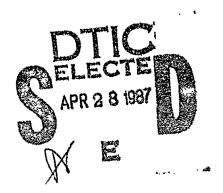


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A Theory of the Blockage Effects on Bluff Bodies and Stalled Wings in a Closed Wind Tunnel

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By E. C. MASKELL.

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Summary.

A theory of blockage constraint on the flow past a bluff body in a closed wind tunnel is developed, using an approximate relation describing the momentum balance in the flow outside the wake, and two empirical auxiliary relations. The theory is well supported by experiment and leads to the correction formula

 $\Delta q/q = cC_D S/C$

where Δq is the effective increase in dynamic pressure due to constraint, and c is a blockage factor dependent on the magnitude of the base-pressure coefficient. The factor c is shown to range between a value a little greater than 5/2 for axi-symmetric flow to a little less than unity for two-dimensional flow. But the variation from 5/2 is found to be small for aspect ratios in the range 1 to 10.

The theory is extended to stalled wings, and an appropriate technique for the correction of wind-tunnel data is evolved.

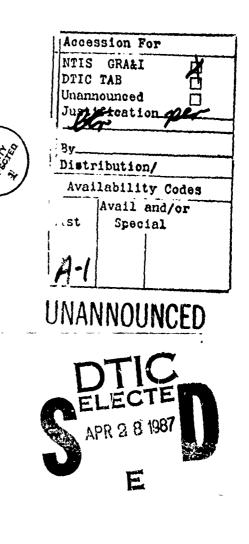
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1. Introduction.

The flow past a symmetrical body immersed in an airstream bounded by rigid walls is subject to what is commonly called blockage constraint. The rigid boundaries prevent a free lateral displacement of the airflow by the body, in the neighbourhood of which velocities are higher than they would be in an unlimited stream. The dominant effect is usually taken to be equivalent to a simple increase in the free-stream velocity, related in part to the volume distribution of the body itself (solid blockage), and in part to the displacement effect of the wake (wake blockage). Appropriate corrections to the observed velocity of the stream can be calculated by standard methods, provided that the given body gives rise to an essentially streamline flow.

Little attention has been given to wall constraint on the non-streamline flow past a bluff body or, more generally, a stalled wing, since Glauert's¹ treatment of the two-dimensional problem in 1933. Glauert's interest in the problem appears to have been stimulated by the experiments of Fage and Johansen^{2,3} on the flow past an inclined flat plate spanning a wind tunnel. He pointed out the nature of the blockage effect associated with the thick bluff-body wake, and his remarks led Fage and Johansen to test several plates of different sizes and hence, by extrapolation to zero chord, to establish the drag coefficient of a two-dimensional flat plate normal to an unlimited stream. Meanwhile Glauert proposed a theory, based in part upon the Helmholtz model of the flow past a bluff body, according to which the drag D_c in an unlimited stream is related to the drag D in the wind tunnel by

$$D_c = D \left(1 - \frac{\eta t}{h} \right)^2$$

where t is the thickness of the bluff base, h the tunnel height, and η an empirical factor. But the presence of the empirically determined η reduces Glauert's formula to an interpolation between known experimental results, which are not sufficiently accurate to give adequate support to the proposed functional dependence upon t/h. The formula seems not to have been widely used, the wake blockage correction

$$\frac{\Delta q}{q} = \frac{1}{2} C_{D0} \frac{S}{C}$$

(where Aq is the effective increment in the dynamic pressure of the undisturbed stream, S the representative area on which the profile drag coefficient C_{D0} is based, and C the cross-sectional area of the tunnel) generally being preferred to it.* However, there is little doubt that the latter correction holds only for streamline flow, and that the bluff-body problem requires a different treatment.

This paper presents a simple theory of the constraint which is well supported by observation. Interest in the problem, especially in its three-dimensional form, was revived when marked differences were noticed in the high-lift characteristics of models of a particular aircraft tested in different wind tunnels. The models in question were basically delta wings of moderately small aspect ratio. And from the onset of stall, which began at the wing tips and then spread inboard with increasing incidence, the different sets of results could be reconciled only through some form of wall interference grossly bigger than those covered by the standard corrections. The purpose of the present investigation, therefore, was to establish the existence of such an interference more convincingly and then to provide appropriate corrections for it.

Since it was evident, from the outset, that the effect was connected with the breakdown of streamline flow over the wing, it seemed worth while to concentrate attention, in the first instance, upon the extreme situation occurring when a wing-like shape—for example, a thin flat plate—is set

3

(90995)

A 2

^{*} See, for example, Pankhurst and Holder⁴.

normal to the windstream. The blockage constraint on this type of bluff-body flow in a wind tunnel with solid walls is therefore the subject of the first part of this paper. A theory is developed which provides an estimate of the effective increase in the dynamic pressure q of the stream, due to the constraint, in the form

$$\frac{q_c}{q} = \frac{k^2}{k_c^2} = 1 + \frac{C_D}{k_c^2 - 1} \frac{S}{C}$$

where $k^2 = 1 - C_{pb}$, C_{pb} is the base-pressure coefficient, and where the suffix *c* refers to effective, or corrected, quantities. The theory is shown to be well supported by experiments on a set of square flat plates in two different wind tunnels. And since it is clear from the work of Fail, Lawford and Eyre⁵, that the base-pressure coefficient for squares, circles and equilateral triangles, is about -0.4, the blockage correction appropriate to this range of three-dimensional shapes follows as $\Delta q/q \doteq (5/2)C_D S/C$, i.e. roughly five times the correction appropriate to the same drag in streamline flow. For the two-dimensional flow studied experimentally by Fage and Johansen, for which the corrected base-pressure coefficient is more nearly -1, the predicted blockage correction is roughly $\Delta q/q = C_D S/C$, i.e. only twice the corresponding correction for streamline flow.

The remainder of the paper is concerned with the extension to stalled wings, with particular reference to wings of moderate to small aspect ratio where both the effect and its practical significance are greatest. To make the extension possible, it is necessary to assume that the breakdown of a three-dimensional streamline flow tends to give rise to discrete regions of nearly axi-symmetric flow, closely similar in structure to the bluff-body wakes previously considered. This assumption is suggested by measurements, by Kirby and Spence⁶, in the wakes behind models of particular delta-wing and swept-wing aircraft. And it is further supported by the work of Fail *et al*, who show that even when a bluff-body wake is far from axi-symmetric near its origin, the subsequent tendency towards axial symmetry is very strong. In consequence of this assumed property of the general flow, the theory developed for non-lifting bluff bodies continues to hold, in principle. And it suggests, further, that the slowly varying factor $1/(k_c^2 - 1)$ in the expression for the blockage correction may usually be replaced by the empirical constant 5/2.

It remains only to identify that part of the measured drag to be included in the blockage parameter $C_D S/C$. For a partially stalled lifting wing there are three contributions to the total drag coefficient: the induced drag C_{Di} ; the profile drag associated with the regions of streamline flow, C_{D0} ; and the profile drag associated with the stalled regions, C_{Ds} . It is, of course, this last contribution that has to be identified. And a composite wake-blockage correction formula is proposed, with the object of ensuring that the high correction appropriate to the effect considered in this paper is applied automatically as the need arises.

The bulk of the work on which this report is based was completed in 1955, and the principal formula derived was given a limited circulation at that time.

2. Bluff Bodies.

2.1. Properties of the Bluff-Body Wake.

Fail, Lawford and Eyre⁵ report detailed measurements in the wakes behind flat plates of finite span set normal to a windstream. Although the flow is highly unsteady, they detect a distinct mean flow structure, which is little affected by aspect ratio (for A < 10) or shape (in the range—circle, square, equilateral triangle). They find a strong tendency towards axial symmetry, with properties

like the drag and base pressure varying very slowly in the range of their experiments. Hence their typical example is the axially symmetric wake formed behind a circular disc. And its main feature is a closed 'bubble', bounded by the stream surface which separates from the sharp edge of the disc, along the forward half of which the static pressure* is constant and equal to the base pressure. A similar picture of the corresponding two-dimensional flow is given by the work of Fage and Johansen^{2,3}.

The classical, discontinuous, model of the bluff-body wake is therefore consistent, qualitatively, with the experimental evidence. It adequately defines the form of the inner boundary condition on the flow external to the wake, in the neighbourhood of the body. And this is sufficient for the present purpose.

It is proposed, accordingly, to represent the wake by the stream surface illustrated in Fig. 1. This extends downstream from the edge of the body, and sustains a constant pressure p_b (the corresponding constant velocity being kU, where U is the velocity of the undisturbed stream) as far as the station 2, where the cross-sectional area of the wake is a maximum. The further development of the wake is of no immediate interest.

The shape of the constant-pressure surface is unknown. And there is no theory available to account for the magnitude of the factor k. Nevertheless the essence of the present problem is to obtain a quantitative estimate of the effect of wall constraint on k. It differs markedly, in this respect, from the superficially similar problem of the blockage effect on the cavitating hydrodynamic flow past a bluff body. For although the same wake model is appropriate in both cases, the pressure p_h is the cavitation pressure in the hydrodynamic problem, and so can be properly regarded as a parameter.

2.2. Invariance under Constraint.

Before proceeding further with the proposed flow model, it is worth while to consider the extent to which wall constraint can be regarded as equivalent to a simple increase in velocity of the undisturbed stream. Exact equivalence implies that the form of the pressure distribution over the body is invariant under constraint: if p is the pressure at any point (y, z) on the surface of the body, and H is the total pressure of the undisturbed stream, then $(p-p_b)/(H-p_b) = f(y, z)$, independent of constraint. And, since $H - p_b = k^2 q$, it follows that

$$\frac{C_D}{k^2} = \text{constant} \tag{1}$$

independent of boundary constraint, where C_D is the drag coefficient D/qS, and S is a representative area of the body. It also follows that the velocity U_c of the unlimited stream which gives rise to a pressure distribution identical to that observed is such that $k_e U_c = kU$. Hence

$$\frac{U_c^2}{U^2} = \frac{k^2}{k_c^2} = \frac{C_D}{C_{Dc}}.$$
 (2)

In order to test the validity of the relation (1), measurements of the drag and base pressure were made, in the 4 ft \times 3 ft and No. 1 11½ ft. \times 8½ ft wind tunnels at the Royal Aircraft Establishment, Farnborough, on a set of geometrically similar sharp-edged square plates, using the technique described by Fail *et al.*

^{*} This pressure is measured just outside the wake, and not strictly on the bubble boundary itself.

TABLE 1

Wind tunnel	$4 \text{ ft} \times 3 \text{ ft}$				No. 1 11 $\frac{1}{2}$ ft × 8 $\frac{1}{2}$ ft		8½ ft
Tunnel cross-sectional area C (ft ²)		11.	55			93.0	
Plate area S (in ²)	5.06	25	50	75	25	75	256
S/C	0.0030	0.0150	0.0301	0.0451	0.0019	0.0056	0.0191
$C_D S/C$	0.00345*	0.0180	0.0376	0.0602	0.0022	0.0066	0.0233*
$\overline{k^2 - 1} = -C_{pb}$	0.375	0.427	0 · 505	0.589	0.386	0.398	0.46
C _D		1.200	1.249	1.335	1.158	1.175	
$\overline{C_D/k^2}$		0.841	0.830	0.840	0.835	0.840	

Experimental Data for Sharp-Edged Square Flat Plates Normal to the Windstream

* C_D estimated from the relation $C_D/k^2 = 0.837$.

The results, recorded in Table 1 and plotted in Fig. 2, are closely represented by

$$\frac{C_D}{k^2} = 0.837$$
 (3)

in formal agreement with the relation (1). Moreover, the results from which equation (3) has been deduced cover a fairly wide range of conditions, the measured base-pressure coefficient C_{pb} ranging from -0.386 to -0.589.

Interpretation of the constraint as an effective increase in stream velocity is, therefore, well supported by these experiments.

2.3. Conservation of Momentum.

Consider, now, the control surface illustrated in Fig. 1. This is formed by the solid walls of the wind tunnel, the surface of the body and the constant-pressure surface bounding the effective wake, and two planes normal to the undisturbed velocity vector—plane 1 lying upstream of the body, and plane 2 located where the cross-sectional dimensions of the bubble are greatest. Let u, v, w be orthogonal components of velocity, with u in the direction of the undisturbed velocity U. And let suffices 1 and 2 denote conditions at the planes 1 and 2 respectively. Then conservation of momentum in the fluid passing through the control surface requires that

$$D + p_b B = \iint_C (p_1 + \rho u_1^2) dy \, dz - \iint_{C-B} (p_2 + \rho u_2^2) dy \, dz \tag{4}$$

where D is the total drag on the body, C the cross-sectional area of the wind tunnel, and B the cross-sectional area of the effective wake at the downstream plane 2.

Since the fluid is wholly outside the wake, Bernoulli's equation gives

$$p_1 + \frac{1}{2}\rho(u_1^2 + v_1^2 + w_1^2) = p_2 + \frac{1}{2}\rho(u_2^2 + v_2^2 + w_2^2) = P + \frac{1}{2}\rho U^2$$

where P, U are the pressure and velocity in the undisturbed stream. And equation (4) may be written

$$D = (P - p_b)B + \frac{1}{2}\rho U^2 B + \iint_{C} \frac{1}{2}\rho u_1^2 dy \, dz - \iint_{C-B} \frac{1}{2}\rho u_2^2 dy \, dz + \\ + \iint_{C-B} \frac{1}{2}\rho (v_2^2 + w_2^2) dy \, dz - \iint_{C} \frac{1}{2}\rho (v_1^2 + w_1^2) dy \, dz \,.$$
(5)

Now the work of Fail *et al* suggests that the wake is likely to be closely axi-symmetric in the neighbourhood of the plane 2, for most three-dimensional bodies of practical interest. Hence, with the plane 1 chosen to lie far upstream, the contribution from the last two integrals in equation (5) is likely to be negligibly small. Since the same conclusion holds equally well for a two-dimensional bluff-body flow, it follows that, for almost all bodies of practical interest, equation (5) can be reduced to

$$D = \frac{1}{2}\rho k^2 U^2 B + \iint_{C} \frac{1}{2}\rho u_1^2 dy \, dz - \iint_{C-B} \frac{1}{2}\rho u_2^2 dy \, dz \tag{6}$$

since

$$p_b + \frac{1}{2}\rho k^2 U^2 = P + \frac{1}{2}\rho U^2.$$

Now write

$$u_1 = U + u_1'$$

$$u_2 = U_2 + u_2'$$

where

$$\iint_{C} u_1' dy \, dz = \iint_{C-B} u_2' dy \, dz = 0$$

so that U is the mean velocity over the plane 1 (i.e. the velocity of the undisturbed stream) and U_2 is the mean velocity over the plane 2 outside the wake. Then, for continuity,

$$UC = U_2(C-B) \tag{7}$$

and equation (6) may be written

$$D = \frac{1}{2}\rho k^2 U^2 B - \frac{1}{2}\rho U^2 B \left(1 - \frac{B}{C}\right)^{-1} + \iint_C \frac{1}{2}\rho u_1'^2 dy \, dz - \iint_{C-B} \frac{1}{2}\rho u_2'^2 dy \, dz \,. \tag{8}$$

Assuming that u_1' and u_2' are sufficiently small for the integrals remaining in (8) to be neglected, the following relation for the drag coefficient is obtained

$$C_D = m(k^2 - 1 - mS/C)$$
(9)

where m = B/S, and where $(mS/C)^2$ is taken to be negligibly small.

Data obtained by Fail *et al* for a circular disc normal to the stream in the 4 ft \times 3 ft wind tunnel are in close agreement with equation (9). With S = 25 in², S/C = 0.015, the measured basepressure coefficient was -0.425 and the drag coefficient 1.18. From measurements of the velocity field in the wake, the radius of the maximum cross-section of the bubble was approximately 4.9 inches, and the displacement thickness of the vortex layer outside the bubble at the same cross-section was approximately 0.08 inches. Hence, since the wake boundary of the mathematical model should coincide with the displacement boundary of the true wake, rather than with the observed bubble boundary, for the present purpose

giving

$$B = \pi 4 \cdot 98^2 \operatorname{in}^2$$

$$\dot{m}=3\cdot\dot{1}2.$$

$$C_D = 3 \cdot 12(0 \cdot 425 - 0 \cdot 047) = 1 \cdot 179$$

compared with the measured $1 \cdot 18$.

2.4. Distortion of the Wake.

A relation between C_{Dc} , k_c and m_c , appropriate to the equivalent unlimited stream, is obtained by putting S/C = 0 in equation (9). Then, using (1) and (9)

$$\frac{C_D}{k^2} = \frac{C_{Dc}}{k_c^2} = \frac{m}{k^2} \left(k^2 - 1 - mS/C \right) = \frac{m_c}{k_c^2} \left(k_c^2 - 1 \right) = \text{const.} = \frac{m^*}{k^2} \left(k^2 - 1 \right).$$
(10)

But the equations (10) are not sufficient to define the blockage effect completely. A further relation is required, to account for the distortion of the wake under constraint. This involves considerations outside the scope of the theory developed so far.

The significance of distortion is easy to demonstrate. So far as the equations (10) are concerned, constraint could give rise solely to distortion of the wake, the pressure distribution over the body, and hence k, remaining invariant. In that case m_c would take the value m^* , and the blockage velocity would be zero. On the other hand, if there were no distortion, the required auxiliary relation would be simply

$$m_c = m$$

which, combined with equations (10), leads to

$$\frac{C_D}{C_{Do}} = \frac{k^2}{k_c^2} = 1 + \frac{C_{Dc}}{k_c^2 - 1} \frac{S}{C}.$$
(11)

This relation can be compared with the data obtained with the set of square flat plates in the 4 ft \times 3 ft wind tunnel, for which extrapolation to S/C = 0 gives $C_{Dc} = 1.139$, $k_c^2 - 1 = 0.361$. Then, according to (11)

$$\frac{C_D}{C_{Dc}} = 1 + 3 \cdot 15 \frac{S}{C},$$

leading to $C_D = 1.142 C_{Dc}$ at S/C = 0.045 (the highest value of S/C reached in the experiments) compared with the measured $C_D = 1.335 = 1.172 C_{Dc}$. Thus it appears that equation (11) underestimates the apparent increase in drag coefficient due to constraint by nearly $20^{\circ}_{.0}$. And in view of the close agreement between experiment and the equations (1) and (9) an attempt to take some account of wake distortion is evidently desirable. To do this theoretically would involve a greater understanding of the internal mechanics of the wake than is available at the present time. The problem, therefore, is to find a suitable empirical relation between m_c and m.

The equations (10) show that $m^* < m$, so that constraint at constant k (and therefore zero blockage velocity) implies a thickening of the wake. But according to the foregoing comparison with experiment, constraint at constant m leads to too small an increase in k, indicating, as might have been expected, that in fact the wake contracts. Thus

$m_c > m > m^*$,

an inequality such that all three of the parameters involved tend to the same value as the ratio $S/C \rightarrow 0$ but, it might be expected, in such a way as to make the ratio $(m-m^*)/(m_c-m^*) \rightarrow 1$. This ratio is therefore expected to behave, in the limit $S/C \rightarrow 0$, in precisely the same manner as the contraction ratio (C-B)/C of the external stream. And it might be profitable to examine the consequences of assuming

$$\frac{m - m^*}{m_c - m^*} = \frac{C - B}{C} = 1 - \frac{mS}{C}$$

which is readily reduced, using equations (10), to

$$\frac{m}{m_c} = 1 - \frac{C_D - C_{Dc}}{(k^2 - 1)(k_c^2 - 1)} \frac{S}{C}$$
(12)

neglecting, as before, terms of $O(S/C)^2$.

The equation (12) is to be regarded, at this stage, as no more than a plausible auxiliary relation. But it is well supported by the experimental data obtained with the series of square flat plates:

п	$(S/C)_n$	C_{Dn}	$k_n^2 - 1$
с	0	1.139	0.361
1	0.015	1.200	0.427
2	0.045	1.335	0.589

for which equation (12) gives

$$\frac{m_1}{m_c} = 1 - 0.0059; \qquad \frac{m_2}{m_c} = 1 - 0.0415$$

whence

$$\frac{m_1 - m_2}{m_c} = 0.0356 \doteqdot \frac{m_1 - m_2}{m_1}$$

Now, taking the wake to be axi-symmetric, with r the mean radius of its maximum cross-section, and l the length of side of the square plate; and taking $r_1 = 4.98$ inches, as for the circular disc (see Section 2.3), then $r_1/l_1 = 0.996$ and

 $\frac{m_1 - m_2}{m_1} \doteq 2 \frac{l_1}{r_1} \left(\frac{r_1}{l_1} - \frac{r_2}{l_2} \right) \doteq 0.0356$ $\frac{r_1}{l_1} - \frac{r_2}{l_2} \doteq 0.018.$

whence

This can be compared with an observed lateral displacement of about 0.02, due to constraint, in pressure distributions through the wakes behind the two plates (see Fig. 3).

2.5. Blockage Correction.

From equations (10) and (12), it follows that

$$\frac{k^2}{k_c^2} = 1 + \frac{C_D}{k_c^2 - 1} \frac{S}{C} + O\left\{ (S/C)^2 \right\}$$
(13)

so that the effect of distortion is to replace C_{Dc} in the correction term of equation (11) by the measured C_D . Alternatively, writing q for the dynamic pressure and using (2), this result may be written

$$\frac{\Delta q}{q} = \epsilon \frac{C_D S}{C} \tag{14}$$

where $\Delta q = q_c - q$ is the effective increase in dynamic pressure of the undisturbed stream due to constraint, $C_D S/C$ is the usual wake blockage parameter, and where

$$c = \frac{1}{k_c^2 - 1} \tag{15}$$

is the so-called blockage factor for the bluff-body flow.

In order to determine e, given measured values of k and C_D , it is necessary to find k_c^2 from equation (13). It is not normally sufficient to replace k_c^2 in (15) by k^2 . An iterative solution of (13) has been found convenient, using the formula

$$(k_c^2)_n = k^2 \left\{ 1 + \frac{1}{(k_c^2)_{n-1} - 1} \frac{C_D S}{C} \right\}^{-1}$$
(16)

where $(k_c^2)_n$ is the *n*th approximation to k_c^2 , and with $(k_c^2)_0 = k^2$.

Measured values of drag and pressure coefficients can now be corrected to the effective dynamic pressure q_o , according to

$$\frac{1-C_p}{1-C_{po}} = \frac{C_D}{C_{Do}} = \frac{k^2}{k_o^2} = \frac{q_o}{q}.$$
(17)

2.6. Discussion.

So far the base pressure has been assumed uniform. But this is not necessary. A mean base pressure can be defined by

$$p_b = \frac{1}{B} \iint p \, dy \, dz$$

the integral being taken over the base of the body and over the surface of the effective wake. With p_b so defined, equation (4) remains unchanged, and equation (9) follows to the same order of approximation as before. It appears that even a substantial non-uniformity in base pressure need not invalidate the theory. It is reasonable to suppose, therefore, that the theory holds for almost all two-dimensional bluff-body flows, and for the wide range of three-dimensional flows for which the wake is closely axi-symmetric at the downstream plane 2.

There is one important possible exception to this rule. An implied assumption in the theory is that the origin of the wake (i.e. boundary-layer separation on the body) is independent of constraint. And so it may be necessary to exclude well-rounded bluff bodies (like the circular cylinder), for which a small change in pressure distribution might lead to a significant movement of the separation front.

With the base pressure uniform, it is evidently possible to determine the blockage factor ϵ from a single measurement of static pressure somewhere on the base of the body. It is then a simple

matter to provide for this measurement in the design of a wind-tunnel model. But since with a non-uniform base pressure it is strictly necessary to measure the detailed pressure distribution over the entire base of the body and over the surface of the wake, it is fortunate that the experimental evidence analysed below suggests that it is probably sufficient, for most practical purposes, to take $\epsilon \neq 1$ for two-dimensional flow and $\epsilon \neq 5/2$ for three-dimensional flow.

2.7. Comparison with Experiment.

2.7.1. A = 1.—The best available test of the theory is provided by the data obtained in the experiments on a set of geometrically similar sharp-edged square plates, recorded in Table 1, to which reference h.s already been made. In these experiments the base pressure was found to be closely uniform, so that determination of the parameter k was straightforward. The principal results have been shown (Fig. 2) to be closely represented by the relation (3), viz. $C_D/k^2 = 0.837$. Only base pressures could be measured on two of the plates (the smallest in the 4 ft \times 3 ft wind tunnel and the largest in the No. 1 11½ ft \times 8½ ft wind tunnel). The corresponding drag coefficients have therefore been estimated from the relation (3), in order to allow subsequent correction of the observed pressures.

Independent solutions of equation (13) provide two groups of corrected base-pressure coefficients, one for each wind tunnel. These are recorded in Table 2.

TABLE 2

Corrected Base Pressure Coefficients for Non-Lifting Square Plates

4 ft × 3 ft wind tunnel			No. 1 11 $\frac{1}{2}$ ft \times 8 $\frac{1}{2}$ ft wind tunnel		
$\frac{C_D S}{C}$	$\begin{array}{c} -C_{pb} \\ (k^2 - 1) \end{array}$	$\begin{array}{c c} -C_{pbc} \\ (k_c^2 - 1) \end{array}$	$\frac{C_D S}{C}$	$\begin{array}{c} -C_{pb} \\ (k^2 - 1) \end{array}$	$\begin{vmatrix} -C_{pbc} \\ (k_c^2 - 1) \end{vmatrix}$
0.00345 0.0180 0.0376 0.0602	0·375 0·427 0·505 0·589	0·362 0·360 0·363 0·360	0.0022 0.0066 0.0233	0·386 0·398 0· 1 60	0·378 0·373 0·375
Me	ean	0.361	M	ean	0.375

The systematic difference between the two groups of corrected results, though difficult to explain,* is not relevant to the present investigation. What matters here is the very close agreement between the results in each group. This strongly supports the theory.

* Great care was taken to avoid significant experimental errors. All the observed results quoted are mean values of several independent observations showing, as a rule, less than $\pm 1\%$ scatter. In particular, each drag coefficient is an average of about ten separate readings of the drag balance, usually taken over a period of several days. Furthermore, each wind tunnel was recalibrated during the investigation, with special reference to the flow in the neighbourhood of the models. The more obvious sources of error therefore appear to be ruled out. There remains a marked difference in turbulence level of the two airstreams: in the 4 ft \times 3 ft wind tunnel, the r.m.s. value of the streamwise component of the turbulent velocity is known to be about 0.01% of the undisturbed velocity, whereas the corresponding figure for the No. 1 11½ ft \times 8½ ft wind tunnel is likely to be neare 0.5%.

The corresponding corrected values of the drag coefficient follow from (17), and are listed in Table 3.

TABLE 3

4 ft \times 3 ft wind tunnel			No. 1 11 $\frac{1}{2}$ ft \times 8 $\frac{1}{2}$ ft wind tunnel		
$\frac{C_D S}{C}$	CD	C _{Dc}	$\frac{C_D S}{C}$	C _D	C _{Dc}
0.0180 0.0376 0.0602	1 · 200 1 · 249 1 · 335	1 · 143 1 · 131 1 · 144	0.0022 0.0066	1 · 158 1 · 175	1 · 151 1 · 154
Me	an	1.139	Me	an	1.152

Corrected Drag Coefficients for Non-Lifting Square Plates

Here again the results support the theory very well. Moreover, it is worth noting that both pairs of mean values defined in the tables satisfy relation (3) almost exactly, viz.

$$\frac{C_{Dc}}{k_{2}^{2}} = \frac{1 \cdot 139}{1 \cdot 361} = \frac{1 \cdot 152}{1 \cdot 375} = 0.837.$$

These mean values, together with the relations (13) and (17), lead to the graphical comparison between theory and experiment shown in Fig. 4.

2.7.2. $A = \infty$.—Data given by Fage and Johansen^{2,3} for a set of four thin flat plates, spanning a 7 ft wind tunnel, provide further support for the present theory. However, the data are rather less complete than for the square plates considered above, since detailed measurements of the flow were made behind only one plate.

Fage and Johansen found the pressure along the surface of the wake to be constant, within the accuracy of measurement, but to be slightly greater than the constant pressure measured on the rear surface of the plate. The pressure p_b appropriate to the theory must consequently be defined according to Section 2.6. It is not, in this case, directly equal to the measured base pressure. The relevant data are: S/C = 0.0715, $C_D = 2.13$, base-pressure coefficient -1.38, mean pressure coefficient along wake boundary -1.30, and the maximum width of the wake 1.85 times the breath of the plate. Hence the mean base-pressure coefficient from which the parameter k is to be determined, is -1.34.

Now, solving equation (13) for k_c , gives

$$\epsilon = 1/1 \cdot 04 = 0 \cdot 962$$

and, by (14) and (17), the set of corrected drag coefficients given in Table 4 are obtained from the measured values given by Fage and Johansen.

The fourth estimate of C_{De} in this set is rather lower than the others, perhaps because at so large a value of the blockage parameter $C_D S/C$ the pressure distribution over the plate becomes distorted. The mean value quoted in the table is therefore based on the first three results. The relations (13) and (17) then lead to the comparison between theory and experiment illustrated in Fig. 5.

$\frac{C_D S}{C}$	C _D	. C _{Dc}
0:0459 0:0976 0:152 0:204	1 • 928 2 • 050 2 • 130 2 • 144	1.845 1.87 1.86 1.79
Me	1.86	

Corrected Drag Coefficients for Non-Lifting Two-Dimensional Plates

2.7.3. The effect of aspect ratio.—It may be inferred that the blockage factor e ranges, in magnitude, from something rather greater than 5/2 for an effectively axi-symmetric flow, to a little less than unity for two-dimensional flow. Moreover, the theory is well supported by experiment at both these extremes. In view of the strong tendency to axial symmetry observed by Fail *et al* in the wakes behind rectangular plates of aspect ratio 1 to 10, almost all bluff-body flows of any practical interest might be expected to fall within the scope of the theory.

This argument justifies the use of the theory by Fail *et al* to correct their observations for blockage. They show that the base pressure is strictly uniform only at the extreme aspect ratios A = 1 and $A = \infty$. Between these limits the pressure distribution varies in the manner shown in Fig. 6, and the parameter k must be determined from the mean base pressure, according to Section 2.6. The resulting blockage factors are tabulated in Table 5 and plotted against 1/A in Fig. 7.

TABLE !	5
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Blockage Factor for Non-Lifting Rectangular Plates

A	6
$ \begin{array}{c} 1 \\ 2 \\ 5 \\ 10 \\ 20 \\ \infty \end{array} $	2.77 2.70 2.41 2.13 1.47 0.96

In the interval A = (1, 10) the blockage factor lies roughly in the range $\epsilon = 5/2 \pm 1/4$. And the constant value $\epsilon = 5/2$ leads to errors of $\pm 0.1\Delta q$ at the extreme points of the range. This amounts to an error of no more than 0.01q, if Δq is not itself allowed to exceed 0.1q. In practice, therefore, $\epsilon = 5/2$ is probably a satisfactory approximation for three-dimensional flow.

3. Stalled Wings.

3.1. Properties of the Wake.

A wing of finite span usually stalls gradually, in the sense that the transition from streamline flow to complete stall can occupy a substantial incidence range. The streamline flow tends to break down first over a limited part of the span, and the stalled region or regions then increase in extent with

increasing incidence until they eventually envelop the entire wing. It is not until this final stage is reached that the flow as a whole closely resembles the bluff-body flow considered so far. And even then there is the additional complication of the lift sustained by the stalled wing, and its possible influence on the wake structure.

However, there is some evidence to suggest that a localised region of stall does not differ materially from a bluff-body flow. And there is evidence, also, to indicate a very strong tendency to axial symmetry in the wakes behind bluff bodies that are themselves far from axi-symmetric. In consequence, there is reason to hope that a simple extension of the foregoing theory might account for the blockage effects on stalled and partially-stalled wings.

The stalled wing of infinite span presents no serious difficulties. Provided that the stall is sufficiently developed for reattachment of the separated boundary layer on to the upper surface of the wing to be impossible, the wake is plainly of the bluff-body type. The presence of lift does not affect the analysis of Section 2.3—there is no induced drag—and the only problem likely to arise is the magnitude to be assigned to the blockage factor ϵ .

3.2. Recommended Forms of Correction.

3.2.1. Finite span.—Assuming that the tendency to axial symmetry in stalled regions of flow is universal—at least within the range of practical wing shapes—and that all such regions are similar in structure to the axi-symmetric bluff-body wake, the blockage factor might be expected to take the value $\epsilon = 5/2$, derived in Section 2.7.3, for most three-dimensional non-streamline flows of aerodynamic interest. But because of the effects of lift and partial stall, the drag coefficient relevant to the blockage parameter $C_D S/C$ cannot correspond to the total measured drag. With lift, the contribution from the last two integrals in equation (5) corresponds to an induced drag D_i , and is non-negligible. And, in addition, there is a momentum defect associated with that part of the wake within the streamline region of flow which corresponds to the conventional profile drag D_0 of streamline flow. The consequential modifications to equation (9) then result in the relation

$$C_{Ds} = C_D - C_{Di} - C_{D0} = m(k^2 - 1 - mS/C)$$

and the formulae (13) and (14) continue to hold provided that the C_p in them is replaced by C_{Ds} .

The problem, now, is to determine the drag coefficient C_{Ds} associated with the stalled regions. There is no way of doing this directly, and the solution depends, in practice, on the choice of a suitable variation of induced drag in the post-stall regime. Great accuracy is not required, and perhaps the most logical course is to define C_{Dl} by extrapolation from the measured properties of the unstalled wing. Visual observation of the flow development—for example, by the surface-oil technique—is a great help in locating the onset of stall. And linear extrapolation of that part of the measured $C_D \sim C_L^2$ relation appropriate to streamline flow—in the manner sketched in Fig. 8— is then probably sufficient for most purposes. This technique ensures that the desired C_{Ds} is zero for the unstalled wing, as it should be.

Once the various components of the measured drag have been identified—there is also a drag D_R associated with the support rig used in a wind-tunnel experiment, and assumed here to correspond to streamline flow—it is possible to formulate the composite correction

$$\frac{q_c}{q} = 1 + \frac{1}{2} \frac{S}{C} (C_{DR} + C_{D0}) + \frac{5}{2} \frac{S}{C} (C_D - C_{Dl} - C_{D0})$$
(18)

which reduces automatically to the correct formula for streamline flow at incidences below the stall, where the last term vanishes. However, inclusion of the second term on the right-hand side of the formula (18) is largely for the sake of completeness. It can usually be ignored in practice. For in most well-designed experiments the blockage corrections are insignificant until the final term in the expression (18) begins to take effect. In consequence there is also little need for precise definition of C_{D0} .

In order to illustrate the effectiveness of the correction formula, Fig. 9 shows the result of applying it to data obtained with two sizes of complete model in the No. 1 $11\frac{1}{2}$ ft $\times 8\frac{1}{2}$ ft wind tunnel. The model in question had a wing of delta planform of aspect ratio 3. And the results from each size of model have been corrected separately, using the technique described above.

3.2.2. Infinite span.—For a stalled wing of infinite span only the magnitude of the blockage factor is in doubt. And perhaps the best procedure is always to measure the pressure distribution over the upper surface of the aerofoil and to derive the appropriate blockage from the general formula (13). In view of the observed variation of base pressure with incidence and the corresponding variation in the blockage effect on two-dimensional flat plates, illustrated in Figs. 10 and 11, ϵ seems unlikely to differ much from unity once the stall is fully developed.*

4. Concluding Remarks.

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Since the factors which control the properties of a bluff-body wake have not been established theoretically, the theory of wind-tunnel constraint developed in this paper necessarily includes a large measure of empiricism. But for non-lifting bluff bodies, at least, the empirical relations used primarily the observation that C_D/k^2 is invariant under constraint and, less critically, the auxiliary relation (12) governing wake distortion—are well supported by experiment. Otherwise the theory depends only on an approximate expression of conservation of momentum in the stream outside the wake. It leads to the surprising, but experimentally confirmed, result that, for a given drag the effect of constraint is greater for a three-dimensional (axi-symmetric) body than for a two-dimensional one by a factor of about $2\frac{1}{2}$, a result that provides striking confirmation of the correctness of the derived dependence of the blockage factor on base pressure.

The crucial assumption underlying the extension to stalled wings is that the properties of the stalled regions of flow are essentially those of axi-symmetric bluff-body wakes. But although none of the known detailed observations are in obvious conflict with this assumption, it cannot be readily established quantitatively. The asymmetric form of a flow pattern associated with lift makes the relevant stream surfaces virtually impossible to define experimentally. And for this class of flows the main support for the theory comes ultimately from tests of the final correction itself.

No doubt there are limits to the range of applicability of the correction formulae. But on the evidence available at present it seems logical to assume that the blockage factor $\epsilon = 5/2^{+}$ is universal for three-dimensional flows. It seems logical, also, to assume that the theory holds for slender wings

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* The pressure coefficients plotted in Fig. 10 correspond to mean base pressures defined according to Section 2.6.

† This is believed to be adequate for most purposes. But slightly different values of the blockage factor might be more accurate for specific cases. For example, $\epsilon = 2.75$ is better than $\epsilon = 2.5$ for the axi-symmetric bluff-body flow.

with vortex breakdown—which gives rise to axi-symmetric regions of flow resembling bluff-body wakes—provided that the breakdown occurs sufficiently well forward of the trailing edge. There is little doubt that, in the absence of vortex breakdown, the flow past a slender wing or body is, for the present purpose, a streamline flow, subject only to the conventional wake-blockage correction.

In many ways the most difficult part of the stalled-wing problem is to identify the induced drag which, by subtraction from the measured drag, effectively defines the component of drag associated with the blockage effect. The empirical extrapolation proposed here is, at best, plausible. It is believed to be adequate, at least for a moderate range of incidence beyond the onset of stall, where practical interest is greatest. But the theory should obviously be applied with judgement. And experiments should be designed so as to make the correction small, and thus to minimize the effects of errors in the empirical parameters that enter the correction formulae. It is also advisable to obtain visual observations of the onset of stall.

SYMBOLS

x, y, 2		Rectangular Cartesian co-ordinates, with z measured in the direction of the undisturbed_stream
u, v, w		Velocity component in the x, y and z directions
U, P, H		Velocity, static pressure and total head of the undisturbed stream
q		Dynamic pressure of the undisturbed stream
Þ		Static pressure
<i>p</i> _b		Base pressure
C_p		Pressure coefficient $(p-P)/q$
D		Drag
D_0, D_i, D_s, D_R		Components of the measured drag (Section 3.2.1)
C_D		Drag coefficient D/qS
k		Base-pressure parameter (Section: 2.1)
S		Reference area of model
С		Cross-sectional area of wind tunnel
В		Cross-sectional area of wake
A		Aspect ratio
'n	R	B/S
m*		A datum value of m (Sec. on 2.4)
£		Blockage factor {equation (14)}
Δ		Operator denoting increment due to constraint
c		Suffix denoting effective, or corrected, values

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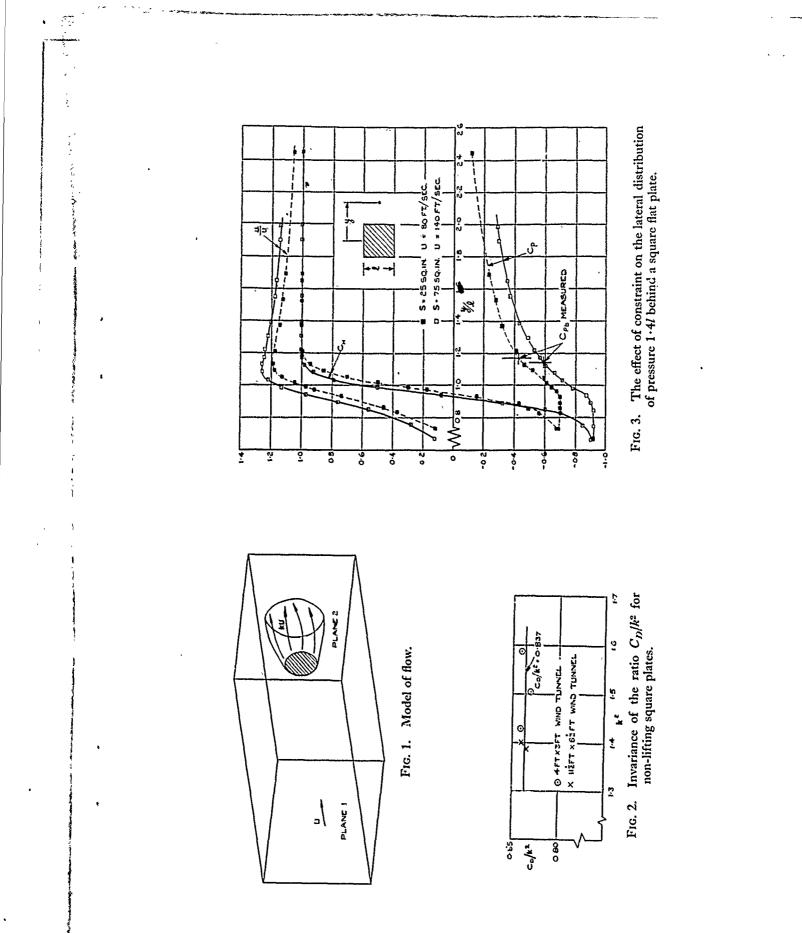
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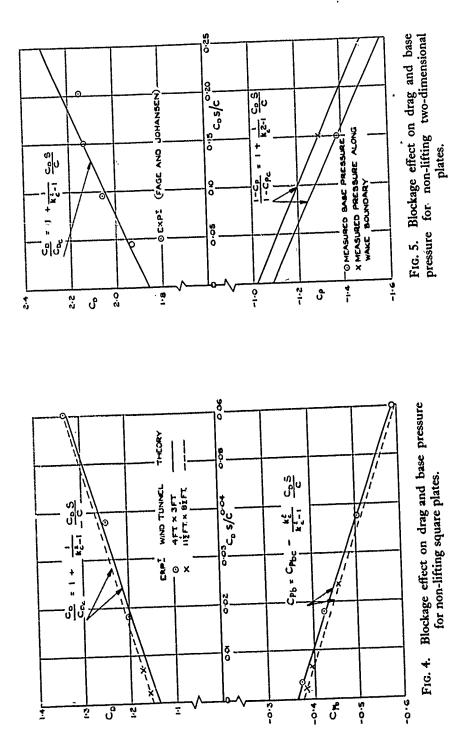


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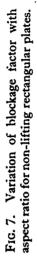
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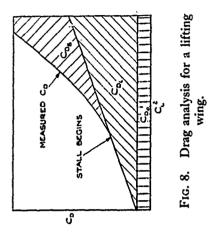
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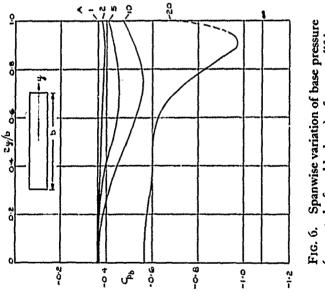
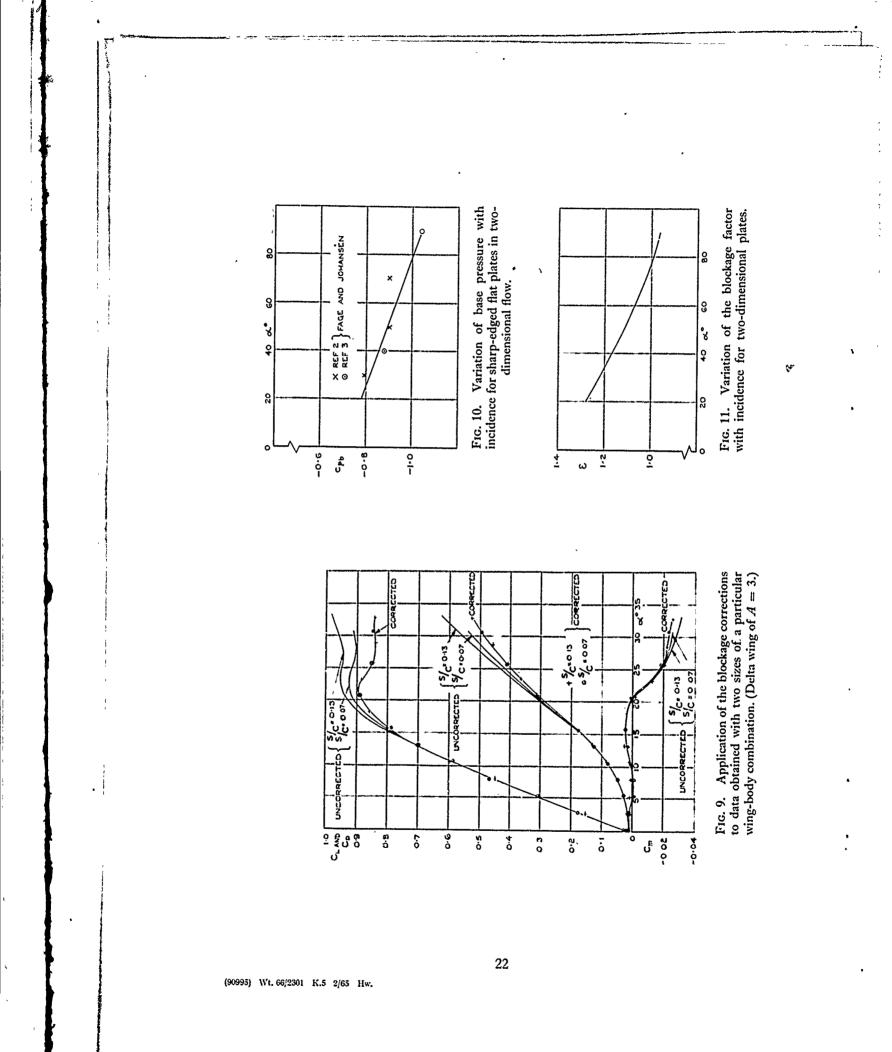


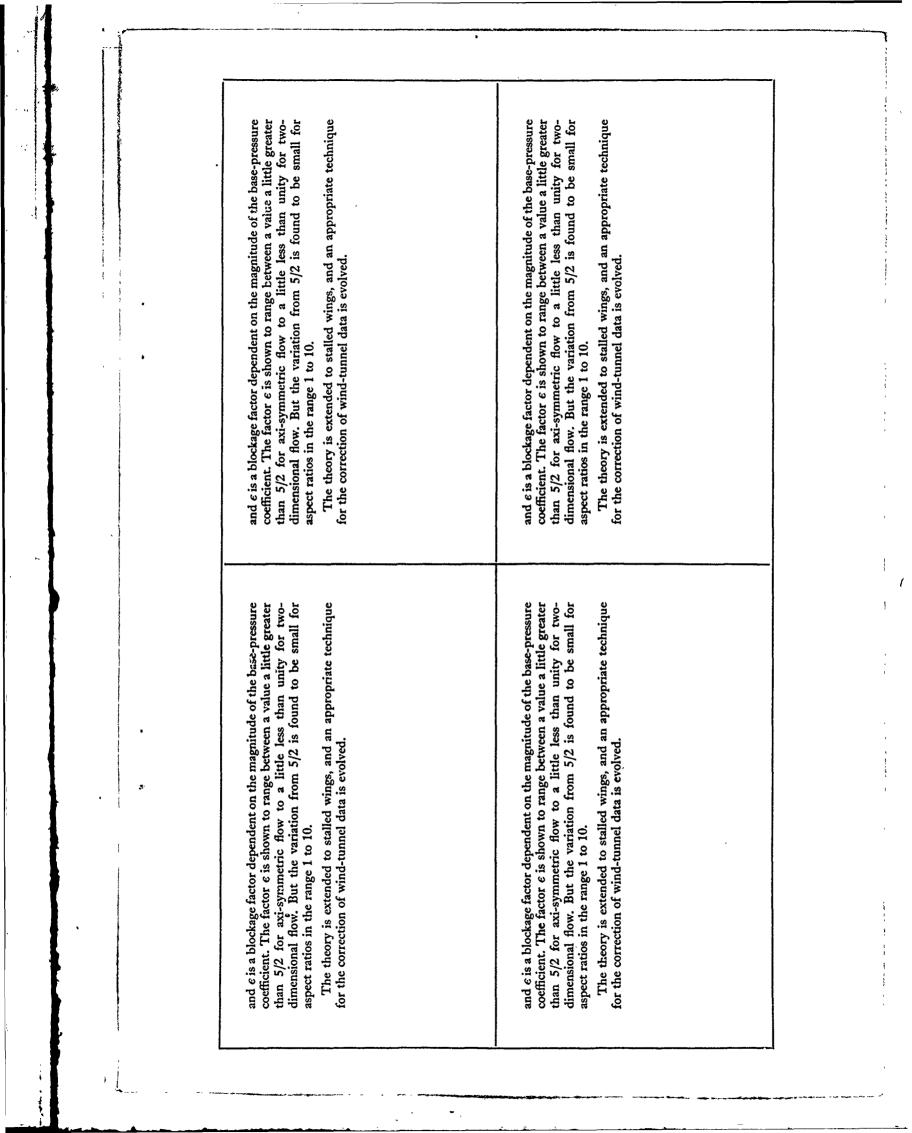
FIG. 6. Spanwise variation of base pressure (corrected for blockage) for non-lifting rectangular plates.



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