THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47 St., New York, N.Y. 10017

89-GT-57

A Theory for Wake-Induced Transition

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Abstract

A theory for transition from laminar to turbulent flow as the result of unsteady, periodic passing of turbulent wakes in the free stream is developed using Emmons' transition model. Comparisons made to flat plate boundary layer measurements and airfoil heat transfer measurements confirm the theory.

Nomenclature

a	turbulent-strip production strength
c	airfoil chord
F(x)	fractional function of x
g	turbulent-spot production function
h	heat transfer coefficient
H	shape factor
I(x)	integer function of x
n	number of rotating spokes or bars
R	influence volume, contains all sources of turbulent spots that will pass over a point on the surface
St	Stanton number
t	time
U_s	local spot- or strip-propagation velocity
U_∞	free stream velocity
U(x)	unit step function of x
V	volume in x, z and t space
x	coordinate on surface in streamwise direction
x_t	transition position
y	coordinate normal to the surface
z	coordinate on surface in transverse direction

Greek

α	one-half propagation angle of turbulent spot or strip
γ	intermittency
$\tilde{\gamma}$	time-averaged intermittency
$\delta(x)$	Dirac delta function of x
θ	momentum thickness
ν	kinematic viscosity
τ	wake-passing period
τ_w	wake duration time over location x_{tw}

Subscripts

L	fully-laminar
n	quantity related to natural transition
T	fully-turbulent
w	quantity related to wake-induced transition

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Introduction

One of the main causes of unsteady flow in gas turbines are the wakes from upstream airfoils or obstructions as shown in Fig. 1. These wakes not only impose a free stream with a periodic, unsteady velocity, temperature and turbulence intensity on the surface boundary layer, but they also produce a convective flow either toward or away from the surface as a result of the velocity deficit associated with them. To date, even though these effects have not yet been satisfactorily investigated, the adverse influence of the unsteady wake on both the performance and life of a gas turbine is well recognized. Of particular concern is the associated increase in heat load to the turbine airfoils where, on the suction side, the wakes cause an unsteady laminar-to-turbulent transition independent of any natural transition which might occur there.

Recently, heat transfer measurements have been made using actual turbine components and facilities that model most of the engine operating conditions. This work was done by Guenette et al. (1985) and Dunn and his co-workers (1986a) (1986b) (1986c) (1988). In general, they obtained either time-resolved or time-averaged heat flux measurements on airfoil, endwall and blade tip surfaces. Dunn et al. (1986b) found that the variations in surface heat flux were large compared to the time-averaged values and that they occurred at the fundamental wake-cutting frequency. In addition, comparisons of the measured time-averaged results with current two-dimensional, boundary layer calculation programs showed that the programs fail to satisfactorily predict the heat load over an airfoil surface (Dunn et al. 1986c and Taulbee et al. 1988).

Other unsteady experiments have been conducted in large-scale, low-speed rotating facilities by Hodson (1984) and Dring et al. (1986). Hodson measured the shear stress and loss on the rotor of a single stage turbine, while Dring et al. obtained interrow aerodynamic data and time-averaged stator and rotor heat transfer data for a one-and-

Presented at the Gas Turbine and Aeroengine Congress and Exposition—June 4-8, 1989—Toronto, Ontario, Canada
This paper has been accepted for publication in the Transactions of the ASME
Discussion of it will be accepted at ASME Headquarters until September 30, 1989

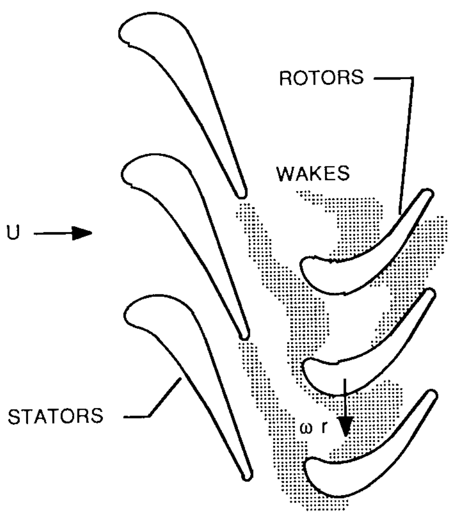

Figure 1. Unsteady wake propagation through a rotor blade row.

a-half stage turbine model. In general, both compared their rotor results to steady flow cascade measurements and found that the unsteady rotor-incident flow affected transition on the suction side. Hodson reported that this caused a 50 percent increase in the time-averaged loss of the rotor. Dring et al. found that it doubled the time-averaged heat load in the laminar region.

The unsteady effects of passing wakes in an airfoil row have been investigated by Ashworth et al. (1985), (1987), Doorly et al. (1985), LaGraff et al. (1988), and Wittig et al. (1988). In each, a rotating wheel of cylinders was used to produce the moving wakes. Doorly et al. measured large unsteady increases in the surface heat flux at the wake cutting frequency and traced the propagation and growth of turbulent regions along the suction surface. All of these investigators found that the wakes directly influenced the onset of transition on the suction side of the airfoil.

In a related investigation, Pfeil and Herbst (1979) and Pfeil et al. (1983) examined the laminar-turbulent transition of a flat-plate boundary layer disturbed by passing wakes. They measured the steady and unsteady velocity components in the flow along a plate positioned downstream of a rotating cylinder of circular spokes aligned parallel to the leading edge of the plate. Besides presenting information about the time-averaged growth of the boundary layer, they showed that the wakes caused the boundary layer to become turbulent during their "impingement" on the plate. This, they pointed out, formed wake-induced transition zones which propagated down the plate such that the time-averaged condition of the boundary layer was composed of an intermittent laminar and turbulent boundary layer. Although not formally stated, they implied that the time-averaged condition of the boundary layer may be obtained from

$$\tilde{f} = (1-\gamma)f_L + \gamma f_T \qquad (1)$$

where f is a boundary-layer flow related quantity, f_L is its laminar value, f_T is its fully-turbulent value, and γ, the intermittency, is the fraction of time the flow is turbulent. The latter varies between zero for laminar flow and unity for fully-turbulent flow. Van Dresar and Mayle (1987) found this expression was also valid for steady flow around a cylinder with an incident wake flow providing the wake intermittency is used.

Recently, Doorly (1987) proposed an intermittency-based model for predicting the effect of a passing wake on the suction side heat flux of a gas turbine airfoil. He assumed that the high turbulence content of the wake produces a turbulent patch in an otherwise laminar boundary layer during each passing. To obtain the time-averaged heat flux, he used eq. (1) and an intermittency determined by computing the temporal position of the wake adjacent to the surface. The intermittency, therefore, depended on the propagation and growth of the wake around the airfoil. A comparison to measurements showed only a qualitative agreement.

In this paper, a model similar to that proposed by Pfeil and Herbst is considered and a theory developed for wake-induced laminar-to-turbulent transition. The theory is based on Emmons' (1951) original idea of natural transition where he considered the propagation and growth of turbulent spots along a surface within a laminar boundary layer. His theory was later extended in light of new measurements by Dhawan and Narasimha (1957) who showed that the intermittency distribution through natural transition is well represented by

$$\gamma_n(x) = 1 - \exp\left[-0.412\left(\frac{x-x_{tn}}{x_{75}-x_{25}}\right)^2\right] \qquad (2)$$

where x is the distance along the surface, x_{tn} is the location of natural transition, and x_{25} and x_{75} are the x positions where $\gamma = 0.25$ and 0.75, respectively. The quantity x_{75}-x_{25} is a characteristic length for transition which must be obtained from either experiments or a correlation.

The extension of Emmons' theory to wake-induced transition invokes the idea that the wake is a production source of turbulent spots so dense that the spots immediately form a turbulent "strip" which propagates and grows along the surface. Therefore, contrary to Doorly's model, the present model assumes that once the turbulent strip is formed by the wake its propagation and growth is independent of the wake. As a result, the theory predicts a wake-induced intermittency distribution without any need to calculate the wake position. Furthermore, it allows one to combine the effects of natural and wake-induced transition.

Theory

Consider a transitional boundary layer on an x-z surface as shown in Fig. 2 where the free stream flow is in the x direction. The fraction of time during which the flow over a point P on a surface is turbulent was shown by Emmons to be given by

$$\gamma(P) = 1- \exp[-\int_R g(P_0)dV_0] \qquad (3)$$

where $\gamma(P)$ is the intermittency at point P, $g(P_0)$ is the rate of turbulent spot production per unit surface area at P_0, dV_0 is an element of volume in an x, z and time t space, and R is an influence volume defined by all the points upstream in the x, z, t space which are sources of turbulent spots that will pass over P. Emmons obtained this expression by considering the propagation and growth of turbulent spots in x, z, t space as shown in Fig. 3. The source at P_0 simply produces a spot that, on the surface (Fig. 2), moves downstream and grows. The conelike volume swept out in x, z, t space depends on the propagation and growth of the spot. The influence volume R in Fig. 3 includes all of the sources affecting point P.

Now consider a production of turbulent spots from two types of sources. In particular, consider that turbulent spots are produced both naturally (natural transition) and by the periodic passing of wakes

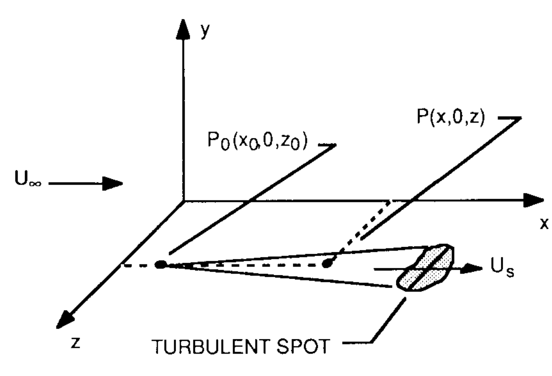

Figure 2. Transitional boundary layer with turbulent spot on x-z surface.

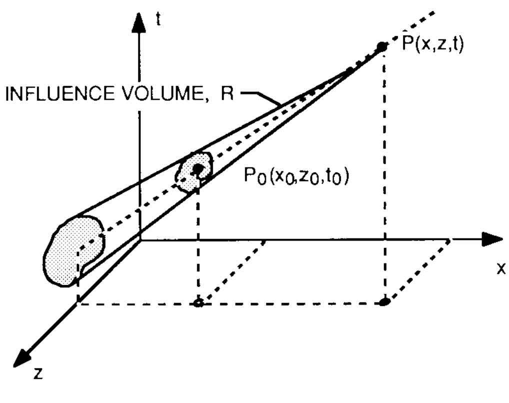

Figure 3. Emmons' x, z, t space and influence volume R for point P.

imbedded in the free stream (wake-induced transition). Assuming that the production function from each source is independent of one another, the function associated with each source can be linearly superposed such that

$$g = g_n + g_w$$

where g_n and g_w are the individual production functions associated with natural and wake-induced transition, respectively. It is not obvious that the turbulent spot production functions are independent. In fact, Pfeil et al. (1983) indicate that a so-called "becalmed region" is formed at the trailing edge of a turbulent spot in which no other spots are produced. This would invalidate a simple superposition model unless its effect is either small or can be somehow incorporated into the production function itself. Without any further information, however, it will presently be assumed that all effects of the wake on transition can be grouped into g_w and that linear superposition provides a good approximation to transition arising from the two sources. Substituting the above expression for g into eq. (3) yields

$$\gamma(P) = 1 - \exp[-\int_{R_n} g_n dV] \exp[-\int_{R_w} g_w dV_0] \tag{4}$$

where the volumes R_n and R_w are now the influence volumes associated with the turbulent spots produced naturally and induced by the wake. These volumes can and, in general, will be different. Consistent with the idea of superposing effects, g_n will be independent of time. Obviously, g_w depends on time. If the wakes pass over the surface with a period equal to τ, the time-averaged intermittency may be obtained by averaging eq. (4) over one period. This provides

$$\tilde{\gamma}(x, z) = \frac{1}{\tau}\int_{t_1}^{t_1+\tau} \gamma(P)dt = 1 - [1-\gamma_n(x, z)][1-\tilde{\gamma}_w(x, z)] \tag{5}$$

where γ_n and $\tilde{\gamma}_w$ are the intermittency functions at the point P associated with the individual source functions for natural and wake-induced transition. These are

$$\gamma_n(x, z) = 1- \exp[-\int_{R_n} g_n dV_0]$$

and (6)

$$\tilde{\gamma}_w(x, z) = 1- \frac{1}{\tau}\int_{t_1}^{t_1+\tau} \exp[-\int_{R_w} g_w dV_0]dt .$$

The expression for γ_n was evaluated by Dhawan and Narasimha and the result has been given above in eq. (2). To arrive at this result, they showed that turbulent spots originate most often at a particular distance downstream on the surface, such that the function g_n is well represented by

$$g_n = (\text{constant}) \times \delta(x-x_{tn})$$

where $\delta(x)$ is the Dirac delta function and x_{tn} is the location where the spots are most often produced (i.e., where natural transition begins). This implies for natural transition that all turbulent spots originate randomly in z and t at x_{tn} such that the only region which contains sources in the influence volume R of x, z, t space (Fig. 3) is on the $x = x_{tn}$ plane.

For wake-induced transition, Pfeil et al. also observed a most probable location for transition, x_{tw}, and that it is in general different than x_{tn}. This implies that all wake-induced turbulent spots originate in x, z, t space on an $x = x_{tw}$ plane. Since the wake induces these spots, spots are produced only when the wake passes over this location. In this sense, the wake acts simply as a switch turning on and off the production of turbulent spots at $x = x_{tw}$. Obviously, if the wake is very weak, i.e., the velocity deficit and turbulence is small, spot production caused by the wake will be less, transition will be more natural-like, and x_{tw} will approach x_{tn}. This implies that both g_w and x_{tw} are functions of the wake strength. In addition, it seems that these quantities may also depend on the wake-passing frequency since a small but finite amount of time is required for the boundary layer to completely respond.

Now assume that the production of wake-induced spots is so intense that these spots coalesce immediately to form a transverse turbulent "strip" across the surface.[1] In x, z, t space then, a wake-induced strip will sweep out a wedgelike, rather than conelike, volume which now depends on the propagation and growth of the strip (not the wake). Furthermore, if the wake is two dimensional, all z = constant planes will be identical and the wake-induced transition problem can be solved in the x-t plane as shown in Fig. 4.

To determine the wake-induced intermittency distribution $\tilde{\gamma}_w$, introduce a new streamwise coordinate x^+, defined as

$$x^+ = \int_{x_{tw}} \frac{dx}{U_s(x)} \tag{7}$$

where $U_s(x)$ is now the local strip-propagation velocity. The quantity x^+ is equivalent to a strip propagation time. If a wake indeed acts as a switch, it may be assumed that the temporal distribution of g_w will be a series of square waves as shown in Fig. 5 where a is the turbulent strip production strength. In this figure, the wakes are passing over the position $x^+ = 0$ with a period τ and with a duration time equal to τ_w. Ex-

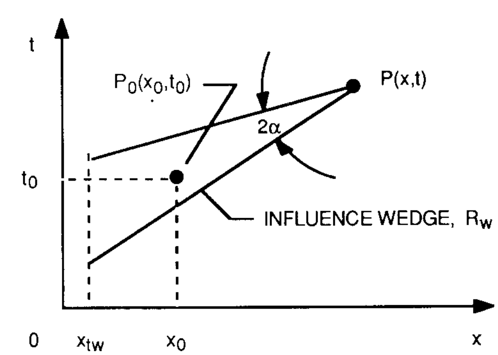

Figure 4. Solution plane x, t for wake-induced transition and influence wedge R_w for point P.

[1] This requires that spots are produced with an average separation distance much less than $2U_s\tau_w tan\alpha$, where U_s is the spot propagation velocity, τ_w is the wake duration time at $x = x_{tw}$, and α is one-half the propagation angle of the spots.

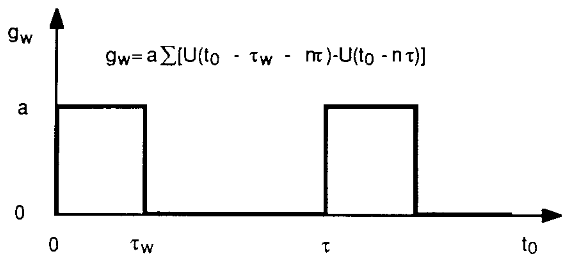

Figure 5. Wake-induced production function for turbulent strips.

pressing this variation in terms of the unit step function, the production function g_w for wake-induced transition may be written as

$$g_w(P_0) = a\delta(x^+) \sum_{n=-\infty}^{\infty}[U(t_0+\tau_w-n\tau) - U(t_0-n\tau)] \tag{8}$$

where $\delta(x^+)$ is the Dirac delta function of x^+, and $U(x)$ is the unit step function of x. Then, from eq. (6), the time-averaged intermittency $\tilde{\gamma}_w$ becomes

$$\tilde{\gamma}_w(x) = 1 - \frac{1}{\tau}\int_{t_1}^{t_1+\tau}\exp\left\{-a\int_{R_w}\delta(x_0^+)\sum_{n=-\infty}^{\infty}[U(t_0+\tau_w-n\tau)-U(t_0-n\tau)]\, dV_0\right\}dt. \tag{9}$$

where a has been taken to be independent of position and time. This expression is evaluated in the Appendix and found to be equal to (see eq. A4)

$$\tilde{\gamma}_w(x) = 1-\exp\left[-b\left(\frac{\tau_w}{\tau}\right)\int_{x_{tw}}\frac{dx}{U_s(x)}\right]$$

where $b = 4a\tan\alpha$ and α is one-half the propagation angle of the turbulent strip. For a constant propagation velocity U_s, $\tilde{\gamma}_w$ is given by the relatively simple expression

$$\tilde{\gamma}_w(x) = 1-\exp\left[-b\left(\frac{\tau_w}{\tau}\right)\left(\frac{x-x_{tw}}{U_s}\right)\right]. \tag{10}$$

A sketch of this function for several values of τ_w/τ is provided in Fig. 6. As might be expected, increasing τ_w/τ produces an earlier time-averaged transition to fully-turbulent flow. A comparison between the distributions for natural transition (eq. 2) and wake-induced transition is shown in the insert.

Substituting eqs. (2) and (10) into eq. (5), yields the time-averaged intermittency (for constant U_s) accounting for both natural and wake-induced transition. This is given by

$$\tilde{\gamma}(x) = 1 - e^{-0.412\left(\frac{x-x_{tn}}{x_{75}-x_{25}}\right)^2} e^{-b\left(\frac{\tau_w}{\tau}\right)\left(\frac{x-x_{tw}}{U_s}\right)} \tag{11}$$

Comparisons of the above expressions for $\tilde{\gamma}_w(x)$ and $\tilde{\gamma}(x)$ to unsteady, wake-disturbed flat plate and airfoil experiments will now be made.

Comparisons to Experiments

The first comparison is made to the data presented by Pfeil and Herbst. They measured the time dependent velocities in a transitional boundary layer on a flat plate positioned behind a rotating cylinder cascade. Measurements were obtained for 0, 3, 9, 18, 36 and 90 spokes in the cascade. Values of $\tilde{\gamma}_w(x)$ were obtained from their reported shape factor distributions. Using eq. (1),

$$\tilde{\gamma}_w(x) = \frac{H-H_L}{H_T-H_L}$$

where H, H_L and H_T are the time-averaged shape factors for transitional, fully-laminar and fully-turbulent flow, respectively. Pfeil and Herbst's data for no spokes was used for the fully-laminar shape factors, while their data for 90 spokes was used for the fully-turbulent values. This is in agreement with their conclusions regarding the state of the time-averaged boundary layer. The results of these calculations are presented in Fig. 7 where $\tilde{\gamma}_w$ is plotted against a modified streamwise distance n(x-0.04). Here, n is the number of rotating spokes. The data was found to collapse best for a shift in the x distance of 0.04 m which consequently corresponds to the beginning of the wake-induced transition. The best fit of this data using eq. (10) is given by

$$\tilde{\gamma}_w = 1- e^{-0.733n(x-x_{tw})} \tag{12}$$

with x_{tw} = 0.04 m. The agreement is excellent. Pfeil and Herbst indicate that the wakes from the individual spokes interfered with one another when n = 90. Hence, $\tau_w/\tau \approx n/90$. Using this and comparing

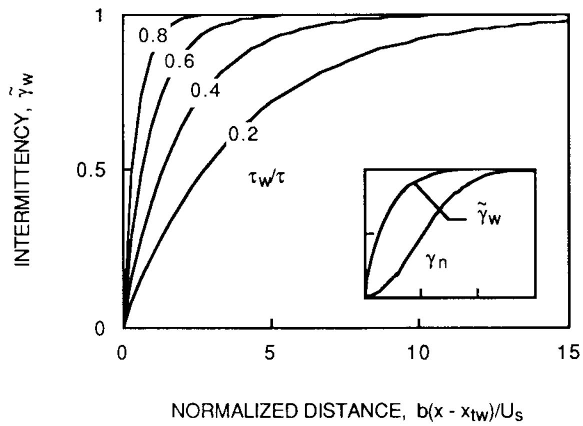

Figure 6. Wake-induced intermittency distribution for several wake widths τ_w/τ and comparison between natural and wake-induced intermittency distributions.

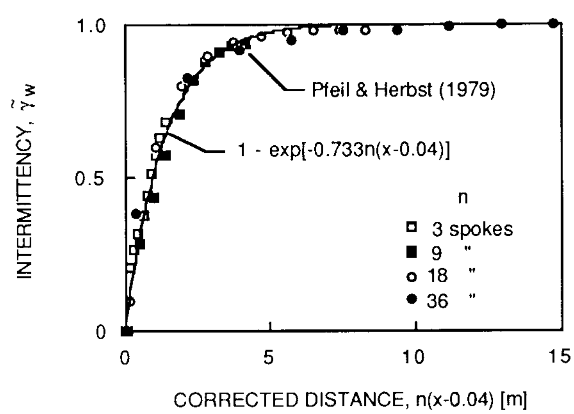

Figure 7. Flat-plate wake-induced intermittency data of Pfeil and Herbst (1979) and comparison to theory.

the exponent in eq. (12) to that in eq. (10), one obtains $b/U_s = 66$ m^{-1}. The momentum thickness Reynolds number at $x_{tw} = 0.04$ m is about 145.

Next, a comparison is made with the airfoil data of Dring et al. (1986). They obtained the time-averaged heat transfer on a first-stage rotor in a large-scale turbine facility. Heat transfer measurements on an airfoil in a steady cascade facility having the same profile were also obtained. Figure 52 in their report contains both the time-averaged and steady results plotted in the form of Stanton number against the distance along the surface of the airfoil. These results are for an airfoil with an axial chord of 161 mm and an inlet velocity of 22.8 m/s. Their figure also contains the predictions for a fully-laminar and fully-turbulent flow over the airfoil. Using eq. (1) and the information in this figure, intermittency can be determined from

$$\gamma(x) = \frac{St-St_L}{St_T-St_L}$$

where St, St_L, and St_T are the transitional, fully-laminar and fully-turbulent Stanton numbers, respectively. Their predicted values of Stanton number were used for the fully-laminar Stanton number St_L and fully-turbulent Stanton number St_T. As pointed out by Sharma (1987), the intermittency calculated from fluid mechanic measurements does not necessarily have to correspond to that calculated from heat transfer results in flows with large pressure gradients. However, if the Pohlhausen pressure-gradient parameter $(\theta^2/\nu)(dU_\infty/dx)$ (where θ is the momentum thickness) is less than 0.005, i.e., the free-stream velocity U_∞ is nearly constant, the difference between the two is negligible. The data examined herein falls into this category.

The results are shown in Fig. 8 where the open symbols correspond to the results from the stationary cascade data and represent γ_n. The closed symbols correspond to those from the rotating turbine facility and represent $\tilde{\gamma}$. The figure shows intermittency plotted against the streamwise distance along the suction side of the airfoil divided by the airfoil's axial chord c. The data for the stationary cascade (open symbols) were used to determine the unknown quantities in eq. (2) for natural transition. The best fit of eq. (2) to this data provides $x_{tn}/c = 0.12$ and $(x_{75}-x_{25})/c = 0.35$ and is shown in Fig. 8 marked as eq. (2). Substituting these values into eq. (11) the best fit to the data for the rotating turbine airfoil is given by

$$\tilde{\gamma}(x) = 1 - e^{-0.412\left[\frac{x/c-0.12}{0.35}\right]^2} e^{-1.62\left[\frac{x-x_{tw}}{c}\right]} \qquad (13)$$

with $x_{tw}/c = 0.05$. This distribution is also shown in Fig. 8 marked as eq. (13). The agreement is good considering the scatter in the data. From Fig. 27a of Dring et al., which provides the turbulence distribution in the wake of the upstream stator row, one finds $\tau_w/\tau \approx 0.18$. Using this and comparing the exponents in eqs. (11) and (13), one obtains $b/U_s = 56$ m^{-1} which is close to that obtained from the flat plate results. In addition, notice that x_{tw} is less than x_{tn}.

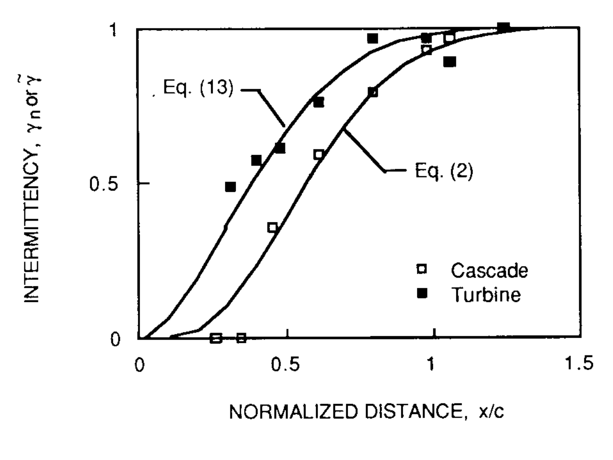

Figure 8. Large-scale turbine intermittency data of Dring et al. (1986) and comparison to theory.

The final comparison is made using data from heat transfer tests on an airfoil in a full-scale, stationary cascade with an unsteady wake generator of rotating bars placed upstream. The apparatus and experimental technique have been completely described by Wittig et al. (1988) where some preliminary results were given for wakes generated by rotating four bars. More recently, measurements of heat transfer on the same airfoil were made for no bars, 4, 7 and 14 bars. Measurements without bars were also made with a turbulence grid placed upstream to force transition to turbulent flow on the suction side of the airfoil. All of the results are shown in Fig. 9 where the distributions of heat transfer coefficient are plotted against the surface distance divided by the airfoil's chord. Each data set in the figure represents an average of four to five test runs with a scatter of less than five percent. For this series of tests, the inlet flow velocity is 80 m/s and the angular velocity of the rotating bars is 262 radians/s. The airfoil chord is 82 mm. The data shown here is just a portion of what will be reported and discussed more completely elsewhere. For the present, however, using eq. (1) and the information in this figure, intermittency can be determined from

$$\gamma(x) = \frac{h-h_L}{h_T-h_L}$$

where h, h_L, and h_T are the transitional, fully-laminar and fully-turbulent heat transfer coefficients, respectively. The values for the fully-laminar heat transfer coefficient h_L were obtained by extending the no bar results further along the laminar-flow, flat-plate, heat transfer correlation (dashed line in the figure), while the results with the upstream grid were used for the fully-turbulent heat transfer coefficient h_T. The intermittency calculated in this manner is plotted in Fig. 10 for the various number of bars. The no bar case shown in this figure corresponds to the intermittency distribution caused by natural transition. The best fit of eq. (2) to this data is obtained using $x_{tn}/c = 0.95$ and $(x_{75}-x_{25})/c = 0.1$ and is shown in Fig. 10 as the line marked eq. (2). Substituting these values into eq. (11), the best fit to the data for the rotating bars is given by

$$\tilde{\gamma}(x) = 1 - e^{-0.412\left[\frac{x/c-0.95}{0.1}\right]^2} e^{-0.08n\left[\frac{x-x_{tw}}{c}\right]} \qquad (14)$$

This distribution is shown in Fig. 10 for the different number of bars as lines passing through the data. The agreement is remarkably good and supports the assumption of superposing natural and wake-in-

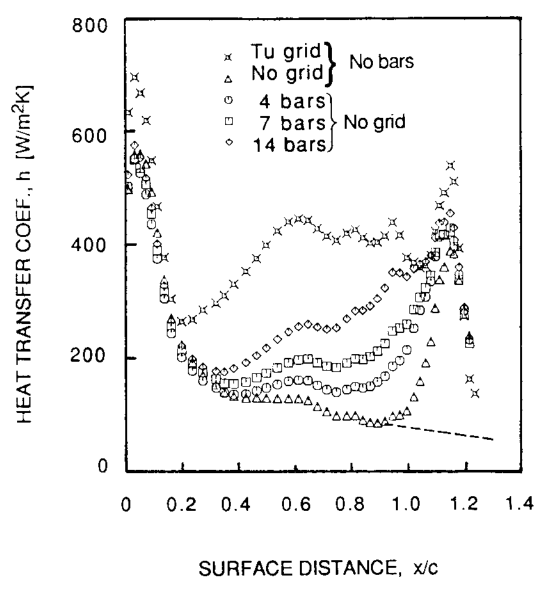

Figure 9. Heat transfer distribution on an airfoil for different number of rods rotating at ω = 262 radians/s.

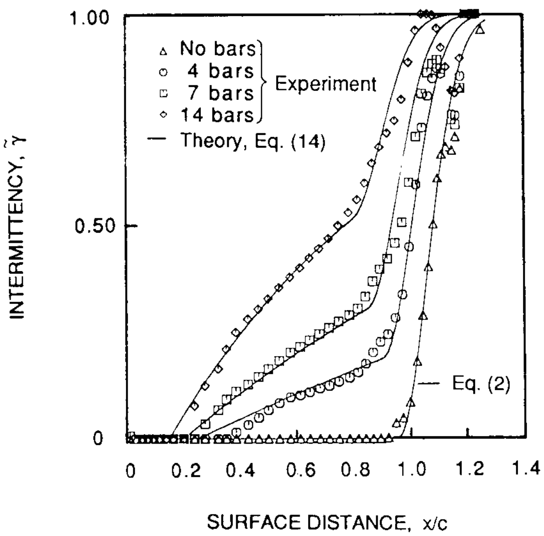

Figure 10. Airfoil wake-induced intermittency distribution for different wake durations and comparison to theory.

duced transitional effects. For these tests, however, it was found that both x_{tn} and x_{tw} changed slightly. The best fit was obtained when $x_{tn}-x_{tw} = 0.65c$ and $x_{tw} = 0.25c$, 0.20c, and 0.15c for 4, 7, and 14 bars, respectively. From hot-wire measurements downstream of the rotating bars in the airfoil's leading edge plane, it was found that $\tau_w/\tau \approx 0.14$ for seven bars. Using this and comparing the exponents in eqs. (11) and (14), one obtains $b/U_s = 49\ m^{-1}$.

The values of b/U_s obtained through the above comparisons are summarized in the following table:

Table of Production Strengths

	b/U_s, m^{-1}
Pfeil and Herbst	66
Dring et al.	56
Present work	49

Since these values depend directly on the value of the wake-duration time-fraction, τ_w/τ, they are influenced by what was used to determine it and the accuracy in measuring it. In the present set of comparisons, τ_w/τ was obtained three different ways; neither of which is presently known to be more correct than the other. But it is suspected that they are nearly consistent with one another such that the resulting values of b/U_s are comparable. If this is so, and until a more complete series of tests are completed, it appears that b/U_s is nearly independent of the test situation and roughly equal to $57\ m^{-1}$.

Conclusions

A model was described and a theory developed to predict wake-induced transition. By assuming that the wake simply increases the production of turbulent spots in a laminar boundary layer such that they merge to form a turbulent strip, the time-averaged intermittency distribution for wake-induced transition was obtained.

In contrast to natural transition where the intermittency depends exponentially on the square of the distance from the inception of transition, the intermittency for wake-induced transition depends exponentially on the distance itself. As may be expected, it also depends on the duration time of the wake in the incident flow. The important wake-induced transition parameter was found to be $\tau_w b(x-x_{tw})/\tau U_s$ where b is related to the turbulent-strip production rate.

Comparing the theory to data showed very good agreement. For all of the tests considered, it appears that the quantity b/U_s is nearly independent of the test situation and roughly equal to $57\ m^{-1}$. For Pfeil and Herbst's data, the momentum thickness Reynolds number corresponding to the beginning of wake-induced transition x_{tw} was found to be about 145. Since it seems, however, that both b and x_{tw} depend at least on the turbulence in the wake, additional data is required before more can be said.

In addition, it was found that the effects of natural and wake-induced transition can be superposed. This provides an important relation for the gas turbine designer who must determine the time-averaged heat load or aerodynamic loss for a turbine airfoil. In either case, the local conditions on the suction side of the airfoil may be obtained from $\tilde{f} = (1-\tilde{\gamma})f_L + \tilde{\gamma}f_T$ where $\tilde{f}$ is the time-averaged, local boundary-layer flow related quantity (such as the heat transfer coefficient), f_L is its laminar value, f_T is its fully-turbulent value, and $\tilde{\gamma}$ is the time-averaged intermittency obtained from eq. (11).

Acknowledgement

One of the authors, R.E. Mayle, is truly indebted to the Alexander von Humboldt Stiftung, Federal Republic of Germany and his host Professor S. Wittig. The theory presented in this paper was developed during his tenure at the Institut für Thermische Strömungsmaschinen, Universität Karlsruhe as a recipient of the Alexander von Humboldt Award for Senior Scientist.

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Appendix

The task is to evaluate the time-averaged, wake-induced intermittency given by eq. (9) in the text, viz.

$$\tilde{\gamma}_w(x) = 1 - \frac{1}{\tau}\int_{t_1}^{t_1+\tau} \exp\left\{-a\int_{R_w}\delta(x_0^+)\sum_{n=-\infty}^{\infty}[U(t_0+\tau_w-n\tau)-U(t_0-n\tau)]\,dV_0\right\}dt\,. \tag{A1}$$

where $dV_0 = dx_0^+ dt_0$ for a two-dimensional source. Define the argument of the exponential function within one period τ as $G(x^+,t)$. Then, integrating with respect to x_0^+, one obtains

$$\tilde{\gamma}_w(x) = 1 - \frac{1}{\tau}\int_{t_1}^{t_1+\tau}\exp(G(x^+,t))dt \tag{A2}$$

with

$$G(x^+,t) = -a\int_A \sum_{n=-\infty}^{\infty}[U(t_0+\tau_w-n\tau)-U(t_0-n\tau)]\,dt_0$$

where A is the region in R_w at $x^+ = 0$, since the only sources of turbulent strips are at $x^+ = 0$. If a turbulent strip spreads in a linear fashion with a half-angle spread of α, A lies on the t axis between $t-x^+(1+2\tan\alpha)$ and $t-x^+(1-2\tan\alpha)$ for a point $P(x^+,t)$ as shown in Fig. A1. The quantity $G(x^+,t)$ can be evaluated by noting that the order of integration and summation may be interchanged since A is not a function of t_0. This provides

$$G(x^+,t) = -a\sum_{n=-\infty}^{\infty}\int_A [U(t_0+\tau_w-n\tau)-U(t_0-n\tau)]\,dt_0\,. \tag{A3}$$

The integral is rather easy to evaluate since the integrand is a single square wave of width τ_w starting at $t_0 = n\tau-\tau_w$. The only time there is a contribution to the integral is when $P(x^+, t)$ is positioned such that region A contains part of this wave. Defining the integral as $G_n(x^+, t)$, one obtains

$$G_n(x^+,t) = \begin{cases} 0 & ;\ t \le n\tau \\ t-n\tau & ;\ n\tau \le t \le n\tau+\{\tau_w\} \\ \{\tau_w\} & ;\ n\tau+\{\tau_w\} \le t \le n\tau+\{4x^+\tan\alpha\} \\ \tau_w+4x^+\tan\alpha-(t-n\tau) & ;\ n\tau+\{4x^+\tan\alpha\} \le t \le n\tau+4x^+\tan\alpha+\tau_w \\ 0 & ;\ t \ge n\tau+4x^+\tan\alpha+\tau_w \end{cases}$$

for $4x^+\tan\alpha \ge \tau_w$. For $4x^+\tan\alpha \le \tau_w$, the quantities τ_w and $4x^+\tan\alpha$ in curly braces should be exchanged. Consider first the case for $4x^+\tan\alpha \ge \tau_w$, i.e., distances not too close to the wake-induced transition point. Substituting the above expressions into eq. (A3) and summing, one obtains

$$G(x^+,t) = -a\sum_{n=-\infty}^{\infty}G_n(x^+,t) = -a[\tau_w I(\frac{4x^+\tan\alpha}{\tau}) + G_I - G_{II}]$$

where I(x) is the integer function of x, i.e., I(2.34) = 2,

$$G_I = \begin{cases} t & ;\ 0 \le t \le \tau_w \\ \tau_w & ;\ \tau_w \le t \le \tau \end{cases}$$

and

$$G_{II} = \begin{cases} 0 & ;\ 0 \le t \le \tau F(\frac{4x^+\tan\alpha}{\tau}) \\ t-\tau F(\frac{4x^+\tan\alpha}{\tau}) & ;\ \tau F(\frac{4x^+\tan\alpha}{\tau}) \le t \le \tau F(\frac{4x^+\tan\alpha}{\tau})+\tau_w \\ \tau_w & ;\ \tau F(\frac{4x^+\tan\alpha}{\tau})+\tau_w \le t \le \tau \end{cases}$$

where F(x) is the fractional function of x, i.e., F(2.34) = 0.34. Substituting this into eq. (A2) yields

$$\tilde{\gamma}_w(x) = 1 - \frac{1}{\tau}\int_{t_1}^{t_1+\tau} e^{G(x^+,t)}dt = 1 - e^{-a\tau_w I(\frac{4x^+\tan\alpha}{\tau})}\frac{1}{\tau}\int_{t_1}^{t_1+\tau} e^{-a(G_I-G_{II})}dt.$$

The integral can be evaluated quickly for $a\tau_w \ll 1$ by expanding the exponential function in a series, keeping only the first two terms, and integrating. This provides, after a some manipulation,

$$\tilde{\gamma}_w(x) = 1 - e^{-4a(\frac{\tau_w}{\tau})x^+\tan\alpha} \tag{A4}$$

Evaluating $a\tau_w$ by comparing the above expression to experimental results (done in the text), it is found that the average $a\tau_w$ for those cases examined is about 0.3. Although not much, much smaller than unity, the error incurred in assuming so is only about fifteen percent and not worth the additional complexity. So the final result is considered to be well represented by eq. (A4).

Now consider the case where $4x^+\tan\alpha \le \tau_w$, i.e., close to the wake-induced transition point. For $4x^+\tan\alpha + \tau_w \le \tau$, the only contribution

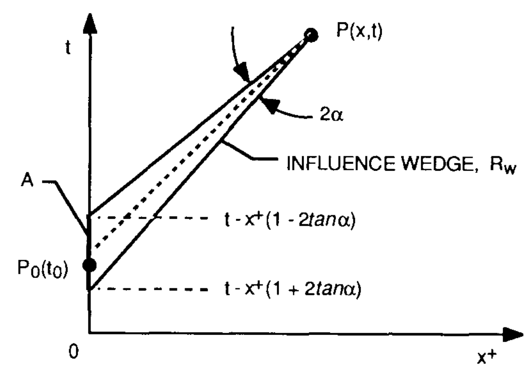

Figure A1. Solution plane for wake-induced transition.