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A Theoretical Model of Primary Frequency Microseisms

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Summary

A comprehensive mechanism for making quantitative estimates of the seismic response associated with quasi-monochromatic system of pressure waves in shallow water is proposed. Based on generalized linear wave theory, it applies to low-frequency components in the microseism spectrum. A model of pressure response function is derived by considering the agitated surface of open sea with wave trains converging towards the shore-line from scattered sources. As expected, this favours low-frequency swell and is able to isolate energy associated with a specified frequency component.

Lastly, an expression for the local energy balance in the seismic wave guide is deduced and the computed results from this theory is tabulated with those from recent measurements. Broadly speaking the two are sufficiently in agreement except at very low frequency range of the spectrum.

Introduction

Microseisms are generally defined as micro-scale oscillations of the solid earth accompanying disturbed weather systems. Spectral analysis of the seismograph records often reveals two distinct types of oscillations, corresponding to two distinct peaks in the spectrum. The low frequency peak has a frequency equal to that of the associated swell (Haubrich, Munk & Snodgrass 1963; Hasselmann 1963; Hindle & Hatley 1965; Darbyshire & Okeke 1969). The upper frequency peak has frequency components equal to twice that of associated free gravity waves (Banerji 1930; Darbyshire 1950; Longuet-Higgins 1950 and, more recently, Hasselmann 1963; Darbyshire & Okeke 1969).

Extensive work has been done by various investigators to explain the origin and mode of propagations of microseisms. Broadly speaking, it is established that they are launched by the action of the pressure field on the sea floor as free gravity waves propagate. This effective pressure field may be of first-order in the form of a system of quasi-monochromatic wave trains and thus has the same frequency as that of associated elastic modes. There is also a second-order pressure effect shown to be identified with the piston-like vertical motion of the water column produced by randomly distributed groups of standing waves which are in phase. This motion couples the centre of gravity oscillations of the water column to waves of compression. This, in turn, is capable of communicating energy in the form of acoustic vibrations to the sea-bed independent of the depth of the water layer, thus initiating double

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frequency microseisms associated with dominant peak in the microseisms spectrum. The two mechanisms may operate simultaneously and, therefore, excite two different types of earth tremor. This theoretical model sets out to re-examine the characteristics of low frequency microseims identified with the low frequency peak in the spectrum. It is intended to obtain, through slightly more involved analysis, a form of energy density spectrum as a function of wave vectors and bottom topography for each frequency component in the spectrum.

The critical role played by bottom topography of a shallow sea in modulating the gravity wave pressure spectrum has recently become evident and is an important parameter in explaining the observed relationship between low frequency microseims and gravity wave components. Previous analyses on this topic have rested on the bases (a) that the bottom contours are parallel to the coast lines, and (b) that the wave trains on the sea-surface approach the beach normally (Darbyshire & Okeke 1969) or at a small angle of incidence (Hasselman 1963). In a complex irregular sea such as the Irish Sea with a small beach gradient and slowly varying bottom topography, it seems likely that there will be considerable contribution to seismic energy from wave trains approaching the beach obliquely. An attempt is therefore made in this model to incorporate this effect by allowing for the angle of incidence to vary between $-\pi/2$ and $\pi/2$.

Finally, a comprehensive model of energy conversion processes associated with the seismic field as a mathematical framework is presented. However, it is acknowledged that, since the geometry of the bottom topography and other relevant physical variables affecting the propagational characteristics of seismic mode can never be specified in detail, a total agreement of theory with observation can hardly be achieved.

Formulation of source function

It is shown that the amplitude of the bottom pressure field associated with free gravity waves of wave number K_0 is proportional to sech $(K_0 h)$ (Lamb 1932) where h is the depth of water layer; this may be approximated to $\exp(-0{\cdot}08K_0 h)$ (Darbyshire & Okeke 1969). However, in this model, the approach adopted is to derive an expression for the linear wave profile using shallow water theory (Stoker 1957) and to convert this to pressure field by hydrostatic assumption.

It is useful first briefly to review the essential equations of hydrodynamics governing shallow water propagations. Take $R = (x^2+y^2)^{\frac{1}{2}}$ where $x = R\cos\theta$ is the co-ordinate axis perpendicular to and $y = R\sin\theta$ parallel to the shoreline; the particle velocity components in R and θ directions are $q = (q_R, q_\theta)$ respectively. Also take $\eta(R, \theta, t)$ as the sea-surface profile.

Thus,

$$\frac{\partial q_R}{\partial t} + q_R \frac{\partial q_R}{\partial R} + \frac{q_\theta}{R}\frac{\partial q_R}{\partial \theta} = -g\frac{\partial \eta}{\partial R} \tag{1}$$

$$\frac{\partial q_\theta}{\partial t} + q_R \frac{\partial q_\theta}{\partial R} + \frac{q_\theta}{R}\frac{\partial q_\theta}{\partial \theta} = \frac{g}{R}\frac{\partial \eta}{\partial \theta} \tag{2}$$

$$\frac{\partial}{\partial R}[Rq_R(\eta+h)] + \frac{\partial}{\partial \theta}[q_\theta(\eta+h)] = R\frac{\partial \eta}{\partial t}. \tag{3}$$

Expressing the physical quantities involved, η, q_R and q_θ, as a linear series expansion in terms of small parameter $\varepsilon = O(Ka)$ with a as the mean wave amplitude in shallow water and K, the modulation wave number, the flow boils down to small changes superimposed on the ambient state. The perturbed linear distribution functions are:

$$q_R = \varepsilon q_R^{(1)} + \dots \tag{4}$$

$$q_\theta = \varepsilon q_\theta^{(1)} + \dots \tag{5}$$

$$\eta = \eta^{(0)} + \varepsilon \eta^{(1)} + \dots \tag{6}$$

and

$$p = p^{(0)} + \varepsilon p^{(1)} \dots \tag{7}$$

for the pressure p.

The equations of order ε are:

$$\frac{\partial}{\partial R}(Rhq_R^{(1)}) + \frac{\partial}{\partial \theta}(hq_\theta^{(1)}) = R\frac{\partial \eta^{(1)}}{\partial t} \tag{8}$$

$$\frac{\partial}{\partial t} q_R^{(1)} + \dots = -g \frac{\partial}{\partial R} \eta^{(1)} \tag{9}$$

$$\frac{\partial}{\partial t} q_\theta^{(1)} + \dots = -\frac{g}{R} \frac{\partial}{\partial \theta} \eta^{(1)}. \tag{10}$$

From (8), (9), (10) and dropping the suffix (1) then

$$\frac{\partial}{\partial R}\left(Rh\frac{\partial}{\partial R}\right)\eta + \frac{\partial}{\partial \theta}\left(\frac{h}{R}\frac{\partial}{\partial \theta}\right)\eta = -\frac{R}{g}\frac{\partial^2 \eta}{\partial t^2}. \tag{11}$$

Usually, consider a beach of constant gradient α' and assume that the parameter η oscillates with frequency ω_0, then let

$$\eta = \eta_0(R)\cos[m\theta]\exp(i\omega_0 t) \tag{12}$$

m being a constant to be determined later; and $\eta_0(R)$ satisfies the equation

$$R^2\frac{d^2\eta_0}{dR^2} + 2R\frac{d\eta_0}{dR} + \left(\frac{R\omega_0^2\alpha'}{g} - m^2\right)\eta_0 = 0. \tag{13}$$

This is a form of spherical Bessel equation with the solution $\eta_0(R) = (d/R)^{\frac{1}{2}}\eta_0'(R)$, d is a constant with dimension of length to be specified and $\eta_0'(R)$ satisfies the equation

$$R^2\frac{d^2\eta_0'}{dR^2} + \frac{Rd\eta_0'}{dR} - \left[(m^2 - \tfrac{1}{4}) - \frac{R\omega_0^2\alpha'}{g}\right]\eta_0' = 0. \tag{14}$$

From (12) and (14), to give a possible form of wave profile, with unit amplitude in deep water, assume $m = \frac{1}{2}$, then

$$\eta_0 = d^{\frac{1}{2}} R^{-\frac{1}{2}} J_0(2\omega_0 \alpha'^{\frac{1}{2}} R^{\frac{1}{2}} g^{-\frac{1}{2}})$$

and

$$\eta(R, \theta, t) = d^{\frac{1}{2}} R^{-\frac{1}{2}} J_0(2\omega_0 \alpha'^{\frac{1}{2}} R^{\frac{1}{2}} g^{-\frac{1}{2}}) \cos(\theta/2)\exp(i\omega_0 t) \tag{15}$$

$$-(\pi/2) < \theta < (\pi/2)$$

and J_0 is a zero order Bessel function of the first kind and R is measured from the generating zone.

The foregoing analysis is true only if the wave length involved is long compared with depth of shallow water. Fortunately, this is satisfied by swell with period greater then 4 s which is usually a source of low frequency microseisms in a wide area of the Irish Sea. Thus, under this condition, the effect of water layer on the wave pressure becomes negligible. We may, therefore, regard the sea-bed as a dynamical system subjected to a stationary and homogeneous progressive pressure field

$$p^{(1)} = P_{33} = d^{\frac{1}{2}} \rho_{\omega} g R^{-\frac{1}{2}} J_0(2\omega_0 \alpha'^{\frac{1}{2}} R^{\frac{1}{2}} g^{-\frac{1}{2}}) \cos(\theta/2) \exp(i\omega_0 t) \quad (16)$$

and ρ_{ω} is sea-water density.

Spectrum of bottom pressure field

In dealing with motion with symmetry, it seems appropriate to use generalized, Fourier–Bessel transform in obtaining amplitude spectrum. Thus, from (16)

$$d^{\frac{1}{2}} R^{-\frac{1}{2}} J_0(2\omega_0 \alpha'^{\frac{1}{2}} R^{\frac{1}{2}} g^{-\frac{1}{2}}) \cos(\theta/2)$$
$$= \tfrac{1}{2} \int_0^{\infty} \sqrt{CH(K', \theta, \omega_0)^2}\, J_0(K'R) K' dK' + \tfrac{1}{2} \int_0^{\infty} \sqrt{SH(K', \theta, \omega_0)^2}\, J_0(K'R) K' dK' \quad (17)$$

where $CH(K', \theta, \omega_0)$ and $SH(K', \theta, \omega_0)$ are the spectral amplitudes of the bottom pressure fields assumed to be symmetric. K' is the wave number in generalized form.

Applying inversion formula to (17) then,

$$CH(K', \theta, \omega_0) = \alpha' d^{-\frac{1}{2}} (K + K_0)^{-\frac{1}{2}} J_0\left(\frac{\omega_0^2 \alpha' g^{-1}}{K + K_0}\right) \cos\frac{\theta}{2} \quad (18)$$

$$SH(K', \theta, \omega_0) = \alpha' d^{-\frac{1}{2}} (K_0 - K)^{-\frac{1}{2}} J_0\left(\frac{\omega_0^2 \alpha' g^{-1}}{K_0 - K}\right) \cos\frac{\theta}{2} \quad (19)$$

$$-(\pi/2) < \theta < (\pi/2)$$

K_0 and K being the wave numbers associated with gravity and seismic wave fields respectively.

From (18) and (19),

(a) At very low frequency, $\omega_0 \sim 0$

$$CH^2(K', \theta, \omega_0) + SH^2(K', \theta, \omega_0) = \alpha'^2 d^{-1} \left[\frac{1}{K_0 + K} + \frac{1}{K_0 - K}\right] \cos^2\frac{\theta}{2}, \quad (19a)$$

(b) As indicated by Hasselmann (1963), the physical factor essential for the generation of primary frequency microseisms in shallow seas is the amplitude modulation usually identified with low phase velocity bottom pressure fields as sea waves with frequency ω_0 propagate through a beach with variable depth configurations. Consequently, the energy of the pressure wave is spread over a wide range of Fourier components with small wave numbers associated with high phase velocities included. Thus, in the range of small-numbers, K_0 matches K and energy conversion from pressure wave to seismic wave fields takes place.

In this situation,

$$CH^2(K', \theta, \omega_0) + SH^2(K', \theta, \omega_0) = \omega_0^{-2} d^{-1} (g\alpha' \cos^2(\theta/2)). \quad (19b)$$

Define the dissipationless power spectrum

$$S\omega_0(K', \theta) = CH^2(K', \theta, \omega_0) + SH^2(K', \theta, \omega_0). \quad (20)$$

since $J_0(x)$ is maximum near $x = 0$, the power spectrum expressed by (20) favours low frequency modes of propagation. It also increases with decreasing wave number, steep breach gradient as already obtained by previous investigators (Darbyshire &

Okeke 1969). With (20) it may be possible to estimate the energy of each component wave train in a frequency-power spectrum, the integration being performed from $-\pi/2$ to $\pi/2$ with respect to direction of approach.

For a swell, the frequency spectrum is narrow and hence, the energy is mainly clustered within an infinitesimal bandwidth δK centred at peak wave number K_m. Thus, the total energy of a wave train

$$E = \frac{2\pi^2}{K_0^2} \rho_\omega g S_{\omega_0}(K_m, \theta)\, \delta K. \tag{20a}$$

And total pressure effect,

$$\frac{2\pi^2}{K_0^2} P_{33} = \frac{2\pi^2 \rho_\omega g \delta K}{K_0^2} \{CH(K_m, \theta, \omega_0) \cos \omega_0 t + SH(K_m, \theta, \omega_0) \sin \omega_0 t\} \cos \frac{\theta}{2} \tag{21}$$

relating two spectral amplitudes.

Seismic response

Introduction

In computing components of ground displacement associated with microseisms, it seems physically more realistic to incorporate into the model the effect of energy dissipation due to wave damping. Since experiments confirmed that this effect is proportional to the wave frequency, in low frequency components, it may be regarded as of second order in importance. However, in the simplest representation, it is often assumed that energy loss is proportional to particle vibrating velocity. But to obtain a more meaningful form of frequency equation, we take, instead, a simplified form of Voigt solid in which (a) the energy dissipation is proportional to the time rate of the change of strain components (Kolsky 1963), (b) if λ and μ are elastic Lame's constants, λ' and μ' are constants identified with the specific energy dissipation, then, $\lambda'/\lambda = \mu'/\mu$ (Quimby 1925; Thompson 1933; Kolsky 1963). In other words, elastic and dissipative behaviour of the material are alike. This introduces only one independent attenuation constant λ' in the boundary equations.

Relevant equations

In the sea bed, let ϕ and ψ be the single-valued displacement potential functions expressing the propagational characteristics of the pressure and shear waves respectively. Introduce the complex frequency $\omega = \omega_0 + i\delta\omega$ for which $\delta\omega \ll \omega_0$, thus, $\phi = \phi_0(R) \exp(-i\omega t)$, $\psi = \psi_0(R) \exp(-i\omega t)$. To allow for attenuation, $\delta\omega$ must be positive and real. In axial symmetric wave motion,

$$\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2} + \frac{\omega^2}{\alpha^2}\right)\phi_0 = 0 \tag{22}$$

$$\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2} + \frac{\omega^2}{\beta^2}\right)\psi_0 = 0 \tag{23}$$

where α and β are phase-velocities of pressure and shear waves respectively.

As a formal integral representation of solutions of (22) and (23) take

$$\phi \exp(+\omega t) = \int_0^\infty A(K)\, J_0(KR)\, K \exp(-K_\alpha Z)\, dK \tag{24}$$

$$\psi \exp(+i\omega t) = \int_0^\infty B(K)\, J_0(KR)\, K \exp(-K_\beta Z)\, dK \tag{25}$$

where

$$K^2 = K_\alpha{}^2 + \frac{\omega^2}{\alpha^2} = K_\beta{}^2 + \frac{\omega^2}{\beta^2} \tag{25a}$$

$A(K)$ and $B(K)$ are amplitude components. Let U_R and U_Z be linear variables representing the components of ground displacements and defined by

$$U_R = \frac{\partial\phi}{\partial R} + \frac{\partial^2\psi}{\partial R\,\partial Z} \tag{26}$$

$$U_Z = \frac{\partial\phi}{\partial Z} - \frac{\partial^2\psi}{\partial R} - \frac{1}{R}\frac{\partial\psi}{\partial R} \tag{26a}$$

J_1 is a first order Bessel function of first kind such that

$$-J_1(R) = \frac{d}{dR} J_0(R). \tag{27}$$

Then, using equations (24)–(27)

$$U_R(R, Z)\exp(+\omega t)$$

$$= \int_0^\infty K^2[-A(K)\exp(-K_\alpha Z) + K_\beta B(K)\exp(-K_\beta Z)]\, J_1(KR)\, dK \tag{28}$$

$$U_Z(R, Z)\exp(+i\omega t)$$

$$= \int_0^\infty K^2[K_\alpha A(K)\exp(-K_\alpha Z) + K^2 B(K)\exp(-K_\beta Z)]\, J_0(KR)\, dK \tag{29}$$

Boundary conditions

At the sea-floor $Z = 0$,

(a) the tangential stress vanishes and this demands that

$$\frac{\partial}{\partial Z}(U_R) + \frac{\partial U_Z}{\partial R} = 0.$$

i.e. from (28) and (29)

$$2K_\alpha A(K) - (2K^2 - \omega^2/\beta^2)\, B(K) = 0. \tag{30}$$

(b) Vertical component of stress is balanced by the high phase-velocity component pressure of free gravity wave,

i.e.

$$\left(\lambda + \rho_s \lambda' \frac{\partial}{\partial t}\right)\nabla^2\phi + 2\left(\mu + \rho_s \mu' \frac{\partial}{\partial t}\right)\frac{\partial U_Z}{\partial Z}$$

$$= \int_0^\infty K P_{33}(K, \omega_0, \theta)\, J_0(KR)\exp(-i\omega t)\, dK \tag{31}$$

$\alpha^2 = (\lambda + 2\mu)/\rho_s$, $\beta^2 = \mu/\rho_s$, ρ_s is the density of sea bed. Assume Poisson's hypothesis, i.e. $\alpha = \beta\sqrt{3}$. After simplifying, we have only one attenuation parameter λ'. From (28), (29) and (31)

$$(\beta^2 + i\omega\lambda')\{(2K^2 - \omega^2/\alpha^2)\, A(K) - 2K^2 K_\beta B(K)\} = \frac{P_{33}}{\rho_s}. \tag{32}$$

And from (30) and (32)

$$B(K) = \frac{2K_\alpha P_{33}}{\rho_s \Delta(K)} \tag{33}$$

$$A(K) = \frac{(2K^2 - \omega^2/\beta^2) P_{33}}{\rho_s \Delta(K)} \tag{34}$$

where

$$\Delta(K) = (\beta^2 + i\omega\lambda')\{(2K^2 - \omega^2/\alpha^2)(2K^2 - \omega^2/\beta^2) - 4K^2 K_\alpha K_\beta\}.$$

Take

$$F(K) = (\beta/\omega)^2 \Delta(K). \tag{35}$$

Hence,

$$U_Z = \int_{-\pi/2}^{\pi/2} \int_0^\infty \frac{K J_0(KR_0) P_{33}\, dK\, d\theta \exp(-i\omega t)}{\rho_s F(K)}. \tag{36}$$

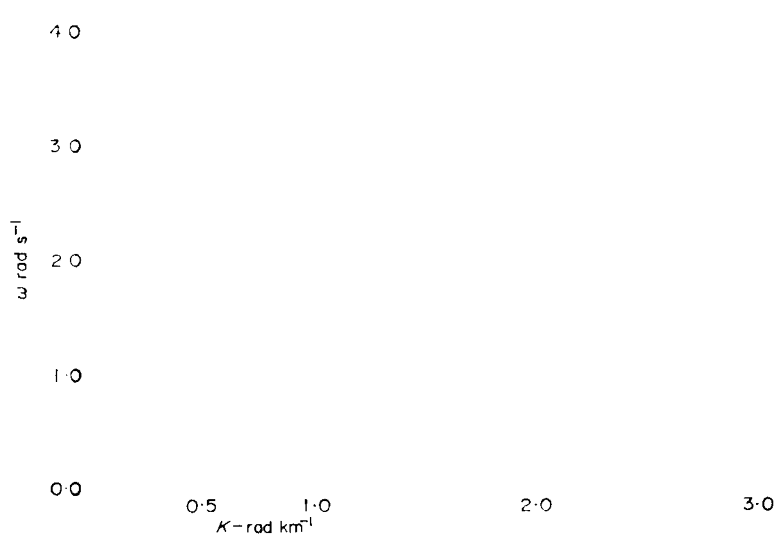
Away from the generating source, the applied pressure is zero.

However, to make the vertical ground displacement still finite and non-zero, $F(K, \omega_0 + i\delta\omega) = 0$ (from this expression, attempt is made to derive dispersion relation shown in the accompanying Appendix and Fig. 1), take $\lambda' = \gamma\beta^{-2}$ then $\delta\omega = 0{\cdot}11\gamma$ and hence $\delta K = \delta\omega/V_g$ where V_g is the propagation velocity of the amplitudes peak. In the frequency range $10^{-3} \leqslant \delta \leqslant 1 c/s$, $\gamma = 2{\cdot}7 \times 10^{-3}$ per s. (Attewell & Ramana 1966) $\delta = \omega_0/2$. Hence from (35)

$$U_Z(R_0, \omega_0) \exp(-0{\cdot}11\gamma t) \cos \omega_0 t = 2 \int_0^\infty \frac{K J_0(KR_0) P_{33}(K, \omega_0) dK}{\rho_s F(K)}. \tag{37}$$

Expressing J_0 in asymptotic form (Jeffreys & Jeffreys 1966)

$$J_0(KR) = \tfrac{1}{2}[H_{s0}(KR) + H_{i0}(KR)] \tag{38a}$$



Secondary and primary frequency dispersion curve

FIG. 1

where

$$H_{s0} = \left(\frac{2}{\pi RK}\right)^{\frac{1}{2}} \exp\left(i(KR-\tfrac{1}{4}\pi)\right) \quad (38b)$$

$$H_{i0} = \left(\frac{2}{\pi RK}\right)^{\frac{1}{2}} \exp\left(-i(KR-\tfrac{1}{4}\pi)\right). \quad (38c)$$

Using (37), (38) on (36)

$$U_Z(R_0, \omega_0) \exp(-0{\cdot}11\gamma t) \cos \omega_0 t$$

$$= \frac{P_{33}(K_m, \omega_0)}{2\rho_s} \sqrt{\left\{\left[\frac{2}{\pi R_0}\right]\right\}} \int_0^\infty \frac{K^{\frac{1}{2}} \cos(KR_0 - \frac{1}{4}\pi)\, dK}{F(K)} \quad (39)$$

K_m is still the wave number identified with the peak of amplitudes spectrum. Since $0 \leqslant \operatorname{Arg} F(K) \leqslant \pi$, the zeros of interest are in upper half plane. As $|K| \to \infty$ so, the integral vanishes as $K^{-7/2}$ where a semi-circular contour is chosen in complex K-place.

Thus, as with similar problems in Seismology (Ewing, Sardetsky & Press 1957)

$$U_Z(R_0, \omega_0) \exp(-0{\cdot}11\gamma t) \cos \omega_0 t = \frac{2P_{33}(K_m, \omega_0)}{\rho_s} \left(\frac{2}{\pi R_0}\right)^{\frac{1}{2}} C_m \quad (40)$$

where

$$C_m = \sum_{n=0}^{N} \left[\frac{\sqrt{K}}{\dfrac{\partial F(K)}{\partial K}}\right]_{K=K_n}. \quad (41a)$$

At resonance

$$K \simeq K_m,\ C_m^2 = \beta^{-2} \sum_{n=0}^{N} \frac{K_n^{-5}}{16} \ (km, s, unit) \quad (41b)$$

and

$$\omega_0^2 \simeq K_\alpha K_\beta \bigg/ \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right). \quad (41c)$$

This theory sets out to analyse the local energy balance in the wave guide. We take R_0 to denote the beach length of gradient α' over which the water layer is negligible; and thus, shallow water gravity waves of frequency ω_0 are associated with appreciable bottom pressure. R_0 is estimated from the relationship $\alpha' R_0 \omega_0 / c_n = O(1)$ where $\alpha' R_0 = h$ (the depth of water layer usually small compared with a given wavelength) and c_n is the characteristic seismic speed in the sea-bed.

Frequency power spectrum

From (39), let

$$U(t)\,\omega_0 = U(R_0, \omega_0) \exp(-0{\cdot}11\gamma t) \quad (42)$$

(42) is a representation of the time series associated with displacement processes of the sea bed. We use frequency power spectrum to obtain the contributions to the total potential energy of the wave fields derived from each frequency in the spectrum. The power per unit frequency associated with the Fourier component with frequency ω_0 is $R_u(\omega_0)$. It is indicated by Darbyshire & Okeke (1969) that with variation of only 1 per cent in wave number, quantitatively,

$$R_u(\omega_0) = \tfrac{1}{2} U(t)_{\omega_0}{}^2 \quad (43)$$

where $U(t)\,\omega_0$ is given by (39) and (42).

Similarly, let $R_p(\omega_0)$ represent the contribution to the energy (per unit frequency) of the bottom pressure field associated with gravity wave of frequency ω_0 and with unit amplitude in deep water. Then, from (20), (21) and (40),

$$\frac{R_u(\omega_0)}{R_p(\omega_0)} = \frac{2\rho_\omega^2 g^2 \gamma^2 S\omega_0(K_m) C_n^2(K_m)}{V_g^2 R_0 K_0^2 \rho_s^2}. \tag{44}$$

Discussion

Here we are interested in gravity waves associated with deep frontal disturbances in sea areas which propagate towards the continental shelf. It seems likely that their sources are not only at the centre of the depression but spread throughout the whole area influenced by severe and prolonged frontal disturbances. Therefore, in computing the amplitude of associated seismic response shown in Table 1, we take into consideration the contributions arising from wave trains approaching the shelf within a wide range of directions. For waves recorded at Rhosneigr, Anglesey used in this model, there were contributions probably from West, North-west, North, South-west and South.

In linear model, appreciable bottom pressure due to individual wave component is of the order of its wave length as re-emphasized in equation (21) using equation (19a) or square of its period using equation (19b). Therefore, crude estimates of bottom pressure were made using the wave refraction diagrams for the Irish Sea. From the same diagrams, we estimated the width d of the breaker-zone for different wave components. These were based on the work of Darbyshire & Okeke (1969).

It appears, therefore, that throughout shallow sea areas considered, swell with period greater than 10 s is identified with appreciable bottom pressure response and also likely to break when d is order of some few metres. However, for components in 5- to 10-s band, the effective depth of water layer is less than 35 m with $d = 0{\cdot}33$ km. Further, consistent with the bottom topography of Irish Sea, we take the beach gradient to be 0·01 when the depth of the sea is less than 35 m and 0·005 otherwise indicating areas of nearly constant depth.

The corresponding results of observation in Table 1 were computed from the records collected by the author on 1967 February 5, with the co-operation of the staff members of Marine Science Laboratories, Anglesey. The microseims and wave

Table 1

Computations of $\frac{R_u(\omega_0)}{R_p(\omega_0)}$ *from equation* (44)

Period in Secs. (s)	Frequency c/sec. (c/s)	Observed $\times 10^{-12}$	Theoretical predictions $\times 10^{-12}$
5	0·20	0·21	0·85
6	0·167	0·65	1·25
	0·150	0·67	1·30 $\alpha' = 0{\cdot}01$
7	0·143	0·65	1·31
8	0·125	0·58	1·10
9	0·110	0·57	1·12
12	0·080	0·25	0·18 $\alpha' = 0{\cdot}01$
15	0·067	0·07	0·02 when $h \leqslant 33$ m
18	0·056	0·06	0·03
20	0·05	0·06	0·02 $\alpha' = 0{\cdot}005$
23	0·043	0·08	0·015 otherwise
25	0·04	0·09	0·013

records were collected from Menai Bridge and Rhosneigr stations respectively. Waves with frequency components less than 0·06 c/s were likely to have penetrated the Irish Sea from the Atlantic and hence, were referred to as Atlantic waves in this analysis.

The type of instruments used in recording, their locations and the processes employed in computing frequency power spectrum from these records were as described by Darbyshire & Okeke (1969).

From Table 1, it appears that the predicted ratios for Irish Sea waves are higher and Atlantic waves lower than observed by about a factor of two. This factor is still lower over the range covered by very low frequency components in the spectrum. To some extent, this may be due to the approximations employed in this model. In addition, probable long periodic fluctuations in the form of background noise associated with thermal turbulence of the air in the vault of seismo-graph described by Hinde & Hartley (1965) might have affected the seismic records used and thus made measurable alterations to the microseisms power spectral densities in the range of very low frequencies.

Nevertheless, the author concludes that the observed and theoretically computed results are sufficiently similar in order of magnitudes and thus, suggests that the method employed is correct to a reasonable extent.

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Appendix

Dispersion equation

$$\frac{\omega^4}{4\alpha^2\beta^2} - \tfrac{1}{2}(\alpha^{-2}+\beta^{-2})\,\omega^2 K^2 + (K^4 - K_\alpha K_\beta K^2) = 0 \qquad (2.1)$$

is the frequency-equation for the whole propagation and may be put in the form $(\omega^2-\omega_1{}^2(K))(\omega^2-\omega_2{}^2(K)) = 0$ where

$$\omega_1{}^2(K) = K^2\left[(\alpha^2+\beta^2) + \left\{(\alpha^4+\beta^4) + \frac{2\alpha^2\beta^2 K_\alpha K_\beta}{K^2}\right\}^{\frac{1}{2}}\right] \qquad (2.2)$$

is the double-frequency branch

$$\omega_2{}^2(K) = K^2\left[(\alpha^2+\beta^2) - \left\{(\alpha^4+\beta^4) + \frac{2\alpha^2\beta^2 K_\alpha K_\beta}{K^2}\right\}^{\frac{1}{2}}\right] \qquad (2.3)$$

is the primary frequency branch.

These two branches are de-coupled, and hence propagate independently.