43.7	4.62	B 7.35	B 9.62	( )
Sperry Rand	10.25	9.62	- 6.1	3.29	8.72	8.33	7.62
Textron, Inc.	13.75	18.00	+30.9	13.62	B14.92	B14.85	( )
TWA	44.25	52.50	+18.0	41.50	45.20	B45.70	( )
Tri-Continental	45.25	35.25	-22.1	44.44	B48.14	B47.70	( )
Unit. Ind. Corp.	4.37	3.00	-31.4	1.63	3.57	3.71	3.18
Univ. Amer. 62	4.00	1.25	-68.7	1.29	3.78	3.53	3.34
Uris Bldg.	8.00	6.00	-25.0	6.68	B10.10	B10.65	( )
Net Total			-325.20	-46.80	-134.60	-105.40	-65.00
Average Percent Change			- 18.07	-46.80	- 19.23	- 9.58	-32.50
Cumulative Balance			172.908	89.144	188.035	184.166	150.989
Ending Value			141.664	47.425	151.876	166.523	101.912

BIndicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1967 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany	5.37	10.12	+88.4	4.12	4.97	5.20	( )
Atlas	1.62	3.87	+138.5	.28	.53	1.43	( )
First National Realty	.31	.87	+178.6	.03	.05	B .47	( )
General Acceptance	4.50	18.25	+305.6	3.56	B 8.02	B 4.64	B 7.21
Indian Head	8.00	26.50	+231.3	4.34	7.33	7.11	7.49
Jeff. Lake Pet.	16.75	48.00	+186.6	13.68	15.93	16.00	( )
Martin Mar.	25.50	21.50	-15.7	18.48	B28.16	22.65	( )
McCrory	3.12	10.62	+240.0	B 3.83	B 6.58	B 5.20	B 7.27
National General	4.25	13.87	+226.5	2.35	4.13	B 5.33	B 4.83
Pacific Pet.	3.25	3.50	+ 7.7	.40	2.95	B 3.57	2.79
Realty Equity	3.37	6.12	+81.5	.29	B 3.91	3.11	( )
Textron	18.00	44.25	+147.2	18.37	B19.44	B18.93	( )
TWA	52.50	36.00	-31.4	51.62	B54.84	B53.79	( )
Tri-Continental	35.25	57.62	+63.5	35.87	B40.07	B40.13	( )
United Ind. Corp.	3.00	4.00	+33.3	.94	2.55	2.13	2.41
Uris Bldg.	6.00	23.00	+283.3	5.94	B 8.79	B 9.02	( )
Net Total			+2163.90	+240.00	+1074.00	+1421.00	+772.10
Average Percent Change			135.24	+240.00	+ 134.25	+ 157.89	+257.37
Cumulative Balance			141.664	47.425	151.876	166.523	101.912
Ending Value			333.250	161.245	355.770	429.446	364.203

BIndicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1968 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany Air	9.87	16.12	+63.3	4.04	6.47	7.00	6.09
Alleghany Corp.	10.12	21.62	+113.6	10.00	B10.50	10.13	( )
Atlas	3.87	5.25	+35.5	1.50	2.50	3.34	3.36
Braniff Air	20.37	27.00	+32.5	7.40	13.55	B22.00	18.14
First National Realty	.87	10.62	+1114.3	neg.	.26	.87	( )
Frontier Air	9.87	7.87	-20.3	6.89	10.09	B12.19	( )
General Acceptance	18.25	43.37	+137.7	14.06	B20.31	B18.91	( )
Indian Head	26.50	24.75	- 6.6	20.02	25.84	B27.45	( )
Lerner Stores	7.75	16.62	+114.5	5.40	B 8.43	B 9.51	B 8.05
McCrory	10.37	16.00	+54.2	8.78	B13.20	B12.97	( )
McCrory (new)	10.62	15.62	+47.1	8.78	B13.20	B13.30	( )
National General	13.87	30.50	+119.8	10.11	13.83	B15.06	( )
Realty Equity	6.12	30.50	+398.0	3.56	6.25	5.96	( )
Textron	44.25	36.50	-18.0	44.37	44.91	44.37	( )
TWA	36.00	30.75	-14.6	28.62	33.20	33.80	( )
Tri-Continental	57.62	72.75	+26.2	57.78	B60.88	59.23	( )
United Ind.	4.00	3.87	- 3.1	1.85	3.88	3.23	3.31
Uris Bldg.	23.00	46.00	+100.0	23.04	B25.18	B25.18	( )
Net Total			+2294.10	none	+593.30	+578.90	+114.50
Average Percent Change			127.45	4.90	84.76	64.32	+114.50
Cumulative Balance			333.25	161.245	355.770	429.446	364.203
Ending Value			757.977	169.146	657.321	705.666	781.215

B Indicates a buy signal. These signals are based on the model value being more than 3 per cent higher than the initial price of the warrant.

1969 DATA

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Alleghany Air	16.12	7.87	-51.2	9.95	13.20	13.77	( )
Alleghany Corp.	21.62	10.25	-52.6	21.62	21.90	21.62	( )
Atlas	5.25	2.62	-50.0	2.48	3.80	4.74	( )
Braniff Air	27.00	10.12	-62.5	15.09	25.67	B30.77	27.77
First National Realty	10.62	3.25	-69.4	8.44	9.78	9.95	( )
Frontier Air	7.87	4.00	-49.2	2.92	4.87	6.48	5.05
Gulf & Western	17.87	6.37	-64.3	11.08	B18.91	B25.83	B 20.61
Indian Head	24.75	11.00	-55.6	20.54	B26.29	B26.50	( )
Leasco Data	26.37	14.87	-43.6	18.05	26.22	B31.24	( )
Lerner Stores	16.62	17.62	+ 6.0	13.07	16.76	B18.26	( )
LTV	40.50	7.75	-80.1	24.55	B41.58	B51.29	B 44.60
Loews Theaters	34.12	16.50	-51.6	20.25	28.72	32.97	( )
LTV Aero	24.37	7.25	-70.3	14.75	21.43	23.91	( )
LTV Ling	7.75	2.50	-67.7	3.15	5.43	7.13	6.05
McCrory	16.00	7.12	-55.5	15.75	B20.75	B21.28	( )
McCrory (new)	15.62	7.62	-51.2	15.75	B20.75	B21.77	( )
National General	30.50	12.12	-60.2	28.37	30.89	30.88	( )
Nat'l Gen. (new)	18.37	6.37	-65.3	11.76	B19.00	B23.80	18.79
Nat'l Industrial	14.50	3.75	-72.5	6.73	10.76	13.75	10.46
Okonite	13.62	3.62	-73.4	7.90	12.85	13.22	12.84
Pac. Southwest Air	16.37	15.00	- 8.4	8.30	12.99	15.47	12.30
Realty Equity	30.50	9.62	-68.4	29.86	31.09	29.87	( )
Textron	36.50	17.50	-52.1	35.37	36.23	35.37	( )
TWA	30.75	13.87	-54.9	21.75	26.97	27.47	( )
Tri-Continental	72.75	68.75	- 5.5	73.53	B76.17	73.53	( )
Uris Bldg.	46.00	29.00	-37.0	46.37	B47.69	46.37	( )
Wilson Co.	13.50	5.12	-62.0	10.15	B16.31	B16.52	B15.98

1969 DATA (CONTINUED)

Listed Warrants	Initial Price	Closing Price	Percent Change	Giguere	Kassouf	Shelton	Empirical Model
Wilson Sporting	11.12	4.12	-62.9	5.64	9.19	9.90	9.21
Net Total			-1491.40	none	-476.50	-534.10	-206.40
Average Percent Change			- 53.26	+6.30	52.94	- 53.41	- 68.80
Cumulative Balance			757.977	169.146	657.321	705.666	781.215
Ending Value			362.863	179.802	309.335	328.770	243.739
Ann. Compound Ret. on Investment			13.8%	6.0%	12.0%	13.0%	9.4%

APPENDIX B

LIST OF OBSERVATIONS USED IN CONSTRUCTING THE CONVERTIBLE BOND MODEL

December 31, 1959

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	54.50	Allegheny Ludlum Steel 4s81	1087.50	990.91	855.00
A	40.37	Atlantic Refining 4 1/2s87	1045.00	761.79	970.00
Baa	33.87	Burroughs Corp. 4 1/2s81	1145.00	869.48	920.00
Baa	41.37	Champion Paper 4 1/2s84	1108.75	897.90	914.00
Baa	28.87	Dresser Industries 4 1/8s77	915.00	525.00	886.00
Baa	65.25	Fansteel Metalurigical 4 3/4s76	1500.00	1468.60	959.00
Baa	50.37	Gardner-Denver 4 1/4s76	1310.00	1356.90	904.00
Baa	63.50	Gen. American Transport 4s81	1685.00	1693.33	855.00
Baa	39.75	Hooker Chemical 5s84	1190.00	883.33	983.00
Baa	30.25	International Min. & Chem. 3.65s77	870.00	560.19	831.00
Baa	42.50	I-T-E Circuit Breaker 4 1/4s82	1150.00	1083.63	885.00
Baa	11.00	Libby McNeil & Libby 5s76	1025.00	743.24	987.00
Baa	40.00	Macy (RH) & Co. 5s77	1250.00	1250.00	986.00
A	46.50	Phillips Petroleum 4 1/4s87	1100.00	930.00	932.00
Baa	66.37	Radio Corp. of Amer. 3 1/2s80	1400.00	1353.49	796.00
Baa	77.37	Richfield Oil Corp. 4 3/8s83	1215.00	1105.36	899.00
Baa	18.50	Royal McBee 6 1/4s77	1155.00	711.54	1045.00
A	78.50	Scott Paper 3s71	1025.00	1019.48	851.00
A	53.25	Sinclair Oil Corp. 4 3/8s86	1022.50	819.23	952.00
Aa	43.87	Standard Oil (Indiana) 3 1/8s82	1070.00	953.80	801.00
			Baa	A	Aa
Estimated December 31, 1959 Bond Yields			5.12	4.69	4.54

December 30, 1960

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	34.50	Allegheny Ludlum Steel 4s81	975.00	627.27	860.00
Baa	60.00	American Optical Co. 4.40s80	1170.00	976.74	913.00
A	43.00	Atlantic Refining 4 1/2s87	1110.00	811.32	994.00
Baa	27.37	Burroughs 4 1/2s81	1097.50	702.64	929.00
Baa	27.00	Champion Paper & Fibre 4 1/2s84	1060.00	585.93	917.00
Baa	24.25	Combustion Engineering Inc. 3 3/8s81	947.50	808.33	781.00
Baa	22.87	Dresser Industries, Inc. 4 1/8s77	930.00	415.91	892.00
Baa	47.62	Fansteel Metallurgical 4 3/4s76	1220.00	1071.91	962.00
Baa	48.75	Gardner-Denver 4 1/4s76	1293.75	1313.13	909.00
Baa	79.75	Gen. American Transport 4s81	2040.00	2126.67	860.00
Baa	34.62	Hooker Chemical Corp. 5s84	1145.00	769.44	985.00
Baa	34.75	International Min. & Chem 3.65s77	925.00	643.52	839.00
Baa	21.62	I-T-E Circuit Breaker 4 1/4s82	950.00	551.38	889.00
Baa	10.50	Libby McNeil & Libby 5s76	1070.00	709.46	988.00
Baa	47.00	Macy (RH) & Co. 5s77	1440.00	1468.75	988.00
A	53.87	Phillips Petroleum 4 1/4s87	1155.00	1077.50	955.00
Baa	88.25	Richfield Oil Corp. 4 3/8s83	1285.00	1260.71	903.00
Baa	13.62	Royal McBee 6 1/4s77	1090.00	524.04	1045.00
A	86.00	Scott Paper 3s71	1111.25	1116.88	872.00
A	38.75	Sinclair Oil 4 3/8s86	1002.50	596.15	975.00
Aa	47.12	Standard Oil (Indiana) 3 1/8s82	1085.00	1024.46	833.00
			Baa	A	Aa
Estimated December 30, 1960 Bond Yields			5.11	4.54	4.33

December 29, 1961

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	75.00	American Optical 4.40s80	1338.75	1209.68	917.00
A	54.50	Atlantic Refining 4 1/2s87	1160.00	1028.30	995.00
Baa	42.50	Burroughs Corp. 4 1/2s81	1300.00	1090.86	927.00
Baa	37.37	Champion Paper 4 1/2s84	1150.00	811.09	920.00
Baa	28.50	Combustion Engineering 3 3/8s81	1131.25	950.00	789.00
Baa	50.75	Continental Baking 4 3/8s83	1080.00	845.83	906.00
Baa	28.50	Dresser Industries Inc. 4s77	988.75	518.18	874.00
Baa	27.75	Fansteel Metallurgical 4 3/4s76	1235.00	964.88	965.00
A	24.12	Flintkote 4 1/2s80	1040.00	536.11	996.00
Baa	164.12	Xerox 4 1/2s81	1690.00	1563.10	927.00
Baa	39.75	Hooker Chemical 5s84	1270.00	883.33	987.00
Baa	18.75	I-T-E Circuit Breaker 4 1/4s82	950.00	478.07	893.00
Baa	13.62	Libby McNeil & Libby 5s76	1160.00	920.61	990.00
Baa	72.50	Macy (RH) & Co. 5s77	2270.00	2265.63	989.00
A	58.62	Phillips Petroleum 4 1/4s87	1230.00	1172.50	958.00
Baa	44.50	Richfield Oil Corp. 4 3/8s83	1310.00	1271.43	906.00
Baa	12.75	Royal McBee 6 1/4s77	1053.75	490.38	1045.00
A	42.62	Scott Paper 3s71	1620.00	1660.50	883.00
A	37.62	Sinclair 4 3/8s86	1025.00	537.50	977.00
Aa	55.00	Standard Oil (Indiana) 3 1/8s83	1200.00	1195.65	829.00
			Baa	A	Aa
Estimated December 29, 1961 Bond Yields			5.10	4.53	4.40

December 28, 1962

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	54.62	Air Reduction 3 7/8s87	1070.00	874.00	859.00
Baa	33.25	Allegheny Ludlum Steel 4s81	1000.00	604.55	891.00
Baa	61.25	American Optical 4.40s80	1220.00	987.90	942.00
A	48.25	Atlantic Refining 4 1/2s87	1121.25	831.90	1022.00
Baa	24.50	Baxter Laboratories, Inc. 4s82	1085.00	644.74	888.00
Baa	28.37	Burroughs Corp. 4 1/2s81	1092.50	728.31	952.00
Baa	25.62	Champion Paper 4 1/2s84	1085.00	556.10	947.00
Baa	25.00	Combustion Engineering 3 3/8s81	1015.00	833.33	816.00
A	56.75	Dow Chemical 3s82	1250.00	1213.90	824.00
Baa	22.50	Dresser Industries 4s77	972.50	409.09	907.00
Baa	17.75	Fansteel Metallurgical 4 3/4s76	1062.50	617.18	985.00
A	17.75	Flintkote 4 1/2s80	1032.50	394.44	1018,00
Baa	157.25	Xerox 4 1/2s81	1650.00	1487.62	952.00
Baa	35.75	Hooker Chemical 5s84	1175.00	794.44	1013.00
Baa	13.75	I-T-E Circuit Breaker 4 1/2s82	950.00	350.99	919.00
Baa	12.62	Libby McNeil & Libby 5s76	1132.50	853.04	1010.00
Baa	23.75	Mac Andrews & Forbes 5s87	1030.00	608.97	1014.00
Baa	39.25	Richfield Oil 4 3/8s83	1230.00	1121.43	932.00
Baa	9.00	Royal McBee 6 1/4s77	1020.00	346.15	1020.00
A	32.00	Scott Paper 3s71	1272.50	1246.59	905.00
Baa	36.12	Sinclair 4 3/8s86	1007.50	516.07	927.00
Baa	63.00	Union Oil Cal. 4 1/4s91	1177.50	988.55	900.00
			Baa	A	Aa
Estimated December 28, 1962 Bond Yields			4.90	4.35	4.25

December 27, 1963

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	55.37	Air Reduction 3 7/8s87	1087.50	886.00	858.00
Baa	37.37	Allegheny Ludlum Steel 4s81	961.25	679.55	895.00
Baa	68.50	American Optical 4.40s80	1250.00	1104.84	944.00
Baa	29.50	Ashland Oil & Ref. 3 7/8s93	1085.00	951.61	841.00
A	56.75	Atlantic Refining 4 1/2s87	1125.00	978.45	1022.00
Baa	26.50	Baxter Lab. 4s82	1125.00	697.37	891.00
Baa	23.62	Burroughs 4 1/2s81	1040.00	606.39	953.00
Baa	32.37	Champion Paper 4 1/2s84	1087.50	702.58	949.00
Baa	49.25	Continental Baking 4 3/8s83	1035.00	820.83	934.00
A	68.62	Dow Chemical 3s82	1471.25	1467.91	830.00
Baa	26.25	Dresser Industries 4s77	955.00	477.27	912.00
Baa	17.50	I-T-E Circuit Breaker 4 1/4s82	935.00	446.20	921.00
Baa	14.00	Libby McNeil & Libby 5s76	1133.75	945.95	1009.00
Baa	26.00	Mac Andrews & Forbes 5s87	1020.00	666.67	1014.00
Baa	75.50	Macy (RH) & Co. 5s77	2340.00	2359.38	1010.00
A	48.00	Phillips Petroleum 4 1/4s87	1115.00	960.00	985.00
Baa	44.00	Richfield Oil 4 3/8s83	1270.00	1257.14	934.00
Baa	44.12	Sinclair Oil 4 3/8s86	1010.00	630.36	929.00
Baa	81.00	Union Oil of Cal. 4 1/4s91	1305.00	1270.99	902.00
			Baa	A	Aa
Estimated December 27, 1963 Bond Yields			4.90	4.41	4.36

December 31, 1964

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	53.87	Air Reduction 3 7/8s87	1080.00	862.00	868.00
Baa	38.50	American Optical 4.40s80	1390.00	1383.40	951.00
Baa	39.25	Ashland Oil 3 7/8s93	1271.25	1266.13	851.00
Baa	30.37	Baxter Lab 4s82	1140.00	799.34	901.00
Baa	35.37	Combustion Engineering 3 3/8s81	1262.50	1179.17	834.00
Baa	55.50	Continental Baking 3 4/8s83	1110.00	925.00	943.00
A	76.50	Dow Chemical 3s82	1635.00	1636.36	821.00
Baa	10.62	Fansteel Metallurgical 4 3/4s76	930.00	369.44	930.00
A	22.87	Flintkote 4 1/2s80	1030.00	508.33	1001.00
Baa	37.50	Hooker Chemical 5s84	1190.00	833.33	834.00
Baa	29.25	Mac Andrews & Forbes 5s87	1035.00	750.00	1020.00
Baa	35.87	Union Oil of Cal. 4 1/4s91	1755.00	1756.86	911.00
Baa	986.25	Xerox 4s84	1341.25	986.25	894.00
			Baa	A	Aa
Estimated December 31, 1964 Bond Yields			4.85	4.49	4.44

December 31, 1965

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	74.50	Air Reduction 3 7/8s87	1270.00	1192.00	846.00
Baa	56.12	Ashland Oil & Refin. 3 7/8s93	1770.00	1810.48	825.00
Baa	48.12	Baxter Lab. 4s82	1382.50	1266.45	883.00
Baa	48.00	Continental Baking 4 3/8s83	1040.00	800.00	922.00
A	77.12	Dow Chemical 3s82	1680.00	1649.73	804.00
A	20.75	Flintkote 4 1/2s80	1073.75	461.11	976.00
Baa	31.12	Grant (WT) Co. 4s90	1220.00	1111.61	853.00
Baa	29.87	Mac Andrews & Forbes 5s87	1050.00	766.03	993.00
Baa	54.00	Macy (RH) & Co. 4 1/4s90	1175.00	900.00	888.00
Baa	51.00	Union Oil 4 1/4s91	2461.25	2497.55	885.00
Baa	202.00	Xerox 4s84	2140.00	2020.00	874.00
			Baa	A	Aa
Estimated December 31, 1965 Bond Yields			5.05	4.73	4.69

December 30, 1966

Rating	Common Price		Bond Price	Conv. Value	Debt Value
Baa	65.25	Air Reduction 3 7/8s87	1078.75	1044.00	736.00
Baa	32.62	Ashland Oil 3 7/8s93	2100.00	2104.84	703.00
Baa	46.00	Continental Baking 4 3/8s83	865.00	707.69	816.00
Baa	20.75	Grant(WT) Co. 4s90	908.75	741.07	733.00
Baa	38.12	Stauffer Chemical 4 1/2s91	950.00	712.61	791.00
Baa	51.25	Union Oil of Cal. 4 1/4s91	2407.50	2330.60	760.00
Baa	197.50	Xerox 44s84	2030.00	1975.00	770.00
Baa	47.37	Combustion Engineering 3 3/8s81	1630.00	1579.17	735.00
A	61.62	Dow Chemical 3s82	1380.00	1318.18	734.00
			Baa	A	Aa
Estimated December 30, 1966 Bond Yields			6.18	5.57	5.39

December 29, 1967

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	80.50	Aluminum Co. of Amer. 5 1/4s91	1125.00	947.06	817.00
Baa	33.25	Amerace Corp. 5s92	1005.00	898.65	784.00
Baa	54.00	Black & Decker 4s92	1285.00	1157.06	666.00
Baa	36.25	FMC Corp. 4 1/4s92	1012.50	873.49	695.00
Baa	34.37	Grant (WT) Co. 4 3/4s87	1245.00	1227.68	777.00
Baa	88.00	Kresge (SS) Co. 4 1/8s92	1320.00	1222.22	681.00
A	47.00	McGraw-Hill 3 7/8s92	1022.50	752.50	694.00
Baa	53.37	Radio Corp. of America 4 1/2s92	1102.50	904.66	725.00
Baa	44.87	Stauffer Chemical 4 1/2s91	1040.00	838.79	730.00
A	67.00	Phillips Petroleum 4 1/4s87	1195.00	1218.18	765.00
			Baa	A	Aa
Estimated December 29, 1967 Bond Yields			6.82	6.37	6.24

December 27, 1968

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	74.62	Aluminum Co. of America 5 1/4s91	1092.50	877.94	780.00
Baa	34.75	Amerace 5s92	1035.00	939.19	748.00
Baa	41.75	Ashland Oil 4 3/4s93	1072.50	835.00	715.00
Baa	66.75	Becton Dickinson 4 1/8s88	1080.00	890.00	676.00
Baa	61.37	Black & Decker 4s92	1310.00	1315.08	636.00
Baa	39.12	FMC 4 1/4s92	1090.00	942.77	664.00
Baa	43.37	Grant (WT) Co. 4s90	1520.00	1549.11	649.00
Baa	40.25	Kresge Co. 4 1/8s92	1682.50	1677.08	650.00
Baa	26.00	Lone Star Cement 5 1/8s93	1090.00	1000.00	758.00
A	38.25	McGraw-Hill, Inc. 3 7/8s92	906.25	612.00	668.00
Baa	71.50	Owens-Illinois, Inc. 4 1/2s92	1225.00	1211.86	692.00
A	46.25	Penney (JC) Co. 4 1/4s93	1110.00	925.00	707.00
Baa	46.00	RCA 4 1/2s92	1040.00	779.66	692.00
Baa	26.00	Ralston Purina 4 7/8s92	1150.00	981.13	734.00
Baa	47.87	Stauffer 4 1/2s91	1060.00	894.86	697.00
Baa	53.62	Trane 4s92	900.00	744.79	636.00
Baa	26.62	Sprague Electric 4 1/4s92	827.50	585.16	664.00
Baa	46.25	White Motor 5 1/4s93	1066.25	848.62	772.00
			Baa	A	Aa
Estimated December 27, 1968 Bond Yields			7.25	6.71	6.52

December 31, 1969

Rating	Common Price	Company	Bond Price	Conv. Value	Debt Value
Baa	71.50	Aluminum Co. of America 5 1/4s91	960.00	841.18	641.00
Baa	21.00	Amerace 5s92	690.00	567.76	612.00
Baa	57.62	Becton Dickinson 4 1/8s88	1171.25	1152.50	560.00
Baa	72.25	Black & Decker 4s92	1500.00	1548.10	516.00
Baa	24.12	FMC 4 1/4s92	720.00	581.33	540.00
Baa	47.25	Grant (WT) 4s90	1690.00	1687.50	531.00
Baa	55.50	Kresge 4 1/8s92	2300.00	2312.50	528.00
Baa	23.12	Lone Star Cement 5 1/8s93	945.00	889.42	619.00
A	28.50	McGraw-Hill, Inc. 3 7/8s92	730.00	456.00	523.00
Baa	62.00	Owens-Illinois, Inc. 4 1/2s92	1075.00	1050.85	564.00
A	49.25	Penny 4 1/4s93	1122.50	985.00	553.00
Baa	34.50	RCA 4 1/2s92	762.50	584.75	564.00
Baa	26.50	Ralston Purina 4 7/8s92	1050.00	1000.00	600.00
Baa	34.12	Stauffer 4 1/2s91	778.75	637.85	570.00
Baa	25.62	Sprague 4 1/4s92	670.00	563.19	540.00
Baa	65.00	Trane 4s92	940.00	902.78	516.00
Baa	30.87	White Motor 5 1/4s93	765.00	566.51	631.00
			Baa	A	Aa
Estimated December 31, 1969 Bond Yields			9.05	8.76	8.40

APPENDIX C

LIST OF OBSERVATIONS USED IN TESTING THE CONVERTIBLE BOND MODEL

1960 DATA

Rating	1-4-60* Common Price	Company	1-4-60 Bond Price	1-2-61 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-4-60 Model Value
Baa	27.87	Combustion Eng. 3 3/8s81	103.00	94.75	-.080	774	929	(1069)**
Ba	32.12	Food Fair Stores 4s79	95.75	99.25	+.037	832	780	961
Ba	44.37	Grace (WR) & Co. 3 1/2s75	97.00	94.75	-.023	802	831	1032B
Baa	10.50	Libby McNeil 5376	102.50	107.00	+.044	987	629	1099B
Ba	38.75	McDermott (J. Ray) & Co. 5s72	98.25	96.12	-.022	964	695	1102B
Ba	16.37	Rohr Aircraft 5 1/4s77	102.50	109.00	+.063	983	885	1198B
A	78.50	Scott Paper 3s71	102.50	111.12	+.084	851	1019	1173B
Ba	19.00	Seiberling Rubber 5s79	93.50	82.25	-.120	935	760	1031B
Ba	15.12	SCM 5 1/4s79	100.25	99.00	-.012	982	644	1099B
Ba	30.62	Vanadium Corp. 4 1/4s76	95.50	83.00	-.131	875	581	982
Net Total					-.160			+.014
Average Change					-.016			+.002
Initial Investment					100.000			100.000
Ending Balance					98.400			100.200
					A	Baa	Ba	B
Estimated December 1959 Bond Yields					4.69	5.12	5.40	5.70

*Prices are taken from the first issue of Barron's and The Commercial and Financial Chronicle each year, and the dates pertain to the issue date of the newspapers.

**Brackets are enclosed on all convertible bonds that are outside the relevant range of the model. Buy signals are not given on these convertibles even though model values exceed the bond's price by more than 3 per cent. Buy signals on the relevant range convertibles are indicated with the letter B.

1961 DATA

Rating	1-2-61 Common Price	Company	1-2-61 Bond Price	1-2-62 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-2-61 Model Value
Baa	34.50	Allegheny Lud. St. 4s81	97.50	110.50	+.133	861	627	998
Ba	36.75	Boeing 4 1/2s80	101.00	119.50	+.183	896	735	1060[B]
Ba	30.87	Copperweld Steel 5s79	95.00	106.50	+.121	950	551	1040[B]
Ba	16.00	Eastern Stainless Steel 5s73	101.50	109.00	+.074	967	593	1069[B]
Ba	31.75	Food Fair Stores 4s79	99.25	123.25	+.242	841	771	963
Ba	20.87	Gen. Amer. Oil 4 3/4s84	96.00	107.50	+.120	918	531	953
Ba	14.75	General Time 4 3/4s79	100.00	106.00	+.060	928	615	1041[B]
B	12.87	Kayser-Roth 5 1/2s80	102.75	170.50	+.659	976	632	1090[B]
B	31.00	NAFI 5 1/4s80	102.87	118.00	+.147	946	873	1163[B]
A	38.75	Sinclair Oil 4 3/8s86	100.25	102.50	+.022	975	517	1053[B]
Net Total					+1.761			+1.266
Average Change					+ .176			+ .181
Cumulative Balance					98.400			100.200
Ending Balance					115.718			118.336
					A	Baa	Ba	B
Estimated December 1960 Bond Yields					4.54	5.10	5.37	5.70

1962 DATA

Rating	1-2-62 Common Price	Company	1-2-62 Bond Price	1-7-63 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-2-62 Model Value
Ba	16.62	CF & I 4 7/8s77	105.00	84.25	-.198	935	519	1016
Ba	39.50	Copperweld Steel 5s79	106.50	103.00	-.028	944	705	1089
Baa	28.50	Dresser Ind. 4 1/8s77	98.87	97.25	-.016	887	518	922
A	24.12	Flintkote 4 1/2s80	104.00	102.00	-.019	988	536	1019
Ba	15.12	General Time 4 3/4s79	106.00	89.00	-.160	916	630	1036
Baa	18.75	I-T-E Cir. Breaker 4 1/4s82	95.00	94.75	-.003	882	424	950
B	18.75	Lionel Corp. 5 1/2s80	106.00	61.50	-.420	961	572	1057
Ba	23.12	National Cylinder 5 1/8s77	108.00	104.00	-.037	961	595	1005
Baa	12.75	Royal McBee 6 1/4s77	105.37	103.00	-.022	1045	447	1100 B
Baa	37.62	Sinclair 4 3/8s86	102.50	100.62	-.018	887	470	958
Net Total					-.921			-.022
Average Change					-.092			-.022
Cumulative Balance					115.718			118.336
Ending Balance					105.072			115.733
					A	Baa	Ba	B
Estimated December 1961 Bond Yields					4.60	5.20	5.50	5.85

1963 DATA

Rating	1-7-63 Common Price	Company	1-7-63 Bond Price	1-6-64 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-7-63 Model Value
Baa	35.87	Allegheny Lud. St. 4s81	101.00	97.50	-.035	891	652	1022
Baa	26.37	Combustion Eng. 3 3/8s81	102.50	107.50	+.049	816	879	1068[B]
Ba	35.00	Copperweld Steel Co. 5s79	103.00	104.00	+.010	983	625	1094[B]
Ba	20.37	Crowell-Collier 4 1/2s81	99.75	90.00	-.098	923	485	995
Ba	23.12	Food Fair 4s79	96.25	94.12	-.022	873	702	959
Ba	16.62	Grand Union 4 1/8s78	96.12	103.50	+.077	891	635	953
Ba	34.00	Hunt Foods 4 3/8s86	102.25	98.00	-.042	895	694	975
Baa	9.50	Royal McBee 6 1/4s77	103.00	105.50	+.024	1030	306	1057
Baa	36.75	Sinclair Oil 4 3/8s86	100.62	101.50	+.009	927	459	992
Ba	15.00	SCM 5 1/4s79	103.00	104.00	+.010	1011	717	1151[B]
Net Total					-.018			+.069
Average Change					-.002			+.023
Cumulative Balance					105.072			115.733
Ending Balance					104.862			118.395
					A	Baa	Ba	B
Estimated December 1962 Bond Yields					4.35	4.90	5.15	5.50

1964 DATA

Rating	1-6-64 Common Price	Company	1-6-64 Bond Price	1-4-65 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-6-64 Model Value
Ba	56.00	Allied Stores 4 1/2s81	103.25	118.75	+.150	925	800	1113[B]
Ba	35.25	American Distilling 4 3/8s86	104.62	101.50	-.030	897	816	1024
Ba	14.12	Automatic Canteen 4 3/4s81	98.50	100.50	+.020	954	438	1012
Ba	37.37	Boeing 4 1/2s80	102.50	150.37	+.567	928	748	1093[B]
Baa	23.75	Burroughs 4 1/2s81	103.12	104.50	+.013	953	610	1062
Baa	49.00	Continental Baking 4 3/8s83	103.12	111.00	+.076	934	817	1052
Baa	14.37	Fansteel Metal. 4 3/4s76	99.12	93.00	-.062	986	324	993
Ba	18.00	Grand Union 4 3/8s78	103.50	121.00	+.169	921	715	1003
Ba	27.87	Hunt Foods 4 3/8s86	98.00	102.62	+.047	897	569	941
Ba	16.25	SCM 5 1/4s79	104.00	115.00	+.106	1011	824	1194[B]
Net Total					+.956			+.723
Average Change					+.096			+.241
Cumulative Balance					104.862			118.395
Ending Balance					114.887			146.928
					A	Baa	Ba	B
Estimated December 1963 Bond Yields					4.41	4.90	5.15	5.50

1965 DATA

Rating	1-4-65 Common Price	Company	1-4-65 Bond Price	1-3-66 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-4-65 Model Value
Ba	17.00	Automatic Canteen 4 3/4s81	100.50	122.00	+.214	961	511	1038[B]
Baa	25.25	Burroughs 4 1/2s81	104.50	142.50	+.364	961	648	1082[B]
Ba	19.00	Collins Radio 4 3/4s83	100.50	182.00	+.811	958	691	1095[B]
Ba	25.12	Crowell-Collier 4 1/2s81	100.37	130.00	+.295	934	598	1041[B]
Ba	21.75	Food Fair Stores 4s79	95.25	97.50	+.024	888	660	958
Ba	12.12	General Time 4 3/4s79	99.00	111.75	+.129	964	505	1039[B]
Ba	30.00	Hunt Foods Ind. 4 3/8s86	102.62	97.75	-.048	906	663	974
B	17.50	LTV 5 3/4s76	92.00	144.25	+.567	920	456	984[B]
Ba	20.12	Stokely-Van Camp 4 1/4s82	102.50	117.50	+.146	903	577	1006
B	30.12	Vornado Inc. 5s82	103.50	168.00	+.623	950	615	1060
Net Total					+3.125			+2.380
Average Change					+ .313			+ .397
Cumulative Balance					114.887			146.928
Ending Balance					150.847			205.258
					A	Baa	Ba	B
Estimated December 1964 Bond Yields					4.50	4.85	5.10	5.45

1966 DATA

Rating	1-3-66 Common Price	Company	1-3-66 Bond Price	1-2-67 Bond Price	Change In Bond Price	Debt Value	Conv. Value	Model Value
B	19.87	Crescent Corp. 5 1/2s80	105.50	81.00	-.232	980	795	1156[B]
Ba	21.87	Food Fair Stores 4s79	97.50	75.25	-.228	875	664	948
Ba	19.62	Amer. Mach. & F. 4 1/2s81	92.62	72.00	-.223	916	315	948
Ba	19.00	Eastern Stainless Steel 5s73	108.00	92.00	-.148	982	777	1075
Ba	28.12	Hunt Foods 4 3/8s86	97.75	77.50	-.207	885	622	944
Ba	32.00	Amer. Distill. 4 3/8s86	104.00	86.00	-.173	885	741	984
B	17.75	CF & I 4 7/8s77	100.00	86.12	-.139	931	555	1024
Baa	48.00	Continental Baking 4 3/8s83	104.00	86.50	-.168	922	738	1012
A	20.75	Flintkote 4 1/2s80	107.37	90.00	-.162	974	461	995
Baa	61.50	Sinclair 4 3/8s86	108.50	101.75	-.062	914	879	1090
Net Total					-1.742			-.232
Average Change					- .174			-.232
Cumulative Balance					150.847			205.258
Ending Balance					124.600			157.638
					A	Baa	Ba	B
Estimated December 1965 Bond Yields					4.75	5.05	5.30	5.70

1967 DATA

Rating	1-2-67 Common Price	Company	1-2-67 Bond Price	1-1-68 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-2-67 Model Value
Ba	18.00	Cluett Peabody & Co. 4 1/4s84	96.25	120.00	+.247	761	871	1025[B]
Ba	14.50	General Time 4 3/4s79	92.00	103.00	+.120	848	604	967[B]
Ba	49.75	Grace (WR) & Co. 4 1/4s90	94.50	85.00	-.101	724	765	945
Ba	44.25	Inter. Min. & Chem. 4s91	96.62	73.87	-.235	690	759	916
Baa	42.37	Macy (RH) Co. 4 1/4s90	93.50	120.50	+.289	764	706	948
Ba	15.25	Outlet (The) Co. 5 1/2s86	100.12	140.00	+.398	885	813	1087[B]
A	49.75	Phillips Petrol. 4 1/4s87	99.50	119.50	+.201	839	829	1059[B]
Ba	50.12	Reynolds Metals 4 1/2s91	103.50	96.25	-.070	750	796	981
Baa	38.12	Stauffer Chemical 4 1/2s91	95.00	104.00	+.095	791	713	965
Ba	40.75	Storer Broad. 4 1/2s86	97.00	97.00	.000	775	799	995
Net Total					+.944			+.966
Average Change					+.094			+.242
Cumulative Balance					124.600			157.638
Ending Balance					136.362			195.786

Estimated December 1966 Bond Yields	A	Baa	Ba	B
	5.75	6.18	6.55	7.00

1968 DATA

Rating	1-1-68 Common Price	Company	1-1-68 Bond Price	1-6-69 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-1-68 Model Value
Baa	33.25	Amerace 5s92	100.50	103.00	+.025	780	899	1057
Ba	46.00	Armstrong Rubber 4 1/2s87	102.50	113.50	+.113	715	902	(1014)
Ba	29.87	Collins & Aik. 4 5/8s91	102.00	148.12	+.452	706	964	(1044)
B	23.50	Fairchild Hiller 4 3/8s92	99.75	95.00	-.048	632	964	(1040)
Baa	36.25	FMC 4 1/4s92	101.25	108.00	+.067	691	873	(1016)
Ba	72.73	Grolier 4 1/4s87	100.25	87.50	-.127	689	827	951
Ba	29.50	Revere 5 1/2s92	100.62	113.75	+.130	801	868	1057
Ba	32.75	Skil Corp. 5s92	100.25	116.75	+.165	744	819	988
Baa	56.00	Trane 4s92	100.00	89.00	-.110	661	778	937
Ba	81.50	United Aircraft 4 1/2s92	99.00	88.00	-.111	687	815	978
Net Total					+.556			+.155
Average Change					+.056			+.077
Cumulative Balance					136.362			195.786
Ending Balance					143.944			210.959
					A	Baa	Ba	B
Estimated December 1967 Bond Yields					6.37	6.85	7.25	7.75

1969 DATA

Rating	1-6-69 Common Price	Company	1-6-69 Bond Price	1-5-70 Bond Price	Change In Bond Price	Debt Value	Conv. Value	1-6-69 Model Value
Ba	39.12	Allied Stores 4 1/2s92	102.00	77.00	-.245	656	879	(996)
Ba	44.25	Belco Petroleum 4 3/4s88	101.75	63.00	-.381	706	770	929
Ba	34.37	Ceco Corp. 4 3/4s88	102.00	65.25	-.360	706	917	(1018)
Ba	30.25	Gidding Lewis 4 5/8s87	102.12	61.12	-.401	700	807	946
Ba	36.00	Green Giant 4 1/4s92	100.00	77.50	-.225	629	818	(939)
Ba	39.75	Gruman Aircraft 4 1/4s92	99.62	71.50	-.282	629	914	(1003)
B	24.37	Helmerich 5s87	102.12	71.00	-.305	694	894	(996)
Baa	47.50	RCA 4 1/2s92	103.00	76.25	-.260	692	805	975
Ba	33.87	Riegel T. 5s93	100.00	57.00	-.430	706	779	939
B	19.87	Standard Pack. 5 1/4s90	101.00	64.50	-.361	700	828	960
Net Total					-3.250			-0-
Average Change					- .325			-0-
Cumulative Balance					143.944			210.959
Ending Balance					97.162			210.959
					A	Baa	Ba	B
Estimated December 1968 Bond Yields					6.75	7.25	7.70	8.25

VITA

Jerry Don Miller was born February 15, 1939 in Waco, Texas. He attended elementary and high school in Chilton, Texas where he was graduated in 1957. Subsequently he attended North Texas State University and was awarded a Bachelor of Science degree, with a major in psychology, from that institution in January 1962. He received a Master of Business Administration from North Texas State University in September 1963 and began a life in college teaching. He attended Louisiana State University during the summer sessions of 1965 and 1966 while employed at West Texas State University. He taught two years at Eastern New Mexico University and returned to Louisiana State University on a full-time basis in 1968. He is a candidate for the degree of Doctor of Philosophy.

EXAMINATION AND THESIS REPORT

Candidate: Jerry Don Miller

Major Field: Business Finance

Title of Thesis: A Theoretical and Empirical Study of Stock Purchase Warrants and Convertible Bonds

Approved:

Donald E. Vaughn
Major Professor and Chairman

Max Goodrich
Dean of the Graduate School

EXAMINING COMMITTEE:

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Date of Examination:

May 14, 1970