

## **A three-dimensional viscoelastic model of a strike slip fault**

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**Summary.** An analytic approximation to the Green's function for the displacements due to a strike slip point source in an elastic layer over a viscoelastic half-space is developed. This approximate Green's function is useful because it can be analytically integrated over the fault surface. Comparison with a numerical integration of the exact solution integral indicates that the approximation is quite good. The approximate Green's function is integrated analytically to obtain the displacements due to a finite rectangular strike slip fault in an elastic layer over a viscoelastic half-space. Ground displacements and angle changes from a model survey net are computed to illustrate the viscoelastic relaxation which follows a fracture in the elastic region.

### **1 Introduction**

Recently there has been a growing interest in the use of static or quasistatic displacements, strains and tilts for the investigation of earthquake-related phenomena. In particular, aseismic horizontal surface displacements due to the San Andreas fault have been studied extensively (Savage & Burford 1973; Thatcher 1974, 1975a, b) by the use of geodetic triangulation techniques. The importance of the San Andreas lies in its well-documented history (Meade 1973) of aseismic surface motions. With this data we can learn much about the processes involved in pre- and post-seismic displacements and in tectonic plate motion.

Observations of surface movements due to the San Andreas can be interpreted by means of mathematical models obtained by idealizing the fault as a vertical fracture in an elastic medium. The models describing near-field displacements are obtained by integration of a Green's function for a strike slip source. For convenience, the source is usually assumed to be located in an elastic half-space whose elastic properties may vary with depth. Although the Green's functions for line sources in layered media have been computed analytically (Rybicki 1971; Chinnery & Jovannovich 1972), the Green's functions for point sources in layered media must at present be integrated numerically (Ben Menahem & Singh 1968; Jovannovich, Husseini & Chinnery 1974a, b). In addition, only the displacements due to an infinitely long fault have been computed analytically (Rybicki 1971). The line source Green's function is really only useful in describing a fault whose dislocation function

changes slowly along its strike. The point source Green's function can be used to describe a finite-dimensional fault with an arbitrary dislocation function.

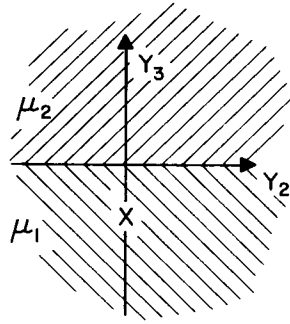
In this paper we construct a model for a rectangular vertical strike slip fault which can be used to explain the postseismic surface displacements occurring after a large earthquake. We propose to represent the fault as a fracture in an elastic layer over a viscoelastic half-space, a model conceptually similar to that proposed by Nur & Mavko (1974) for infinitely long thrust faults. Unlike the Nur–Mavko model however, ours describes a vertical, finite-dimensional strike slip fault. In both models, the displacements from a fracture in an elastic layer overlying an elastic half-space are first computed, and the correspondence principle (Biot 1954; Lee 1955; Fung 1965) is then used to introduce Newtonian viscoelastic properties into the half-space.

We base our model on an approximate representation of the Green's function for horizontal displacements due to a strike slip point source in an elastic layer over an elastic half-space. Although the exact expression for the Green's function has been found (Ben-Menahem & Singh 1968), it is in the form of an integral over a Fourier wave number  $k$ . In our model, three additional integrations of this Green's function are needed: two over the fault surface and an inverse Laplace transform from use of the correspondence principle. These four integrals can be done numerically, but since near-field displacements are desired over a wide range of times, small mesh sizes are needed in the integrations. The approximate Green's function described below is accurate, easy to use and has the advantage that all the succeeding integrations can be done analytically. Thus the inverse problem (Rundle & Jackson 1977a, b) can be done simply using analytically computed partial derivatives.

## 2 An approximate image technique

The following is concerned with the computation of the Green's function for a horizontal double-couple point source. The double couple will be located at  $\mathbf{y}$  and produces displacements in the  $i$  direction at the observation point  $\mathbf{x}$ . We compute this Green's function by considering first the displacements in the  $k$  direction at  $\mathbf{y}$  due to a point force in the  $i$  direction at  $\mathbf{x}$ . We emphasize the distinction between source point and observation point by writing the coordinates as (observation point, source point). It can be shown that the stress  $\tau_{jk}^i(\mathbf{y}, \mathbf{x})$  at  $\mathbf{y}$  due to the force at  $\mathbf{x}$  is itself proportional to a displacement field  $g_{jk}^i(\mathbf{x}, \mathbf{y})$  at point  $\mathbf{x}$  (see, e.g. Steketee 1958). That is, the  $jk$  stresses at  $\mathbf{y}$  due to a force in the  $i$  direction at  $\mathbf{x}$  are proportional to the displacements in the  $i$  direction at  $\mathbf{x}$  due to the  $jk$  nucleus of strain (Love 1944) at point  $\mathbf{y}$ . To compute the displacements due to a distribution of nuclei of strain over a fault surface, it is necessary to integrate the Green's function  $g_{jk}^i(\mathbf{x}, \mathbf{y})$  over a fault surface defined by limits on the  $\mathbf{x}$  variable. The displacements at  $\mathbf{y}$  in direction  $i$  due to a dislocation  $\Delta U_j(\mathbf{x})$  over  $\Sigma(\mathbf{x})$  with unit normal  $\nu_k$  are the result. The interchange of indices and coordinates is somewhat confusing, and this basic outline of the procedure should be kept in mind. A more complete discussion can be found in Steketee (1958).

Ishii & Takagi (1967) solved a two-dimensional problem for the displacements from a line force located at the coordinates  $x_3, x_2 = 0$ , in an infinite elastic medium whose rigidity changes abruptly at  $y_3 = 0$  (Fig. 1). Rybicki (1971) showed that the equations of elasticity for this case reduce to Poisson's equation. He was thus able to use the method of images (Fig. 2) to find the displacements from a line source in and below a horizontal layer over a half-space. For an infinite medium whose  $y_1 - y_2$  plane divides the space into two parts, one of rigidity  $\mu_1$  and another of rigidity  $\mu_2$ , we can write the Green's function for the displace-



X Source point at X<sub>3</sub>

Figure 1. Coordinate system used in finding the displacements due to a line dipole of force in the y<sub>1</sub> direction.

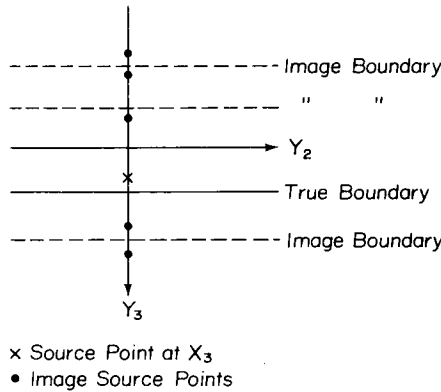


Figure 2. Coordinate system used to find the displacements due to a line dipole of force in the y<sub>1</sub> direction for a medium consisting of an elastic layer overlying an elastic half-space.

ments due to a line force at (x<sub>2</sub>, x<sub>3</sub>) of unit magnitude in the y<sub>1</sub> direction at a point (y<sub>2</sub>, y<sub>3</sub>)

$$K_1^1(y, x) = -\frac{1}{2\pi\mu_1} \left[ \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} + \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \log [(y_2 - x_2)^2 + (y_3 + x_3)^2]^{1/2} \right]$$

for y<sub>3</sub> ≥ 0, x<sub>3</sub> ≥ 0

$$K_1^1(y, x) = -\frac{1}{\pi(\mu_1 + \mu_2)} \left[ \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} \right]$$

for y<sub>3</sub> ≥ 0, x<sub>3</sub> ≤ 0 and y<sub>3</sub> ≤ 0, x<sub>3</sub> ≥ 0

$$K_1^1(y, x) = -\frac{1}{2\pi\mu_2} \left[ \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} + \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \log [(y_2 - x_2)^2 + (y_3 + x_3)^2]^{1/2} \right]$$

for y<sub>3</sub> ≤ 0, x<sub>3</sub> ≤ 0

where μ<sub>1</sub> is the rigidity for y<sub>3</sub> < 0 and μ<sub>2</sub> is the rigidity for y<sub>3</sub> > 0. The displacements at x due to an arbitrary stress distribution σ(y) over the surface Σ(y) are found by multiplication of (1) by σ(y) and integration over Σ(y) as in Rybicki (1971). The superscript of K<sub>1</sub><sup>1</sup> indicates the coordinates along which the displacement appears, and the subscript indicates the direction along which the force acts. The bold variable denotes a vector.

It has been shown (Rybicki 1971) that the displacements  $V_{12}^1$  at  $(x_2, x_3)$  due to an elementary single-couple line dislocation  $U_0 dS$  at  $(y_2, y_3)$  across a vertical plane can be obtained from  $K_1^1(y, x)$  by the operation

$$V_{12}^1(x, y) = U_0 dS \mu \frac{\partial}{\partial y_2} K_1^1(y, x)$$

$$\mu \equiv \begin{cases} \mu_1, \nu_3 < 0 \\ \mu_2, \nu_3 > 0 \end{cases} \quad (2)$$

where  $U_0$  is the magnitude of the dislocation and  $dS$  is the area over which it occurs. As mentioned previously, the displacements  $V_{12}^k(x, y)$  at  $x$  in direction  $k$  due to a strike slip source at  $y$  are proportional to the horizontal (12) shear stresses at  $y$  due to a unit force at  $x$  in direction  $k$ .

We would like to find the displacements due to a line dislocation in a layered half-space. Using the coordinate system of Fig. 2, one can superpose image line forces to satisfy the requirements of a traction-free surface at  $y_3 = 0$  and continuity of the solution and its consequent stresses at  $y_3 = H$ . The displacements from a line force in a layered half-space whose layer thickness is  $H$  (Rybicki 1971) is

$$G_1^1(y, x) = -\frac{1}{2\pi\mu_1} \left\{ \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} + \log [(y_2 - x_2)^2 + (y_3 + x_3)^2]^{1/2} \right.$$

$$+ \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ \log [(y_2 - x_2)^2 + (y_3 - 2mH - x_3)^2]^{1/2} \right.$$

$$+ \log [(y_2 - x_2)^2 + (y_3 - 2mH + x_3)^2]^{1/2}$$

$$+ \log [(y_2 - x_2)^2 + (y_3 + 2mH - x_3)^2]^{1/2}$$

$$\left. \left. + \log [(y_2 - x_2)^2 + (y_3 + 2mH + x_3)^2]^{1/2} \right] \right\} \text{ for } y_3 \leq H, \quad x_3 \leq H$$

$$G_1^1(y, x) = -\frac{1}{\pi(\mu_1 + \mu_2)} \left\{ \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} + \log [(y_2 - x_2)^2 + (y_3 + x_3)^2]^{1/2} \right.$$

$$+ \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ \log [(y_2 - x_2)^2 + (y_3 - 2mH - x_3)^2]^{1/2} \right.$$

$$\left. \left. + \log [(y_2 - x_2)^2 + (y_3 + 2mH + x_3)^2]^{1/2} \right] \right\} \text{ for } y_3 \leq H, \quad x_3 \geq H \quad (3)$$

$$G_1^1(y, x) = -\frac{1}{2\pi} \left\{ \frac{1}{\mu_2} \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2} \right.$$

$$+ \frac{1}{\mu_2} \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \cdot \log [(y_2 - x_2)^2 + (y_3 - 2mH + x_3)^2]^{1/2}$$

$$+ \frac{4\mu_1}{(\mu_1 + \mu_2)^2} \log [(y_2 - x_2)^2 + (y_3 - x_3)^2]^{1/2}$$

$$\left. + \frac{4\mu_1}{(\mu_1 + \mu_2)^2} \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \cdot \log [(y_2 - x_2)^2 + 2mH + x_3)^2]^{1/2} \right\}$$

for  $y_3 \geq H, \quad x_3 \geq H$

where  $\mu_1$  is the layer rigidity and  $\mu_2$  is the half-space rigidity. The operator  $U_0 dS \mu (\partial/\partial y_2)$  can again be used to obtain the displacements at  $(x_2, x_3)$  due to a line strike slip source at  $(y_2, y_3)$ . Chinnery & Jovanovich (1972) have extended (3) to a many-layered half-space.

Equation (3) satisfies the conditions which uniquely define the Green's function for the linear differential operator, the Navier equation of elastostatic equilibrium, and the boundary conditions of the problem (Courant & Hilbert 1953, p. 353):

(i) For fixed  $x_2$  and  $x_3$ ,  $G_1^1(y, x)$  has a singularity in the stress at  $y_2 = x_2, y_3 = x_3$ . Thus in two dimensions  $G_1^1(y, x)$  is proportional to the logarithm of the distance between observation and source points.

(ii) The displacements  $G_1^1(y, x)$  and tractions on any horizontal plane are everywhere continuous except at  $y_2 = x_2, y_3 = x_3$ .

(iii)  $G_1^1(y, x)$  satisfies the boundary conditions. These three requirements imply that for the Green's function considered here, the reciprocal theorem holds:  $G_1^1(y, x) = G_1^1(x, y)$ .

Since we are interested in calculating displacements due to a finite-dimensional strike slip fault in an elastic layer over a viscoelastic half-space we must find the Green's function for a strike slip point source. Let us consider the situation in which a double-couple source at  $y_3$  is in the layer and attempt to write down an approximate Green's function. The approximation must satisfy conditions (i)–(iii) as well as possible. For example, one can show that condition (ii) can be satisfied using any function whose  $y_3$  and  $x_3$  arguments appear only in the form  $(y_3 - x_3)^2, (y_3 + x_3)^2, (y_3 \pm 2mH \pm x_3)^2$ , with the function occurring in the same term-by-term arrangement as the logarithms in equation (3). This suggests that we can apply the form of (3) to our three-dimensional problem.

We already know the exact analytic solution for the displacements  $u_{12}^1(x, y)$  from a double-couple force in an infinite medium (Steketee 1958)

$$\begin{aligned}
 u_{12}^1(x, y) &= \frac{U_0 dS}{8\pi} \left[ 2(1-\alpha) \frac{x_2}{R^3} + 6\alpha \frac{x_1^2 x_2}{R^5} \right] \\
 u_{12}^2(x, y) &= \frac{U_0 dS}{8\pi} \left[ 2(1-\alpha) \frac{x_1}{R^3} + 6\alpha \frac{x_1 x_2^2}{R^5} \right] \\
 u_{12}^3(x, y) &= \frac{U_0 dS}{8\pi} \left[ 6\alpha \frac{x_1 x_2 (x_3 - y_3)}{R^5} \right].
 \end{aligned} \tag{4}$$

The double-couple source is located at  $(0, 0, y_3)$  and the observation point is at  $x$ .  $U_0 dS$  is the elementary displacement dislocation corresponding to the double-couple force,  $\alpha = \lambda + \mu/\lambda + 2\mu$ , and  $R = [x_1^2 + x_2^2 + (x_3 - y_3)^2]^{1/2}$ . Again the superscript indicates the displacement direction, and the subscripts correspond to the fact that the  $u_{12}^k$  are derived by finding the horizontal shear stresses at  $y$  due to a point force at  $x$  along coordinate  $k$ . Using the fact that equation (3) represents horizontal displacements due to horizontal forces, we can try the following as an approximate Green's function

$$\begin{aligned}
 W_{12}^1(x, y) &= \frac{U_0 ds}{8\pi} \left( 2(1-\alpha) \frac{x_2}{R^3} + \frac{6\alpha x_1^2 x_2}{R^5} + 2(1-\alpha) \frac{x_2}{S^3} + \frac{6\alpha x_1^2 x_2}{S^5} \right. \\
 &+ \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ 2(1-\alpha) \frac{x_2}{R_{--}^3} + \frac{6\alpha x_1^2 x_2}{R_{--}^5} + \frac{2(1-\alpha)x_2}{R_{-+}^3} + \frac{6\alpha x_1^2 x_2}{R_{-+}^5} \right. \\
 &\left. \left. + \frac{2(1-\alpha)x_2}{R_{+-}^3} + \frac{6\alpha x_1^2 x_2}{R_{+-}^5} + \frac{2(1-\alpha)x_2}{R_{++}^3} + \frac{6\alpha x_1^2 x_2}{R_{++}^5} \right] \right) \text{ for } x_3 \leq H, \quad y_3 < H
 \end{aligned}$$

$$\begin{aligned}
 W_{12}^1(x, y) = & \frac{2\mu_1}{\mu_1 + \mu_2} \frac{U_0 ds}{8\pi} \left( \frac{2(1-\alpha)x_2}{R^3} + \frac{6\alpha x_1^2 x_2}{R^5} + \frac{2(1-\alpha)x_2}{S^3} + \frac{6\alpha x_1^2 x_2}{S^5} \right. \\
 & + \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ \frac{2(1-\alpha)x_2}{R_{--}^3} + \frac{6\alpha x_1^2 x_2}{R_{--}^5} + \frac{2(1-\alpha)x_2}{R_{++}^3} \right. \\
 & \left. \left. + \frac{6\alpha x_1^2 x_2}{R_{++}^5} \right] \right) \quad \text{for } x_3 \leq H, \quad y_3 \geq H \tag{5}
 \end{aligned}$$

where

$$S^2 = x_1^2 + x_2^2 + (x_3 + y_3)^2; \quad \alpha = \frac{\lambda + \mu_1}{\lambda + 2\mu_1}$$

$$R_{++}^2 = x_1^2 + x_2^2 + (x_3 + 2mH + y_3)^2$$

$$R_{+-}^2 = x_1^2 + x_2^2 + (x_3 + 2mH - y_3)^2$$

$$R_{-+}^2 = x_1^2 + x_2^2 + (x_3 - 2mH + y_3)^2$$

$$R_{--}^2 = x_1^2 + x_2^2 + (x_3 - 2mH - y_3)^2$$

where again  $\mu_1$  is the layer rigidity and  $\mu_2$  is the half-space rigidity.

This trial Green's function has a singularity at the proper point; its contribution to displacement is continuous at  $x_3 = H$ ; and its contribution to shearing stress,  $\mu(\partial W_{12}^1/\partial x_1)$ , and normal stress,  $\lambda(\partial W_{12}^1/\partial x_3)$  is continuous at  $x_3 = H$  if  $\lambda_1 = \lambda_2$ , which we shall henceforth assume. However, it is evident that there still exists a normal stress at  $x_3 = 0$ , and thus the surface is not stress-free. We can remedy this defect by adding to each source and its image a term corresponding to the displacements that result from superposing a normal stress to cancel the normal stress mentioned above

$$\begin{aligned}
 g_{12}^1(x, y) = & \frac{U_0 ds}{8\pi} \left\{ \frac{x_2}{r^4} \left( A + \frac{x_1^2}{r^2} B \right) + \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ \frac{x_2}{r^4} \left( A_+ + \frac{x_1^2}{r^2} B_+ \right) + \frac{x_2}{r^4} \left( A_- + \frac{x_1^2}{r^2} B_- \right) \right] \right\} \\
 & \text{for } x_3 = 0, \quad \text{source at } (0, 0, y_3), y_3 \leq H \tag{6a}
 \end{aligned}$$

$$\begin{aligned}
 g_{12}^1(x, y) = & \frac{2\mu_2}{\mu_1 + \mu_2} \frac{U_0 ds}{8\pi} \frac{x_2}{r^4} A + \frac{x_1^2}{r^2} B + \sum_{m=1}^{\infty} \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \frac{x_2}{r^4} A_+ + \frac{x_1^2}{r^2} B_+ \\
 & \text{for } x_3 = 0, \quad \text{source at } (0, 0, y_3), y_3 \geq H \tag{6b}
 \end{aligned}$$

where

$$A = -4y_3 + 2\rho + 2 \frac{y_3}{\rho}; \quad \rho^2 = x_1^2 + x_2^2 + y_3^2$$

$$B = 16y_3 + 6\rho - 48 \frac{y_3^2}{\rho} + 38 \frac{y_3^4}{\rho^3} - 12 \frac{y_3^6}{\rho^5}$$

$$A_+ = -4(y_3 + 2mH) + 2\rho_+ + 2 \frac{(y_3 + 2mH)^2}{\rho_+};$$

$$\rho_+ = x_1^2 + x_2^2 + (y_3 + 2mH)^2$$

$$B_+ = 16(2mH + y_3) + 6\rho_+ - 48 \frac{(2mH + y_3)^2}{\rho_+} + 38 \frac{(y_3 + 2mH)^4}{\rho_+^3} - 12 \frac{(y_3 + 2mH)^6}{\rho_+^5}$$

$$A_- = -4(2mh - y_3) + 6\rho_- + 2 \frac{(2mH - y_3)^2}{\rho_-};$$

$$\rho_- = x_1^2 + x_2^2 + (2mH - y_3)^2$$

$$B_- = 16(2mH - y_3) + 6\rho_- - 48 \frac{(2mH - y_3)^2}{\rho_-} + 38 \frac{(2mH - y_3)^4}{\rho_-^3} - 12 \frac{(2mH - y_3)^6}{\rho_-^5}$$

$$r^2 = x_1^2 + x_2^2.$$

Note that each term of the form  $(U_0 dS/8\pi) (x_2/r^4) [A_{\pm} + (x_1^2/r^2)B_{\pm}]$  is the Green's function for a strike slip point source at depth  $2mH \pm y_3$  in a homogeneous half-space for observation points at the free surface  $x_3 = 0$  (Steketee 1958). Thus for observation points  $x_3 > 0$ ,  $g_{12}^1(x, y)$  can be written out by substituting for each Green's function for  $x_3 = 0$  the corresponding Green's function for arbitrary  $x_3$  as found by Steketee (1958).

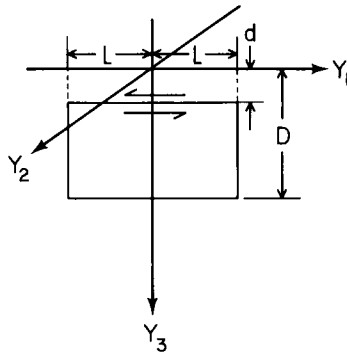
The surface  $x_3 = 0$  is now stress-free, but  $g_{12}^1(x, y)$  and its associated contribution to the stresses are no longer continuous at  $x_3 = H$ . We are missing a set of terms which 'fixes up'  $g_{12}^1(x, y)$  and makes it smoothly continuous across the boundary between layer and half-space. Since we are limited to the observation of surface motion however, it is necessary only to insure that this approximate solution is nearly equal to the real solution for observation points at  $x_3 = 0$ .

We shall postpone further discussion of the accuracy of the approximate Green's function to a later Section. Instead we complete the development of the model by integrating (6a) over a rectangular fault surface and introducing viscoelastic properties into the half-space.

### 3 The finite fault

The solution for horizontal displacements due to a finite fault can be found by integrating (6a) over the area of the fault according to the well-known Volterra relation (Steketee 1958)

$$u^k(y) = \iint_{\text{fault surface}} \Delta u_i(x) g_{ie}^k(x, y) n_e dS(x) \tag{7}$$



**Figure 3.** Coordinate system for the integration of the Green's function for a strike slip point source. The relative displacement across the rectangular fault surface shown is indicated by the two parallel half-arrows. After Chinnery, M., *Bull. seism. Soc. Am.*, 53, p. 9, 1963, copyrighted by the Seismological Society of America.

where  $n_e$  is the outward normal to the fault surface and  $\Delta u_i$  is the displacement jump. Both bold and index notation are used to denote vectors, and the summation convention is assumed. An upper carat denotes a unit vector.

Note that the observation point is now located at  $\mathbf{y}$ , and the integral is performed over the variable  $\mathbf{x}$ . Performing (7) over a rectangular strike slip fault of semilength  $L$  with top at depth  $d$ , bottom at depth  $D$  and with  $\Delta u_i = U_0 \hat{x}_i = \text{constant}$  is particularly easy. The result can be taken from the literature (Chinnery 1961, 1963) using the coordinate system of Fig. 3. For  $y_3 = 0, x_3 \leq H$ , the case of interest here,

$$\begin{aligned}
 u_1 = & \frac{U_0}{4\pi} \left( y_2 t \alpha \left( \frac{1}{S_1(S_1 + q)} + \frac{(1+c)S_2 + (1-b)p + q}{S_2(S_2 + p)^2} - \frac{(p^2 - q^2)(2S_2 + p)}{2S_2^3(S_2 + p)^2} \right) \right. \\
 & + \tan^{-1} \frac{qt}{y_2 S_1} + \tan^{-1} \frac{pt}{y_2 S_2} + \sum_{m=1}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \left[ y_2 t \alpha \left( \frac{1}{S_{1+}(S_{1+} + q_+)} \right. \right. \\
 & + \left. \frac{(1+c)S_{2+} + (1-b)p_+ + q_+}{S_{2+}(S_{2+} + p_+)^2} - \frac{(p_+^2 - q_+^2)(2S_{2+} + p_+)}{2S_{2+}^3(S_{2+} + p_+)^2} \right) + \tan^{-1} \frac{q_+ t}{y_2 S_{1+}} \\
 & + \tan^{-1} \frac{p_+ t}{y_2 S_{2+}} + y_2 t \alpha \left( \frac{1}{S_{1-}(S_{1-} + q_-)} + \frac{(1+c)S_{2-} + (1-b)p_- + q_-}{S_{2-}(S_{2-} + p_-)^2} \right. \\
 & \left. \left. - \frac{(p_-^2 - q_-^2)(2S_{2-} + p_-)}{2S_{2-}^3(S_{2-} + p_-)^2} \right) + \tan^{-1} \frac{q_- t}{y_2 S_{1-}} + \tan^{-1} \frac{p_- t}{y_2 S_{2-}} \right] \Bigg\} \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 u_2 = & \frac{U_0}{4\pi} \left( (1 - \alpha) \ln(S_1 + q) + (1 - \alpha - \alpha c) \ln(S_2 + p) - \alpha \left( \frac{(b+c)p - q}{S_2 + p} + \frac{(p^2 - q^2)}{2S_2(S_2 + p)} \right) \right. \\
 & - y_2^2 \alpha \left( \frac{1}{S_1(S_1 + q)} + \frac{(1+c)S_2 + (1-b)p + q}{S_2(S_2 + p)^2} - \frac{(p^2 - q^2)(2S_2 + p)}{2S_2^3(S_2 + p)^2} \right) \\
 & + \sum_{m=0}^{\infty} \left( \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^m \times \left[ (1 - \alpha) \ln(S_{1+} + q_+) + (1 - \alpha - \alpha c) \ln(S_{2+} + p_+) \right. \\
 & - \alpha \left( \frac{(b+c)p_+ - q_+}{S_{2+} + p_+} + \frac{(p_+^2 - q_+^2)}{2S_{2+}(S_{2+} + p_+)} \right) - y_2^2 \alpha \left( \frac{1}{S_{1+}(S_{1+} + q_+)} \right. \\
 & + \left. \frac{(1+c)S_{2+} + (1-b)p_+ + q_+}{S_{2+}(S_{2+} + p_+)^2} - \frac{(p_+^2 - q_+^2)(2S_{2+} + p_+)}{2S_{2+}^3(S_{2+} + p_+)} \right) + (1 - \alpha) \ln(S_{1-} + q_-) \\
 & + (1 - \alpha - \alpha c) \ln(S_{2-} + p_-) - \alpha \left( \frac{(b+c)p_- - q_-}{S_{2-} + p_-} + \frac{(p_-^2 - q_-^2)}{2S_{2-}(S_{2-} + p_-)} \right) \\
 & \left. \left. - y_2^2 \alpha \left( \frac{1}{S_{1-}(S_{2-} + q_-)} + \frac{(1+c)S_{2-} + (1-b)p_- + q_-}{(S_{2-} + q_-)^2} - \frac{(p_-^2 - q_-^2)(2S_{2-} + p_-)}{2S_{2-}^3(S_{2-} + p_-)^2} \right) \right] \Bigg\} \quad (9)
 \end{aligned}$$

$$\text{for } x_3 \leq H, \quad y_3 = 0.$$



where

$$p = x_3 + y_3$$

$$p_+ = x_3 + y_3 + 2mH$$

$$p_- = x_3 - y_3 + 2mH$$

$$t = x_1 - y_1$$

$$q = x_3 - y_3$$

$$q_+ = x_3 - y_3 - 2mH$$

$$q_- = x_3 + y_3 - 2mH$$

$$\alpha = \frac{\lambda_1 + \mu_1}{\lambda_1 + 2\mu_1}$$

$$b = \frac{\lambda_1 - \mu_1}{\lambda_1 + \mu_1}$$

$$c = \frac{\mu_1 \lambda_1}{(\lambda_1 + \mu_1)^2}$$

$$S_1^2 = t^2 + y_2^2 + q^2$$

$$S_{1+}^2 = t^2 + y_2^2 + q_+^2$$

$$S_{1-}^2 = t^2 + y_2^2 + q_-^2$$

$$S_2^2 = t^2 + y_2^2 + p^2$$

$$S_{2+}^2 = t^2 + y_2^2 + p_+^2$$

$$S_{2-}^2 = t^2 + y_2^2 + p_-^2.$$

The notation  $\parallel$  is taken from Chinnery (1961, 1963) and has the meaning

$$f(x_1, x_3) \parallel = f(+L, D) - f(+L, d) - f(-L, D) + f(-L, d).$$

#### 4 Introduction of time dependence

Having found the displacements due to a finite-dimensional fault in an elastic layer of rigidity  $\mu_1$  over an elastic half-space of rigidity  $\mu_2$ , we modify the solution to add Newtonian viscous properties to the half-space. It is not our purpose here to enter the debate upon the rheological properties of the Earth's constitutive material. We remark, however, that a Newtonian flow law may be justified for the shallow viscoelastic regions in the Earth (McKenzie 1968) although there is evidence for nonlinear rheology as well (Post & Griggs 1973; Stocker & Ashby 1973). In addition, it has been shown that under some conditions a Newtonian rheology produces flow structures very similar to a rheology in which the strain rate depends upon the cube of the deviatoric stress (Parmentier, Turcotte & Torrance 1976). Thus we conclude that the use of a Newtonian flow rheology for our half-space is justified provided it is recognized that the viscosity may be an 'average' in the sense suggested by Parmentier *et al.* (1976).

It may also be that mineral rheology plays little role in the stress relaxation process. Since the zone of brittle fracture ends at a relatively shallow depth on the San Andreas (Brace & Byerlee 1970), an alternative source of the viscous properties of the lower viscoelastic zone may be transient flow of water in porous rock (e.g. Nur & Schultz 1973). Darcy's law, which governs hydrogeologic flow, implies that the stress relaxation process is essentially linear. Thus to a good approximation, the half-space can be assumed to be a linear viscoelastic solid.

To add viscous properties to the half-space, we use the correspondence principle (Biot 1954; Lee 1955; Fung 1965) as outlined in the introduction. This principle states that if the elastic solution to a problem is known and the inertial forces are negligible, the quasi-static solution for a linear viscoelastic medium is obtained by replacing all time-dependent quantities by their Laplace transforms and then taking the inverse transform of the resulting expression. In the case of (6a) or (8), we replace  $\mu_2$  by  $\mu_2(s)$  and  $U_0H(t)$  by  $U_0/s$ , where  $H(t)$  is the Heaviside step function,  $t$  is time and  $s$  is the conjugate Laplace variable.

We choose the properties of the half-space to be those of a Maxwell viscoelastic solid whose elastic constants are the same as those of the layer. This choice of Maxwell properties is motivated by the observation that for times short compared to a year the Earth behaves elastically, while for longer times permanent non-elastic deformations are observed, for example in areas of crustal rebound (McConnell 1965; Crittenden 1963; Peltier 1974) and in tectonically active fault zones during recent geologic history.

The Maxwell solid is the simplest kind of medium to exhibit this dual behaviour. A Maxwell element is shown in Fig. 4; if a constant displacement is suddenly applied to the

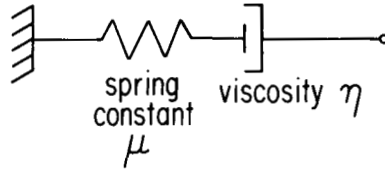


Figure 4. A Maxwell element.

free end, the spring will immediately stretch. In the course of time, the spring will gradually unstretch as the piston in the dashpot moves. The constitutive equation for a Maxwell solid is

$$\frac{\partial}{\partial t} \sigma_{ij} + \gamma \sigma_{kk} \delta_{ij} + \frac{2\mu_0}{\eta} \sigma_{ij} = \frac{\partial}{\partial t} (\lambda e_{kk} \delta_{ij} + 2\mu_0 e_{ij}) \tag{10}$$

where  $\sigma_{ij}$  is the stress tensor,  $e_{ij}$  is the strain tensor,  $\delta_{ij}$  is the Kronecker delta,  $\mu_0$  is a constant rigidity and  $\eta$  is the viscosity. The constants  $\lambda$  and  $\gamma$  are related to compressional stresses which are assumed not to relax.

Upon performing the Laplace transform on (10), we can set

$$\mu_2(s) = \frac{\mu_0 S}{S + 2\mu_0/\eta} = \frac{\mu_0 S}{S + 2/\tau} \tag{11}$$

$$\mu_1(s) = \mu_0 \quad \tau = \eta/\mu_0$$

$\tau$  is a characteristic time constant for the system. Since the rigidity of the Maxwell medium has been chosen equal to that of the layer, only the effective viscosity  $\eta$  remains unknown.

After insertion of (11) in (8) and (9) we must find the inverse Laplace transform  $L^{-1}[\ ]$  of quantities such as

$$\frac{1}{s} \left[ \frac{\mu_0 - \mu_0 s / [(s + 2)/\tau]}{\mu_0 + \mu_0 s / [(s + 2)/\tau]} \right]^m = \frac{1}{\tau^m} \frac{1}{s} \left( \frac{1}{s + 1/\tau} \right)^m \tag{12}$$

From Abramowitz & Stegun (1970) we find

$$L^{-1} \left[ \frac{1}{\tau^m} \frac{1}{s} \left( \frac{1}{s + 1/\tau} \right)^m \right] = H(t) \left[ 1 - \exp(-t/\tau) \sum_{n=0}^{m-1} \left( \frac{t}{\tau} \right)^n \frac{1}{n!} \right] \tag{13}$$

Upon insertion of (13) into (8) and (9) we get expressions of the form

$$\begin{aligned} u_1(y) &= U_0 H(t) F_1(y) \left[ 1 + \sum_m \left[ 1 - \exp(-t/\tau) \sum_{n=0}^{m-1} \left( \frac{t}{\tau} \right)^n \frac{1}{n!} \right] (A_{m+}(y) + A_{m-}(y)) \right] \\ u_2(y) &= U_0 H(t) F_2(y) \left[ 1 + \sum_m \left[ 1 - \exp(-t/\tau) \sum_{n=0}^{m-1} \left( \frac{t}{\tau} \right)^n \frac{1}{n!} \right] (B_{m+}(y) + B_{m-}(y)) \right] \end{aligned} \tag{14}$$

where the  $A_m(y)$  and  $B_m(y)$  can be found from (8) and (9) and the  $F_i(y)$  are the uniform elastic half-space solutions.

Upon examination of (14) it is clear that the first response of the system is entirely elastic. As time proceeds, successive terms under the summation on  $m$  become progressively more important. In the limit  $t \rightarrow \infty$ , the exponential dies away and we get

$$\begin{aligned}
 u_1(y) &= U_0 H(t) F_1(y) \left[ 1 + \sum_m (A_{m+}(y) + A_{m-}(y)) \right] \\
 u_2(y) &= U_0 H(t) F_2(y) \left[ 1 + \sum_m (B_{m+}(y) + B_{m-}(y)) \right].
 \end{aligned}
 \tag{15}$$

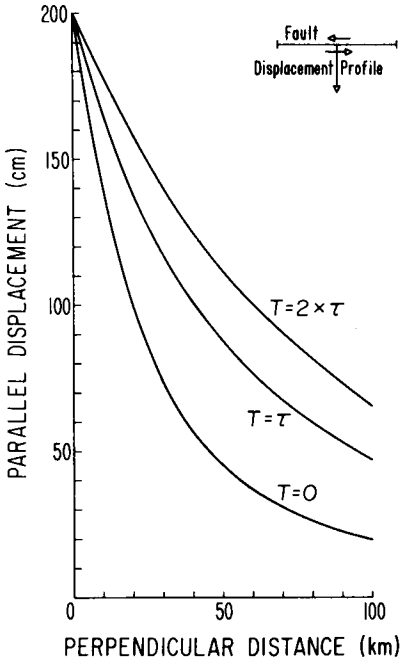


Figure 5

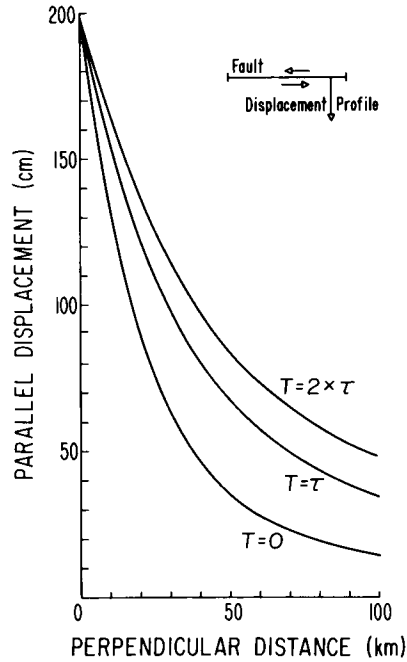


Figure 6

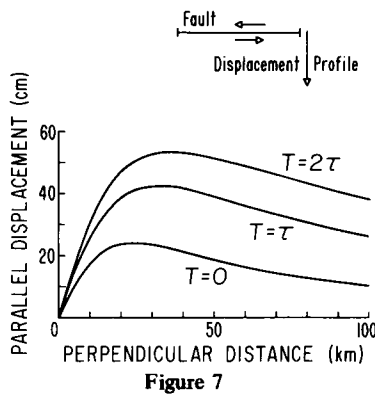


Figure 7

**Figures 5–7.** Surface displacements parallel to a rectangular fault in an elastic layer over a viscoelastic half-space calculated using the approximate Green's function. The displacements were calculated along the horizontal profile shown at the top of each figure where the double arrows lie along the fault trace. The layer thickness is 20 km and the fault fractured the entire thickness of the layer. The relative displacement along the fault is 4 m.

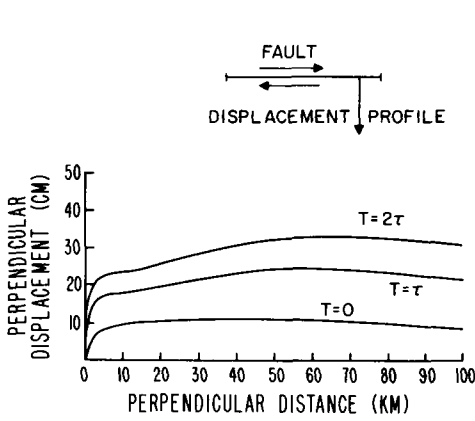


Figure 8

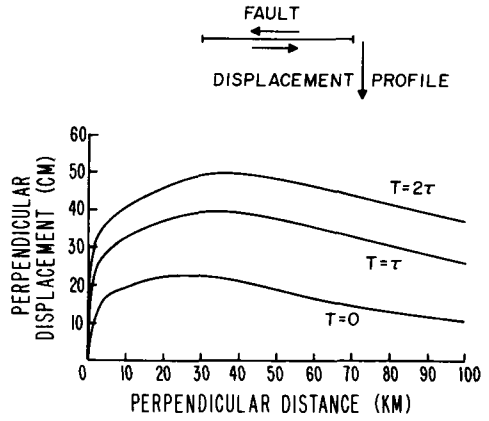


Figure 9

Figures 8–9. Surface displacements perpendicular to a rectangular fault in an elastic layer over a viscoelastic half-space. All parameters are the same as for Figs 5–7.

Numerical tests indicate that only the first five terms need be retained under the sum in (14) for convergence accuracy within 1 per cent for  $t < 2\tau$ . Since  $A_{m\pm}(y)$  and  $B_{m\pm}(y)$  have the same sign as  $F_1(y)$ ,  $F_2(y)$  respectively, we arrive at the conclusion that with the viscoelastic model (14) the quasistatic stress relaxation tends to increase the displacements found immediately after the fracture.

For a long fault which breaks entirely through the elastic layer, the post-fracture viscoelastic displacements can be an appreciable fraction of the initial movement (Figs 5–9). Within 10 km of the fault shown, there is a relatively small amount of post-fracture movement compared to the rupture-induced displacements. Between approximately 10 and 50 km, the post-fracture displacements increase slowly with distance at any given time. At distances of more than 30 km from the fault, most of the total movement is accounted for by viscoelastic effects rather than by the initial elastic response. The result is a surface strain profile which increases near the rupture surface but spreads out from the fault as time progresses. Observations of strains localized near the San Andreas fault following 1906 led Thatcher

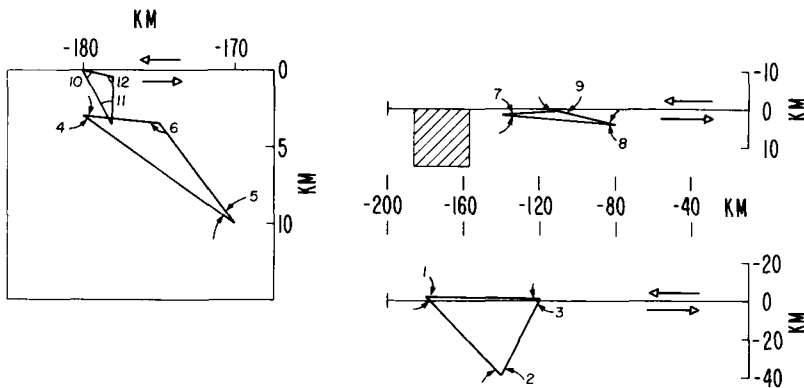


Figure 10. Location of some illustrative model survey triangles relative to a rectangular fault in an elastic layer over a viscoelastic half-space (see Table 1). The fault is indicated by the double arrows, and the insert is a blow-up of the cross-hatched region. All angle changes were computed using the approximate Green's function.

**Table 1.** Comparison of the coseismic elastic and postseismic viscoelastic angle changes for two different ruptures, each in an elastic layer overlying a viscoelastic half-space (see Fig. 10). For case I, the layer thickness is 20 km, the fault length is 400 km; the displacement dislocation across the fault face is 4 m and the fault has fractured the entire thickness of the layer. For case II, the layer thickness is 5 km, the fault length is 400 km, the displacement dislocation across the fault face is 4 m and the fracture extends from the surface to 3 km depth.

Angle	Case I		Case II	
	Coseismic	Postseismic viscoelastic change ( $t = 2\tau$ )	Coseismic	Postseismic viscoelastic change ( $t = 2\tau$ )
No.	(s)	(s)	(s)	(s)
1	-6.16	-1.83	-3.47	-1.10
2	-13.45	-0.98	-9.84	-1.12
3	9.59	2.60	6.40	1.44
4	3.56	-2.03	5.58	-1.97
5	3.87	-1.71	4.50	-1.37
6	-7.43	3.75	-10.09	3.35
7	-0.03	-0.04	-0.01	-0.02
8	0.11	-0.13	0.67	-0.19
9	-0.07	0.18	-0.66	0.22
10	-12.32	4.60	-0.47	10.55
11	16.09	-5.85	57.86	-13.25
12	-3.77	1.24	-10.86	2.70

(1975a) to postulate an aseismic sliding model, but as shown here such a strain distribution can also be explained by anelastic adjustment.

As another illustration, angle changes for the model triangulation networks in Fig. 10 have been computed from equations (8), (9) and (14) and the results are shown in Table 1. The most important factor determining transient motion due to viscoelastic adjustment within each time interval  $\Delta t = \tau = \eta/\mu_0$  is the relationship of the elastic layer thickness  $H$  to the fault-plane height  $D - d$ . If  $H \approx D - d$ , the amount of post-rupture movement will be large; if  $H \gg D - d$ , the motion will be small. A typical value of  $D - d$  for the San Francisco earthquake of 1906 is 10 km (Thatcher 1975a), while  $H$  could be as small as 20 km (Brace & Byerlee 1970). We should therefore expect that viscoelastic relaxation plays a significant role in postseismic surface displacements.

### 5 Accuracy of the Green's function

To determine the accuracy of  $g_{12}^1$ , we can compare the surface displacements computed from the approximate Green's function, equation (6), with those obtained from a numerical integration of the exact solution (Ben-Menahem & Singh 1968). Note the error in equation (11-57) of Ben-Menahem & Singh (1968), which is corrected in Jovannovich *et al.* (1974a). The integration was performed using the IBM-supplied Scientific Subroutine Package.

In presenting the results of the surface displacement calculations we adopt the Jovannovich *et al.* (1974b) convention of  $U_0 = 1$  km and  $dS = 1$  km<sup>2</sup>. The worst agreement between equation (6) and the exact solution is expected when the source point is near the layer/half-space boundary, because the displacements and stresses implicit in the approximate Green's function are not continuous there. Tables 2 and 3 and Figs 11 and 12 are examples of comparisons between exact and approximate Green's functions.

For Fig. 11 and Table 2, use was made of Thatcher's (1975a) result that the fracture of the 1906 San Francisco earthquake extended to a depth of 10 km. The elastic layer was

**Table 2.** See Fig. 11. In column 2 are the horizontal surface displacements calculated from the approximate Green's function. Column 3 is the displacements computed with the approximate Green's function minus the displacements computed by a numerical integration of the exact Green's function.

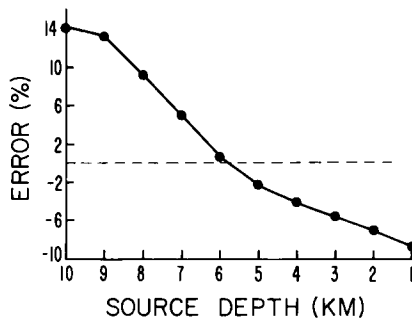
Source depth (km)	Approximate displacements (cm)	Displacement difference (cm)	Error (per cent)
10	87.15	12.36	14.18
9	91.47	11.99	13.11
8	100.52	9.13	9.08
7	114.20	5.60	4.90
6	132.20	0.60	0.45
5	153.52	-3.49	-2.27
4	176.28	-7.37	-4.18
3	197.35	-11.01	-5.58
2	212.66	-15.01	-7.06
1	217.80	-19.09	-8.77
Totals	1483.15	-16.29	-1.1

**Table 3.** See Fig. 12. Columns 2-4 are the same as in Table 2.

Layer thickness (km)	Approximate displacement (cm)	Displacement difference (cm)	Error (per cent)
10	87.15	12.47	14.31
15	52.04	-0.75	-1.44
20	48.00	-2.37	-4.94
25	47.14	-1.72	-3.65
30	46.85	-1.18	-2.52

therefore constrained to be at least 10 km thick. Since the fault plane extends from the surface to 10 km depth, it is natural to check the approximate Green's function for  $H = 10$  km, since as stated above, the approximation is expected to be worst for this case.

In Fig. 11 and Table 2, the source depth is varied from 10 to 1 km in 1-km increments. The point of observation was located at a radial distance of 10 km from the epicentre of the

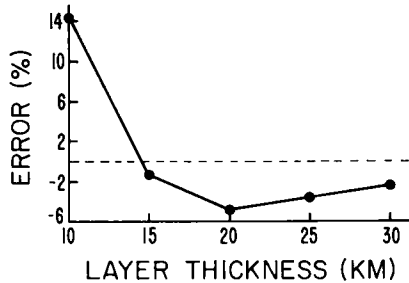


**Figure 11.** Plot of the differences in the predicted horizontal surface displacements due to using the approximate Green's function in place of a numerical integration of the exact Green's function for a point source in an elastic layer over an elastic half-space.  $H$  is fixed at 10 km and the rigidity of the layer is one-tenth the rigidity of the half-space. The point of observation is at a radial epicentral distance of 10 km. See Table 2.

source. The error is defined as [(approximate surface displacement – exact surface displacement)/(approximate surface displacement)] × 100 per cent. It is evident that the greatest errors in the approximate Green’s function occur when the source is near the layer/half-space boundary. Above and below a source depth of about 5 km the difference between exact and approximate Green’s functions has the opposite sign. Upon integrating the Green’s function over the entire 10 km depth, the errors tend to compensate in this case.

To estimate the error in the displacements from a finite, rectangular fault obtained using equation (6), we can sum the entries in column 2 of Table 2 and divide them by the sum of the entries in column 3. The result of this calculation indicates that the discrepancy in the displacements caused by using (8), (9) and (14) in place of the exact solution is small.

Fig. 12 and Table 3 illustrate the effect of a changing elastic layer thickness  $H$  upon the surface displacements due to a source fixed at 10 km depth. The point of observation was



**Figure 12.** Plot of the differences in the predicted horizontal surface displacements due to using the approximate Green’s function in place of a numerical integration of the exact Green’s function for a point source in an elastic layer over an elastic half-space. The rigidity of the layer is one-tenth the rigidity of the half-space and the point of observation is at a radial epicentral distance of 10 km. The source depth is fixed at 10 km. See Table 3.

again located at an epicentral distance of 10 km, while  $H$  was varied from 10 to 30 km in 5-km increments. As shown, the error in the surface displacements decreases sharply as the interface is removed to a greater depth. For thicknesses  $H$  greater than 15 km, the error is never greater than 5 per cent in magnitude.

Note that equation (6) is essentially a perturbation expansion in the quantity  $(\mu_1 - \mu_2)/(\mu_1 + \mu_2) \equiv \xi$  about the homogeneous half-space Green’s function for  $y_3 < H, x_3 < H$ , and is a similar expansion multiplied by the constant  $2\mu_2/(\mu_1 + \mu_2)$  for  $x_3 < H, y_3 > H$ . From the observation that successive terms represent more deeply buried ‘sources’ of the same strength, thus giving rise to smaller contributions to the surface displacements, we can write the series in the form

$$g_{12}^1(x, y) = F_1(x, y) \left( 1 + \sum_{m=1}^{\infty} \xi^m [A_m(x, y) + B_m(x, y)] \right) \quad y_3 < H, \quad x_3 < H \tag{16}$$

$$g_{12}^1(x, y) = \frac{2\mu_2}{\mu_1 + \mu_2} F_1(x, y) \left( 1 + \sum_{m=1}^{\infty} \xi^m [A_n(x, y)] \right) \quad y_3 \geq H, \quad x_3 < H$$

where  $F_1(x, y)$  is the Green’s function for a source in a homogeneous half-space, and  $A_m$  and  $B_m$  are defined as in (6). Note that  $A_m(x, y)$  and  $B_m(x, y)$  are positive definite since they are derived from terms under the summation in (6) which all have the same sign as  $F_1(x, y)$ .  $A_m(x, y)$  and  $B_m(x, y)$  are also bounded, and because  $|\xi| < 1$  for all physical earth models, both series in (16) converge (e.g. Whittaker & Watson 1927, p. 11).

We can show that the representation (6) has the correct limiting properties. For the case

$\mu_1 = \mu_2$ ,  $g_{12}^1$  and  $g_{12}^2$  reduce to the Green's function for the homogeneous half-space. Also, we know that integration of the solution for a point source along a line gives rise to the solution for a line source, so integration of the infinite series of point sources represented by (6) along a line parallel to the  $x_1$  axis yields Rybicki's (1971) Green's function. Finally, as we let  $H$  go to infinity in (6a), the terms under the summation vanish, whereas if  $H$  approaches zero in (6b) we have

$$\begin{aligned} \lim_{H \rightarrow 0} g_{12}^1(x, y) &= \frac{2\mu_2}{\mu_1 + \mu_2} F_1(x, y) \left( 1 + \sum_{m=1}^{\infty} \left( \frac{\mu_1 + \mu_2}{\mu_1 + \mu_2} \right)^m \right) \\ &= \frac{2\mu_2}{\mu_1 + \mu_2} F_1(x, y) \frac{1}{1 - (\mu_1 - \mu_2)/(\mu_1 + \mu_2)} = F_1(x, y). \end{aligned} \tag{17}$$

The fact that the solution to this problem can be approximately represented as a rapidly converging series in the parameter  $\xi$  suggests a method for the evaluation of exact integral solutions for problems of this type. At present, the integration of the exact Green's function has been done numerically by approximating the denominator as a finite series of exponentials (Ben-Menahem & Gillon 1970). Upon examination of the exact Green's function integral, equation (11-56) of Ben-Menahem & Singh (1968), we see that if we write

$$\left. \begin{aligned} \xi &= \frac{1 - \nu}{1 + \nu} \mu_2 = \nu \mu_1 \\ \nu &= \frac{1 - \xi}{1 + \xi} \delta_2 = \frac{\lambda + \mu_1 \nu}{\lambda + 2\mu_1} = \frac{[\lambda + \mu_1(1 - \xi/1 + \xi)]}{[\lambda + 3\mu_1(1 - \xi/1 + \xi)]} \end{aligned} \right\} \tag{18}$$

we can expand the integrand in powers of  $\xi$ . The convergence properties of the series can be investigated and if (6) is an indication, term-by-term integration can probably be carried out. The advantage of this procedure is that it may well put the integral in a form which is more amenable to analytic solution. We know in advance that the first term must be the Green's function for the homogeneous half-space and thus its algebraic representation is known (Steketee 1958). In addition, the modelling of an elastic layer over a viscoelastic half-space via the correspondence principle is particularly easy due to the isolation of the elastic constants in the factor  $\xi$ . One can extend this procedure to half-spaces of many layers by using multiple sums of several parameters  $\xi_i = \mu_i/\mu_0$  where  $\mu_0$  is the rigidity of a particular reference layer.

**Conclusions**

We constructed an accurate analytic approximation to the Green's function for a vertical strike-slip fault in an elastic layer over an elastic half-space. To model the San Andreas fault, we integrated the Green's function over a rectangular surface and introduced linear viscoelasticity into the half-space by means of the correspondence principle. Examples of viscoelastic relaxation in the form of post-fracture surface displacements and angle changes from a model triangulation net were computed.

**Acknowledgments**

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