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A thermal history model for the Earth with parameterized convection

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Summary. A thermal history model for the Earth is described in which the energetically important effects of convection are parameterized through the Nusselt number. The validity of the resulting quasi-steady-state thermal model is shown to depend upon the separation of two time-scales – a dynamic time-scale associated with the overturn time for an assumed mantle-wide convective circulation, and a thermal time-scale associated with the cooling of the planet. Provided the initial thermal state of the Earth was 'hot', the assumption of a time-scale separation can be shown under certain conditions, to be valid throughout the Earth's history. In this connection, the temperature-dependent mantle rheology plays a key role in regulating the thermal history. It is shown that the present-day, gross thermal structure of the Earth can be understood within the context of a quasi-steady-state model which is driven mainly by primordial heat. The notion of whole-mantle convection is shown to be consistent with several additional observational constraints, including the observed mean lithospheric thickness and the mean plate velocities. We briefly consider the extension of the parameterized thermal model to Venus.

1 Introduction

The first serious attempt to link geological observation with the thermal state of the Earth's interior was made by Lord Kelvin (1882). Accepting the Laplacian condensation hypothesis which was then the generally accepted explanation for the formation of the solar system, he supposed that the Earth condensed at high temperatures as a liquid and subsequently cooled rapidly to the solid state. There were, of course, no radioactive heat sources in this model and it was therefore implied that the entire thermal history was determined at formation. Subsequent to Kelvin's classic investigation there have been many noteworthy attempts to deduce a thermal history based upon the assumption that ordinary conduction is the only heat transport mechanism. Jeffreys (1924) and Slichter (1941), for example, emphasized the long time-scale associated with the diffusion of heat in the interior. Urey (1952)

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consider the importance of radioactive heating and Jacobs & Allen (1954) carried out the first detailed numerical calculations which accounted for the previously-neglected time dependence of radioactive heat production. The non-linear contribution to the thermal conductivity due to radiative heat transfer was studied by Clark (1956) and Lubimova (1967).

With the observation by Birch (1958) that the present-day rate of heat production for an earth model possessing chondritic abundances of radioactive isotopes closely approximates the actual surface heat flow, it was suggested that either the radioactive elements were concentrated near the surface or that the thermal conductivity increased with depth by several orders of magnitude. Both these possibilities were investigated numerically by Macdonald (1959) and later by Elsasser (1963) and Birch (1965). However, analytical (McConnell *et al.* 1967) and experimental studies (Fujisawa *et al.* 1968; Schatz & Simmons 1972) suggest that radiative conductivity should not significantly reduce the long time constant for heat transport through the Earth.

Models for the Earth's thermal history reached a new level of sophistication with the attempt to simulate melting and its consequences. Reynolds, Fricker & Summers (1966) developed procedures which attempted to simulate both melting and fluid convection. Subsequent finite difference solutions to the diffusion equation (Fricker, Reynolds & Summers 1967; Lee 1968) also included schemes for the redistribution of radioactive isotopes. While increasingly elaborate subroutines continued to be appended to these conduction models, a common characteristic which remained was that the thermal state of the interior, below a depth of about 10^3 km could not 'communicate' with the surface on a time-scale comparable to the Earth's age. In a series of articles, Tozer (1965, 1972, 1977) questioned the relevance of thermal histories based solely upon solutions to the diffusion equation. He argued on the basis of the Rayleigh number criterion for stability that proposed conduction temperature profiles for the mantle were significantly unstable to convection and hence would not be physically realized.

Mantle convection was not a new idea. It was considered as early as 1921 by Bull and was proposed by Holmes (1931) as the driving mechanism for continental drift. Early studies of this process were described by Pekeris (1935) and Hales (1936) although the idea was certainly not fashionable at the time and had few adherents. The importance of mantle convection to thermal history models becomes readily apparent when one realizes that if the observed lithospheric plate velocities are representative of convection velocities at depth, then the transport of heat by convection will completely dominate other modes of heat transport. This importance would be enhanced in the past, when convection is believed to have been more vigorous than it is at present.

While the driving mechanism responsible for the relative motion of the lithospheric plates is almost universally agreed to be associated with some sort of mantle convection, there still exist significant differences of opinion concerning important details of this convection. It is not known whether the heating mechanism responsible for driving convection is due to the decay of radioactive mantle heat sources or to the escape of primordial core heat, or some combination of the two. The extent to which the mantle participates in convection is also not known. Some authors (McKenzie, Roberts & Weiss 1974; Richter 1978) favour the hypothesis that convection is confined to the upper mantle or at least that the upper and lower mantles are filled by distinct convective circulations (McKenzie & Weiss 1975). Others prefer the notion of whole mantle convection (Peltier 1972, 1976; O'Connell 1977).

One might expect that these problems could be resolved through a direct numerical solution of the full, coupled thermo-mechanical equations which govern convection. For such a solution to be physically relevant to the mantle, it would have to be fully three

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dimensional, time dependent, and non-Boussinesq. Notwithstanding the practical difficulties which preclude the construction of such a solution at present, in particular the spatial resolution required to resolve the thermal boundary layers at the anticipated large Rayleigh numbers (see Busse (1979) for a discussion of further problems), the physical significance of such solutions will be limited by our present level of understanding of the required thermodynamical and rheological constitutive relations.

In constructing a thermal history model for the Earth we are therefore confronted with a dilemma. Recognizing that one is concerned with the dynamics of the mantle convective circulation only in so far as the radial transport of heat from the interior is concerned, we may enquire into the conditions under which a restricted solution to the problem may be obtained in which the gross internal energy balance is treated correctly, without requiring an explicit solution for the dynamics. This enquiry has led us to the construction of thermal history models in which the effects of mantle convection are 'parameterized', and the purpose of the present paper is to describe the results which we have obtained with such models and the conditions under which their application is valid.

The use of such models is, of course, not without precedent in the general area of geophysical fluid dynamics. Perhaps the best example in which an equivalent approach is adopted is the model of global atmospheric climate described by North (1975). In this model, the important poleward heat transport effected by stationary and transient baroclinic waves is parameterized in terms of the baroclinicity (pole to equator temperature gradient) of the basic state. The parameterization is in terms of an enhanced thermal conductivity, which leads to an increased heat flux for a given baroclinicity beyond that which would be affected either by conduction or radiation. The validity of this parameterization relies upon a separation of time-scales in the problem, the dynamical processes being characterized by a time-scale which is short compared to the time-scale over which significant changes in climate occur.

As will become clear in the course of the subsequent discussion, the validity of our mantle 'climate' models depends upon a similar separation between the dynamic and thermal time-scales. The short time-scale refers to the convective overturn time for the mantle, and the long time-scale is determined by the rate at which the mean mantle temperature changes. We will also see that the validity of these quasi-steady-state models requires an initially 'hot' thermal state for the Earth. In Section 2 we present the development of our assumed formation model as well as some observational 'evidence' which seems to favour an initially 'hot' thermal state.

Finally, we may consider which questions concerning the Earth's thermal history can be addressed within the context of our radially symmetric, parameterized convection model. Assuming that the interior of the Earth is totally depleted in radioactive heat sources, and that whole-mantle convection has persisted throughout the history of the Earth, the relevance of the quasi-steady-state model may be studied from the standpoint of the present-day gross internal structure and thermal state of the Earth. We may further investigate to what extent the thermal history is constrained by the present-day mean mantle viscosity of 10^{22} P which has been inferred from studies of postglacial rebound.

The use of a radially symmetric, quasi-steady-state convection model is particularly appropriate to investigating these problems as we will be maximizing the convective transport of heat from the interior. Therefore, as regards the relevance of primordial heat to the presently observed thermal characteristics of the Earth, we may consider that we are testing a 'worst case' and hence are examining an important end member in a class of possible thermal history models. The demonstration of the effects of a temperature-dependent viscosity on the results will be of particular importance. Finally we will show that

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by interpreting the oceanic plates as the cold thermal boundary layer of a mantle-wide convective circulation, we are led to a thermal history model which is, at present, fully self-consistent with the observed plate velocities and horizontal scales, and the inferred boundary layer thickness.

The analytical formulation of the quasi-steady-state model, and the conditions under which it is expected to be valid are discussed in Section 3. We also consider an alternate parameterization scheme for solid-state convection, which is based upon the use of an enhanced thermal conductivity in the standard diffusion equation. The resultant thermal history models for both approaches are discussed in Section 4. In Section 5 we summarize our main findings and suggest that while the simple, quasi-steady-state model only represents a first step at understanding the impact which convection may have had on the Earth's thermal evolution, the results may nevertheless serve as a useful guide in understanding the predictions of models based upon complete solutions to the Navier–Stokes equations.

2 The formation model

Inasmuch as we are attempting to solve an initial value problem for which the initial conditions are unknown, we cannot separate the problem of the thermal evolution of a planet from the problem of its formation – unless, of course, the system should conveniently exhibit a fading memory of its initial state. The most popular formation model at present is the gradual accretion of solid material from dispersed matter (grains and dust). While the origins of this suggestion may be traced to the nineteenth century, it has recently been the subject of renewed discussion, partly because of the fact that the surfaces of the Moon, Mars, Mercury, Phobos and Deimos have been observed to be saturated with craters. However, it must be remarked that the accretion process has never been demonstrated or observed to occur. Until such time as planetary formation can be directly witnessed, this model must remain speculative!

We shall assume for simplicity that the Earth accreted from a homogeneous mixture of iron and silicates. In view of the observational 'constraints' which suggest that the fluid iron core/solid silicate mantle structure is an early feature (early magnetic field, Hanks & Anderson 1969; early solid mantle, Ringwood 1960; Ulrych & Russell 1967; Buolos & Manuel 1971), such a model would require that the bulk of the Earth passed through an early molten stage, followed by rapid mantle solidification. As we shall see in Section 2.1.1, the action of fluid convection during the formation process is capable of removing the enormous quantities of heat associated with accretion and core–mantle segregation on a sufficiently fast time-scale so as to permit the above 'constraints' to be satisfied.

2.1 OUTLINE OF THE ACCRETION MODEL

In this section we briefly outline the physical basis of the accretion model. A more detailed description is given in Sharpe (1977).

Several authors have suggested on the basis of geochemical and geophysical data that the initial temperature of the Moon was strongly peaked towards the surface, resulting possibly in crustal formation (Ringwood 1970; McConnell & Gast 1972; Wood 1972). If we assume that accretion occurred in a radially symmetric fashion from a cloud of particulate matter, then by employing a simple energy balance relation which must be continuously satisfied at the accreting surface, we may enquire into the energetics of the accretion process and the characteristics of the 'primordial cloud' required to achieve near-surface melting of the Moon (Mizutani, Matsui & Takeuchi 1972). Assuming next that the Earth accreted from a cloud of

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similar characteristics, consisting of material having a basic chondritic composition, we can also determine its initial uncompressed, undifferentiated thermal state. The resultant thermal profile is shown in Fig. 1(a) and was obtained from the following energy balance equation,

$$\rho\left[\frac{GM(r)}{r}+\frac{v_\infty^2}{2}\right]\frac{dr}{dt}=\epsilon\sigma\left[T^4(r)-T_a^4\right]+\{\rho C_p\left[T(r)-T_b\right]+\rho L\}\frac{dr}{dt} \tag{1}$$

where $M(r)$ and $T(r)$ are the mass and temperature respectively of the planet at radius r, ρ is the density, ϵ the opacity, C_p the heat capacity at constant pressure, L the latent heat of fusion, T_a the temperature of free space, T_b the base temperature of the accreting material (usually $= T_a$) and v_∞ is the velocity of the accreting material far from the planet. The assumed minimum efficiency for conversion of the energy of the impacting material to locally heating the *in situ* material, as deduced from a consideration of the lunar accretion is 55 per cent. The other main parameters are the mean cloud density (10^{-9} gm cm^{-3}), and the most probable particle speed in an assumed Maxwell–Boltzmann velocity distribution (10 m/s). The corresponding total time for accretion is 1.0×10^5 yr.

We note from Fig. 1(a) that the initial thermal state of the Earth is such that at the end of accretion it is molten for all radii greater than about 1470 km. This implies that not only is a continual supply of material available during formation, but that the growth rate of the 'embryo' is sufficiently rapid to satisfy the instantaneous minimum conversion efficiency required for melting, which diminishes from 55 per cent at 1470 km to 2 per cent at the final radius. As a result, if the Earth accreted from material which was derived from a primitive solar nebula, then either condensation of this nebula was complete before accretion began, or the rate of condensation was comparable to the rate of growth.

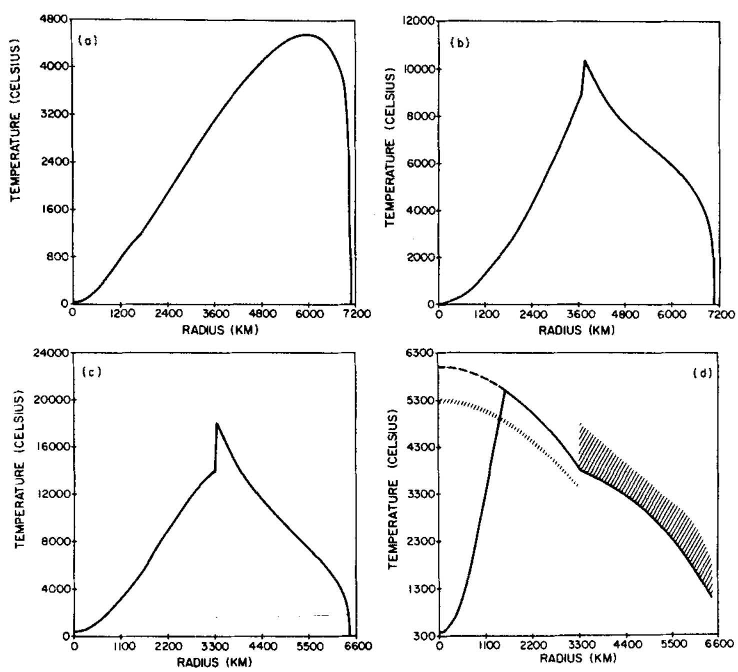

Figure 1. Stages in the development of the initial temperature profile: (a) following homogeneous accretion but prior to differentiation and compression; (b) following core–mantle segregation; (c) following adiabatic compression; (d) following the action of fluid convection. Lower and upper boundaries of the hatched regions are the solidus and liquidus curves respectively.

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While much more sophisticated, and probably more physically realistic dynamical models exist for this stage of the accretion process (Giuli 1968; Safronov & Zvjagina 1969; Alfvén & Arrhenius 1970; Ip 1974; Weidenschilling 1974, 1976; Mizutani *et al.* 1972; Wetherill 1976; Kaula 1979), some of which attempt to understand planetary spin, we feel that the subsequent modifications to the geotherm in Fig. 1(a), due to core–mantle segregation, adiabatic compression and finally fluid convection (see Fig. 1(b), (c), (d)), did not warrant a more detailed treatment. Furthermore, as we are only interested in deducing the formation model in so far as it provides a 'hot' initial thermal state with which to commence the evolutionary model, we again feel that the present simple model is sufficient. As will be seen in Section 3, any 'hot' initial model will be sufficient to justify the 'separation-of-time-scales' assumption required in the early history of the model. The present formation model has the added advantage that the somewhat speculative 'constraints' on the early history of the Earth are also satisfied (see Section 2.1.1).

Early heating effects other than accretion and gravitational differentiation, such as those due to short-lived radioactive isotopes and electromagnetic induction have been ignored, not because we consider them irrelevent, but because we wish to study the consequences of 'primordial' heat associated solely with the formation process itself.

2.1.1 *Fluid convection*

The four processes shown in Fig. 1 actually occur concurrently as the planet grows. Consequently, the extremely high temperatures of Fig. 1(b), (c) are never actually realized. As the Earth accretes, we must expect that at some time during the formation process the temperature at the instantaneous core–mantle boundary will exceed that at the accreting surface. As the mantle is completely molten at this stage, it can easily be shown (see Section 3.1 and Sharpe 1977) that the convective overturn time is small compared to the accretion rate, so that the immediate establishment of a fluid adiabat down to the core–mantle boundary, consistent with the instantaneous surface temperature, can be assumed. The temperature at the core–mantle interface determined in this fashion will result in a steep, superadiabatic gradient at the outer boundary of the core. Consequently a fluid adiabat should also develop in the core. It is based upon the temperature at the core–mantle boundary established by mantle convection, and is constructed back until it intersects either that part of the initial temperature profile which is rising steeply from the inner core into the lower regions of the molten outer core, or until the core solidus itself is intersected. The excess heat is transported to the top of the core where it is 'picked up' by mantle convection and then removed to the surface. It is in this manner that the formation model prevents the enormous quantities of heat produced by iron-silicate segregation and adiabatic compression from ever creating internal temperatures significantly exceeding those in Fig. 1(a).

As the radius of the accreting Earth approaches its final value, it is seen in Fig. 1(a) that the instantaneous surface temperature decreases rapidly. At some time prior to the completion of formation, the resultant mantle adiabat will intersect the mantle solidus. If the solidus is superadiabatic everywhere, the intersection will first occur at the core–mantle boundary. As the instantaneous surface temperature continues to decrease during the final stages of accretion, consecutive mantle adiabats will intersect the solidus at progressively greater radii. When the accretion process has been completed, the initial temperature profile for the mantle will be coincident with the solidus except at the surface, where the temperature will equal that of free space. Since the entire mantle is now 'solid', as well as superadiabatic, it is 'potentially' unstable to solid-state convection. Our choice for the solidus and liquidus curves employed in Fig. 1(d) will be discussed in Section 3.

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2.2 DISCUSSION

An examination of Fig. 1(d) reveals that the initial internal structure and thermal state of the Earth are almost indistinguishable from the known present-day conditions. In this sense, the initial conditions are the same as the final ones, so that the thermal history simulation reduces to a determination of those critical physical parameters which permit such a situation to prevail for at least 4.5×10^9 yr. This will form the focal point of Section 4.

An important consequence of the formation model is the depletion of radioactive heat sources in the interior. This may be understood in terms of an initially molten mantle which solidifies from the base outwards. The residual liquid which is successively squeezed upwards would be progressively enriched in the incompatible radioactive isotopes U^{IV}, Th^{IV} and K^{I}, due to the inability of these ions to enter the major phases during the primary crystallization of the mantle (Ringwood 1960; Taylor 1964). By the end of accretion, the interior of the Earth will essentially be depleted in heat sources (except for the possibility of potassium in the core – see Section 4.1.5), the bulk of them being concentrated in an outer 'crustal' radioactive surface layer.

While recognizing that the heat generated by the decay of these isotopes is presently considered by some authors to be the main energy source driving mantle convection, we feel that the lack of constraining data for this important thermal parameter, justifies the consideration of an 'end member' thermal history model which neglects mantle heat sources. While the success of a thermal model driven mainly by primordial heat does not preclude the existence of deep-mantle sources, it does condition arguments which insist unequivocally on the dominance of radioactive heating.

Finally we consider the nature of the solid inner iron core. Fig. 1(d) suggests that by the end of accretion, the Earth should possess a small, cold, inner core consisting of undifferentiated, primordial material. This feature is a consequence of employing the Moon as a 'control point', a planet which itself may be the 'inner core' for a larger planet which never formed due to a lack of material. Scenarios whereby this primitive core may be replaced by iron and subsequently preserved until the present day are discussed in Sharpe (1977).

An equally plausible model for the inner core assumes that, initially, the core geotherm was everywhere above the liquidus, and that subsequently as the core cooled, a super-adiabatic iron solidus was first intersected at the Earth's centre, with continued solidification then proceeding outwards (Jacobs 1953; Safronov 1972). This scenario is demonstrated in Section 4. Since the inner core is so small, its formation and evolution will in general have little impact on the main thermal features for the rest of the Earth. Accordingly, to simplify the treatment of the evolutionary model presented in the next section, we have somewhat arbitrarily continued the initial outer-core geotherm down to the centre of the Earth along an adiabat (dashed line in Fig. 4(d)).

3 Parameterized convection models

3.1 FORMULATION OF THE PARAMETERIZATION SCHEMES

3.1.1 Quasi-steady-state thermal models

The application of a steady-state heat transport model to a study of the Earth's thermal history at first appears inconsistent. Whether the Earth's heat engine is driven by primordial heat, or by the decay of radioactive heat sources, over a sufficiently long time-scale, the

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interior must cool, eventually attaining an isothermal state in which the temperature everywhere equals the surface temperature. It would seem, then, that we should rather consider that the Earth cools via a sequence of *quasi* steady states. The validity of such an assumption would then depend upon the demonstrable separation of time-scales of simultaneous thermal processes.

To see this more clearly, consider a simplified earth model in which the fluid core of radius r_c is characterized by a mean temperature $\bar{T}(t)$, and for which vigorous, whole-mantle convection is assumed to be occurring. If the heat flux across the core–mantle boundary is q_c, and that at the surface due only to the secular cooling of the mantle is q_m, then it follows that,

$$\frac{d\bar{T}}{dt} = -\frac{3q_c}{r_c\bar{\rho}_c C_{p_c}} \approx -\frac{6q_m}{[(r_p^3 - r_c^3)/r_p^2]\bar{\rho}_m C_{p_m}} \tag{2a}$$

where $\bar{\rho}_c$ and $\bar{\rho}_m$ are the mean densities of the core and mantle respectively and C_{p_c} and C_{p_m} are the corresponding specific heats. Furthermore we also have that,

$$q_s = q_c(\mathscr{A} + (r_c/r_p)^2) \tag{2b}$$

where

$$\mathscr{A} \equiv \frac{1}{2}\frac{C_{p_m}}{C_{p_c}}\frac{\bar{\rho}_m}{\bar{\rho}_c}\left(\frac{r_p}{r_c} - \left(\frac{r_c}{r_p}\right)^2\right)$$

and q_s is the surface heat flux. We may further relate q_c to the vigour of the mantle convection by extending results which have been obtained for steady-state, plane-layer convection at high Rayleigh and high Prandtl numbers. The steady-state, two-dimensional boundary layer theory of Turcotte & Oxburgh (1967, 1972) leads to the following predictions:

$$Nu \sim \left(\frac{Ra}{Rc}\right)^{1/3} \tag{3a}$$

$$u \sim w \sim 0.15\, Ra^{2/3}\, \kappa/d \tag{3b}$$

$$\delta \sim d(Rc/Ra)^{1/3} \tag{3c}$$

where u and w are typical horizontal and vertical velocities in the flow, d is the thickness of the convecting layer, δ is the boundary layer thickness and Ra and Rc are the actual and critical Rayleigh numbers respectively. The Nusselt number, Nu, expresses the ratio of the total heat transported across the convecting layer, to the heat which could be conducted across for the same boundary temperatures. For convection driven by heating from below, the Rayleigh number is given by

$$Ra = \frac{g\alpha d^3 \Delta T}{\kappa\nu} \tag{4}$$

where g is the acceleration due to gravity, α is the thermal expansivity, and κ and ν are the thermal diffusivity and kinematic viscosity respectively. When compressibility effects are important, the temperature drop across the layer is replaced by the adiabatic excess.

While equation (3a) is known to fit a wide range of experimental data for steady-state, plane-layer convection (Globe & Dropkin 1959; Rossby 1969), and is also demonstrable from similarity theory under the same conditions (Kraichnan 1962), the relevance of the system of equations (3) to convection in a spherical shell remains to be shown. However assuming

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this validity, we may relate q_c to the vigour of the overlying mantle convection as follows:

$$Nu \sim q_s/(K\bar{T}/d)$$

$$\rightarrow q_c \sim \left(\frac{K\bar{T}}{d}\right)\left(\frac{Ra}{Rc}\right)^{1/3}\left(\mathscr{A}+\left(\frac{r_c}{r_p}\right)^2\right)^{-1} \tag{5}$$

where K is the mean mantle lattice conductivity. In writing equation (5) we have assumed a mantle surface temperature effectively zero (0 K), and have also assumed a uniform heat flux through the mantle, whereas this flux should be corrected by a geometric factor. Since we are only estimating the gross energetics in this example, these secondary effects are ignored here, but are included later in this section in a more rigorous analysis.

Substituting equation (5) into equation (2) leads to the following O.D.E. for $\bar{T}$,

$$\frac{d\bar{T}}{dt} = -a\bar{T} \tag{6}$$

where

$$a \equiv \frac{3\kappa(Ra/Rc)^{1/3}}{d\cdot r_c}\left(\mathscr{A}+\left(\frac{r_c}{r_p}\right)^2\right)^{-1}.$$

From equation (6) we may identify the characteristic time-scale for the cooling of the planet as the time required for $\bar{T}$ to decrease by a factor 'e'

$$t_{1/e}\,|_{\text{planet}} = \frac{r_c\cdot d}{3\kappa(Ra/Rc)^{1/3}}\left(\mathscr{A}+\left(\frac{r_c}{r_p}\right)^2\right). \tag{7}$$

An additional time-scale, related to the convective overturn time for the mantle, t_{ov}, may be obtained from equation (3b), assuming steady state,

$$t_{ov} = \frac{\pi d}{u} \sim \frac{\pi d^2}{0.15\,Ra^{2/3}\kappa}. \tag{8}$$

The validity of the quasi-steady-state thermal history model requires that,

$$t_{1/e}\,|_{\text{planet}} \gg t_{ov}, \tag{9}$$

so as to enable a mantle adiabat, consistent with the lower boundary temperature to be continuously maintained during the secular cooling of the planet. An additional restriction is,

$$t_{ov} \ll t_{age} \tag{10}$$

since otherwise it will not be possible to time step the problem via the construction of a sequence of quasi-steady-state geotherms.

Some support for this assumption of quasi-steady-state may be obtained by employing the steady-state equations (3b) and (3c) to estimate u and d. For a Rayleigh number of 10^7, obtained by assuming whole-mantle convection and a mean viscosity of 10^{22} P, consistent with the post-glacial rebound data (Cathles 1975; Peltier 1974, 1976; Peltier & Andrews 1976; Peltier, Farrell & Clark 1978), we obtain a boundary layer thickness $\delta \sim 100$ km and a plate velocity $u \sim 10$ cm/yr. The value of δ agrees with the lithospheric thickness which is required to fit post-glacial rebound observations and the value of u agrees with the observed speeds of the surface plates. While the significance of these results may be questioned because we have applied a two-dimensional theory to what is intrinsically a three-dimensional problem, we feel that the resultant correlation is nevertheless encouraging.

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Table 1. *Physical parameter values utilized in the simulation.*

Parameter	Value	Reference
Mass	5.97×10^{27} gm	Allen (1963)
Radius	6.371×10^{8} cm	Allen (1963)
Density		
silicates	3.3 gm cm^{-3}	Ringwood (1966)
iron	7.9 gm cm^{-3}	Ringwood (1966)
Heat of fusion		
silicates	4×10^{9} erg/gm	Birch, Schairer & Spicer (1942)
iron	2.72×10^{9} erg/gm	Bolze & Tuve (1970)
Expansivity		
silicates	3×10^{-5} K^{-1}	Stacey (1977)
iron	7×10^{-5} K^{-1}	Basinski, Hume-Rothery & Sutton (1955)
Specific heat		
silicates	10^{7} erg/(gm °C)	Birch *et al.* (1942)
iron	0.6×10^{7} erg/(gm °C)	Birch *et al.* (1942)
Thermal conductivity		
silicates	5×10^{5} erg/(s cm °C)	Schatz & Simmons (1972)
iron	5×10^{6} erg/(s cm °C)	Powell, Ho & Lilly (1966)
Crustal heat sources		
granite	5.43×10^{-5} erg cm^{-3} s^{-1}	MacDonald (1965)
basalt	7.92×10^{-6} erg cm^{-3} s^{-1}	MacDonald (1965)

Since the Rayleigh number is inversely proportional to the mean viscosity, which itself has an exponential dependence on temperature, it is necessary as the simulation proceeds to check *a posteriori* whether equation (9) is satisfied at each time step. This emphasizes the importance of starting with 'hot' initial conditions since, for such a model, the Rayleigh number would have been continuously decreasing to its present-day value of 10^7. This rate of decrease, and hence the extent to which the thermal and dynamical time scales remain separated, is particularly sensitive to the rheology.

We proceed now to develop in detail our quasi-steady-state thermal model. The results of the simulation are discussed in Section 4.

Steady-state convection in a plane layer heated from below is observed experimentally and numerically to be characterized by a slowly rotating isothermal core, with thin conductive thermal boundary layers adjacent to the bounding plates. Where the length scale of the system is sufficiently large that compressibility effects are important, the mean, horizontally averaged temperature away from the boundary layers will lie close to an adiabat. In our thermal history models, it must be understood that the temperature at any radial location actually represents the mean temperature averaged over a spherical surface having the specified radius.

In Fig. 2 we present the geometric setting of the problem. The radius to the core–mantle boundary is r_1, and that to the surface is r_2. The heat flux into the base of the mantle, which is at temperature T_1, is Q_1 and the temperature at the top of the lower boundary layer, having thickness δ_1, is T_{δ_1}. The heat flux into the base of the upper thermal boundary layer is Q_2. The temperature at its base is T_{δ_2} and its thickness is δ_2. Embedded within this layer is a shell of thickness d adjacent to the surface, which is assumed to contain all the radioactive heat sources, consistent with the formation model of Section 2. The heat source generation rate per unit volume is $\hat{Q}$.

In the definition of the Nusselt number, the conductive heat flux refers to that flux which would be realized under conductive steady-state conditions, with the same boundary temperatures as for the convective problem. Therefore as a first step towards constructing

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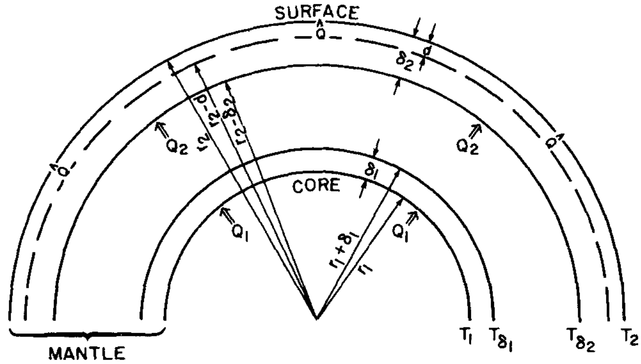

Figure 2. Geometric setting for the quasi-steady-state thermal history model. See text for the definition of the symbols.

the quasi-steady, convective, mantle geotherm, we must construct the steady-state, *conductive* mantle geotherm which is consistent with the 'instantaneous' lower boundary temperature T_1, and the fixed, upper boundary surface temperature T_2.

We have then (for $\hat{Q} = 0$),

$$T(r) = -\frac{a}{r} + b \qquad r_1 \leqslant r \leqslant r_2 \tag{11}$$

where

$$a \equiv \left(\frac{T_2 - T_1}{r_2 - r_1}\right) r_1 r_2, \qquad b \equiv T_1 + \frac{a}{r_1}.$$

The steady-state conductive flux through the surface follows,

$$Q_{\text{cond.}} = \bar{K}\left(\frac{r_1}{r_2}\right)\left(\frac{T_2 - T_1}{r_2 - r_1}\right) \tag{12}$$

where $\bar{K}$ is the mean-mantle thermal conductivity. From the definition of the Nusselt number, we therefore have that,

$$Q_2 = Nu \cdot \bar{K} \cdot \left(\frac{r_1}{r_2}\right) \cdot \left(\frac{T_2 - T_1}{r_2 - r_1}\right). \tag{13}$$

For steady-state convection, the geometry of the problem requires that,

$$Q_1 = Nu \cdot \bar{K} \cdot \left(\frac{r_1}{r_2}\right) \cdot \left(\frac{T_2 - T_1}{r_2 - r_1}\right) \cdot \left(\mathscr{A} + \left(\frac{r_c}{r_p}\right)^2\right)^{-1} \tag{14}$$

The specification of Q_1 in equation (14) permits the steady-state conductive geotherm in the lower thermal boundary layer to be constructed,

$$T(r) = \frac{Q_1}{\bar{K}} r_1 \left[1 - \frac{r_1}{r}\right] + T_1 \qquad r_1 \leqslant r \leqslant r_1 + \delta_1. \tag{15}$$

Also, with Q_2 given by equation (13) and specifying $\hat{Q}$ and d, we can construct the upper

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boundary layer temperature profile,

$$T(r) = \frac{Q_2}{\bar{K}}(r_2 - \delta_2)^2\left[\frac{1}{r_2} - \frac{1}{r}\right] + \frac{\hat{Q}}{3\bar{K}}\left[\tfrac{1}{2}(r_2^2 - (r_2 - d)^2) - \frac{d}{r_2}(r_2 - d)^2\right] \tag{16a}$$

$(r_2 - \delta_2) \leqslant r \leqslant (r_2 - d)$

$$T(r) = \frac{\hat{Q}}{6\bar{K}}(r_2^2 - r^2) + \hat{\alpha}\left(\frac{1}{r_2} - \frac{1}{r}\right) + T_2 \qquad (r_2 - d) \leqslant r \leqslant r_2 \tag{16b}$$

where

$$\hat{\alpha} \equiv \frac{Q_2}{\bar{K}}(r_2 - \delta_2)^2 + \frac{\hat{Q}}{3\bar{K}}(r_2 - d)^3.$$

The geotherm between the boundary layers is constrained to lie along an adiabat

$$T(r) = T_{\delta_2} \exp \overline{(\alpha g/C_p)}\,[(r_2 - \delta_2) - r] \qquad (r_1 + \delta_1) \leqslant r \leqslant (r_2 - \delta_2) \tag{17}$$

where we have assumed that the quantity $(\alpha g/C_p)$ is constant across the mantle. In Section 3.3 we will remove this restriction and investigate more general variations of (α/C_p).

Combining equations (15) and (16a) with equation (17), we obtain the following transcendental equation for the boundary layer thicknesses δ_1 and δ_2,

$$-\frac{Q_2}{\bar{K}}\left(\frac{\delta_2}{r_2}\right)(r_2 - \delta_2) + \frac{\hat{Q}}{3\bar{K}}\left[\tfrac{1}{2}(r_2^2 - (r_2 - d)^2) - \frac{d}{r_2}(r_2 - d)^2\right]$$
$$= \left[\frac{Q_1}{\bar{K}}\,\frac{r_1\delta_1}{r_1 + \delta_1} + T_1\right] \exp\left[-\overline{(\alpha g/C_p)}\right]\left[(r_2 - r_1) - (\delta_1 + \delta_2)\right] \tag{18}$$

where

$$Q_2 = Q_1\left(\frac{r_1}{r_2 - \delta_2}\right)^2.$$

From geometrical considerations the thicknesses of the two thermal boundary layers are related as follows,

$$\delta_1 = \delta_2\left(\frac{r_1}{r_2 - \delta_2}\right)^2. \tag{19}$$

While the validity of this relationship can only be checked by solving the complete problem we found that the results presented in Section 4 were relatively insensitive to the precise form of equation (19). Substituting equation (19) in (18), we obtain a single equation for the upper boundary layer thickness δ_2. With δ_2 specified, we can then use equations (14)–(17) to construct the quasi-steady-state mantle geotherm which is consistent with the 'instantaneous' lower boundary temperature T_1 and core heat flux Q_1.

We may now outline the algorithm employed for performing the thermal history simulation. At any given time we examine the mantle geotherm away from the boundary layers and obtain an estimate for $\bar{\nu}$ (see Section 3.3.4 for the temperature-dependent rheology model), from which Ra and Nu may be computed (equations (3a) and (4)). Given Nu now, we may compute Q_1 and Q_2 from equations (13) and (14) and hence evaluate

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δ_2 from equation (18). From equations (15)–(17) the complete mantle geotherm can be constructed. To time step the problem, we base the time step, Δt, on the overturn time (equation (8)). During this interval, we hold the surface heat flux fixed and determine the corresponding total heat loss from the planet. Assuming the Earth cools via a sequence of quasi-steady-states, we then estimate the new core–mantle boundary temperature, T_1, by requiring that the total integrated heat difference between the previous geotherm and the one corresponding to the new estimate of T_1, should equal $Q_2 \cdot 4\pi r_2^2 \cdot t_{\mathrm{ov}}$. Allowance is made for the latent heat of fusion should the core geotherm drop below the liquidus. Since Q_2 is a function of T_1, the assumption that Q_2 is fixed during Δt while T_1 changes is not rigorously correct. An important constraint on Δt then, is that during any time step, only 'small' changes in T_1 are admissable. For Δt given by equation (8), it was found that consecutive changes in T_1 did not exceed 0.3 per cent.

The quasi-steady-state thermal history model is based upon the validity of the boundary layer theory for convection at high Rayleigh and Prandtl numbers extended to a spherical shell. The basic assumption in the construction of this boundary layer theory is that there is a balance within the thermal boundary layers between the horizontal advection of heat and its vertical diffusion (Turcotte & Oxburgh 1967, 1972). Application of this simplest-possible boundary layer theory to whole-mantle-convection implies specific interpretations which are not universally accepted. For instance, we are by implication assuming that the lithospheric plates are active parts of the mantle general circulation, and owe their rigidity to the fact that they are within the cold boundary layer of this convectively driven flow. If we further assume that the lithospheric thickness (δ) is equal to the boundary layer thickness, then since δ is a rheological observable (e.g. post-glacial rebound), we then have a direct observation of the single most important thermal characteristic of the convecting mantle. We further require that the time-scale of the convective circulation is in no way impeded by the high viscosity of its outer boundary layer, but rather is determined by the mean viscosity in the interior of the convecting region.

This view of the basic properties of mantle convection has the advantage of providing an immediate explanation for several basic observations. Because the circulation extends to the core–mantle boundary, the large horizontal scale of the lithospheric plates is immediately explicable in terms of aspect-ratio-one convection, which is the usual preferred mode. Because the theory is based upon boundary layer ideas, the observed linear variations of heat flow and topography on the ocean floor with the square root of the age is also predicted (Sclater & Francheteau 1970; Turcotte & Oxburgh 1972). Indeed, these observations directly argue in favour of the interpretation of the oceanic lithosphere as the boundary layer of the convective circulation, and against the view that the oceanic lithosphere is decoupled in any way from the underlying mantle.

3.1.2 Models employing the modified diffusion equation

From the definition of the Nusselt number, one may consider that a convenient way to parameterize convection in a thermal history model would be to define an enhanced thermal conductivity, which would be the product of the usual lattice thermal conductivity and Nu. The equation governing the thermal history simulation would then be the modified diffusion equation,

$$\rho C_{\mathrm{p}} \frac{\partial T}{\partial t} = \nabla \cdot [(NuK)\nabla T]. \tag{20}$$

In a recent article (Sharpe & Peltier 1978), the conditions under which this empirical

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relation may be valid were studied. Starting with the non-dimensional equation governing the internal energy balance for a moving fluid heated from below (Peltier 1972), and ignoring viscous dissipation, it can be shown that,

$$\frac{\partial \bar{T}}{\partial t} = \overline{\nabla \cdot (K \nabla T)} \left[\frac{-\overline{w(\partial T/\partial r + \tau T)} + \overline{\nabla \cdot (K \nabla T)}/Ra\bar{\rho}}{\overline{\nabla \cdot K \nabla T}} \right] \tag{21}$$

where w is the radial component of velocity, $\tau = g\alpha d/C_p$, the function K measures the deviation of thermal conductivity from a reference state value, and the overbars indicate averages over concentric spherical surfaces. The quantity in square brackets may be interpreted as a local Nusselt number, and if it is assumed constant over the convecting region, then equation (21) reduces to equation (20). The method employed for the solution of equation (20) is based upon the Crank–Nicolson algorithm (Von Rosenberg 1975). This is an implicit, second-order accurate scheme for which the solution is stable for all ratios of the radial grid size to the time step. By writing all finite differences about the point $(r_i, t_{n+\frac{1}{2}})$, which is halfway between the known (t_n), and unknown (t_{n+1}) time levels, we can obtain the numerical analogue to equation (20).

$$\begin{bmatrix} b_1 c_1 0 \ldots \\ a_2 b_2 c_2 0 \\ \ldots \\ \ldots 0\, a_i b_i c_i\, 0 \\ \ldots \\ \ldots 0\, a_I b_I \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \ldots \\ T_i \\ \ldots \\ T_I \end{bmatrix}_{n+1} = \begin{bmatrix} d_1 \\ d_2 \\ \ldots \\ d_i \\ \ldots \\ d_I \end{bmatrix}$$

where

$$a_i \equiv \frac{1}{2\Delta r} \left[\frac{\beta}{\Delta r} - \frac{\alpha}{2} \right]_{i-1}$$

$$b_i \equiv - \left[1 + \frac{\beta}{\Delta r^2} \right]_i$$

$$c_i \equiv \frac{1}{2\Delta r} \left[\frac{\beta}{\Delta r} + \frac{\alpha}{2} \right]_{i+1}$$

$$d_i \equiv -a_i T_{i-1,n} + \left[1 - \frac{\beta}{\Delta r^2} \right]_i T_{i,n} - c_i T_{i+1,n} - \gamma_{i,n}$$

$$\gamma_{i,n} \equiv \Delta t \left. \frac{Q}{\rho C_p} \right|_{i,n}$$

$$\alpha_i \equiv \frac{\Delta t}{\rho C_p} \left[\frac{\partial (Nu \cdot K)}{\partial r} + \frac{2(Nu \cdot K)}{r} \right]_i$$

$$\beta_i \equiv \frac{\Delta t}{\rho C_p} (Nu \cdot K) |_i$$

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and $i = 1, I$, $n = 0, N$ where I and N are the number of spherically concentric shells and numbers of time steps respectively. By using a central difference formula for $\partial(Nu \cdot K)/\partial r|_i$, the discontinuous changes in the Nusselt number at the boundaries of the convecting regions are 'smoothed', and hence do not present any difficulty. Analytically, however, these 'steps' would introduce unacceptable delta functions in equation (20). The resultant coefficient matrix is tri-diagonal, and hence can be solved efficiently using the Thomas algorithm (Von Rosenberg 1975). The boundary conditions employed are zero heat flux at the planet's centre and constant surface temperature. The increments in radius and time were chosen to guarantee that the finite difference solution to equation (20) closely approximated the exact solution in situations for which such (analytic) solutions exist (Carslaw & Jaeger 1959; Urey 1952; Slichter 1941). It was found that agreement between numerical and analytic solutions for the radial temperature profile over the history of the Earth was achieved to within a few per cent, when the sphere was approximated by 50 uniform concentric shells and for 100 time steps. In fact, we actually used 100 evenly spaced radial steps to achieve a greater spatial resolution. It was not possible to use a finer grid because of storage limitations. Because of the enhancement of the thermal conductivity by the Nusselt number, it was necessary to allow for a self-adjusting time step as the simulation progressed, to enable the thermal consequences of the convection to be accurately tracked.

There is, however, a serious problem inherent in utilizing equation (20) to parameterize the convective enhancement in the radial flow of heat. This problem relates to the inability of the modified diffusion equation to represent properly the physics of high Rayleigh number convection, where two distinct time-scales should explicitly govern the thermal history. Instead, the 'diffusion' thermal history always operates on a single time-scale, which is that given by the modified thermal time constant of order $d^2/(\kappa \cdot Nu)$. By assuming that the 'local Nusselt number' of equation (21) is constant over the convecting region, we are implicitly assuming that at any given time mantle convection is quasi-steady. The time derivative of the temperature field, on the left side of equations (20) and (21), could then be interpreted as representing the 'long time-scale' of the thermal history, and one could argue that this method for parameterizing convection does, in fact, make provision for a separation of time-scales at high Rayleigh number (quasi-steady-state). However, the response time of the mantle to changes in the lower boundary conditions is of the order $d^2/(\kappa \cdot Nu)$, which even for Rayleigh numbers of 10^7–10^8 is comparable to the age of the Earth. On the other hand, the response time (overturn time) for quasi-steady-state convection at these high Rayleigh numbers is very short, on the order of 10^8 yr. The mantle 'diffusion' geotherms are therefore evolving on a time-scale which neither corresponds to the fast time-scale associated with the overturn time, nor the long time-scale as determined by the e-folding time for the loss of primordial heat. We will consider this parameterization scheme further in Section 4.2 where a specific thermal history model will be presented.

3.2 DATA SELECTION

The main physical parameters which explicitly control our evolutionary model are the melting curves, α, C_p and K, the distribution of 'crustal' radioactives and the mantle rheology. Most of these quantities are pressure dependent and some are also functions of temperature. Both theory and experiment are still in an early stage of developing a consistent understanding for the properties of solids and liquids at the high pressures and temperatures of the Earth's interior. Consequently, our objective is not to determine a specific set of values for these parameters, since our present knowledge is simply inadequate to the task, but rather to demonstrate in light of the observational constraints, the

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characteristic control which the main parameters exercise on the final results. In this way we hope to identify an acceptable *class* of models. The next few subsections briefly outline our choices for the various parameters.

3.2.1 Solidus and liquidus curves

Experimental data for the pressure dependence of the melting point are only available to about 200 kbar for iron (Liu & Bassett 1975) and to about 60 kbar for silicates (Williams & Kennedy 1970). On considering the present state of understanding for the melting process, Birch (1972) concludes that it is not possible to estimate the melting point in the core to an accuracy better than 700 K (see also Boschi 1974a, b). It would appear that at least the same uncertainty also applies for the mantle (Kennedy & Higgins 1972).

For the mantle, we will adopt the solidus and liquidus curves of Kennedy & Higgins (1972), since they attempt not only to consider the mantle as a multicomponent eutectic system, but in addition to incorporate the effect of pressure on the deepening of the eutectic trough. For the core, rather than following a specific theory, we first assume the lower temperature bound of 5000 K estimated by Verhoogen (1973) for the inner core–outer core interface, and then *construct* the core solidus to be exactly adiabatic. This is done since at present it is not possible to determine if the actual conditions are otherwise. In constructing the core solidus, we have assumed that it consists of pure iron. As a result, the liquidus should be nearly coincident with the solidus. We have, arbitrarily then, constructed the core liquidus to be exactly 100° above the solidus. The presence of a light alloying element, such as sulphur, could dramatically lower the solidus as well as affecting the radial variation of the liquidus (Murthy & Hall 1972). Since the main objective of our thermal history simulation is to investigate the ramifications of convection, we are not as concerned with the actual values of the melting curves as we are with their relationships to the local adiabat.

3.2.2 α, C_p, K

The thermal expansivity α, and the heat capacity C_p are important to the simulation because they enter the evaluation of the adiabatic gradient (as α/C_p) and the estimation of the Rayleigh number (as $\alpha \cdot C_p$). While α is known to increase with temperature, its pressure dependence is less certain and it may, in fact, show a negative $\partial\alpha/\partial p$ (Jacobs 1953; Stacey 1977). While C_p is thought to be less variable, the uncertainties in α and C_p for the P and T regime encountered in the Earth are sufficient to significantly affect the geotherms in the thermal history model (Stacey (1977); and see Frazer (1973) for a summary of the available bounds on α/C_p). Rather than specifying particular P–T relations for (α/C_p), we shall study in Section 4.1.6 the consequences of various assumed radial variations of (α/C_p) for the mantle. These can be directly incorporated into our analytical formulation, and may provide important insights into the semi-quantitative behaviour of (α/C_p) required from a consideration of the observational constraints and the variation of other mantle parameters. Specific models for $\alpha/C_p(P, T)$ could then be constrained by these results. In the core we assume a constant representative value for (α/C_p) (see Table 1).

The thermal conductivity K (or thermal diffusivity $\kappa = K/\rho C_p$) enters our study in the estimation of the Rayleigh number and in construction of the thermal boundary layers. The expected variation of the thermal conductivity through the mantle is sufficiently small so as to have a relatively unimportant effect on the quantities of interest (Schatz & Simmons

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1972). Accordingly, we will assume a constant mean-mantle conductivity. The values assumed for these parameters, and others relevant to our study are presented in Table 1.

3.2.3 Crustal radioactives

We have seen in Section 2 that an important consequence of the formation model is the early concentration of the bulk of the Earth's radioactive material in an outer 'crustal' layer. This layer has been accommodated in the analytical formulation of Section 3.1, and is characterized by thickness d and uniform heat-production density $\hat{Q}(t)$. The recent work of Pollack & Chapman (1977) suggests that this heat production drops off exponentially away from the surface with an e-folding depth of about 9 km. Since the total heat production integrated under this curve is approximately the same as for a shell 9 km thick having a uniform heat-production density equal to the surface value, we may employ the results of Pollack & Chapman in a simple way to study the effects of such a radioactive layer, assuming it to be in a steady state with the instantaneous rate of heat production. Two extreme chemical compositions were studied, granitic and oceanic basalt (see Table 1). We also investigate the effects of varying the 'crustal' layer thickness d.

3.2.4 Mantle rheology

Over geologic time-scales the Earth's mantle is generally considered to deform as a viscous fluid. Modelling the mantle as a Maxwell solid, Peltier (1974) has shown that for viscosities on the order of 10^{22} P, the transition time between the asymptotic behaviours of Hookean elastic solid and Newtonian viscous flux is a few hundred years. The creep mechanism is generally considered to be either of two possible types: diffusion creep which leads to the definition of a Newtonian viscosity (Herring 1950; Gordon 1965) and/or dislocation glide in which the 'effective viscosity' depends on the average stress level (Weertman 1970; Weertman & Weertman 1975). At present, it is not possible to ascertain whether or not the creep process associated with convection is Newtonian (see Carter 1976, for a review).

Regardless of the rheological details, as creep is an activated process, the effective viscosity will show an exponential dependence on temperature. Basing his qualitative thermal models on this fact, Tozer (1965, 1972) argued that for any given rheological model, a quasi-steady-state geotherm would eventually be attained such that the mean viscosity was $0(10^{22}-10^{21})$ P. This self-regulation of the convection by the negative feedback resulting from a temperature-dependent viscosity is implicit in our parameterization schemes where the Nusselt number $\propto 1/\nu^{1/3}$. To study the impact of such a rheology in as simple a fashion as possible, we have employed the semi-empirical rheological model proposed by Weertman (1970) and Weertman & Weertman (1975),

$$\nu(T) = \nu_0 \exp(gTm/T) \tag{22}$$

where g is a dimensionless constant which controls the rate at which the material stiffens and is approximately equal to 18 for metals and 30 for silicates. This model implicitly incorporates the effect of pressure on the activation parameters by including the radial variation of the solidus, Tm.

For the purpose of computing the Rayleigh number, we will evaluate $\nu(T)$ at a specific radial location near the centre of the convecting zone. Our models were not sensitive to the precise location chosen provided it was not near the boundary layers. So we choose $\nu_0 = 10^{18}\,\mathrm{cm^2\,s^{-1}}$. Since our starting geotherm is coincident with the solidus, the corresponding Rayleigh number is so high that the initial overturn time is $0(10^4\,\mathrm{yr})$ (see equation (8)).

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This guarantees a complete separation of time-scales at the start of the simulation, and supports the construction of the initial steady-state mantle geotherm.

4 Model results

We will here present representative thermal history models for the parameterized convection schemes which were developed in the last section. Section 4.1 discusses the quasi-steady-state model, while the results obtained by application of the modified diffusion equation are briefly indicated in Section 4.2. In Section 4.1 we have restricted ourselves to the somewhat artificial situation in which $\mathscr{A} \equiv 0$ in equation (2b). This is equivalent to assuming that the mantle has zero heat capacity and was motivated for two reasons. Firstly, we wish to isolate the extent to which core heat can contribute to the success of our maximum cooling model in satisfying the main constraints. Secondly, when the secular cooling of the mantle is included, it is not clear if a heated-from-below Rayleigh number (equation (4)) remains appropriate for describing the convection, since in this case one may consider there to be an effective 'internal' heat source. Further, even the precise form of the *Nu–Ra* relationship is uncertain. By neglecting the heat capacity of the mantle, we will be significantly overestimating the rate at which the planet cools. The consequence of this assumption, in light of the results to follow, will be discussed in Section 5.

4.1 RESULTS OF THE QUASI-STEADY-STATE THERMAL HISTORY MODEL

4.1.1 Mantle rheology

In Fig. 3(a) we present a thermal history model in which the mean-mantle viscosity has been held constant at 10^{22} P. We note that within 10^9 yr the core has completely solidified, so that all subsequent geotherms must be considered invalid as the quasi-steady-state assumption is no longer applicable. Fig. 3(b) presents the same thermal model except that the temperature-dependent rheology of equation (22) has been introduced with $g = 22$. The significance of this feedback effect for both the thermal histories of the mantle and core is obvious. Within the first few hundred million years the mantle viscosity stiffens to $\sim 10^{23}$ P and over the next several billion years increases to $\sim 10^{25}$ P. While this significantly restricts the flow of heat from the core, it is the gradual decrease in the total heat content of the core which permits the mantle to stiffen to such an extent. The assumption of a separation of time scales remains valid for this model.

We note from Fig. 3(b) that core solidification occurs shortly after 3×10^9 yr. This does not present as serious a problem however as in Fig. 3(a) since we have several options available to prevent such an occurrence. Consistent with the notion of the effect of a light alloying element in the core, such as sulphur (Murthy & Hall 1972), we could lower the solidus by 1000°. Alternatively, increasing the assumed value for the heat of fusion would also delay solidification. Finally, increasing g from 22 to 28 results in an acceptable model as seen in Fig. 3(c). This is due to the fact noted by Tozer (1965, 1972), that the vigour of mantle convection is sensitive to the mean viscosity rather than the actual temperature. Increasing g in equation (22) results in an enhanced value for the quasi-steady-state mean mantle temperature, but the mean viscosity of 10^{25} P remains the same as for Fig. 3(b), being fixed by the 'gross' energetics of the problem.

The models of Fig. 3(b) or (c), while satisfying the constraints on the gross thermal structure of the Earth (the formation of the solid inner core does not present a problem and is discussed in Section 4.1.7) are too 'cold'. The present-day, mean-surface heat flow is

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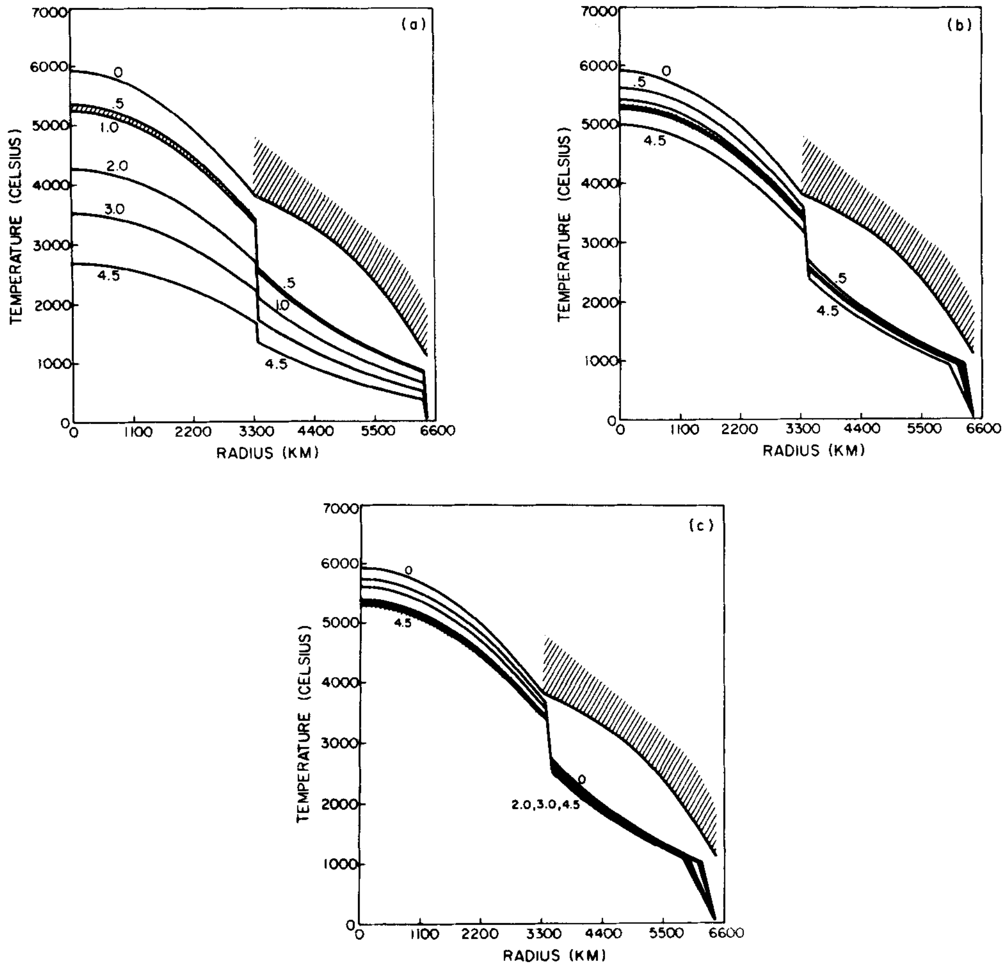

Figure 3. Quasi-steady-state thermal history models which demonstrate the importance of a temperature-dependent viscosity: (a) constant mean-mantle viscosity (10^{22}P); (b) $\nu(T) = \nu_0 \exp(gTm/T)$, $g = 22$; (c) $g = 28$.

about 11 mW m^{-2}, a factor of 4 to 5 too low, the mean-mantle viscosity is high by 3 orders of magnitude, and the upper thermal boundary layer is too thick. Of course, in constructing steady-state thermal models, we are maximizing the cooling rate of the Earth, so that these results could be interpreted in favour of a significant time lag in the cooling rate, if we neglect mantle heat. However, in keeping with our main purpose, which is to study the relevance of a quasi-steady-state thermal model driven entirely by core heat, we will now consider various methods whereby the mantle geotherms may be 'propped up', thus enabling the additional constraints outlined above to be satisfied as well.

4.1.2 Crustal radioactives

One may consider that the incorporation of radioactive heat sources in an outer 'crustal' layer would lead to the defining of a higher 'effective' surface temperature, resulting in enhanced mean-mantle temperatures and lower mean viscosities. In Fig. 4 we have investigated the consequence of distributing these sources in a crustal layer of varying thickness d.

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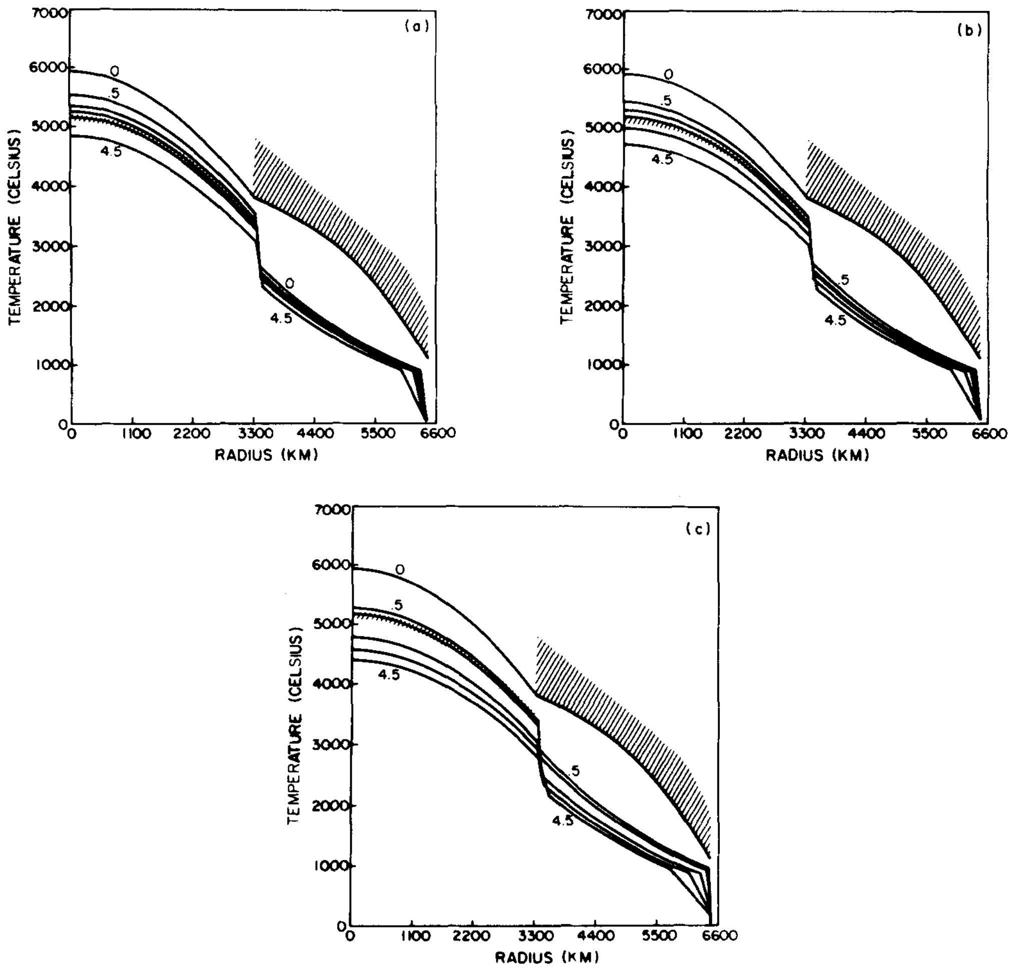

Figure 4. Quasi-steady-state thermal history models which demonstrate the importance of the depth, d, over which the 'crustal' radioactives are distributed: (a) d = 0; (b) d = 10 km; (c) d = 20 km.

The composition is granitic (see Table 1) and recall from Section 3.2.3 that the data of Pollack & Chapman (1977) suggest that d = 10 km for the continental crust.

Comparing Fig. 4(a) (d = 0) and (c) (d = 20 km) we note that the presence of crustal sources does in fact lead to hotter mantle geotherms during the first two billion years (mainly due to K^{40}). However, because of the corresponding enhanced vigour of mantle convection, the heat content of the core is rapidly diminished, resulting in early core solidification and present mantle geotherms which are colder than in the no-heat-source model. The present-day, mean-mantle heat flow for Fig. 4(c) (6 mW m^{-2}) is about one half of that of Fig. 4(a) (11 mW m^{-2}). Also the present-day, mean-mantle viscosity has increased by an order of magnitude. While these results are more representative for subcontinental geotherms, the spherically symmetric nature of our models did not permit incorporation of the lateral inhomogeneity in the crustal radioactivity which is known to exist. Nevertheless, it is apparent that on time-scales on the order of several billion years, crustal radioactivities will result in a lower total heat content for the Earth relative to a model depleted in such sources (d = 0).

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In all subsequent models we will assume $d = 10$ km, consistent with the discussion in Section 3.2.3. The corresponding contribution to the present-day, mean-surface heat flow is 35 mW m^{-2}.

4.1.3 Enhanced initial core heat

An alternate possibility for delaying the cooling of the core, and hence also the mantle, is to increase the initial heat content of the core. In describing the formation model, we suggested that the fluid mantle would cool along adiabats, solidifying from the base outwards for a superadiabatic solidus. The value of the solidus at the base of the mantle was then used in constructing the initial core geotherm, thus minimizing the initial heat content of the core. However, as the fluid mantle cools, we may expect that the top and bottom of the vigorously convecting region will be characterized by thin, well-developed, thermal boundary layers. While the initial mantle geotherm will still be coincident with the solidus, the starting core adiabat should then be based upon a core–mantle temperature about 1000° above the mantle solidus at that point. This results in an increase in the total initial heat content of the core of about 40 per cent.

Computations show that this enhancement in the initial core heat is negated within the first 2×10^9 yr due to the corresponding enhancement in the vigour of mantle convection, resulting from the warmer mean temperatures. After this time, the thermal history is essentially the same as for the model in Fig. 4(b). It would therefore appear that on time-scales of a few billion years, the quasi-steady-state thermal history model is relatively unaffected by changes in the initial core heat which are on the order of 50 per cent or less.

4.1.4 Subadiabatic lower mantle solidus

In Fig. 5 we demonstrate the consequence of constructing the lower third of the mantle solidus to be subadiabatic. At present, the precise character of the solidus in this region is not known, but the work of Kennedy & Higgins (1972) suggests that this is a likely possibility. We note the presence of a sizeable, initial melt zone in the lower mantle. The geotherm in this region is a fluid adiabat based upon the value of the solidus at the radial location at which the solidus becomes precisely adiabatic.

Comparing the thermal models of Figs 5 and 4(b), it is apparent that by reducing the initial depth of penetration of the mantle convection, we have made some progress in achieving a 'warmer' present-day interior. The partially molten core is preserved, the mean-mantle flux has increased to 17 mW m^{-2}, and the mean-mantle viscosity is reduced from 10^{25} P to 10^{24} P. This result is due primarily to a reduction in the initial convective length scale. Since through our parameterization scheme the Nusselt number is directly proportional to this length scale, we have significantly reduced the cooling rate of the core in the first 2×10^9 yr, during which time the bottom of the convecting layer approaches the core–mantle boundary. At present, the geotherms are still 'catching up' to those of Fig. 4(b), unlike the thermal model presented in the last section in which the 'catching up' was complete by 2×10^9 yr.

We considered a further reduction in the assumed vigour of mantle convection by decreasing the value for the power-law exponent in equation (3a). While boundary layer theory suggests that the cube-root dependence is applicable for high *Ra* #, high *Pr* # steady-state convection in a plane layer, a similar analysis for convection in a spherical shell is not available. However, even for the case of steady two-dimensional convection, experimental studies indicate that the cube-root exponent may be too high by as much as 10 per cent

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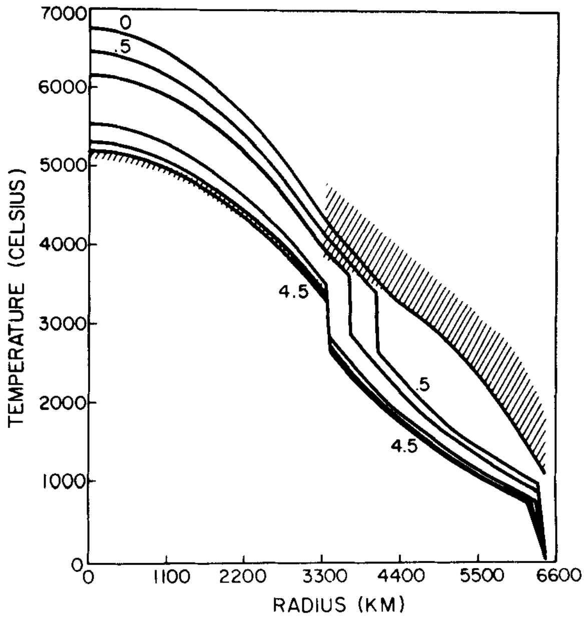

Figure 5. A quasi-steady-state thermal history model which demonstrates the consequence of a sub-adiabatic lower mantle solidus – *cf.* the thermal model of Fig. 4(b).

(e.g. Rossby 1969). Reducing the exponent by only 10 per cent produced significant changes in the model of Fig. 5. The present-day core geotherm was everywhere above the liquidus, the mean-mantle heat flux increased to 25 mW m^{-2}, about one-half of the presently observed value, and the mean-mantle viscosity decreased from 10^{24} P to 10^{23} P.

The previous subsections indicate that within the context of a quasi-steady-state thermal history model with mantle-wide convection, which includes a temperature-dependent rheology, and which cools from an initially 'hot' state, the constraint of a present-day molten core–solid mantle structure can easily be met without invoking any mantle heat or radioactive sources. Up to 50 per cent of the presently observed mean-mantle heat flux can also be understood with this model. However, if we invoke the additional constraint of a present-day, mean-mantle viscosity of 10^{22} P, as inferred from the glacial rebound data (Peltier 1974, 1976; Peltier & Andrews 1976), then we are confronted with a serious problem. This is due to the fact that the heat content of the core is insufficient to sustain the vigorous level of quasi-steady-state convection required by 10^{22} P over time-scales on the order of the Earth's age for the assumed rheology model. Had a rheology been assumed which stiffened sufficiently slowly so that the present-day, mean-mantle viscosity was 10^{22} P (the corresponding mean surface heat flux would then be 50 mW m^{-2}) the resultant decrease in the mean core temperature would have been ~ 3000° rather than ~ 1500°, which was obtained on the average for the models presented in this section. To prevent core solidification would require an initial magnitude of core heat about 3 times that inferred for the present thermal models. While it is difficult to conceive of a formation model partitioning such a quantity of heat to the core, it is interesting to note that such a model, driven entirely by primordial core heat, would be consistent with all the constraints. Alternatively, the effective heat content of the core may be enhanced by increasing the assumed value for the latent heat of fusion of iron. Values for the heat of fusion higher than those

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employed in the present study (see Table 1) have in fact been suggested (Gschneider 1964; Stacey 1977).

4.1.5 Core heat sources

Deviating somewhat from our basic premise of considering only heat associated with the formation of the Earth, but remaining within the restriction of no mantle heat, we will consider here briefly the consequences of adding K^{40} to the core. On the basis of cosmochemical models for the formation of the solar system, and the chalcophilic behaviour of potassium at high pressure, it has been suggested (Goettel & Lewis 1973; Goettel 1976) that, relative to solar or chondritic abundances, 80–90 per cent of the Earth's potassium may be in the core. This would result in a concentration of from 0.1–0.2 per cent. Bukowinski (1976) has also suggested that at high pressures potassium may behave more like a transition metal and hence become soluble in iron.

Fig. 6 demonstrates the consequence of introducing potassium into the core for the thermal model of Fig. 4(b). Fig. 6(a) is for a concentration of 0.1 per cent and Fig. 6(b) is for 0.2 per cent. In Fig. 6(b) the present-day, mean-mantle heat flow is 42 $mW\,m^{-2}$ (recall that in addition to this, the crustal radioactives for an assumed granitic composition contribute 37 $mW\,m^{-2}$, for a total of 79 $mW\,m^{-2}$), and the present-day mantle viscosity is 5×10^{22} P. This model must therefore be considered acceptable from the standpoint of all imposed observational constraints. Note also from Fig. 6(b) that the core warms during the first 2×10^9 yr, as mantle convection is not sufficiently vigorous to 'dump' the additional core heat at the rate at which it is being generated.

4.1.6 Relative curvatures of the mantle adiabat and solidus

In addition to the constraint on the present-day, mean-mantle viscosity, post-glacial rebound studies also suggest that the viscosity variations down to the core–mantle boundary do not exceed about an order of magnitude (Cathles 1975; Peltier 1974, 1976; Peltier & Andrews 1976; Peltier *et al.* 1978). Given the mantle geotherms of Fig. 6(b) and the semi-empirical

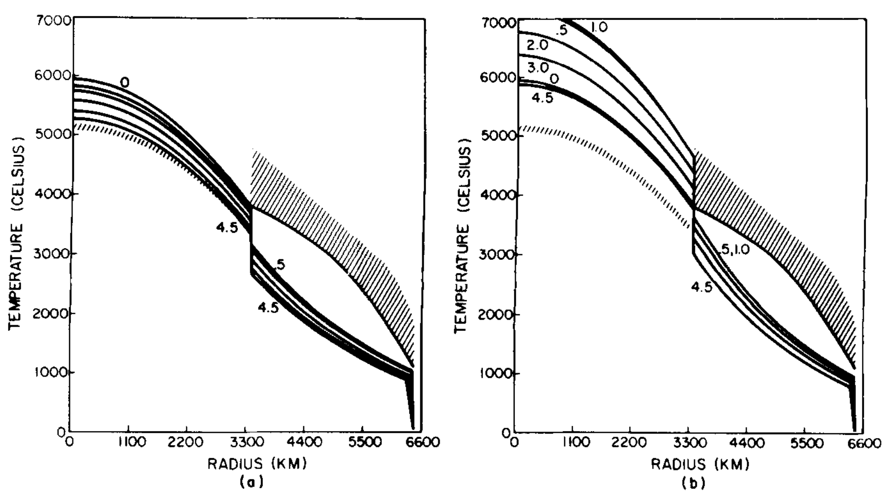

Figure 6. A quasi-steady-state thermal history model which demonstrates the consequence of introducing K^{40} into the core: (a) $[K] = 0.1$ per cent; (b) $[K] = 0.2$ per cent – *cf.* the thermal model of Fig. 4(b).

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rheological model of Sammis *et al.* (1977), there is no difficulty in choosing appropriate, realistic profiles for the activation parameters which will guarantee a corresponding uniform viscosity profile through the mantle. However, if we restrict ourselves to the rheological model of Weertman (1970) and Weertman & Weertman (1975) (equation (22)), in which the local viscosity depends upon the ratio Tm/T, then it is clear that the geotherms in Fig. 6(b) will violate the uniform viscosity constraint. These geotherms would suggest a low viscosity at the top and bottom of the mantle, and a relatively high viscosity in between. While this may be inconsistent with the rebound data, such a variation would be compatible with the inferred Q structure for the mantle (Anderson & Hart 1978).

If we accept the Weertman empirical model, and the curvature of the solidus (concave to the origin), neither of which are generally agreed upon, we may enquire into the restrictions on the adiabatic parameters required to satisfy the uniform viscosity constraint. Obviously we will be concerned with reversing the curvature of the mantle geotherms, to 'mimic' that of the solidus. To ensure that the mantle adiabat remains everywhere concave to the origin, i.e. $d^2T/dr^2 < 0$, $r \in$ the mantle, it can easily be shown that the following condition must be satisfied,

$$\left(\frac{\alpha}{C_p}\right)_r < \frac{1}{1/(\alpha/C_p)_0 + \bar{g}(r_2 - r)}, \tag{23}$$

recalling that r_2 is the radius of the Earth. Furthermore, from physical arguments we must also have,

$$\left(\frac{\alpha}{C_p}\right)_r > 0. \tag{24}$$

In all the previous thermal models presented in this study, (α/C_p) has been assumed constant at 3.0×10^{-12} gm/erg (see Table 1) to facilitate the analytical solutions for the mantle geotherms.

The restriction imposed by equation (23) requires that all mantle profiles for (α/C_p) lie below the boundary indicated in Fig. 7. Assuming for example a linear decrease with depth, i.e.

$$\left(\frac{\alpha}{C_p}\right)_r = a(r - r_2) + \left(\frac{\alpha}{C_p}\right)_0 \qquad a > 0 \tag{25}$$

it can be shown that equation (23) leads to the following condition on the constant a,

$$a = \chi \max \left\{ \frac{(\alpha/C_p)_0}{(r - r_2)} \left[\frac{1}{1 + (\alpha/C_p)_0 \bar{g}(r_2 - r)} - 1 \right] \right\} \tag{26}$$

where $\chi > 1$ and $r \in$ the mantle.

Inserting equation (26) into equation (25), and applying equation (24) leads to an upper bound on χ, namely $\chi < 1.033$. The (α/C_p) profile corresponding to $\chi = 1.030$ is shown in Fig. 7. The corresponding thermal history model is presented in Fig. 8(a), and is otherwise the same as the model of Fig. 6(b). We note that while the mantle geotherm is everywhere concave to the origin, it fails to parallel the solidus and hence will not yield a uniform mantle viscosity within the context of the Weertman model. Furthermore, $(\alpha/C_p) \sim 0$ at the core–mantle boundary.

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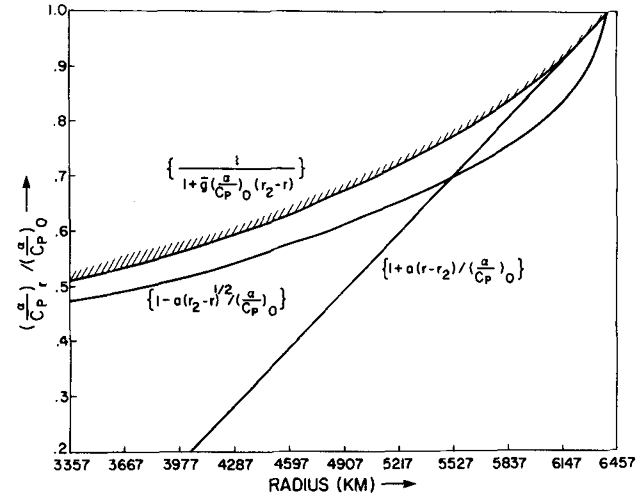

Figure 7. Mantle profiles for (α/C_p) such that the resultant mantle geotherms (adiabats) remain concave to the origin-of-coordinates at all radii – the hatched curve indicates the upper boundary for acceptable profiles.

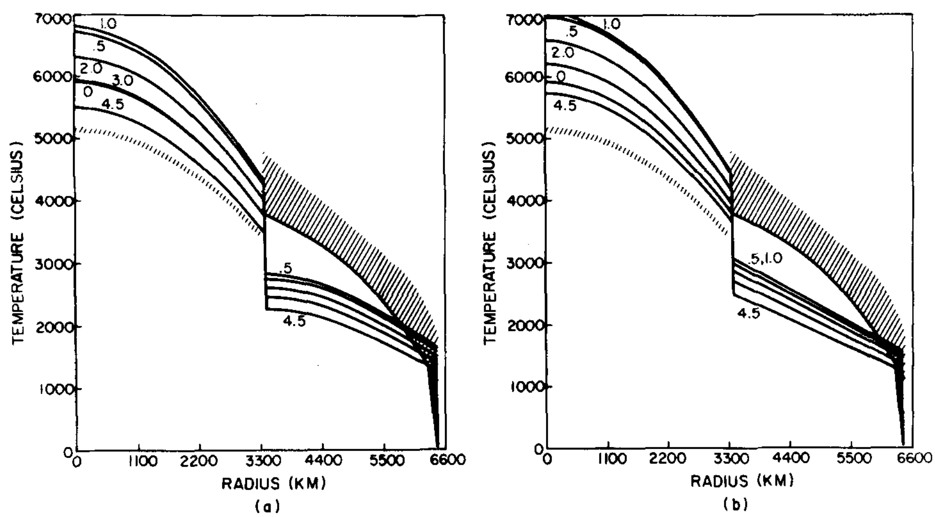

Figure 8. Quasi-steady-state thermal history models which demonstrate the consequence of constraining the mantle geotherm to be concave to the origin-of-coordinates at all radii: (a) $(\alpha/C_p)_r = (\alpha/C_p)_0 - a.(r_2 - r)^{1/2}$; (b) $(\alpha/C_p)_r = a(r - r_2) + (\alpha/C_p)_0$ – *cf.* the thermal model of Fig. 6(b).

It is recognized from this example that to parallel the solidus more closely, it is desirable to keep the value of (α/C_p) at the base of the mantle as large as possible. Accordingly, a functional form for $(\alpha/C_p)_r$ which does not deviate significantly from equation (23) should produce the 'best' thermal model. Consider the following alternative to equation (25),

$$\left(\frac{\alpha}{C_p}\right)_r = \left(\frac{\alpha}{C_p}\right)_0 - a(r_2 - r)^{1/2} \qquad a > 0. \tag{27}$$

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Equation (23) leads to the following condition on a,

$$a = \chi \max \left\{ \frac{(\alpha/C_{\mathrm{p}})_0}{(r_2 - r)^{1/2}} \left[1 - \frac{1}{1 + (\alpha/C_{\mathrm{p}})_0 \bar{g}(r_2 - r)} \right] \right\} \tag{28}$$

where $\chi > 1$ and $r \in$ the mantle.

Inserting equation (28) in (27) and applying equation (24) results in the additional condition that $\chi < 2$. Equation (27) is plotted in Fig. 7 for $\chi = 1.10$ and the corresponding thermal model is shown in Fig. 8(b). While this model represents an improvement over that of Fig. 8(a), the mantle geotherms still show a significant radial variation in (Tm/T).

These results depend primarily on the value of $(\alpha/C_{\mathrm{p}})_0$ which is relatively well constrained (in addition to the assumption of chemical homogeneity for the mantle). Therefore, *if* we were to accept both the Weertman rheology model as well as the apparent rebound restrictions on the radial variation of the mantle viscosity, then we would be compelled to conclude that the mantle solidus must be in error. If, however, the mantle solidus were everywhere *convex* to the origin, then the thermal model of Fig. 6(b), for which (α/C_{p}) was assumed constant, could satisfy the additional rebound constraint without any further modification. But this does not seem to be a likely possibility!

Finally we note that the mantle profile for (α/C_{p}) as determined by Stacey (1977), deduced in part from the earth model of Dziewonski, Hales & Lapwood (1975), falls everywhere below the upper boundary curve in Fig. 7, and hence should result in a mantle adiabat which is everywhere concave to the origin of coordinates.

4.1.7 The inner core

As noted earlier, the core solidus is constructed to be precisely adiabatic. Since the core cools along adiabats, it should solidify uniformly. The thermal model in Fig. 9 is exactly the same as in Fig. 6(b) except that the core solidus has now been constructed to be 30 per cent superadiabatic. We note the formation of a small, solid, inner iron core about 0.5×10^9 yr ago. The size of the solid core corresponds to that which is presently observed, and is surrounded by a partially molten shell about 500 km thick, which could be interpreted as the F-layer (see, e.g. Bullen 1965). The most important conclusions from this model are that the formation of a solid inner core is not incompatible with the present thermal model, and also that the influence of the inner core on the rest of the Earth's thermal history is negligible.

4.2 A THERMAL HISTORY USING THE MODIFIED DIFFUSION EQUATION

Employing the same methods for estimating the Nusselt number in equation (20) as were used in the quasi-steady-state models, a 'diffusion' thermal history model is presented in Fig. 10 which otherwise is directly comparable to the thermal model in Fig. 3(b). While the mantle geotherms are totally dissimilar, the core histories are essentially the same, and the mean-mantle heat fluxes and mean-mantle viscosities are also of the same order.

As discussed at the end of Section 3.1.2, this parameterization scheme implicitly assumes that at any given time, mantle convection should be in a quasi-steady-state, consistent with the 'instantaneous' lower boundary temperature. However, inspection of the mantle geotherms in Fig. 10 reveals that this is not the case. A representative steady-state mantle geotherm, constructed on the basis of the core–mantle temperature and the surface temperature is shown as the dashed curve in Fig. 10. There is no tendency to develop either

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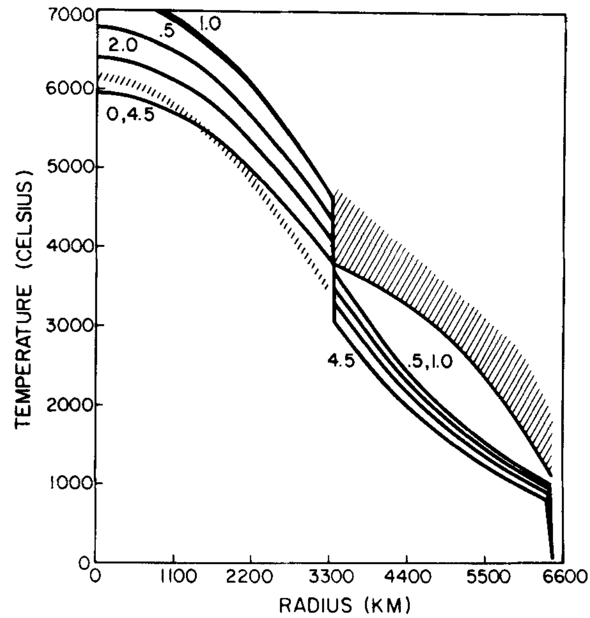

Figure 9. A quasi-steady-state thermal history model which demonstrates the consequence of a super-adiabatic core solidus. Note the formation of the small, solid, inner iron core – *cf.* the thermal model of Fig. 6(b).

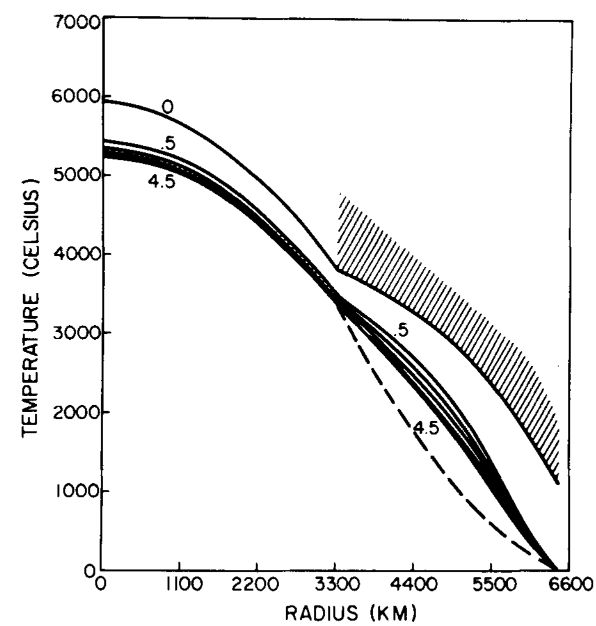

Figure 10. A 'diffusion' thermal history model in which the effects of convection are parameterized by enhancing the standard lattice conductivity in the diffusion equation with the Nusselt number – *cf.* the thermal model of Fig. 3(b). The dashed curve shows a 'steady-state' diffusion geotherm.

an adiabatic central region or upper and lower thermal boundary layers. Neither would we expect solutions of the diffusion equation to display such features. The parameterization of convection through the use of the modified diffusion equation is therefore inferior compared to the direct use of the quasi-steady-state boundary layer solutions.

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The similarity of the core geotherms in the thermal models of Figs 3(b) and 10 is not surprising. For the standard lattice conductivity, the heat flux into the base of the mantle in Fig. 3(b) is determined mainly by the steep temperature gradient in the lower boundary layer. In the absence of such a boundary layer in Fig. 10, the heat flux is controlled by the enhanced thermal conductivity (NuK). Both models give similar values for the heat flux at the core–mantle boundary because they both depend upon the Nusselt number, either directly or indirectly, for estimating this flux.

4.3 DISCUSSION

Fig. 11 summarizes several important characteristics of the quasi-steady-state thermal models presented in this section. The time dependence of the Rayleigh number is illustrated in Fig. 11(b) and emphasizes an important consequence of the assumed 'hot' initial state. Because of the extremely high Rayleigh numbers early in the history of the Earth, the separation of time-scales between the convective overturn time and the cooling rate of the planet is complete during this period. As the system cools and the Rayleigh number decreases, the quasi-steady-state assumption remains valid at any time, because the system has been cooling via a sequence of quasi-steady-states. As a result of the temperature-dependent rheology, the system stiffens rapidly, entering a quasi-steady operating level consistent with the mean heat content of the core. The corresponding time dependences of the mean-mantle viscosity and mean-mantle heat flows are shown in Fig. 11(a) and (c) respectively.

The profiles of Fig. 11 emphasize the key role which the thermal inertia of the core plays in controlling the cooling rate of the mantle. The strong coupling between the thermal histories of the core and the mantle, which is a fundamental characteristic of the quasi-steady-state model, suggests that evolutionary models for the mantle which ignore this coupling would significantly overestimate the cooling rate of the mantle.

It is interesting to note the effects of incorporating K^{40} in the core. Referring to the profiles for the thermal model of Fig. 6(b), the first 0.5×10^9 yr are characterized by a warming trend, as mantle convection is not sufficiently vigorous to keep pace with the heating of the core. During the next 10^9 yr, the entire Earth is in a true steady state, as mantle convection cools the core precisely at the rate of internal heat generation. Subsequent to this time, the thermal history is characterized by a slow cooling.

The development of the upper thermal boundary layer for this model is also of interest. From equation (3c), for steady two-dimensional, high $Pr\#$, high $Ra\#$ convection, the present-day thermal boundary layer should be on the order of 100 km thick. This agrees well with the data in Fig. 12 for the thermal model of Fig. 6(b), and thus corroborates in part, *a posteriori,* the assumptions of the steady-state model. Furthermore, it is interesting to note that the present-day mantle geotherm, as deduced by Stacey (1977) from a consideration of observational data, does in fact exhibit a sharp gradient near the core–mantle boundary which is suggestive of a lower thermal boundary layer. Of course, the relevance of employing a steady two-dimensional boundary layer theory to convection in a spherical shell remains to be demonstrated.

Finally, we will briefly consider the application of our parameterized convection model to the thermal history of Venus. The dashed curves in Fig. 11 present the results of a Venusian evolutionary model for which the values of all the main parameters correspond to those of the earth model in Fig. 6(b), except for the surface temperature which is held fixed at the present-day Venusian value of 750 K. The most significant observation is that the core of Venus cools faster than the Earth's core, as a result of the enhancement in the vigour of mantle convection, due to consistently higher mean-mantle temperatures throughout the history of Venus. In fact, the model suggests that the molten iron core should have

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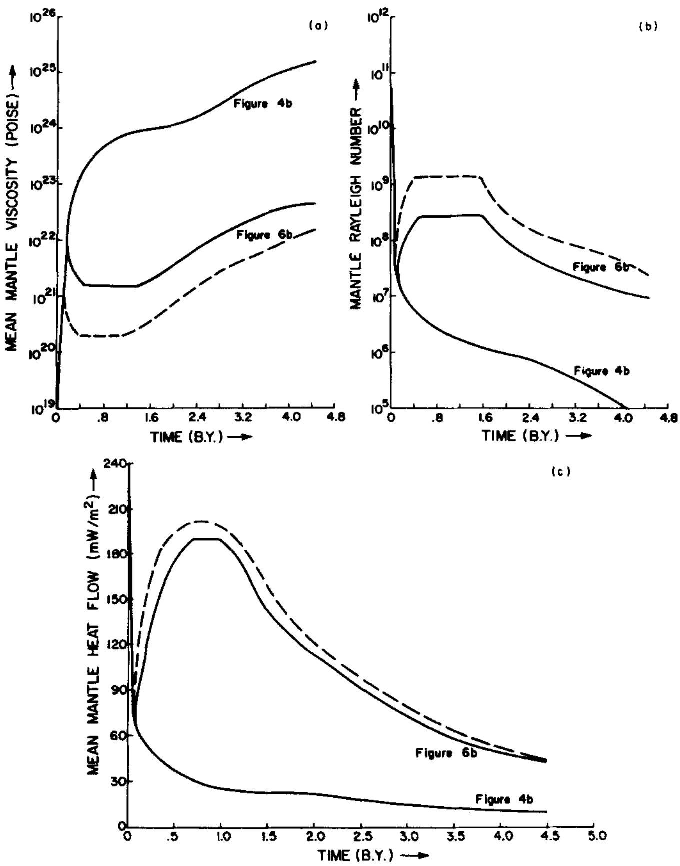

Figure 11. Summary of the evolution of several important thermally related characteristics of the quasi-steady-state thermal models: (a) mean-mantle viscosity; (b) mantle *Ra* # (whole-mantle convection); (c) mean-mantle heat flow. Dashed curves show corresponding results for Venus.

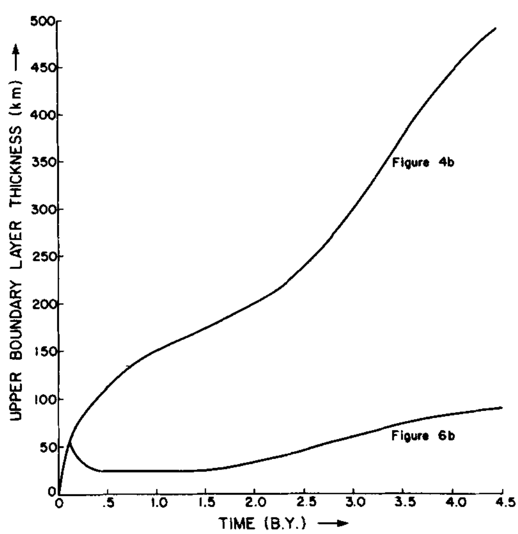

Figure 12. Evolution of the upper boundary layer thickness for the quasi-steady-state thermal models of Figs 4(b) and 6(b).

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solidified about 1.5×10^9 yr ago. This interesting result could explain the observation that, while the Earth and Venus have similar physical characteristics, the latter does not appear to possess a significant magnetic field. One would also expect that if the high surface temperature of Venus has prevailed throughout most of its history, an important assumption which has yet to be substantiated, that relative to the Earth, Venus should have more extensively outgassed by the present time.

5 Conclusions

The purpose of the present study was to develop a rational thermal history model for the Earth which could explain, as simply as possible, its gross internal structure and thermal state. To avoid the complications involved in having to solve explicitly for the dynamics of the mantle convective circulation, we assume a quasi-steady-state convective model in which the enhancement in the radial flow of heat due to convection is parameterized through the Nusselt number.

The results indicate that if quasi-steady-state, whole-mantle convection has persisted throughout the history of the Earth then, while the fluid core could still be preserved until the present, primordial core heat would only account for about 20 per cent of the presently observed mean-mantle heat flow, and the mantle itself would be much stiffer (colder) than is presently suggested from the rebound data. We must recognize, however, that these are 'worst case' results since the model maximizes the cooling rate of the Earth. If we are willing to allow that mantle convection did not initially penetrate down to the core–mantle boundary, and/or that convection in a spherical shell is less vigorous than would be suggested from plane-layer convection at the same Rayleigh number, and/or that radioactive heat sources may be present in the core, then all relevant observational constraints can be met within the context of a self-consistent thermal model. We do not wish to suggest from these results that the mantle is depleted in heat sources, but rather want to emphasize that it is not essential to invoke such sources to understand the main thermal features of the Earth. Further, as indicated earlier, our model has also overestimated the cooling of the Earth because of our neglect of the 'primordial' heat content in the mantle. Since $\mathscr{A} \sim 0(1)$ (see equation (2b)), the secular cooling of the mantle will contribute substantially to the mean-mantle heat flow (by perhaps as much as 80 per cent) in a quasi-steady-state model. This may obviate the necessity of placing potassium in the core and suggests that a thermal history driven *entirely* by primordial heat may yet satisfy all the main constants.

The substantiation of this qualitative conclusion would require a proper formulation of the Rayleigh number for a slowly cooled, vigorously convecting, heated-from-below system. In addition, the relevant form of the *Nu–Ra* relationship must also be determined so as to ascertain if the secular cooling of the mantle affects the parameterization scheme which has been employed in this paper.

A further interesting result of this study is the demonstration of the key role played by the temperature-dependent mantle rheology in regulating the coupled, thermal histories of the core and the mantle. The relevance of these results to the actual physical problem remains to be demonstrated in an evolutionary model in which the full Navier–Stokes equations are solved explicitly; however, we feel that the present work suggests itself as a useful guide.

Acknowledgments

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