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A THEORETICAL APPROACH TO HIGH - ENERGY PION PHENOMENA

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ABSTRACT

A theoretical analysis of the main features of high-energy interactions is performed. First of all, by applying dispersion theory to the elastic scattering amplitude for high - energy and low-momentum transfer we have obtained expressions for the shape of diffraction scattering and for the total cross-sections.

The form of those results has suggested a definite model for the most important inelastic processes. In this model a well-defined group of Feynman graphs, the "most peripheral" ones, give the dominating effect in high-energy collisions.

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I. INTRODUCTION

A great effort, both experimental and theoretical, has recently been devoted to the study of strong interactions at high energy.

The theoretical investigations have followed many different directions.

First of all, several estimates of cross-sections, multiplicities, etc., have been carried out on the basis of the statistical model[1] which is essentially based on the idea that a very complicated interaction takes place in a certain volume V and that the probabilities for the different final states are determined by the available phase space.

A different and complementary approach is the peripheral one[2] which studies those events in which only one pion is exchanged between the two colliding particles.

A very interesting point of view has recently been developed by Chew and Frautschi[3]: if one considers the elastic scattering amplitudes at high energy it is possible to make use of the Mandelstam[4] representation which gives an unambiguous prescription to locate the singularities of the amplitude.

In addition to that, it is possible by using unitarity to obtain explicit formulae for the singularities located near the physical region in the so-called "strips".

The "strip approximation" approach tries to evaluate the effect of peripheral collisions on elastic scattering. It has the advantage on the Drell - Salzman method that, due to the possibility of writing down a complete spectral representation, a greater control on the different approximations is possible.

On the other side, the amount of information on high-energy physics which one obtains from the Chew and Frautschi programme is limited to diffraction scattering and to the total cross-section.

In this paper we wish to propose a general model for high-energy processes suggested by dispersion theory.

As in Chew and Frautschi, we shall start by considering elastic scattering, and in particular the absorptive part of the elastic amplitude for large energies and small momentum transfer. This amplitude is particularly interesting because:

1) the forward amplitude is related to the total cross-section by the optical theorem;

2) the angular dependence of this amplitude is mainly due to the absorption of the incident wave by inelastic processes and it will depend in an essential manner on the way this absorption takes place. Let us discuss a qualitative picture of the high-energy scattering. The main experimental features are the following:

   i) the total cross-section has an approximately constant value of the order of the square of the pion Compton wavelength;

   ii) the scattering angular distribution is concentrated in a small region of the momentum transfer of some units of the square of the meson mass.

It will be shown that unitarity gives a simple relation between the size of the total cross-section and the width of the elastic diffraction peak.

The general features i) and ii) show that the effective radius of the target nucleon from high-energy process is of the order of the pion Compton wavelength $1/\mu$.

We can get the following picture of the high-energy collision. The nucleon can be visually divided into two parts: a core with a radius of the order $1/3\mu$, and the external meson cloud.

About the core we know very little; its structure is very complicated since it depends on states with several virtual particles. Very probably it will act like a completely absorptive sphere; in any case because of the geometrical limitations on cross-section it can only contribute to a small fraction of the total cross-section.

The outer part of the nucleon plays a very important role in the process (as shown by the large size of the total cross-section); it acts as a semi-transparent medium whose absorption will decrease with the cloud density(*) and thus with the distance from the centre.

Our approach is based on the classification of the high-energy phenomena into those due to the cloud and to the core.

On the basis of the geometrical considerations discussed previously, the first ones are expected to be the most frequent and will be the object of our theoretical investigation.

In Section II a general scheme is introduced to study the absorptive part of the scattering amplitude. Using the Mandelstam representation in a manner similar to Chew and Frautschi, a non-linear integral for the amplitude is obtained.

In Section III a method of solution is discussed. The solution is obtained in the form of a sequence of terms in which the only external parameters are the low-energy scattering cross-sections. In this manner definite expressions are obtained both for the total cross-sections and for the scattering amplitudes(**).

As already discussed earlier, the form of the diffraction pattern is determined by the manner in which the absorption mechanism takes place. In this manner our result for diffraction and for the total

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(*) This is confirmed experimentally by the fact that σ_{el} is smaller than σ_{react}.

(**) An independent investigation on high-energy diffraction scattering has been recently carried out by C. Goebel (see reports of the CERN Conference on Theoretical Aspects of High-Energy Phenomena) leading to physical conclusions very similar to ours.

cross-sections will enable us (Section IV) to construct a general model for all inelastic processes. This model has a very simple physical interpretation in terms of the Feynman graphs implying that all final particles are produced directly either by the two colliding particles or by the pion which is exchanged between the incoming particles. The only parameters entering in the model are low-energy parameters, like the position and width of the pion-nucleon and pion-pion resonances.

II. THE INTEGRAL EQUATION FOR THE ABSORPTIVE AMPLITUDE

In the following we shall deal only with no-spin-flip, no-isospin-flip amplitudes: this is because it is intuitively clear (and experimentally verified where experiments are available) that diffraction shall mainly be present in this amplitude. This will allow us to forget all the complications which arise from the spin and isospin of the particles.

We shall treat the high-energy elastic scattering in which strongly interacting particles are involved. We shall denote by X and Y the two colliding particles and define as usual:

$$\begin{aligned} s &= (q_x + q_y)^2 \\ t &= (q_x + p_x)^2 \\ \bar{t} &= (q_x + p_y)^2 \end{aligned} \quad . \tag{1}$$

q,p are the initial and final X and Y momenta:

$$s + t + \bar{t} = 2\,M_x^2 + 2\,M_y^2 \; . \tag{1'}$$

The imaginary part $A_{xy}(s,t)$ of the scattering amplitude is expected to be the most important contribution to high-energy scattering, since it is directly connected through unitarity to the inelastic processes. This has been verified in (πN) scattering already around 1.5 GeV.

In the forward direction the $A_{xy}(s,t)$ is related to the total cross-section by the optical theorem[5]:

$$A(s,o) = 2\,q\,W\,\sigma_T(s). \tag{2}$$

As a consequence of the optical theorem we want to show that, at high energy, A(s,t) must have a strong and narrow peak in the forward direction, whose width is roughly inversely proportional to the total cross-section. The total elastic cross-section satisfies the obvious inequality:

$$\sigma_{el}(s) = \frac{1}{s} \int \left|\frac{F(s,t)}{8\pi}\right|^2 d\Omega \geq \frac{1}{\pi s} \int_{t_{max}}^{0} \left(\frac{A(s,t)}{8q}\right)^2 dt \tag{3}$$

where $t_{max} = -4q^2$.

On the other hand, using the optical theorem we can write:

$$A(s,t) = 2\,q\,W \sigma_T(s)\,U(s,t) \tag{4}$$

where the form factor $U(s,t)$ is normalized to:

$$U(s,o) = 1.$$

Substituting Eq. (4) into Eq. (3) we get:

$$< t > \lesssim 16\,\pi\,\frac{\sigma_{el}}{(\sigma_T)^2} \tag{5}$$

where

$$< t > = \int_{t_{max}}^{0} |U(s,t)|^2\, dt \tag{5'}$$

is the width of the forward diffraction peak. Eq. (5) gives a very strong limitation on $< t >$.

The relation (5) is roughly satisfied with the equality sign. In fact, taking as an example the πN scattering in the two GeV regions we have

$$\sigma_T \sim \frac{1.5}{\mu^2}, \quad \sigma_e \sim \frac{1}{3}\,\sigma_T$$

and so $< t > \lesssim 10\,\mu^2$ which is of the same order of the width of the diffraction peak. This means that the low momentum transfer scattering is so important to saturate the unitarity condition, and to leave very little place for the rest.

Let us discuss now the information one can get on A(s,t) from dispersion theory. The Mandelstam representation allows us to write the following fixed energy spectral representation:

$$A(s,t) = \frac{1}{\pi}\int_{a_1}^{\infty} dt' \, \frac{\rho_1(s,t')}{t' - t} + \frac{1}{\pi}\int_{a_2}^{\infty} dt' \, \frac{\rho_2(s,t')}{t' - \bar{t}}. \qquad (6)$$

The real functions ρ_1 and ρ_2 are the two-dimensional spectral functions introduced by Mandelstam. The lower limits a_1 and a_2 depend on the nature of the particles exchanged in the (X,Y) scattering: a_1 is always of the order of $4\mu^2$; a_2 is $\sim 4\mu^2$ for $(\pi\pi)$ and (NN) scattering and is M^2 for (πN) scattering.

Here we are interested in the features of the forward difraction peak; this will lead us to study the amplitude only for small ($\lesssim 9\mu^2$) positive or negative values of t. Because of Eq. (1′) for energy high enough, the singularities in $\bar{t}$ are very far from the values of t in which we are interested and will have a very small influence on the amplitude A.

We shall therefore only take into account the effect of $\rho_1(s,t)$. Let us discuss now the different contributions to ρ. Using unitarity in the channel $X + \bar{X} \to Y + \bar{Y}$ (see Appendix) we can decompose $\rho(s,t)$ as a sum of contributions coming from the different groups of particles which can be exchanged between X and Y. The variable t represents the square of the "mass" (i.e., the total c.m. energy) of each group of particles.

Therefore the larger the number of exchanged particles, the more distant from the physical region will be the singularities of A(s,t). On the other hand, we have already seen that A(s,t) has a very large peak of width $\sim 10\,\mu^2$ in the forward direction and is almost zero everywhere else. This strongly suggests that the region of singularities for $4\mu^2 \leq t \leq 16\mu^2$ (the so-called "strip") gives the dominating contribution to the integral (6).

Therefore the contribution of the two-pion states will be the most important since it gives the only contribution to $\rho(s,t)$ for $4\mu^2 \leq t \leq 16\mu^2$ for $\pi\pi$ and πN and $4\mu^2 \leq t \leq 9\mu^2$ for NN.

This result is the dispersive analogue of the geometrical argument which had led us to believe that the external part of the nucleon structure gives the largest effect to the high-energy cross-sections. Let us now consider in detail the two-pion term. In the Appendix it is shown that $\rho_{\pi\pi}(s,t)$ can be written in the form:

$$\rho_{\pi\pi}(s,t) = \frac{1}{\pi^2} \iint ds_1\, ds_2\, K_0(s\,t;\, s_1 s_2) A^*_{x\pi}(s_1,t) A_{y\pi}(s_2,t) \tag{7}$$

where

$$K_0(s\,t;\, s_1 s_2) = 0 \quad \text{for} \quad t_0 > t \quad \text{and} \quad \sqrt{s} < \sqrt{s_1} + \sqrt{s_2}$$

and

$$K_0(s\,t;\, s_1 s_2) = - \frac{1}{16\sqrt{f(s,s_1 s_2)}} \frac{1}{\sqrt{t[t - t_0(s,s_1 s_2)]}} \quad \text{for } t \geq t_0 \text{ and } \sqrt{s} \geq \sqrt{s_1} + \sqrt{s_2} \tag{8}$$

where

$$t_0 = \frac{g(s,s_1 s_2)}{f(s,s_1 s_2)}; \quad f(s,s_1 s_2) = \left[s - \left(\sqrt{s_1} + \sqrt{s_2}\right)^2\right]\left[s - \left(\sqrt{s_1} - \sqrt{s_2}\right)^2\right]$$

$$g_{\pi\pi}(s,s_1 s_2) = 4\left\{\mu^2 s^2 + s\left[s_1 s_2 - 2\mu^2(s_1 + s_2)\right] + \mu^2(s_1 - s_2)^2\right\}$$

$$g_{\pi N}(s,s_1 s_2) = 4\left\{\mu^2 s^2 + s\left[s_1 s_2 - 2 s_1\mu^2 - s_2(M^2 + \mu^2)\right] + \mu^2(s_1 - s_2)^2 + \right.$$
$$\left. + \left[(s_1 - s_2)(\mu^2 - M^2) + (\mu^2 - M^2)^2\right] s_2\right\} \tag{8'}$$

$$g_{NN}(s,s_1 s_2) = 4\left\{\mu^2 s^2 + s\left[s_1 s_2 + (M^2 - \mu^2)^2 - (M^2 + \mu^2)(s_1 + s_2)\right] + \right.$$
$$\left. + M^2(s_1 - s_2)^2\right\}$$

and $A_{x\pi}(s,t)$ and $A_{y\pi}(s,t)$ are the absorptive parts of the $(X\pi)$ and $(Y\pi)$ scattering amplitudes. The physical meaning of Eq. (7) is illustrated in Fig. 1. $\rho_{\pi\pi}(s,t)$ is related to all inelastic processes in which only one pion is exchanged between X and Y. $\sqrt{s_1}$ and $\sqrt{s_2}$ are the energies of the two groups of particles produced in such a collision. $A_{x\pi}$ and $A_{y\pi}$ are the elastic amplitudes for the two subprocesses contained in the two black boxes. The function $K(s,t;s_1 s_2)$ is the Mandelstam spectral function of the perturbation graph in Fig. 2, and represents the weight with which the different groups of particles in Fig. 1 contribute to the elastic diffraction scattering. Eqs. (8) and (8′) show that the spectralfunction is different from zero only for $t \geq t_0(s;s_1 s_2)$; in particular, we must always have $\sqrt{s} \geq \sqrt{s_1} + \sqrt{s_2}$. The boundary curves $t = t_0$ are called the Landau curves; it is easy to verify that we can have $t_0 < 16\,\mu^2$ only when $\sqrt{s_1} + \sqrt{s_2} << s$. In other words, all values of s_1, s_2 for which $\sqrt{s_1} + \sqrt{s_2} \leq \sqrt{s}$ are kinematically possible, but $\sqrt{s_1}$ and $\sqrt{s_2}$ must both be rather small as compared to $\sqrt{s}$ in order to have contributions from the strip.

We understand here that the process of diffraction begins at energies for which some Landau curves have already entered into the strip. It is easy to verify that for πN scattering the forward peak appears near 900 MeV which is just the energy for which the first Landau curve has entered into the strip. This fact shows, for example, that the pion-pion scattering must be of some importance near its Landau curve (characterized by $s = 4\,\mu^2$), i.e., that $(\pi\pi)$ scattering must have its importance even at low energies (far before the region around $s = 20 \div 30\,\mu^2$ at which the $T = J = 1$ resonance is expected to appear).

Eq. (7) is the starting point for our investigation of diffraction scattering. It gives a relation between the absorptive part at an energy s and those at the lower energies s_1 and s_2 [6]. In the next section we shall take advantage of the fact that s_1 and s_2 must be smaller than s in order to obtain an approximate solution of Eq. (7).

III. EXPRESSIONS FOR THE ABSORPTIVE AMPLITUDES

In order to solve the self-consistent problem given by Eqs. (6) and (7), we want first to write down an explicit expression for the absorption amplitude valid at all energies.

Of course, at low energy in the resonance region the important contribution will not come from the two-pion singularities which become important only in the diffraction region.

We shall therefore write

$$A_{xy}(st) = A^R_{xy}(st) + \frac{1}{\pi^3}\iiint dt' ds_1 ds_2 \, \frac{K(s,t';s_1 s_2)}{t' - t} \, A^*_{x\pi}(s_1,t') A_{y\pi}(s_2,t') \tag{9}$$

where $A^R_{xy}(st)$ is the contribution to the absorptive part of the amplitude coming from the $(\pi\pi)$ or (πN) resonances and bound states. Therefore it will be very important at low energy and then drop to zero very rapidly.

Since for low energy the singularities in t are far, both from the physical region and from the strip, one can safely write[7]

$$A^R_{xy}(st) \approx A^R_{xy}(s,o) = 2q\sqrt{s}\,\sigma^R_{xy}(s). \tag{10}$$

Let us now consider the meaning of Eq. (9) for the different scattering processes. In the case of $(\pi\pi)$ scattering, Eq. (9) is a non-linear equation involving only the $(\pi\pi)$ amplitude. The πN equation is a linear equation whose kernel contains $(\pi\pi)$ amplitude. Finally the (NN) amplitude can be obtained directly once the (πN) amplitude is known.

This corresponds to the usual classification of the different processes given by the Mandelstam representation.

We see that the only inputs one has to insert in Eq. (9) are the values of low-energy pion-pion and pion-nucleon cross-sections.

Let us now solve Eq. (9) by iteration: we obtain

$$\begin{aligned} A_{xy}(st) = {} & A^R_{xy}(s) + \frac{1}{\pi^3}\iiint dt' ds_1 ds_2 \, \frac{K_0(s,t';s_1 s_2)}{t' - t} \, A^R_{x\pi}(s_1)\, A^R_{y\pi}(s_2) + \\ & + \frac{1}{\pi^4}\iiiint dt\beta ds_1 ds_2 ds_3 \, \frac{K_1(s,t';s_1 s_2 s_3)}{t' - t} \, A^R_{x\pi}(s_1) A^R_{\pi\pi}(s_2) A^R_{y\pi}(s_3) + \\ & + \ldots\ldots\ldots \end{aligned} \tag{11}$$

where

$$\begin{aligned} K_1(s,t';s_1 s_2 s_3) = \frac{1}{\pi^2}\iint \frac{dt'' \, d s_A}{t'' - t'} \Big\{ & K_0(s\,t';s_1 s_A) K_0(s_A t'';s_2 s_3) + \\ & + K_0(s\,t';s_A s_3) K_0(s_A t'';s_1 s_2) \Big\} \end{aligned} \tag{12}$$

etc. ......

It can immediately be seen that

$$K_{i-2}(s,t;\ s_1,s_2\ldots s_i) \neq 0 \text{ only for } \sqrt{s} > \sum_{j=1}^{i} \sqrt{s_j} \tag{13}$$

This implies that since $|s_j| > 4\,\mu^2$ for each finite value of s the interaction sequence will only have a finite number of term, so that Eq. (11) will indeed represent a solution of Eq. (10).

We have, however, still to verify that the solution (11) is a physically acceptable one in which the maximum contribution comes from the integration range inside the strip.

This is clear for the first term since only small values of s_1, s_2 appear in the sum and the integral in t' is rapidly convergent.

On the other hand, it is very difficult to obtain a direct estimate of the higher order terms since different kernels K_i are oscillating functions.

Therefore in order to be able to study the form and the consistency of the expansion (11) we shall transform the different terms of this sum into integrals on a positive definite function.

Let us first of all recall that $K_0(s\,t,\ s_1 s_2)$ is the Mandelstam spectral function of the perturbation graph of Fig. 2. Now it is shown in Appendix 2 that the functions $K_{i-2}(s,t;s_1 s_2\ldots s_i)$ are the Mandelstam spectral functions of the perturbation graphs of Fig. 3.

Let us now rewrite Eq. (11) in the form

$$\begin{aligned} A(s,t) = A^R(s) &+ \frac{1}{\pi^2}\iint ds_1 ds_2\ I_0(s,t;s_1 s_2)\ A^R(s_1)\ A^R(s_2)\ + \\ &+ \frac{1}{\pi^3}\iiint ds_1 ds_2 ds_3\ I_1(s,t;s_1 s_2 s_3)\ A^R(s_1)\ A^R(s_2)\ A^R(s_3)\ + \\ &+ \ldots\ldots\ldots \end{aligned} \tag{14}$$

where

$$I_{i-2}(s,t;s_1 s_2\ldots s_i) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} dt'\ \frac{K_{i-2}(s,t';s_1 s_2\ldots s_i)}{t'-t} \tag{15}$$

The new functions $I_{i-2}(s,t;s_i)$ are the absorptive parts in the s channel of the elastic amplitudes given by the graphs in Fig. 3.

Those quantities can be computed directly by using unitarity in the channel s, i.e., by cutting vertically the graphs of Fig. 3. Thus we can write[8)]

$$I_{i-2}(s,t;s_1 s_2 \dots s_i) = \frac{1}{2(2\pi)^{3i-4}} \int \frac{\delta(k_1^2 - s_1)d^4k_1 \; \delta(k_2^2 - s_2)d^4k_2 \dots}{(u_{x1} - \mu^2)(v_{x1} - \mu^2)\dots} \frac{\delta(k_i^2 - s_i)d^4k_i \; \delta^4(q_x + q_y - \sum_{j=1}^{i} k_j)}{(u_{xi-1} - \mu^2)(v_{xi-1} - \mu^2)} \qquad (16)$$

$$u_{x1} = (q_x - k_1)^2 \quad ; \quad v_{x1} = (p_x - k_1)^2$$
$$u_{xr} = \left(q_x - \sum_{j=1}^{r} k_j\right)^2 \quad ; \quad v_{xr} = \left(p_x - \sum_{j=1}^{r} k_j\right)^2 . \qquad (17)$$

Let us now discuss the properties of the integral (16). The integrand is always positive definite and the largest contribution will come from those integration regions for which all u_{xj} and v_{xj} are small. We remark here that:

1) all denominators can be simultaneously small only for $\Sigma\sqrt{s_j} \ll s$;

2) the integral I is a rapidly decreasing function of the momentum transfer t ; this means that the most important contribution to the spectral integrals in Eq. (11) do indeed come from the strip.

Let us now use Eq. (14) for a simple comparison with experiment. Let us consider (πN) scattering in the GeV region. Only the first term of the series is important:

$$A_{\pi N}(s,t) = \frac{1}{\pi^2} \iint ds_1 ds_2 \; I_0(s,t;s_1 s_2) \, 2q_1\sqrt{s_1} \; \sigma^R_{\pi N}(s_1) \, 2q_2\sqrt{s_2} \; \sigma^R_{\pi\pi}(s_2) \qquad (18)$$

where

$$I_0(s,t;s_1 s_2) = -\frac{1}{8\pi} \frac{1}{\sqrt{f(s,s_1 s_2)}} \frac{1}{t_0(s,s_1 s_2)} F\left(\frac{|t|}{t_0}\right) \qquad (19)$$

where $$F\left(\frac{|t|}{t_0} \equiv x^2\right) = \frac{1}{x\sqrt{1+x^2}} \log\left(x + \sqrt{1+x^2}\right) . \tag{20}$$

For $\sqrt{s} \gg \sqrt{s_1} + \sqrt{s_2}$ t_0 will be very near to its lowest value $4\mu^2$. Therefore we can get approximately

$$A(s,t) \simeq A(s,o)\, F\left(\frac{t}{4\mu^2}\right) . \tag{21}$$

This shows that, at least at intermediate energies, the shape of the diffraction pattern depends weakly both on the process and on the incident energy.

This is very reasonable from the physical point of view and in good agreement with experiment.

In Figs. 4a) and 4b) a comparison with experimental data at 1.8 [5)] and 5 GeV [6)] is reported. At 5 GeV the theoretical curve is less peaked than the experimental one; the dotted curve represents the result of an estimate of the higher order terms.

IV. A MODEL FOR HIGH-ENERGY INTERACTION

In the preceding section we have obtained expressions for the absorptive part of the scattering amplitude. This is all that one can obtain directly on the basis of a dispersion theory of elastic scattering. We want now to show that the form of our result suggests strongly a definite model about the inelastic processes responsible for the absorption process. Let us write explicitly the expression for the total cross-section given by Eqs. (14) and (16):

$$\sigma(s) = \sum_i \sigma_i(s) \tag{22}$$

where

$$\sigma_i(s) = \frac{1}{2q\sqrt{s}}\, \frac{1}{\pi^i} \iint\cdots\int ds_1 ds_2 \ldots ds_i \, \times$$

$$\times\, 2q_1\sqrt{s_1}\, \sigma^R(s_1)\, 2q_2\sqrt{s_2}\, \sigma^R(s_2) \ldots 2q_i\sqrt{s_i}\, \sigma^R(s_i)\, I_{i-2}(s;s_i) \, . \tag{23}$$

where $$I_{i-2}(s;s_i) = \frac{1}{2(2\pi)^2}\int\cdots\int F(q_x,q_y;k_i,s_i)\times$$

$$\times\,\delta(k_1^2-s_1)d^4k_1\;\delta(k_2^2-s_2)d^4k_2\ldots\delta(k_i^2-s_i)d^4k_i\;\delta^4(q_x+q_y-\Sigma\,k_i) \tag{24}$$

$$F(q_x,q_y;k_i,s_i)=\frac{1}{(2\pi)^{3(i-2)}}\,\frac{1}{[(q_x-k_1)^2-\mu^2]^2[(q_x-k_1-k_2)^2-\mu^2]^2\ldots[(q_x-k_1\ldots k_{i-1})^2-\mu^2]^2} \tag{25}$$

It is very reasonable to consider the different terms σ_i in the sum (22) as different partial cross-sections corresponding to the production of i groups of particles.

The important contributions to the integral (23) come from the energies of the different groups of particles corresponding to a large scattering cross-section. In other words the dominating effect will be the production of several $(\pi\pi)$ or (πN) isobars. Let us now examine the form for σ_i given by Eqs. (23), (24) and (25). Also in this case one can reasonably interpret the integrand $F(q_xq_y;k_i)$ as the angular distribution of the different isobars in the high-energy collisions[11].

We are therefore led to the following model illustrated in Fig. 5.

1) The incoming particles exchange a single pion.

2) The produced particles are emitted either by the initial particles or by the intermediate pion. In other words the emissiom must take place either from one of the two vertices or by the meson propagation.

3) In each single act of emission either one particle or a group of low-energy particles will be produced.

4) The differential cross-section for production is given by

$$d\sigma = \frac{1}{16\,q\sqrt{s}}\,\frac{1}{\pi^{i+2}}\,F(q_xq_y;k_i,s_i)\,2\,q_1\sqrt{s_1}\;\sigma^R(s_1)\delta(k_1^2-s_1)d^4k_1\;\ldots$$

$$2\,q_i\sqrt{s_i}\;\sigma^R(s_i)\delta(k_i^2-s_i)d^4k_i\,\delta^4(q_x+q_y-\Sigma\,k_i). \tag{26}$$

Of course, Eq. (26) could have been derived more directly by summing the series of Feynman graphs shown in Fig. 5 [12]. However, it would have been difficult in that case to have any estimate of the order of magnitude of the neglected graphs and of the error introduced in the different extrapolations in the mass variable.

The dispersion analysis gives very strong arguments implying that the graphs selected in Fig. 5 are the dominating effect in high-energy collisions.

We want to discuss now some qualitative features of the model. Let us consider a fixed incident energy. For each pair of pions one wants to produce one has to add one extra propagator $1/(t-\mu^2)^2$. The average multiplicity will be therefore determined by the competition of the graphs with different number of denominators. For a small number of mesons produced it will be possible kinematically to make all denominators of the order of a few μ^2. For a larger number if will no longer be possible to have all small denominators; those terms will become therefore less and less important. We have thus to expect an average multiplicity much lower than that predicted by the statistical model.

About the angular distributions in a particular production process, the most probable configurations will be those corresponding to the lowest value for the product of all denominators. This happens first of all when all transverse momenta are very small. In the c.m. system we have to expect two jets of particles collimated in the forward and in the backward direction. An approximate calculation shows that the different groups of particles in the two beams will have different energies decreasing rapidly from the maximum available energy to zero.

V. CONCLUSIONS

Let us review briefly the different points which have been raised in this paper.

The empirical basis for the model proposed here comes from the fact that the high-energy total cross-section has a constant and rather large value. This implies that the collisions with large impact parameters give the most important effect. This fact is expressed in a more precise form by showing [see Eq. (4)] that a large cross-section implies a very narrow peak in A(s t) and therefore the two-pion singularities give the dominating effect.

An evaluation of the two-pion exchange effect in terms of the low-energy $(\pi\pi)$ and (πN) cross-section has allowed us to obtain definite expressions for the total cross-sections and for the absorptive part of the scattering amplitude.

The form of this result has suggested to us that a well-defined group of Feynman graphs, namely the "most peripheral", give the dominating effect in high-energy collisions. The reason why the graphs not included in the model are not expected to give a large contribution at high energy, is unitarity, and its limitations on high-energy amplitudes. It is, however, not easy to express those limitations directly for the inelastic graphs.

The model gives unambiguous prediction for all features of high-energy collision; work is now in progress to evaluate numerically the consequences of the model. A detailed comparison between theory and experiment will be most interesting.

One point which might look somewhat surprising is the fact that the model contains only low-energy parameters. Thus, by looking at the general features of high-energy collisions, one does not learn anything essentially new as compared to low-energy physics.

The following example might be useful to clarify the situation. Consider the elastic scattering of high-energy electrons by protons and suppose one asks very general questions such as: in what direction is the majority of the electron scattered and with what cross-section? The answer is that the cross-section is dominated by a Rutherford denominator and that the great majority of the electrons are scattered around the forward direction with a cross-section which depends only on the renormalized charge e which is certainly a low-energy parameter [of course much better known than low-energy $(\pi\pi)$ interaction!].

If one wants to get new and interesting information about the nucleon form factors one has to lose a large factor in intensity and to look for the rare events which take place at large angles.

In the same manner if one wants to test the internal part of the nucleon by means of strong interactions one has to look for those collisions with large momentum transfer which lie outside the range of our model.

Also in this case one has probably to lose an appreciable factor in intensity.

One other important point is that, as pointed out by Chew and Frautschi, our understanding of high-energy phenomena might improve the status of our theories of low-energy scattering.

One can get important information about low-energy scattering by means of the customary dispersion relations at fixed momentum transfer.

Our model allows to obtain the imaginary part of the scattering amplitudes at high energy and one will not be obliged, as in the past, to cut the dispersion integrals above the (3,3) resonance energy. This will probably lead to a reduction of the number of independent parameters one had to introduce in order to explain the main features of low-energy scattering.

*

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APPENDIX

Let us consider the absorptive part of the scattering amplitude in the channel $X + \bar{X} \to Y + \bar{Y}$.

Using the unitarity condition and taking as intermediate states the two pions only we have

$$A_{x\bar{x}}(s,t) = \frac{1}{2(2\pi)^2} \int d^4 q \, F^*_{x\pi}[(q - q_x)^2, t] \, F_{y\pi}[(q + q_y)^2, t] \, \delta(q^2 - \mu^2) \times \times \, \delta[(q_x + p_x - q)^2 - \mu^2] \, . \tag{A.1}$$

Using the fixed momentum transfer dispersion relation we have:

$$F_{x\pi}(s,t) = \frac{1}{\pi} \int \frac{A^1_{x\pi}(s' t)}{s' - s} ds' + \frac{1}{\pi} \int \frac{A^2_{x\pi}(t', t)}{\bar{t}' - \bar{t}} d\bar{t}' \tag{A.2}$$

and substituting in Eq. (A.1) we shall obtain four terms. We are only interested in the singularity of the absorptive part in the s variable as explained in the text, i.e.,

$$A_{x\bar{x}}(s,t) = \frac{1}{\pi} \int \frac{\rho_{\pi\pi}(s' t)}{s' - s} ds' \tag{A.3}$$

and we can drop the crossed terms which gives singularity in $\bar{t}$.

The two remaining terms give contribution to $\rho_{\pi\pi}(s t)$. In our case in which we neglect isotopic dependence, the two intermediate pions are identical and also the two terms give identical contribution. Therefore we shall insert in Eq. (A.1) only the first term of Eq. (A.2). The factor 2 is compensated by the normalization of the two identical particle phase spaces. We obtain:

$$A_{x\bar{x}}(s,t) = \frac{1}{\pi^2} \iint ds_1 ds_2 \, A^*_{x\pi}(s_1, t) \, A_{y\pi}(s_2, t) \, I_{x\bar{x}}(s, t; s_1 s_2) \tag{A.4}$$

where

$$I_{x\bar{x}}(s,t;s_1 s_2) = \frac{1}{\pi} \int \frac{K_0(s', t; s_1 s_2)}{s' - s} ds' \tag{A.5}$$

and

$$K(s,t;s_1 s_2) = -\frac{1}{4}\int d^4q\ \delta(q^2-\mu^2)\,\delta[(q_x+p_x-q)^2-\mu^2]\,\delta[s_1-(q-q_x)^2]\ \times$$
$$\times\ \delta[s_2-(q+q_y)^2]\ . \qquad \text{(A.6)}$$

From Eqs. (A.3), (A.4) and (A.5) we get

$$\rho_{\pi\pi}(s,t) = \frac{1}{\pi^2}\iint ds_1 ds_2\ A^*_{x\pi}(s_1,t)\ A_{y\pi}(s_2,t)\ K(s,t;s_1 s_2)\ . \qquad \text{(A.7)}$$

We note that K is the Mandelstam spectral function of the 4° order graph. This can be seen by direct inspection of Eq. (A.6) or simply by using the general formula (A.7) and substituting for $A(s,t) = \pi\,\delta(s-\bar{s})$ which is the 2° order absorptive part.

This iteration procedure to construct the spectral function of the 4° order graph using the unitarity condition and the analiticity properties (which was first worked out by Mandelstam) can be used in order to obtain the spectral functions of higher-order graphs. For the 6° order diagram in which we are interested, substituting in Eq. (A.7) the absorptive part of the amplitude up to the 4° order

$$A(s,t) = A_2(s,t) + A_4(s,t) = \pi\,\delta(s-\bar{s}) + \frac{1}{\pi}\int \frac{K_0(\bar{s}\,t')}{t'-t}$$

and collecting the crossed terms $A_2^* A_4 + A_4^* A_2$ we get

$$\rho_6(s\,t) = \frac{1}{\pi^2}\iint \frac{ds_A\,dt'}{t'-t}\Big\{K_0(s,t;s_1 S_A)K_0(s_A t';s_2 s_3) +$$
$$+ K_0(s,t;s\ s_3)K_0(s\ t';s_1 s_2)\Big\}\ .$$

In this manner we have verified the K_1 in the text is the 6° order spectral function. In a similar way it is possible to prove that the K are the Mandelstam spectral function of the (n+2)° order graphs.

* * *

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2) S. Drell, Phys.Rev.Letters 5, 342 (1960).
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G.F. Chew and S.C. Frautschi, "A Dynamical Theory for Strong Interaction at Low Momenta Transfers but Arbitrary Energies", UCRL-9510.

4) S. Mandelstam, Phys.Rev. 112, 1344 (1958); Phys.Rev. 115, 1741 and 1752 (1959).

5) Our amplitudes F(s,t) correspond to the Feynman ones multiplied by a factor $\sqrt{2M}$ for each external nucleon line.

6) One might be tempted to approximate the integrals in Eq. (7) by taking in the r.h.s.

$$A(s\,t) = A(s\,o) = 2\,q\,W\,\sigma_T\,. \qquad \text{(I)}$$

One would obtain

$$A(s,t) = \frac{4}{\pi^3}\iiint dt' ds_1 ds_2\, \frac{K(s,t';s_1 s_2)}{t'-t}\, q_1 W_1 \sigma_T(s_1) q_2 W_2 \sigma_T(s_2) \quad \text{(II)}$$

and

$$\sigma_T(s) = \frac{2}{qW\pi^3}\iiint \frac{dt'}{t'} ds_1 ds_2\; K(s,t';s_1 s_2)\; q_1 W_1 \sigma_T(s_1) q_2 W_2 \sigma_T(s_2). \quad \text{(III)}$$

Equation (III) corresponds to what one would obtain by integrating over all energies and angles the peripheral formula of Salzman and Salzman. Now the approximation (I) is only justified when in the integral (7) the variables s_1 and s_2 are so small that the sinularities in t are far from the strip.

We know that Eq. (III) grossly overestimates the total cross-sections at high energies. On the other hand, Eq. (II) for A(s,t) would give a shape for diffraction scattering in complete disagreement with experiment.

7) The reasoning leading to Eqs. (9) and (10) can also be understood in the following manner. Consider for sake of definiteness the case of $\pi\pi$ scattering. We can separate the T matrix into four parts, $T = T_1 + T_2 + T_3 + T_0$, where T_1, T_2, T_3 have the two-pion singularities between $4\mu^2$ and t_{max} (which may be as usual larger than the inelastic singularities at $16\,\mu^2$) in the variables $s, t, \bar{t}$ and T_0 has no low singularities in any variables. In the spirit of the strip approximation one can disregard the effect of T_0; T_1, T_2, T_3 can be computed once the singularities in the strip are

known. Taking the imaginary part (in the s channel) of T we get:

$$A = A_1 + A_2 + A_3 .$$

A_2 and A_3 are the contributions to A given in Eq. (6), A_1 corresponds to A^R of Eq. (9) which has to vanish for s outside the elastic region.

We see that it is in principle possible in terms of elastic scattering to compute the t dependence of $A^R(s\,t)$ and therefore to improve on the simple approximation (10).

8) Here we are making use of the important fact that a perturbation Feynman graph can always be computed by using unitarity both in the s and in the t channel.

For the ladder graphs of Fig. 3 the use of unitarity in channel t allows us to consider only graphs with only four external lines and therefore to use at each step the Mandelstam representation. From the computational point of view, however, one has the big disadvantage of the possibility of many energy denominators which can have both signs giving rise to oscillations.

On the other hand, unitarity in the s channel introduces immediately the amplitudes for the inelastic processes. So its use is at the moment impossible in the framework of a general theory of strong interaction since we have not yet any general representation (like the Mandelstam one) for inelastic amplitudes. However, if we are able, as in our case, to reduce the whole problem to the computation of the <u>well-defined perturbation graphs</u> of Fig. 3, then the unitarity in the s channel has an enormous advantage since it has to be used <u>only once</u> and gives rise to well convergent integrals on a positive definite integrand.

9) R.C. Whitten and M.M. Block, Phys.Rev. **111**, 1676 (1958).

10) R.G. Thomas, Jr., Phys.Rev. **120**, 1015 (1960).

11) An important feature of the model is that all integrations on s_i are limited to the resonance region. This automatically ensures [see Eq. (25)] that the important contribution comes from small values of the different pion momentum transfer t_i. This is not equivalent to the procedure of cutting-off the integrations to small values. Using that procedure and choosing a sufficiently high value of the initial energy, one would still have integrations on large values of s_i.

12) Multijet graphs of the kind shown in Fig. 5 have been first considered by V.B. Berestetsky and Ya. Pomeranchuk, Nucl. Physics **22**, 629 (1961).

* * *

FIGURE CAPTIONS

Fig. 1 : Graphs contributing to Eq. (7).

Fig. 2 : The perturbation graph corresponding to $K_n(s,t;s_1 s_2)$.

Fig. 3 : The perturbation graphs corresponding to $K_i(s,t;s_1 s_2 \ldots s_n)$.

Fig. 4a) : πN scattering angular distribution taken from reference 5) compared with the theroetical diffraction curve [Eq. (21) of Section III] .

Fig. 4b) : πN scattering angular distribution taken from reference 6) compared with the theoretical diffraction curve [Eq. (21) of Section III] and with the improved theory (broken curve).
The theoretical curves are normalized to the first point.

Fig. 5 : Diagrammatic representation of the production process in:

a) $\pi\pi$ interaction; the masses $s_1 \ldots s_i$ will be of the order of $30\ \mu^2$ corresponding to the $\pi\pi$ resonance in the J = T = 1 state or $\sim 4\mu^2$ if there is a strong s-wave interaction at threshold.

b) πN interaction; the mass s_i connected with the nucleon will likely be of the order of W_R^2 corresponding to the (3,3) resonance. The other masses will be similar to those of the $\pi\pi$ case.

c) NN interaction; the two masses s_1 and s_i connected with the nucleon will be $\approx W_R^2$, the others like the $\pi\pi$ case.

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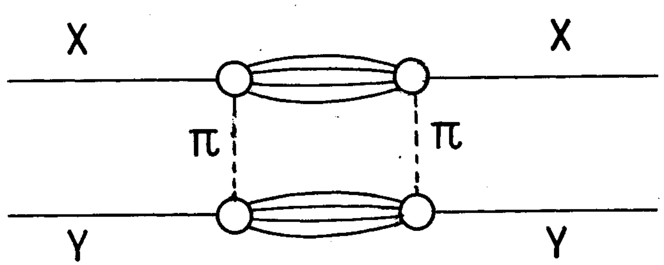

FIG. 1

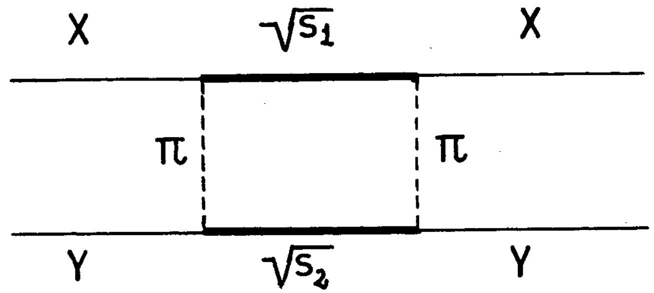

FIG. 2

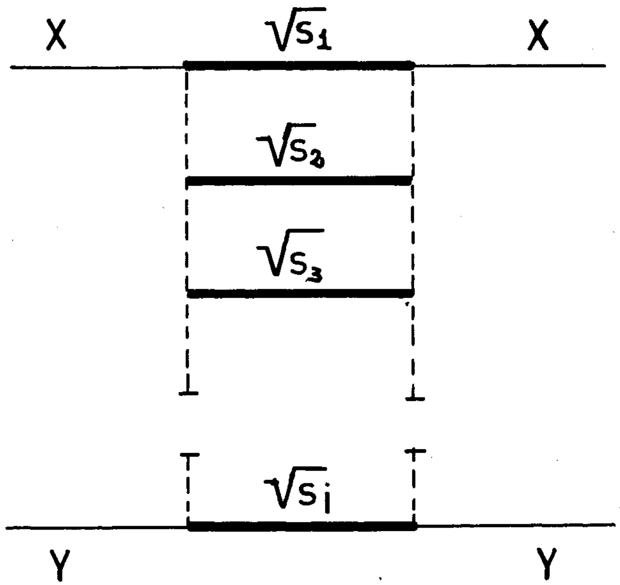

FIG. 3

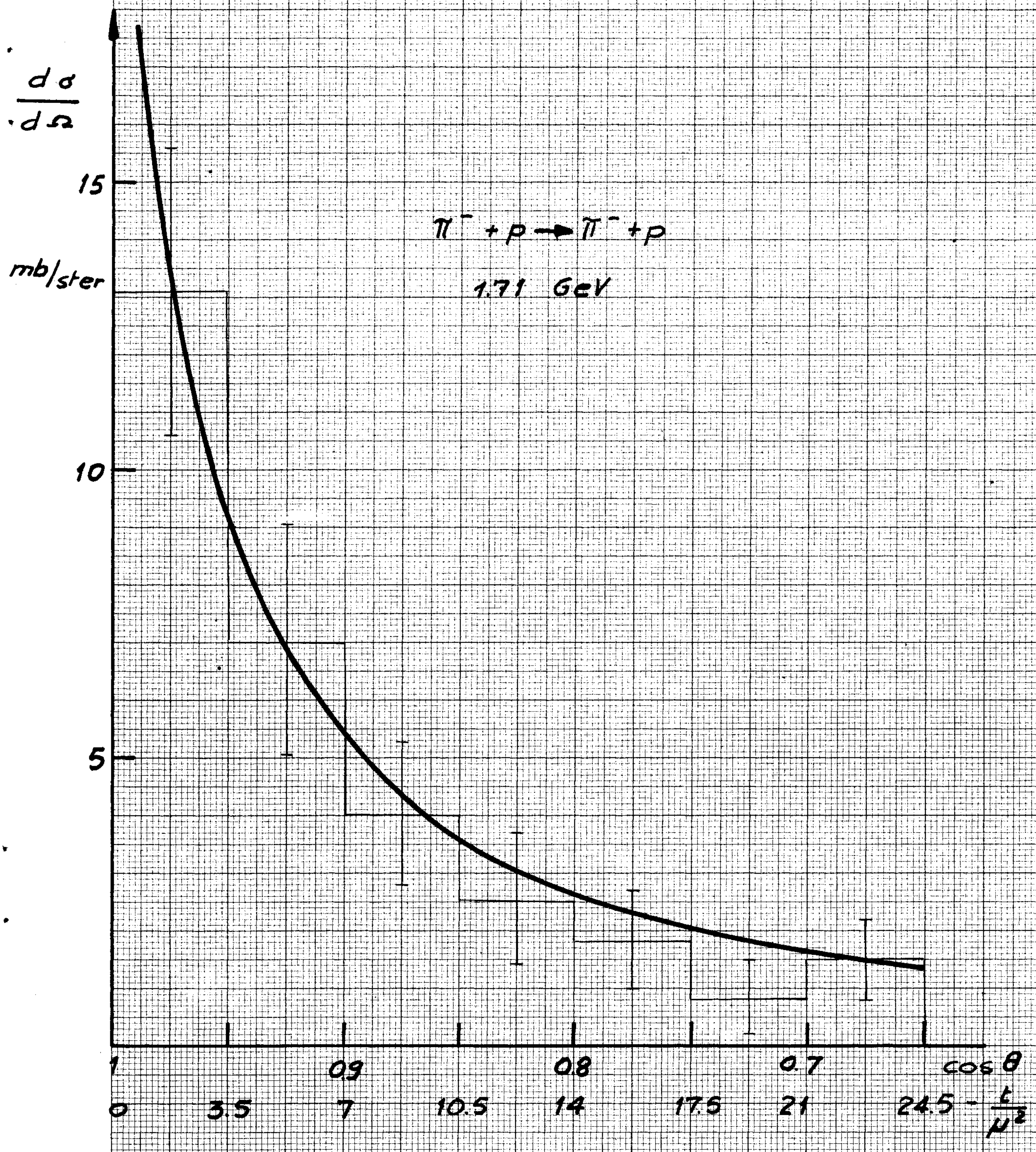

Fig. 4a

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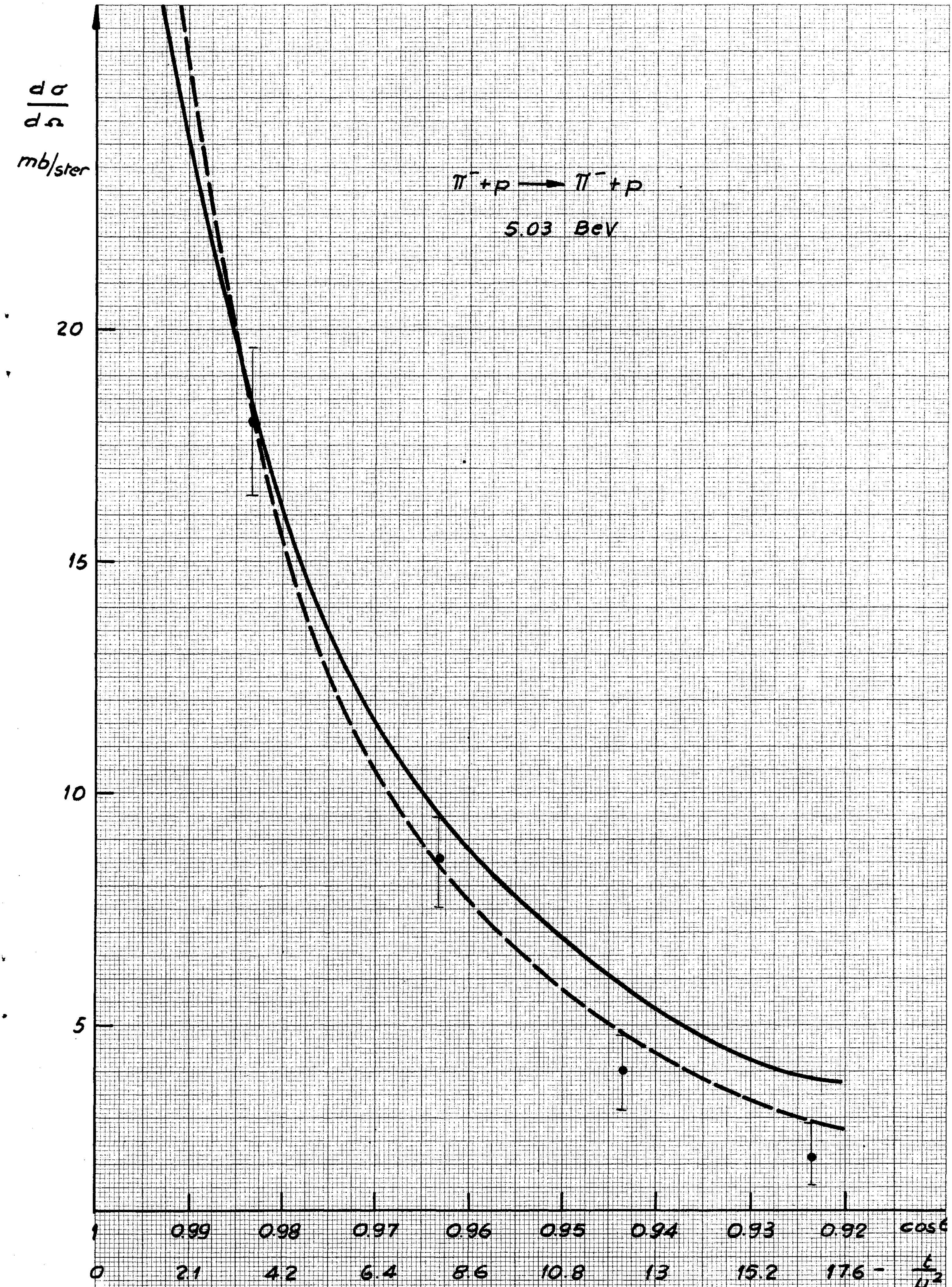

dσ / dΩ
mb/ster
π⁻ + p → π⁻ + p
5.03 BeV
20
15
10
5
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
cosθ
0
2.1
4.2
6.4
8.6
10.8
13
15.2
17.6
− t/μ²

Fig. 4b

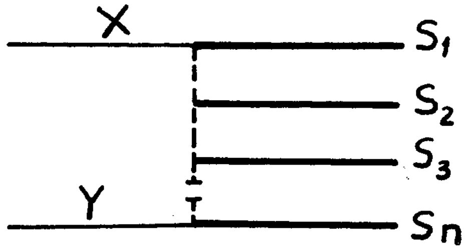

Fig. 5