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A Theory of Geomagnetic Micropulsations II

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Abstract

The amplitude spectrum of micropulsation signal at the surface of the earth is essen-tially determined by the resonances of the transmission path. On the other hand the power spectral density depends both on the squared amplitude of transmission and the occurrence or persistence probability. This latter probability is a function of the sensitivity
of the overall transmission on the variation of the path length in the outer magnetosphere.
A crude analysis based on this sensitivity yields spectral peaks at time periods of approxi-
mately 2.5, 6, 11 and 20 seconds with a slope of 6 db/octave at long time periods only.

1. Introduction

A theory of geomagnetic micropulsations was proposed by Sen (1968), henceforth referred to as paper I. It was shown that hydromagnetic waves generated by an instability on the boundary of the magnetosphere are transmitted through the magnetosphere and received on the surface of the earth as micropulsations. The characteristic wavelengths of the unstable waves are determined by the propagation of these waves on the bounded curved surface on the magnetospheric boundary. This and the minimum hydromagnetic scale length given by ion gyro-radius, yield a bandwidth of micropulsations at the source to be a few tenths of a second to hundreds of seconds. The transmission of micropulsations through the entire path in the magnetosphere from its outer boundary to the surface of the earth was discussed in terms of hydromagnetic transmission lines.
It was shown that the resonances of the lower exosphere and upper ionosphere are usually masked out by the slightest variation in the path length in the outer magnetosphere. This variation is normally expected due to variable solar wind conditions. The most likely resonances in the transmission were found to be due to the path in the outer magnetosphere and to have time periods of approximately 935, 311, 134, 103 and 71 seconds. Less likely resonances due to the path in lower exosphere and upper ionosphere were found to have time periods of approximately 39, 14, 9, 7 and 4 seconds.
We use the same symbols as used in paper I and repeat below the key analytical results
of paper I which will be used in this paper. The propagation path of micropulsations was considered in terms of a layered model. The five layers were considered as five pieces of hydromagnetic transmission lines. The overall transmission coefficient To was found in Eqs. (13) and (14) of paper I as

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T0=IT0' T2; T0' T3 T4 T5 (1)

where the subscripts 1 through 5 stand for the designation of the layer. TO' represents the transmission through the large non-dissipative portion of the path and 1T21 represents the attenuation through the lower ionosphere. We single out the expression of 751 from Eq. (2)) in paper I and repeat it below for our present purpose.

IT5) \/5((X42+R42+I52)+{(X42+R42+i2)2-(2Z5R4)2]1/2sin(2a515/T+V)} 1/2
where CV=tan-1[(i 52 R42-X42)/(25X4)} (2)

and as in paper I T without any subscript stands for time period of micropulsations. The long period resonances, when the transmission of lower exosphere and upper ionosphere I T3 and I T4 are off-resonance, were found with the help of Eq. (22) as

IT01Imax'" T5max (X42+R42+152)'/2/R4
at T given by tan(a5l5/T)5/X2 (3)

The short period resonances of the lower exosphere and upper ionosphere I T3 and I T4 pro-duce significant transmission, when these coincide with the "spatial" resonance of IT5 of the outer magnetosphere. According to Eqs. (24) and (25) this is given by

ITO'Isp. max T3 T4 (X42+R42+i752)1/2/R4

at 15 given by 15=T tan-1-25X 4 (4)2 a5 i52-X42-R42

where the value of I T31I T4I is taken from Fig. 5 of paper I
As in paper I, rationalized MKS units are used throughout this paper.

2. Further Study of Transmission Characteristics

2.1 Sensitivity of Transmission on the Path Length in the Outer Magnetosphere
We have already demonstrated in Figs. 7 and 8 of paper I the extreme sensitivity of the overall transmission on the variation of the path length in the outer magnetosphere. Now to be quite specific, let us calculate a definite measure of the sensitivity of overall transmission I To' on l5 as follows. The percentile variation % in l5 required to reduce TO' by a factor of N can be deduced from Eqs. (1), (2) and (4) to be

1%=(T/14.6)cos-1[l+2(1-N)(-5R4)2/(X42+R42+i152)2] (5)

For Ri and XZ in the above equation Eqs. (15) through (18) of paper I have to be used. Fig. 1 shows a plot of o % versus T for N=10. It is clear from this figure that a variation of the order of only 1% in the path length of the outer magnetosphere can completely quench the reception of micropulsation signals on the surface of the earth. As pointed out in paper I the physical origin of this sensitivity lies in the extremely narrow bandwidth of such sharp resonances and the path length variations equivalent to these. Furthermore, smaller the value of o%, higher will be the probability of quenching of the micropulsation signal of the corresponding time period. In other words d% is a measure of the persistence or oc-currence probability of micropulsations. With this interpretation of o%, the sharp peaks

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in Fig. 1 clearly indicate that the occurrence probability of micropulsations of time periods 2.5, 6, 11 and specially 20 seconds should be much higher than others. The peaks of % corresponding to these time periods are so sharp as to suggest almost single frequency sinusoidal signals, which is a characteristic of micropulsation reception. Because of sharply higher occurrence frequency, one should expect spectral lines or peaks at time periods 2.5, 6,11 and specially 20 seconds in the power spectrum of micropulsations. This is in striking agreement with Davidson (1964) who finds a first group of spectral peaks at T-5 and 10 seconds and a second group at T-17 to 70 seconds. In the second group, the daytime peaks occur mostly at T-26 seconds and sometimes at T-45 seconds. As our calculations are based on the dayside of the magnetosphere, we can only compare the daytime peaks at T-26 and 45 seconds with our estimate of T-20 seconds.
It is pointed out that similar effects will be caused by variations in the length of the other parts of the transmission path. Though this certainly is the case with the path lengths 14 and 13 in the lower exosphere and upper ionosphere, to some degree, the effect is negligible for the path length t2 in lower ionosphere. But the variations in the path length 15 in the outer magnetosphere are certainly most frequent and most drastic because of the direct effect of the variations in the solar wind conditions.
2.2 Effect of Ionospheric and Other Attenuation

Fig. 1 Sensitivity of overall transmission on the path length in the outer magnetosphere. Percentile variation o% of the path length 15 in the
outer magnetosphere required to reduce the overall transmission T011
by a factor of 10, is plotted vs. time period T. Eq. (4) has been used.

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At this point we want to include the attenuation T2 through the ionosphere and other factors in the estimation of the overall transmission. T2 can be calculated from Eq. (16) of paper I, but for the sake of simplicity we will make a drastic approximation of this as

T2 T/100 for 1T<100
1 for T>100

The approximation is not incorrect in the order of magnitude sense. Furthermore, we will consider an additional attenuation, say of 1/3, for loss through mode conversion at all T. For T>100 there will be another additional attenuation because of loss of wave energy due to lack of guidance along the geomagnetic field, as the wave lengths tend to be comparable with the radius of curvature of field lines. We shall roughly estimate this to be inversely proportional to T. Hence the total effective attenuation I Tatt can be very crudely depicted as in Fig. 2. It is recalled that Eq. (4) gives the amplitude of transmission at any time period under the condition of "spatial" resonance of the path in the outer magnetosphere. Hence it represents the maximum possible transmission at any time period, which can be achieved by proper adjustment of the path length in the outer magnetosphere. As such Eq. (4) gives the greatest upperbound on the transmission and consequently the transmission obtained from the attenuation of Fig. 2 and Eq. (4) will be designated ITolgub and expressed as

ITQlgub-ITattIIT31IT4 (X42+42 52)12/R4 (6)

For computing Ri and Xi in the above equation, Eqs. (15) through (18) of paper I have to be used. This is plotted in Fig. 3 as a function of time period T. As explained above, this

Fig. 2 Approximate total attenuation due to ionospheric and other factors. The total attenuation ITattl is plotted vs. time period T.

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quantity is quite different from the transmission coefficient discussed by other authors like Greifinger and Greifinger (1965), Field and Greifinger (1965) and Prince and Bostick (1964), One of the possible meanings which can be attached to this quantity is that it is the cut-off point at the high end of the probability distribution function of the transmission.

3. On the Spectrum of Micropulsations

3.1 Amplitude Spectrum
Now we will attempt to estimate the spectrum of micropulsations at the surface of the earth. We note that the transmission path of the micropulsations discussed in paper I is a linear system. Hence, using the concept of transfer function of a linear system, we can write in terms of Fourier Transforms

1earthUW) T0(J)I source(J) (7)

where

Fig. 3 The greatest upperbound of the overall transmission of micropulsations. The greatest upperbound I,T0abs max of the overall transmission through
the entire path down to the surface of the earth is plotted vs. time
period T. Figures 2 of this paper and 6 of paper I and Eq. (5) of
paper I have been used.

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Hearth(W)=+Hearth(t)e (0tdt; Hsource(w)-+Hsource(t)e tdt

-oo-oo

T0(jw)= Transfer function of the transmission path Transmission coefficient To defined in section 2.

The quantity of interest is the amplitude spectrum Hearth(W)1 j,at the surface of the earth, which is easily found from Eq. (7) as

Hearth(iw) J TQ Hsource(jw) J (8)

We have discussed ToJ in paper I and section 2.1 of this paper in great detail. It now remains to specify Hsource(jw) J, before we can determine the amplitude spectrum at the sur-face of the earth.

In paper I we have discussed the unstable waves on the magnetospheric surface as the source of micropulsations. There we have indicated that the entire curved bounded surface will be covered with surface waves of a variety of modes through mode coupling. Hence, the spectra at the source, though discrete, may be quite dense. Furthermore, due to strong mode coupling the spectra may not be far from uniform. Lastly, the bandwidth of the spectrum at the source has been estimated to be a few tenths of a second to hundreds of seconds. Hence, it is not unreasonable to assume that the spectrum Hsource(jw) isdiscrete, dense, essentially uniform and bandlimited. We now recall from paper I that the overall transmission coefficient JToJ has widely separated, extremely narrow and very high reso-nances. In comparison, the off-resonance values of J TO) are very low. Therefore the right hand side of Eq. (8) can be looked upon as dense uniform spectra JHsource(jw) I modulated by slowly varying JToJ. It essentially picks out a few of the spectral components of the source and elevates these to the levels of the resonances of the transmission path. JToJ is off-resonance for most of the spectral components of the source and these will not appear in the spectrum at the surface of the earth. Hence, we conclude that the basic structure of the spectrum of micropulsations received on the surface of the earth will be that of the transmission coefficient TO) within the bandwidth of a few tenths of a second to hundreds of seconds. The fine structure of this spectrum will be obviously due to the detailed dis-tribution of the dense spectra at the source (and also possibly various sources of noise in the transmission path). In view of our discussion in section 2.1, we would expect a pre-dominance of micropulsations of a few frequencies in the vicinity of each time period T-935, 311,134,103 and 71 seconds and less probably in the vicinity of T-39, 14, 9, 7 and 4 seconds. The last statement does not of course apply, when the source is located in the lower exosphere or ionosphere. This will be so for micropulsations caused by energetic particle precipitation (see section 4.2). In that case the resonances at T-39, 14, 9, 7 and 4 seconds will be dominant.

It is pointed out that the amplitude spectrum discussed above is the Fourier Transform
of the received signal, whereas power spectral density is the squared magnitude of the Fourier Transform of the auto-correlation function of the signal. All the authors on the subject including Prince and Bostick (1964), Greifinger and Greifinger (1965), and Field and

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Greifinger (1965, 1966) present the squared amplitude spectrum (obtained as the squared transmission coefficient) as the power spectral density. This is strictly correct, if and only if the source is white noise, and the transmission path has no statistical variations. The assumption about the source may not be bad for some classes of micropulsations. But it will be shown below that a certain slight statistical variation in the transmission path will have profound influence on the power spectral density, causing it to be far different from the squared magnitude of the transmission coefficient. Hence, the comparisons between the theoretical resonances of transmission and the observed spectral peaks made by Prince and Bostick, (1964), Greifinger and Greifinger (1965), Field and Greifinger (1965, 1966) are somewhat misleading. All one could expect at the resonances of transmission is a measure of relative prevalence, but not necessarily spectral peaks. In this restricted sense one can say that our estimate of micropulsations at T'4, 7 and 9 seconds is consistent with the observation of Yanagihara (1963), those at T'7, 9, 14, 71 and 103 seconds with the findings of Maple (1959), those at 4, 7, 9, 14, 71,103, and 134 seconds with the observation of Benioff (1960). Our estimate of giant pulsations at T134, 311 and 935 seconds is corroborated by the observations of Veldkamp (1960) and Nagata, Kokburn and Iijima (1963).
3.2 Power Spectral Density
Power spectral density gives the statistical distribution in frequency of the power of the signal. Consequently, in general it is impossible to determine it without very complex statistical analysis involving the stochastic behavior of the source mechanism and the transmission path. Without embarking on such a nearly impossible task, we can neverthe-less demonstrate a significant feature of the power spectral density by the following simple analysis. The signal at the source in our theory is essentially deterministic, although admittedly with some statistical fluctuations due to some randomness inherent in solar wind conditions. Leaving this aside, we intend to show the strong effect of the slight statistical behaviour of the transmission path on the power spectral density on the surface of the earth. The statistical behaviour in question is the fluctuation of the path length in the outer magnetosphere caused by the fluctuations in the solar wind conditions. We have seen in section 2.1 that the transmission of micropulsations is extremely sensitive to the slightest variation of this path length. Hence, the statistical behaviour and specifically the power spectral density of micropulsation signal at the surface of the earth will be strongly affected by this. We have already indicated in section 2.1 that sharp peaks in persistence probability or occurrence probability as reflected by dominant peaks in o% may correspond to spectral lines at T-2.5, 6,11 and 20 seconds in the power spectral density at the surface of the earth. Now we wish to find the broad structure as a function of time period. We note that the value of power spectral density at a certain frequency is proportional to the product of occurrence probability of that frequency (of which d% is a measure) and the square of the amplitude of transmitted signal at that frequency. Hence the power spectral density P(w) can be very roughly written as proportional to the product

P(w) To 2gubo (9)

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Substituting Eqs. (5) and (6) in the
above equation, we obtain some
rough features of P(w) as shown in
Fig. 4. The slope of 7 db/octave of
the power spectral density for large
T is remarkably close to the observed value of 6 db/octave (Santirocco and Parker, 1963 and Davidson, 1964). For small T the slope is 42 db/octave and the agreement is poor.
A better measure of the occur-

rence probability for the present purpose will be the variation in the path length related to the probable amplitude of the received signal. That is, we define a new measure d'%, the percentile variation in the path length (l5) in the outer magne-tosphere required to reduce the overall transmission coefficient to a maximum possible value of A from its greatest upper bound. A is an appropriately small quantity for
which the reception of micropulsa-
tions can be considered negligible.
From Eqs. (1), (2) and (4) this can
be deduced to be

%=T cOS-1 1-2,52 IT3I2IT4I2Tattl2 R42 (10)14.6 X 42-+42+X52 A2 X42+R42+52

where Tattl is taken from Fig. 2. A plot of 8'% versus time period T is shown in Fig. 5 for
A=1. Now the power spectral density P(w) is roughly written as proportional to the product
P(w) To2gubo (11)

Using Eqs. (6) and (10) in the above equation, we obtain the broad structure of P(w) versus
time period T as shown in Fig. 6. Again the slope of the power spectral density of Fig. 6 for large T is 6 db/octave which is exactly the same as that observed by Santirocco and Parker (1963) and Davidson (1964). However for shorter T the slope is 28 db/octave and the agreement is poor. As pointed out at the outset of this section, Figs. 4 and 6 are not

Fig. 4 To l2gubo, a quantity roughly indicative of the
power spectral density P(w), is plotted vs. time
period T. Eqs. (4) and (5) have been used.

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intended to be plots of theoretical power spectral density in any sense. Nevertheless, our crude analysis makes some general points about its trend with increasing time period, especially at large T.

4. Other Characteristics of Micropulsations

4..1 Nighttime Micropulsations
The theory presented here relies on the unstable waves generated on the dayside of the magnetospheric boundary and consequently their propagation to the dayside of the earth. Hence, the detailed calculations are directly relevant to the daytime micropulsations. In principle the nighttime micropulsations can be explained by simple extensions of the mechanisms proposed here. These are caused by the unstable waves generated on the surface of the magnetospheric tail (Sen, 1965) and/or the same from the dayside of the magneto-spheric boundary propagating through mode coupling partly in the east-west directions to

Fig. 5 Sensitivity of the amplitude of micropulsation signal received on the surface of the earth, on the path
length in the outer magnetosphere. Percentile varia-
tion 5'% of the path length 15 in the outer magneto-
sphere required to reduce the overall transmission coefficient to a maximum possible value of 1/10 from
its greatest upper bound given in Fig. 3, is plotted vs time period T. Figures 5 of paper I and 2 of this paper are used.

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the nightside and then descending
along the geomagnetic field lines to the
surface of the earth.

4.2 Other Mechanisms Responsible
for Micropulsations
It is pointed out that the theory proposed here is only one of several possible mechanisms responsible for micropulsations. The others are: high frequency hydromagnetic emissions by charged particle bunches oscillating along geomagnetic field lines (Went-worth and Tepley, 1962; Jacobs and Watanabe,1963); hydromagnetic whis-tlers (Obayashi, 1965) which includes the effect of wave dispersion in the hydromagnetic emission mechanism mentioned above; the effect of pulsa-ting current systems in the ionosphere set up by periodic precipitation of particles initiated by the periodic agita-tion of the geomagnetic field by solar disturbance (Campbell and Matsushita, 1962); and lastly, the result of transient disturbances of the geomagnetic field.
The first two mechanisms listed above

are for high frequency emissions whose
frequencies fall in the Pcl range and
beyond. We have excluded the con-
sideration of these emissions, which fall
anyway at the high frequency end of the bandwidth of micropulsations estimated in paper I, and some of it may properly be part of the ELF noise spectrum.
As discussed by Campbell and Matsushita, the third mechanism presupposes a sustained periodic oscillation of the distant geomagnetic field whose origin and nature are left un-specified. Furthermore, as the particle precipitation in this theory is connected with the auroral zone electrons, one should use the recent idea of auroral phenomena in terms of particle acceleration by an instability of the neutral sheet in the geomagnetic tail. As a matter of fact, instabilities of the neutral sheet may set up characteristic oscillations on the geomagnetic lines leading to the auroral zones and provide a mechanism for periodic precipitation of particles required by the theory of Campbell and Matsushita (1962). This possibility should be looked into.

Fig. 6 j To 2gub6', another quantity roughly indicative
of the power spectral density P(w), is plotted
vs. time period T. Eqs. (5) and (9) have been
used.

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A wide range of micropulsations may be due to the last mechanism. Some of the variety of causes of disturbance to the geomagnetic field are sudden commencement, substantial precipitation of high energy particles due to natural causes, and man-made nuclear explosions in the magnetosphere. The characteristics of the micropulsations due to these transient causes will be different from those of our steady state theory in two im-portant respects. The first is that the spectrum of the transient source will influence the spectrum of the received signal much more strongly than that of our steady state theory or the common assumption of white noise at source (Jacobs and Watanabe, 1962; Prince and Bostick, 1964; Greifinger and Greifinger, 1965 and Field and Greifinger, 1965). The second is the manner in which the transfer function of the transmission path influences the spectrum of the received signal in contrast to that in our steady state theory or the consid-eration of steady state transmission coefficient (Jacobs and Watanabe, 1962; Prince and Bostick, 1964; Greifinger and Greifinger, 1965 and Field and Greifinger, 1965, 1966). We can briefly indicate these differences in principle. An examination of Eq. (7) immediately reveals that in a steady state theory the frequencies of the received signal are the same as those of the source, only the magnitudes and phases of various spectral components are altered by the transfer function of the transmission path. In a transient situation, the frequencies of the received signal are determined both by the spectral content of the source and the transfer function. That is, the frequencies of the received signal contain the fre-quencies of the transient disturbance as well as the natural modes of the transmission path. The natural modes of the transmission path appeared to play a significant role in our steady state theory, only because these were the highly discriminating pass bands of the effective filtering action of the transmission path and the nature of our source as discrete, dense and essentially uniform. In the case of general transient source, this will not be the case as explained above. In a future paper we would discuss this situation analytically
Lastly, the suggestion of Herron (1966) that most micropulsations are interference signals that propagate with a phase velocity that increases with decreasing period, is an important one and should be further explored.
4.3 Correlation with Various Geophysical Phenomena
It has been shown in paper I and elsewhere (Sen, 1965) that the spatial extent of the instability on the dayside of the magnetospheric boundary is almost completely independent of the speed and density of the solar wind. (But the same is critically dependent on the angle between the geomagnatic field and the compressed interplanetary field at the bound-ary.) The stable regions on the nightside of the boundary are somewhat dependent on solar wind conditions, but parts of it are always unstable. Hence, it is clear that the source of micropulsations in this theory is hardly correlated with K or Kp indices. Furthermore, in section 2.1 we have shown that the reception of micropulsations on the surface of the earth can be completely quenched by the slightest variation of the path length in the outer mag-netosphere. As the slight variations in the path length necessary for such quenching can always be caused by the slight fluctuations normally expected in the solar wind at all times, it appears that the transmission probability or occurrence probability of micropulsations

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are not dependent on K or Kp indices. Hence in our theory neither the source mechanism nor the transmission characteristics (with one exception described below) are substantially correlated with K or Kp indices.
Only in one respect will there be a relationship between K and Kp indices and micro-pulsations in our theory. Under highly enhanced solar wind, the path length in the outer magnetosphere will be considerably shortened. In paper I we have seen that the longest period resonances of the transmission path are determined by this path length, and these decrease with decreasing value of the path length. Hence one can expect a somewhat nega-tive correlation of very long period micropulsations with K or K indices. In section 4.2 we have noted that most shorter period micropulsations may have their origin in a mecha-nism different from the one proposed in this paper. Therefore we must exclude these from our consideration at the moment. Hence, in conclusion, one can surmise that long period micropulsations have essentially no correlation with K or Kp indices, with the exception of the very long period oscillations which can have a somewhat negative correlation. This is in general agreement with the observations of Maple (1959).
Now we can discuss the case of shorter period pulsations. In section 4.2 we have in-dicated the possibility that most shorter period micropulsations may have their origin in the transient disturbance of the geomagnetic field due to precipitation of energetic particles. The events of energetic particle precipitation are obviously related to K and Kp indices. Furthermore, auroral events are certainly caused by energetic particle precipitation. Hence, the class of shorter period micropulsations generated by energetic particle precipitation should be correlated with not only K and Kp indices, but also with auroral zone bremsstrah-lung, ionospheric absorption of cosmic noise and negative bay disturbance. This positive correlation has been observed by Benioff (1960), Sechrist (1962), Chen et al (1962), and Campbell and Matsushita (1962).
4.4 Fine Structure of Micropulsations
In the theory presented here we have attempted to derive only the broad features of the received micropulsation signals. That is, we have found the dominant frequencies and some general behaviour of the power spectral density. The variety of fine structures found in micropulsations has not been considered here. Nevertheless, from our observations in section 3.1 we can state that the fine structure will be determined by the dense spectra of our source and also possibly various sources of noise in the transmission path. For the shorter period micropulsations caused by energetic particle precipitation, the spectrum of the source will play the decisive role in determining the fine structure.
4.5 Latitudinal, Diurnal and Seasonal Variations
We have considered these variations as manifestations of detailed behaviour of micro-

pulsations dependent on a variety of geophysical phenomena and have not attempted to explain these in the framework of our theory. Nevertheless one can make the following conjectures.
The unstable waves on the boundary of the magnetosphere, discussed in paper I, evanesce towards the earth (Sen, 1963). The e-folding distance of evanescence is of the

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order of the wavelength and the unstable waves will be restricted to a layer of thickness of this order on the magnetospheric boundary. (This is not to be confused with the usual boundary layer.)
The geomagnetic lines of force from this layer descend to the surface of the earth at higher latitudes, specifically in the auroral zones. Therefore it appears reasonable that both the occurrence frequency and amplitude of pulsations will be maximum in the auroral zones. For various reasons this has been suspected by other authors and is in accordance with the findings of Jacobs and Sinno (1959, 1960). The variation of the period of micropulsation with latitude is probably a more complex phenomenon. The realistic transmission problem will certainly involve mixed mode latitude-dependent propagation and the resulting trans-mission resonances will reflect latitude dependence. Moreover, in our theory the source mechanism itself dictates a certain latitude dependence of the time period. This is due to the fact that the unstable waves are restricted in a thick layer of evanescence. From our discussion in paper I it is obvious that the period of the longest period wave generated will decrease as one moves deeper into the layer toward the earth. As the geometric lines of force deeper in the layer connect to lower latitudes it appears that the time period of pulsa-tions may decrease with decreasing latitude. This is in accordance with the observations of Obayashi and Jacobs (1958).
The diurnal variation in terms of the difference between daytime and nighttime micro-pulsations is probably due to the daynight asymmetry of the magnetosphere and the day to night variation in the ionization level of the ionosphere. The continuous dependence of the occurrence frequency on the geomagnetic local time can be explained in the framework of our theory. We have indicated in paper I that the mode on the boundary of the mag-netosphere which may account for micropulsations is the one parallel to the local geomag-netic field. On the other hand the most unstable mode is the one parallel to the local tangential streaming velocity of the solar wind on the boundary surface. Hence the most favorable situation for the occurrence of micropulsation will be the one when the tangential streaming velocity of the solar wind is nearly parallel to the local geomagnetic field. This is so in the vicinity of the local noon meridian of the hemispherical boundary surface shown in Fig. 1 of paper I. Hence, one may expect that the ocurrence frequency of pulsations on the surface of the earth is maximum near the local noon. This is not contrary to the ob-servations of Troitskaya (1953) and specially Jacobs and Sinno (1959).
Lastly, the seasonal variation may be attributed to the variation in the angle of attack
of the solar wind with respect to the dipole axis of the earth and the variation in the ioniza-tion level of the ionosphere. It is understood that in any season there will be some fluctua-tion in the angle of attack, but with changing season there will be a systematic change of the average angle of attack. It is this latter change which may play a role in the seasonal variation in micropulsations.
4.6 Polarization

All the authors on this subject including the present one have considered single mode propagation of circularly polarized longitudinal or transverse waves. In reality the propaga-

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tion will be in mixed mode, oblique to the geomagnetic field and hence the polarization of the signal at the surface of the earth will be, in general, elliptical. The polarization would be quite variable due to variability of mode coupling in the transmission path and abrupt changes in the conductivity at the surface of the earth. (Dawson and Sugiura, 1963; Pope et al, 1963; Santirocco and Parker, 1963.)
4.7 Probable Origin of Class Distinctions of Pulsations
Some of the various classes of micropulsations may have somewhat different origin. The classes Pc's and Pi's (Jacobs et a1, 1964) are certainly due to different mechanisms. In view of our discussion in section 4.2 we can only conjecture the following. Pcl may be due to hydromagnetic emissions (Wentworth and Tepley, 1962; Jacobs and Watanabe, 1963) or hydromagnetic whistlers (Obayashi, 1965). Some of Pct and Pc3 may be related to pulsating current systems in the ionosphere postulated by Campbell and Matsushita (1962), or to transient disturbance of the geomagnetic field due to sudden commencement, or to precipi-tation of energetic particles through other mechanisms. Most of Pc4 and Pc5 and at least some of Pc2 and Pc3 may be due to the mechanism proposed in the present theory. Lastly, Pil and Pi2 are probably due to transient disturbance of the geomagnetic field initiated by precipitation of energetic particles through a variety of geophysical causes.

5. Conclusions and Discussion

In this section we draw comprehensive conclusions and give discussions based on both paper I and the present paper.
5.1 Conclusions
A theory of one of several possible mechanisms responsible for micropulsations is pre-sented in this paper. A source mechanism, its strong effect on the bandwidth and relatively small effect on the spectrum, the important role of the transmission path in determining the spectrum and the crucial role of the variation of the path length in the outer magneto-sphere on the spectrum of the received signal are discussed. Specifically, the following conclusions are drawn.

(i) A source mechanism at the interface between solar wind and the magnetosphere is proposed. It is shown that the hydromagnetic surface waves generated by an instability on this interface are transmitted through the magnetosphere and received on the surface of the earth as micropulsations.
(ii) The surface waves suffer strong reflections due to the difference in their velocity
and the Alfven velocity in the regions where the geomagnetic lines leave the interface and/or the strong variation of the surface wave velocity on the interface. These set-up characteristic wave lengths and the resulting bandwidth is found to be a few tenths of a second to hundreds of seconds. The spectral lines at the source may be very dense and have the relatively small effect of probably controlling the fine structure of the spectrum at the surface of the earth.

(iii) The amplitude spectrum of micropulsation signal at the surface of the earth is essentially determined by the resonances of the transmission path. The transmission of

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micropulsations through the entire path in the magnetosphere from its outer boundary to the surface of the earth is discussed in terms of hydromagnetic transmission lines. It is found that the most likely resonances of transmission are due to the path in the outer mag-netosphere and are approximately at T-935, 311, 134, 103 and 71 seconds. The resonances of the lower exosphere and upper ionosphere are usually masked out by the effect of the path in the outer magnetosphere, because the overall transmission is extremely sensitive to the variation in the path length in the outer magnetosphere. This variation is always to be expected due to the variable solar wind conditions. In the exceptional situation when the

"temporal" resonances of the lower exosphere and upper ionosphere coincide with the "spatial" resonances of the outer magneto sphere, we again find significant resonances of
the overall transmission. Under those improbable conditions, we find resonances at ap-proximately T-39, 14, 9, 7 and 4 seconds.
For the class of micropulsations caused by a source mechanism in the lower exosphere
or ionosphere, the group of resonances at T-39, 14, 9, 7 and 4 seconds will be dominant in determining the amplitude spectrum.
(iv) Giant pulsations of very long periods and very high amplitudes are well explained
by the theory.
(v) The persistence or occurrence probability is a function of the sensitivity of the overall transmission on the variation of the path length in the outer magnetosphere. Hence the sharp peaks in this sensitivity at T'2.5, 6, 11 and specially 20 seconds correspond to spectral peaks in the power spectral density. The power spectral density depends both on the occurrence probability and the squared amplitude of transmission. Hence the broad features of the spectrum are reflected in the product of this sensitivity of the overall trans-mission and the squared amplitude of transmission. At large T this reveals a slope of ap-proximately 6 db/octave.
(vi) The long period micropulsations due to the mechanism proposed here should have essentially no correlation with K or Kp indices, with the exception of the very long period pulsations which can have a somewhat negative correlation.
5.2 Discussion
Now we discuss the limitations and reservations of the theory presented in this paper.
(i) The source mechanism proposed here is only one of several possible mechanisms responsible for various classes of micropulsations. The other possibilities have been dis-cussed in section 4.2.

(ii) The coupled modes of wave motion caused by the instability on the curved bounded surface of the magnetosphere are not analytically discussed with any rigour. Furthermore, the finite amplitude of these waves needs to be determined through non-linear analysis. Only then the complete spectrum at source can be found and consequently the actual spec-trum and its fine structure at the surface of the earth can be determined.

(iii) Our discrete layered model of the transmission path is obviously a drastic simplifi-cation of the real continuous problem. The latter, however, necessitates extensive numerical solutions.

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(iv) All the authors on this subject including the present one have considered single mode propagation of circularly polarized longitudinal or transverse waves. In our case it is a little more justified, because our source mechanism will tend to direct hydromagnetic energy mostly in the Alfven mode (linearly polarized) which is the low frequency limit of the circularly polarized transverse wave. Even then, in reality, the propagation will be somewhat oblique to geomagnetic field in mixed mode. It is clear that only a mixed mode analysis will reveal latitude dependence and polarization of micropulsation received at the surface of the earth.

(v) A rigorous and comprehensive derivation of power spectral density is lacking in
all work on the subject including the present one.
(vi) The explanations of correlations with various geophysical phenomena discussed
in section 4.3 are probably reasonable conjectures. But the discussions on fine structure, latitudinal, diurnal and seasonal variations, polarization and distinctions of various classes of micropulsations in sections 4.4 through 4.7 are quite incomplete and hypothetical.
Since the preparation of this paper, Parks, McPherron and Anderson (1966) have dem-onstrated correlations between micropulsations and energetic electron precipitation and auroral substorms, as briefly mentioned in section 4.3. Heacock (1966) has found a band of micropulsations at T-4 seconds, which is in remarkable agreement with our estimate of exactly the same period in paper I and section 3 above. Lastly, Fernando and Kannanga a (1966) have found pulsations of time period 38+5 and 60*5 seconds which can be compared with our estimates of T-39 and 71 seconds, respectively, in paper I and section 3 above.

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