A Three-Stage Stochastic Facility Routing Model for Humanitarian Logistics Planning

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Abstract

This paper presents a humanitarian logistics decision model to be used in the event of a disaster. The operations under consideration span from opening of local distribution facilities and initial allocation of supplies, to last mile distribution of aid. A mathematical model is developed aiming to enable efficient decision making, maximizing the utility of distribution of aid amongst beneficiaries. This model is formulated as a three-stage mixed-integer stochastic programming model to account for the difficulty in predicting the outcome of a disaster.

Accessibility of new information implies initiation of distinct operations in the humanitarian supply chain, be it facility location and supply allocation, or last mile distribution planning and execution. The realized level of demand, in addition to the transportation resources available to the decision maker for execution of last mile aid distribution and the state of the infrastructure, are parameters treated as random due to uncertainty.

An assessment of the applicability and validity of the stochastic program is made through extensive computational testing based on several test instances. The results show that instances of considerable size are challenging to solve due to the complexity of the stochastic programming model but, still, optimal solutions may be found within a reasonable time frame. Moreover, findings prove the value of the stochastic programming model to be significant as compared with a deterministic expected value approach. Finally, the model is also applied to a case study based on the earthquake that hit Haiti in January of 2010, showing how it could be used in a realistic operation framework.

Keywords: Disaster management, stochastic programming, humanitarian logistics.

1 Introduction

Natural disasters such as droughts, earthquakes, hurricanes and floods have proven a global challenge in their unpredictable nature and potential scale of impact represented by fatalities and

social, environmental and economic costs. The Haiti earthquake of 2010 efficiently demonstrated the potential severity of events following a natural disaster. It killed 222 570 people and affected a total of 3.9 million others. The disaster caused an estimated US$ 8.0 billion worth of damages, and led to the collapse of around 70 per cent of buildings and homes (Guha-Sapir et al., 2011).

In 2011, 302 natural disasters at large claimed over 29 780 lives worldwide, affected nearly 206 million others and caused record economic damages of US$ 366 billion (Guha-Sapir, 2012). In 2012, over 11 000 lives lost and US$ 140 billion damages have been estimated (Swiss Re, 2012). These figures substantiate the importance of providing aid of the appropriate kind and amount to those affected in the most efficient and effective way possible, to prevent loss and suffering (Christopher and Tatham, 2011). The number, magnitude and economical impact of disasters are on the increase along with the overall size of the global population, and advances in the management of disaster operations are imperative, and will contribute to an improvement in readiness, increase response speed, ease recovery and provide institutional learning over time (Altay and Green III, 2006; Christopher and Tatham, 2011; de la Torre et al., 2012; Thomas and Kopczak, 2005).

The function concerning the process of distributing required aid and supplies in disaster relief situations is often referred to as humanitarian logistics. This term is widely used, and covers operations ranging from supply chain strategies and processes to technologies which will maintain the flow of goods and materials required by the humanitarian agencies (Baldini et al., 2011). At its simplest, the operations involve procurement, dispatch of aid for shipment to the beneficiary region, storage in either national or regional warehouses, and eventually transport to the extended and final distribution points where the aid is handed to the beneficiaries (Maspero and Ittmann, 2008). Movement of goods and people accounts for up to 80 % of the costs in any disaster relief operation, making it the most expensive part of the operation (Clark and Culkin, 2013; Sangiamkul and Hillegersberg, 2011).

Uncertainty and unpredictability characterizes the surroundings of disasters, and the logistical activities are to be performed in rapidly changing environments. Knowledge of timing and location of events is substantially difficult, if not impossible, to predict with any significant degree of certainty (Christopher and Tatham, 2011). The same applies to the total magnitude of events immediately following a disaster. This translates into a number of different aspects: uncertainty regarding the nature of demand, capacity of facilities to be used in the distribution process, the transportation capacity, and amount of supply available for the decision maker, along with several other factors. Adding to this is the implicit urgency of need. Not only is the decision maker in charge of the distribution process required to make decisions based on limited and unreliable information, he must also make these at the earliest possible point in time following a disaster in order to prevent lives from being lost (Altay and Green III, 2006; de la Torre et al., 2012).

Against a backdrop of uncertainty, the aim of this paper is to present an optimization model for use in catastrophic events which incorporates stochastic aspects. A three-stage stochastic model is proposed, capturing uncertainty in demand, capacity of the vehicle fleet and the state of the infrastructure. The initial stage concerns facility location decisions, whereas the last two stages involve last mile distribution decisions. The model captures the flow of multiple commodities using multiple modes of transportation, and the goal is to maximize the utility

provided by covering the demand for different commodities.

There has been a recent increase in the amount of research performed in relation to humanitarian logistics. For an up-to-date view on the literature on decision aid models for disaster management and emergencies, we refer to (Vitoriano et al., 2013), and in particular (Ortuño et al., 2013) which covers deterministic models for humanitarian logistics and (Liberatore et al., 2013) which covers uncertainty in humanitarian logistics.

The remainder of the paper is organized as follows. Section 2 gives a detailed description of the disaster response problem approached, whereas a three-stage stochastic programming model for this problem is presented in Section 3. An extensive computational study and an application of the model to a case study based on the Haiti earthquake of 2010 is presented in Section 4, while concluding remarks are given in Section 5.

2   Problem Description

This section describes a planning problem for a humanitarian supply chain in the event of a disaster. Although readily applicable to a wider range of disasters, owing to the generality of the problem, it will deal in particular with humanitarian response in relation to earthquakes. The main task is to establish at which drop-points supplies should arrive and be managed for further distribution, and the international depots from which these supplies should originate. The planning problem includes creation of a distribution plan for the available vehicle types and commodities, from point of supply to point of consumption via local distribution centers, in order to meet the immediate needs of the affected population. The problem is complicated by limited information and uncertain systems, resulting in distribution planning activities of high complexity.

2.1   Contributors

There are several contributors across the different tiers in the humanitarian supply chain. The Federal Emergency Management Agency (FEMA) (2008) and the International Federation of Red Cross and Red Crescent Societies (IFRC) (2012) give descriptions of central actors, out of which mainly three are relevant for the problem considered in this paper. They comprise the International Central Depots (ICDs), the Local Distribution Centers (LDCs) and the different Points Of Distribution (PODs).

The ICDs serve as the initial source of supply. They deliver stock, warehousing and fleet services, as well as general logistics support to operations. The key objective of each ICD is to be able to deliver relief items globally to 5,000 families within 48 hours of a request, and to a further 15,000 families within two weeks.

The LDCs are classified as one of two types; temporary or permanent. The permanently established LDCs serve as year-round emergency supply storages, and are prepared to handle minor seasonal natural disasters such as floods and hurricanes on their own. LDCs of this type also serve as distribution centers should major catastrophes occur, but will under these circumstances require additional supply from an ICD. The fact that they hold a certain level of stock at all times, enables them to provide initial help faster. The stationary LDCs are

thus subject to accompanying holding and operating costs (Zhu et al., 2008). The temporary LDCs on the other hand, are non-stationary and non-operational in times of no crisis. They are commonly located at airports, train stations, harbors or other sites adequate for handling large inflows and outflows of goods and personnel, and can serve as drop points. LDCs of this type are provisional and do not have an explicit holding cost due to the temporary nature of the supply (Zhu et al., 2008). There is an impending risk that possible sites for LDCs are destroyed during the disaster. Ascertaining feasible destinations for LDCs prior to the crisis is thus a crucial element in ensuring efficient allocation of aid (Federal Emergency Management Agency (FEMA), 2008).

The Federal Emergency Management Agency (FEMA) (2008) describes a POD as a centralized location at which the public can collect life sustaining commodities. Shelf stable food, bottled water and limited amounts of ice, tarps, and blankets exemplify commodities of this nature. The actual amount and type of commodities sent to the different PODs are determined at the LDCs, as is the type and location of PODs to activate. The LDCs are also in charge of operation and eventually demobilization of the PODs.

2.2 Course of Events

Shortly after an earthquake has occurred, aid agencies or the government will decide whether or not it is necessary to initiate emergency response. If so, local agents will start gathering information about the consequences of the disaster. To manage the emergency response and distribution of goods efficiently and effectively, new or updated information has to be gathered continuously. The local agent sends a team of experts into the area of relevance to complete an initial assessment of the extent of the disaster and the needs of the people affected. The assessment will serve as a basis for an appeal that lists specific items and quantities needed to provide immediate relief to the affected populations (International Federation of Red Cross and Red Crescent Societies (IFRC), 2008).

Based on identification of the location of the demand points, the team of experts informs the local agent where to inaugurate PODs. The location of the PODs, together with knowledge of the state of the infrastructure prior to the earthquake, serve as the basis for determination of the LDCs to open and operate.

If the local agent managing the LDCs stands in need of supplies from the ICDs, an account of the distribution plan and the associated costs will be required. As goods are sent from the ICDs, medical teams, vehicles and volunteers are engaged to the operating LDCs. Seeing as how the number of vehicles and volunteers to arrive on time is uncertain, the knowledge of exact transportation capacity at each LDC is not realized until packing of the vehicles that have actually arrived, is started.

Once an initial distribution plan is designed at an LDC, the vehicles are packed and dispatched according to this plan. In some cases, the vehicles will be prevented from completing their initial routes due to infrastructure damage. For those to which this applies, the local agent will be consulted and an alternative route will be generated based on updated information about the network. As time passes, accrued amounts of information about the state of the infrastructure will become available via satellite pictures and first-hand experience provided by drivers and local reports. Some vehicles will reach their planned destination PODs, whilst others may

have to change destination along the way because of obstacles. Either way, their duties are considered completed when the commodities they carry are delivered.

2.3 Sources of Uncertainty

There are several factors that influence the strategies of the local agents and restrict the number of alternatives that may be considered. First, demand is uncertain. The districts in need may be situated in remote areas, and the disaster site might be in a state of chaos making a complete overview impossible to achieve (Thomas and Kopczak, 2005). Second, the size of the vehicle fleet and the available medical personnel are highly uncertain factors in humanitarian logistics. The consequences of not reaching a POD are severe, and uncertainty in the state of the infrastructure is the third main element of uncertainty considered in this study. To the extent that this study concerns uncertainty, its focus lays mainly on these three aspects and the consequences that they entail.

Unpredictable demand patterns increase the complexity of the distribution plan (Balcik et al., 2008). Demand can fluctuate unexpectedly due to a number of reasons. These reasons include after-shock damages, people returning to greater self-sufficiency, beneficiaries moving between different areas in hopes of find greater relief, or unexpected challenges such as outbreak of disease epidemics (de la Torre et al., 2012).

Volunteer organizations state that they generally do not possess their own vehicle fleet. The implication of this fact is that multiple independent local drivers and vehicles need to be hired and managed internally (de la Torre et al., 2012). This in turn, complicates prediction of the size of the vehicle fleet, its total capacity, the experience and knowledge held by local drivers and the employable technology in the vehicles. In case of insufficiency of available vehicles, agencies may be required to import the amount needed. The ease of importing vehicles for a short time period and the ability to transport these to the requiring LDCs, influence the final vehicle capacity. An additional limitation arise from regular cost-benefit analysis. Engaging an excessive number of vehicles, especially high technology vehicles, will be very expensive and a waste of resources. The engaged amount should thus, to the extent possible, correlate with the amount needed in order to perform distribution.

Within the first 72 hours after impact, the vehicle fleet and the number of drivers available at the LDCs will be more or less confirmed, and an initial distribution plan will be determined. At this point in time we assume that the team of experts have more precise information concerning the level and nature of demand at each POD, which will naturally affect the distribution plan.

The initial distribution plan is subject to alternation because of the potentially severe damage typically caused to the local distribution network by an earthquake. Roads, bridges and airports are often destroyed. The accessibility of the different recipients may be reduced accordingly depending on their location relative to the earthquake's epicenter, and the quality of the infrastructure connecting the LDCs and the PODs. The attributes of a vehicle may in addition restrict it from using certain paths in the infrastructure. The cause of failures in parts of the infrastructure may be due to factors such as natural gas explosions, consequent fires, building, bridge or road collapse or road blockage (Günneç and Salman, 2007). As a result, operable infrastructure needs to be estimated based on the decision maker's judgment and experience, and the distribution plan and emergency strategy based on this input may in effect not be applicable.

2.4 Fairness in Distribution of Aid

Clark and Culkin (2013) propose three principles said to define humanitarianism: humanity, impartiality and neutrality. In short, these state that suffering should be alleviated wherever it is found, giving priority to the most urgent needs without discrimination. The concept of fairness is vital to consider when distributing emergency supplies. The Sphere Handbook (The Sphere Project, 2011) argues that agencies should provide aid impartially and in accordance with need. Generally, the needs of the most vulnerable sub-groups of the population, such as wounded, children, pregnants and women, is prioritized. Consequently, the marginal utility of the delivered items diminishes in line with reception of aid by the most indigent at each POD. Even though need for relief may still exist in an area, helping people of higher levels of distress in other regions before continuing to deliver to the initial region might be of greater utility. The requirement of prioritizing is usually caused by shortage in supply or capacity, or damages in the distribution network.

3 Three-Stage Stochastic Programming Model

The aim of stochastic programming is to determine optimal decisions in problems involving uncertain data, as the one considered in this paper. The decision maker is restrained by limited information, but seeks to produce optimal distribution plans. For a thorough introduction to stochastic programming in general, we refer to Birge and Louveaux (2011) and Higle (2005). In what follows, we first give a general overview of the proposed three-stage stochastic programming model and then proceed to describe the model in full detail.

3.1 Assumptions and limitations of the model

While the proposed model takes into account uncertainty, the available budget and supplies, among others, are still treated as deterministic parameters. In real life, the budget could be stochastic, since it may depend on donations and other uncertain contributions. However, although it is normally restrictive, it must be large enough to at least satisfy the demand of highest importance. On the other hand, supply at the ICDs is normally certain, but due to potential delays, supply at the LDCs might be stochastic. Even so, the proposed model will treat both the supply and the budget as deterministic elements.

A general challenge in stochastic programming is the potentially immense number of possible realizations of the stochastic elements. The number of realizations, translated into an equivalent number of future scenarios, is the product of possible outcomes for each distinct random value of a parameter. Demand, size of the available vehicle fleet, utilities and state of the last-mile transportation network comprise the stochastic parameters modeled. To overcome the challenge of the number of realizations, it is assumed that a discrete approximation of the possible outcomes in stage 2 and stage 3, along with respective probability distributions, can be constructed.

After the disaster happens and as the situation progresses, some beneficiaries may return to greater self-sufficiency, whilst others may relocate to different areas in hopes of finding greater relief. Besides, the arise of unexpected challenges such as disease epidemics or changes in the

conflictive zones can also bring about change in the level of demand at different PODs. This argument serves as the grounds for considering demand as a stochastic parameter. The demand at a POD is differentiated by the type of commodity and the severity of the need. For each combination of commodity and severity, there is a specified utility resulting from the delivery of one unit of the commodity. This utility is also treated as a stochastic parameter.

Allocation of vehicles can often prove troublesome due to limited availability of vehicles and difficulty in predicting the actual level of availability. The range of different types of vehicles accessible to the decision maker will be considered known in advance, whereas the number of vehicles per type at disposal at each LDC will be subject to uncertainty. Vehicle capacity is hence modeled as a stochastic parameter in terms of the number of disposable vehicles, exclusively.

The allocation of commodities and other necessary items to LDCs, as well as selection of which LDCs to open, are modeled as decisions made in stage 1. The first reception of updated information, mainly concerning the demand at PODs and the vehicles that have already arrived at the LDSs, marks the transition from stage 1 to stage 2. Demand information is now assumed to be known as certain, as well as information concerning utilities and the number of vehicles available of each type. When this information is available, the decision maker will be able to generate initial routes, based on an anticipation of the state of the network; the infrastructure state is now the uncertain parameter at stage 2, since the disaster may have damaged some roads and information about their state is not known yet. In accordance with this plan, vehicles are packed and dispatched from the LDCs as soon as they are loaded.

The third stage corresponds to recourse actions for the vehicles that have been sent out towards the different PODs. If a planned route proves fully operative and can be executed according to the plan, the delivery will be made according to the second stage decisions. Should, however, a vehicle be unable to follow its planned route, due to infrastructure being unavailable, it will need to follow a modified route. Since the information about the infrastructure is uncertain when the vehicles leave the LDCs, they have to follow their planned route until an obstacle is met, and will only be rerouted when and if this happens.

Further assumptions concern repacking of vehicles at PODs during last mile distribution in order to combine loads. Vehicles are permitted to travel empty to be able to relieve other vehicles, when necessary. All vehicles end their routes at the final destination POD, as defined by their designated distribution routes generated in stage 2, given that they are able to reach the POD in question according to plan and delivering to another POD does not provide an increase in the level of utility. However, if the given destination POD is inaccessible to the vehicle type in question, this vehicle is free to serve other PODs. As a direct consequence of this, the assigned load will not go to waste and contribute to satisfaction of demand.

In order to enable control of inflow versus outflow of commodities and vehicles at the LDCs, a dummy node system in which each LDC is provided with a corresponding dummy LDC has been introduced. By prohibiting inflow to the original LDC from all nodes apart from the ICDs, but allowing both inflow and outflow to the dummy LDCs, we allow for transshipment at the LDCs. The LDCs and their respective dummy LDCs are connected to the identical set of nodes, but not to each other. The arcs connected to the original LDC are only outgoing, whereas the arcs connected to the respective dummy LDC are symmetric. The dummy LDCs, as opposed to their respective original LDCs, are empty nodes. They are neither provided with any supplies

or vehicles, nor do they experience any demand.

The dummy LDCs will be utilized should a vehicle need to return to the node of origin, or use an LDC for transshipment purposes. Even though the LDC and corresponding dummy LDC are represented by two different nodes in the transportation network, they are in reality situated at the exact same location. If an LDC is to be used for transshipment only, opening costs should not be induced. Transshipment will thus take place at the dummy LDC, which will not be subject to opening costs.

A decisive and significant attribute of the proposed model is the enablement of initialization of routes prior to reception of information regarding the final state of the distribution network. The dispatched vehicles will thus need to be forced to follow the planned routes to the extent possible. They will either be able to complete the given route, or need to be given an alternative plan of attack should an obstacle hinder this route from being carried through. In order to tackle the mathematical challenge of modifying a route, two almost identical networks has been created. The original network utilized in stage 2 includes the nodes representing LDCs, dummy LDCs and PODs. The mathematically extended network builds upon this network, but also includes a duplicated network consisting of the the dummy LDCs and PODs only. As long as the arcs to be traversed exist in stage 3, flow will occur in the original network according to the initially generated distribution plan. If an obstacle impedes this route from being followed, the original network will be abandoned and the vehicle in question will proceed in the duplicated network according to a modified route. Flow on links connecting corresponding nodes of the original and duplicated network represent artificial flow created for modeling purposes only, as to indicate a change of network and transition to stage 3.

3.2 Formal Definitions

This section gives a formal definition of the sets, indices, constants and variables used in the mathematical formulation of the stochastic programming model proposed in this paper. Sets are denoted by calligraphic upper-case letters, indices and variables by standard lower-case letters and constants by standard upper-case letters.

Network Quantities

Q_I, Q_L, Q_P, Q_N - total number of ICDs, LDCs, PODs and nodes in the original network, respectively.

Main Sets

$\mathcal{B}$ - set of commodity types
$\mathcal{E}$ - set of scenario tree nodes in the scenario tree
$\mathcal{K}$ - set of utility intervals
$\mathcal{N}$ - set of nodes, $\mathcal{N} = \{1, \dots, Q_I + 2Q_L + Q_P = Q_N\}$
$\mathcal{T}$ - set of stages, $\mathcal{T} = \{1, 2, 3\}$
$\mathcal{V}$ - set of vehicle types

Indices of Main Sets

b	- commodity type	$b \in \mathcal{B}$
i, j, j'	- node	$i, j, j' \in \mathcal{N}$
k	- utility interval	$k \in \mathcal{K}$
n	- scenario tree node	$n \in \mathcal{E}$
t	- stage	$t \in \mathcal{T}$
v	- vehicle type	$v \in \mathcal{V}$

Derived Sets

$\mathcal{A}$ - subset of arcs which vehicles traveling between ICDs and LDCs
are allowed to traverse

$\mathcal{A}_{vn}$ - subset of arcs between LDCs (including dummy LDCs) and PODs that vehicle type v
is allowed to traverse in scenario tree node n

$\mathcal{A}'_{vn}$ - subset of arcs between LDCs (including dummy LDCs), PODs, duplicated LDCs
(including dummy LDCs) and duplicated PODs that vehicle type v is allowed to traverse
in scenario tree node n

$\mathcal{E}_t$ - set of scenario tree nodes at stage t, $\mathcal{E}_t \subset \mathcal{E}$

$\mathcal{I}$ - set of ICDs, $\mathcal{I} \subset \mathcal{N}$

$\mathcal{L}$ - set of LDCs, $\mathcal{L} \subset \mathcal{N}$

$\mathcal{L}^D$ - set of dummy LDCs, $\mathcal{L}^D = \{i + Q_L | i \in \mathcal{L}\} \subset \mathcal{N}$

$\mathcal{P}$ - set of PODs, $\mathcal{P} \subset \mathcal{N}$

$\mathcal{L}^{D'}$ - set of duplicated dummy LDCs, $\mathcal{L}^{D'} = \{i + 2Q_L + Q_P | i \in \mathcal{L}^D\}$

$\mathcal{P}'$ - set of duplicated PODs, $\mathcal{P}' = \{i + 2Q_L + Q_P | i \in \mathcal{P}\}$

$\mathcal{V}_{jn}$ - subset of vehicle types allowed to travel into node j in scenario tree node n, $V_{jn} \subseteq \mathcal{V}$

Indices of Subsets

(i, j) - arc, $(i, j) \in \mathcal{A} \cup \mathcal{A}_{vn} \cup \mathcal{A}'_{vn}$

$a(n)$ - predecessor scenario tree node, $a(n) \in \mathcal{E} \backslash \mathcal{E}_3$
of scenario tree node n

Deterministic Parameters

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C^I	- unit capacity of vehicles traveling between ICDs and LDCs
C_v^L	- unit capacity of vehicle type v
C_j^C	- unit capacity at an LDC j
E_b^C	- unit cost of commodity type b
E_{ij}^I	- cost associated with traveling from ICD i to LDC j
E_i^O	- cost associated with opening an LDC i
F_i^I	- total available number of vehicles at ICD i
H^B	- available budget
H^T	- upper convoy time limit
M^B	- utility factor for residual budget
P_n	- probability of scenario tree node n occurring
Q_b^C	- unit size of commodity type b
Q_v^V	- unit size of vehicle type v
S_{ib}	- supply of commodity type b at ICD i
T_{ij}^I	- time spent traveling from ICD i to LDC j

Stochastic Parameters

C_{ijn}^A	- unit capacity of arc (i, j) in scenario tree node n
D_{jbn}	- demand of commodity type b at POD j in scenario tree node n
E_{ijvn}^L	- cost associated with traveling from node i to node j for vehicle type v in scenario tree node n
F_{ivn}^L	- total expected number of vehicles of vehicle type v at LDC i in scenario tree node n
M_{jbkn}^D	- utility factor for satisfied demand at POD j of commodity type b in utility interval k in scenario tree node n
T_{ijvn}^L	- time spent traveling from node i to node j for vehicle type v in scenario tree node n
U_{jbkn}	- size of utility interval k for commodity type b at POD j in scenario tree node n

First Stage Variables

l_i	$= \begin{cases} 1, \text{ if LDC } i \text{ is opened} \\ 0, \text{ otherwise} \end{cases}$
x_{ij}^I	- number of vehicles to travel from ICD i to LDC j
y_{ijb}^I	- amount of commodity type b sent from ICD i to LDC j
z_{ij}^I	$= \begin{cases} 1, \text{ if a vehicle traverses arc } (i, j) \\ 0, \text{ otherwise} \end{cases}$

Second and Third Stage Variables

d_{jbkn} - amount of satisfied demand of commodity type b at POD j in utility interval
        k and scenario tree node n

x_{ijvn}^{L} - number of vehicles of type v to travel from node i to node j in scenario tree
        node n

y_{ijbvn}^{L} - amount of commodity type b sent from node i to node j with vehicle type v
        in scenario tree node n

z_{ijvn}^{L} $= \begin{cases} 1, \text{ if vehicle type } v \text{ traverses arc } (i,j) \text{ in scenario tree node } n \\ 0, \text{ otherwise} \end{cases}$

w_n - level of residual budget in scenario tree node n

3.3   Model Formulation

Objective Function

The objective function of the model must consider both effectiveness and fairness of distribution. Effectiveness is measured in terms of cost, while the aspect of fairness relates to level of urgency in reception of aid.

$$\max \sum_{j \in \mathcal{P}} \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{E} \backslash \mathcal{E}_1} P_n M_{jbkn} d_{jbkn} + \sum_{n \in \mathcal{E} \backslash \mathcal{E}_1} P_n M^B w_n \tag{1}$$

The proposed objective function (1) maximizes utility accumulated over the scenario tree nodes, n, of stage 2 and 3. The two terms constituting the utility function express utility in terms of level of demand fulfillment and residual monetary budget respectively. The objective function is summing up the utility from both stage 2 and stage 3. The reason for including both stages is the desire to produce efficient decisions for both the planned and the realized distribution plan. These decisions, planned as well as realized, should be made according to the level of urgency in need at the PODs.

Constraints Stage t = {1}

The initial decisions concern which LDCs to open and the amount of supply to provide these LDCs with. The 1st stage decisions are made under uncertainty regarding the capacity of the vehicle fleet performing last mile distribution, the level of final demand and the state of the distribution network.

$$\sum_{j\in\mathcal{L}} y_{ijb}^I \le S_{ib} \qquad\qquad i\in\mathcal{I}, b\in\mathcal{B} \qquad (2)$$

$$\sum_{i\in\mathcal{I}}\sum_{b\in\mathcal{B}} Q_b y_{ijb}^I - C_j^C l_j \le 0 \qquad\qquad j\in\mathcal{L} \qquad (3)$$

$$\sum_{j\in\mathcal{L}} x_{ij}^I \le F_i^I \qquad\qquad i\in\mathcal{I} \qquad (4)$$

$$x_{ij}^I - F_i^I z_{ij}^I \le 0 \qquad\qquad (i,j)\in\mathcal{A} \qquad (5)$$

$$\sum_{b\in\mathcal{B}} Q_b y_{ijb}^I - C^I x_{ij}^I \le 0 \qquad\qquad (i,j)\in\mathcal{A} \qquad (6)$$

$$z_{ij}^I - x_{ij}^I \le 0 \qquad\qquad (i,j)\in\mathcal{A} \qquad (7)$$

The Supply Constraints (2) ensure that the amount of commodities dispatched from the ICDs is kept within the amount available at the ICDs. Constraints (3) prohibit violation of the capacity restrictions for the initialized LDCs, in addition to preventing inflow of commodities to the LDCs which are excluded from the distribution process. Constraints (4) limit the outbound number of vehicles at the ICDs according to the size of the available vehicle fleet, while constraints (5) limit the number of vehicles traversing an arc between an ICD and an LDC. Constraints (6) ensure vehicle capacity compliance, whereas Constraints (7) assure coherence between the integer variable defining quantity of vehicles traversing an arc and the corresponding binary variable.

Constraints Linking Stage t={1} and t={2}

The transition from stage 1 to stage 2 takes place as the LDCs have received their designated amounts of supply from the ICDs, in addition to information about vehicle fleet capacity and demand. The 2nd stage decisions indicate the *expected* last mile distribution routes, and represent the planned routes. They are generated based on uncertain information regarding the state of the infrastructure.

$$\sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{A}_{vn}} y_{ijbvn}^L - \sum_{(j,i)\in\mathcal{A}} y_{jib}^I \le 0 \qquad\qquad i\in\mathcal{L}, b\in\mathcal{B}, n\in\mathcal{E}_2 \qquad (8)$$

Constraints (8) control the planned number of commodities to be used to serve final demand, and are intended to balance in and outgoing flow of supplies at the LDCs chosen for operation. These constraints ensure that the amount of commodities dispatched from each LDC, does not exceed the number of commodities received from the ICDs.

Constraints Stage t = {2}

$$\sum_{v \in \mathcal{V}_{jn}} \sum_{(i,j) \in \mathcal{A}_{vn}} y^L_{ijbvn} - \sum_{v \in \mathcal{V}_{jn}} \sum_{(j,i) \in \mathcal{A}_{vn}} y^L_{jibvn} - \sum_{k \in \mathcal{K}} d_{jbkn} = 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_2 \quad (9)$$

$$\sum_{k \in \mathcal{K}} d_{jbkn} \leq D_{jbn} \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_2 \quad (10)$$

$$d_{jbkn} \leq U_{jbkn} \qquad \begin{aligned} &j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, \\ &n \in \mathcal{E}_2 \end{aligned} \quad (11)$$

$$\sum_{(i,j) \in \mathcal{A}_{vn}} x^L_{ijvn} - F^L_{ivn} l_i \leq 0 \qquad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{E}_2 \quad (12)$$

$$x^L_{ijvn} - \left(\sum_{j' \in \mathcal{L}} F^L_{j'vn} \right) z_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \quad (13)$$

$$\sum_{b \in \mathcal{B}} Q^C_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \quad (14)$$

$$\sum_{(j,i) \in \mathcal{A}_{vn}} x^L_{jivn} - \sum_{(i,j) \in \mathcal{A}_{vn}} x^L_{ijvn} \leq 0 \qquad j \in \mathcal{L}^D \cup \mathcal{P}, n \in \mathcal{E}_2, v \in \mathcal{V}_{jn} \quad (15)$$

$$\sum_{v \in \mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+Q_L)jvn}) \leq C^A_{ijn} \qquad n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{L}, j \in \mathcal{N} \backslash \mathcal{I} \quad (16)$$

$$\sum_{v \in \mathcal{V}} Q^V_v x^L_{ijvn} \leq C^A_{ijn} \qquad n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{P}, j \in \mathcal{N} \backslash \mathcal{I} \quad (17)$$

$$\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn} : i \in \mathcal{L}} y^L_{ijbvn} - \sum_{j \in \mathcal{P}} \sum_{k \in \mathcal{K}} d_{jbkn} = 0 \qquad b \in \mathcal{B}, n \in \mathcal{E}_2 \quad (18)$$

$$\sum_{(j,i) \in \mathcal{A}_{vn}} y^L_{jibvn} - \sum_{(i,j) \in \mathcal{A}_{vn}} y^L_{ijbvn} \leq 0 \qquad j \in \mathcal{L}^D \cup \mathcal{P}, b \in \mathcal{B}, v \in \mathcal{V}, n \in \mathcal{E}_2 \quad (19)$$

Constraints (9) define the level of demand fulfillment for each demand point, whereas Constraints (10) restrain demand satisfaction from exceeding the actual demand and Constraints (11) register the level of demand satisfaction achieved within each utility interval representing a specific level of urgency of reception of a given commodity. Combined with the first term of the Objective Function (1), Constraints (11) ensure that distribution is performed according to urgency of need. Constraints (12) ensure that the number of vehicles departing from an LDC does not exceed the total available quantity, while Constraints (13) ensure that the number of vehicles traversing an existing arc does not exceed the total number of available vehicles across all LDCs. Constraints (14) keep total load assigned to the different vehicles within the vehicle's capacity, whereas Constraints (15) represent the vehicle flow balance constraints. Constraints (16)-(17) enforce arc capacity compliance by considering the size and number of vehicles seeking to traverse an arc. Due to the dummy node system, the outgoing arcs of an LDC equal those of the respective dummy LDC, consequently rendering it necessary to consider them concurrently. Finally, constraints (18) ensure that the amount of commodities initially planned for dispatch from the LDCs contribute in satisfying final demand, whereas constraints (19) balance inflow versus outflow of commodities at the dummy LDCs and PODs.

Constraints Linking Stage t={2} and t={3}

$$\sum_{j\in\mathcal{P}}\sum_{k\in\mathcal{K}}d_{jbkn}-\sum_{j\in\mathcal{P}}\sum_{k\in\mathcal{K}}d_{jbka(n)}\le 0 \qquad\qquad b\in\mathcal{B}, n\in\mathcal{E}_3 \qquad (20)$$

$$\sum_{k\in\mathcal{K}}d_{jbka(n)}-\sum_{k\in\mathcal{K}}d_{jbkn}$$
$$+(\sum_{(i,j)\in\mathcal{A}'_{vn}}\sum_{v\in\mathcal{V}_{jn}}y^L_{ijbvn}-\sum_{(i,j)\in\mathcal{A}_{va(n)}}\sum_{v\in\mathcal{V}_{ja(n)}}y^L_{ijbva(n)})\le 0 \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_3 \qquad (21)$$

$$\sum_{(i,j)\in\mathcal{A}'_{vn}}x^L_{ijvn}-\sum_{(i,j)\in\mathcal{A}_{va(n)}}x^L_{ijva(n)}\le 0 \qquad i\in\mathcal{L}, v\in\mathcal{V}, n\in\mathcal{E}_3 \qquad (22)$$

$$x^L_{ijva(n)}-x^L_{ijvn}+(\sum_{(j',i)\in\mathcal{A}_{vn}}x^L_{j'ivn}-\sum_{(j',i)\in\mathcal{A}_{va(n)}}x^L_{j'iva(n)})\le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad (23)$$

$$x^L_{ijvn}-x^L_{ijva(n)}\le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad (24)$$

$$\sum_{(i,j)\in\mathcal{A}'_{vn}}y^L_{ijbvn}-\sum_{(i,j)\in\mathcal{A}_{va(n)}}y^L_{ijbva(n)}\le 0 \qquad i\in\mathcal{L}, b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3 \qquad (25)$$

$$y^L_{ijbva(n)}-y^L_{ijbvn}+(\sum_{(j',i)\in\mathcal{A}_{vn}}y^L_{j'ibvn}-\sum_{(j',i)\in\mathcal{A}_{va(n)}}y^L_{j'ibva(n)})\le 0 \qquad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3,$$
$$(i,j)\in\mathcal{A}_{vn} \qquad (26)$$

$$y^L_{ijbvn}-y^L_{ijbva(n)}\le 0 \qquad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3,$$
$$(i,j)\in\mathcal{A}_{vn} \qquad (27)$$

Constraints (20) prevent total planned demand satisfaction in stage 3 from exceeding the total actual demand satisfaction of the planned stage across all demand points. In stage 2 the vehicles are packed according to realized capacity and demand, and tentative vehicle routes are generated. As the different vehicles start their designated routes generated in stage 2 before stage 3 is initialized, we are prevented from repacking the vehicles, and are in effect unable to add additional cargo to increase total demand satisfaction in stage 3. The decision maker should however be allowed to vary the amount to be delivered at the different destinations between stage 2 and 3. This is provided for by Constraints (21), which enable the model to change the level of demand fulfillment at a demand point should it prove costly to reach or inaccessible, or provide less utility than expected. These constraints also imply that PODs can receive more than planned such that available commodities are not wasted.

The planned and realized last mile distribution decisions made in the 2nd and 3rd stage, respectively, require coordination. We thus need to enforce 3rd stage adherence to the initial plan developed and initialized in the 2nd stage, and ensure that it is followed to the extent feasible. In that regard, Constraints (22) make sure that the number of vehicles upon which the realized distribution plan of stage 3 is based, does not surpass the loaded and potentially dispatched number of vehicles of stage 2.

The network of stage 3 will at best equal that of stage 2, and thus constitutes a subset of the operable arcs of stage 2. An enforcement of the initial distribution plan is hence exclusively viable for their mutual arcs. Constraints (23) require the realized vehicle quantity traversing an arc in the final stage to equal that of stage 2, given that the flow of vehicles into the start node of that arc does not diverge between stages. If however, the number of vehicles able to reach

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the start node is less in the final stage than in stage 2, the constraints allow for a reduction in the number of vehicles traversing that arc accordingly.

The vehicles are forced over in the duplicated network should a change of route prove necessary, and Constraints (24) are included to prohibit increased vehicle flow in the original network. Flow from the original into the duplicated network thus indicates that recourse actions were required due to an obstacle in the original network. Finally, The commodity flow linking constraints (25) - (27) merge the decisions made in the 2nd and 3rd stage concerning the amount of a commodity type to traverse an arc in a specific vehicle type.

Constraints Stage t = {3}

The majority of constraints describing stage 2 also relate to stage 3, and thus the 3rd stage constraints for which this applies are modified versions of constraints also stated for stage 2. The most intuitive modification is that of the different networks they consider.

$$\sum_{v \in \mathcal{V}_{jn}} \sum_{(i,j) \in \mathcal{A}'_{vn}} y^L_{ijbvn} - \sum_{v \in \mathcal{V}_{jn}} \sum_{(j,i) \in \mathcal{A}'_{vn}} y^L_{jibvn} - \sum_{k \in \mathcal{K}} d_{jbkn} = 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_3 \quad (28)$$

$$\sum_{k \in \mathcal{K}} d_{jbkn} \leq D_{jbn} \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_3 \quad (29)$$

$$d_{jbkn} \leq U_{jbkn} \qquad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, \\ n \in \mathcal{E}_3 \quad (30)$$

$$\sum_{(i,j) \in \mathcal{A}'_{vn}} x^L_{ijvn} - F^L_{iva(n)} l_i \leq 0 \qquad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{E}_3 \quad (31)$$

$$x^L_{ijvn} - (\sum_{j' \in \mathcal{L}} F^L_{j'va(n)}) z^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \quad (32)$$

$$\sum_{b \in \mathcal{B}} Q_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \quad (33)$$

$$z^L_{ijvn} - x^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \quad (34)$$

$$\sum_{(j,i) \in \mathcal{A}'_{vn}} x^L_{jivn} - \sum_{(i,j) \in \mathcal{A}'_{vn}} x^L_{ijvn} \leq 0 \qquad j \in \mathcal{L}^{D'} \cup \mathcal{P}', n \in \mathcal{E}_3, v \in \mathcal{V}_{jn} \quad (35)$$

$$\sum_{v \in \mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+Q_L)jvn} + x^L_{(i+2Q_L+Q_P)(j+2Q_L+Q_P)vn}) \leq C^A_{ijn} \qquad n \in \mathcal{E}_3, \\ (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{L}, j \in \mathcal{N} \backslash \mathcal{I} \quad (36)$$

$$\sum_{v \in \mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+2Q_L+Q_P)(j+2Q_L+Q_P)vn}) \leq C^A_{ijn} \qquad n \in \mathcal{E}_3, \\ (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{P}, j \in \mathcal{N} \backslash \mathcal{I} \quad (37)$$

$$\sum_{(j,i) \in \mathcal{A}'_{vn}} y^L_{jibvn} - \sum_{(i,j) \in \mathcal{A}'_{vn}} y^L_{ijbvn} \leq 0 \qquad j \in \mathcal{L}^{D'} \cup \mathcal{P}', b \in \mathcal{B}, v \in \mathcal{V}, n \in \mathcal{E}_3 \quad (38)$$

Constraints (28) - (30) correspond to (9) - (11) of stage 2, while constraints (31) - (33) and (35) correspond to (12) - (15) of stage 2. Besides, constraints (34) are aimed to force the binary flow variables indicating whether or not an arc is subject to passage to zero if a vehicle type does not traverse a given arc.

Constraints (36) - (37) correspond to (16) - (17) of stage 2. However, a duplication of the distribution network enabling in transit modification of the initially planned route is introduced

to the 3rd stage. The arcs signifying connection between an LDC and the remaining nodes in the mathematical original network will in consequence be subject to a threefold duplication. The arcs connecting a POD and the remaining nodes in the original network are similarly subject to a twofold duplication. While considering this, Constraints (36) and (37) ensure arc capacity compliance for arcs having LDCs and PODs as points of departure, respectively.

Finally, constraints (38) correspond to constraints (19) of stage 2, with the addition of also applying to the LDCs and PODs constituting the mathematical duplicated network.

Efficiency Constraints Applied to All Stages

$$\sum_{i \in \mathcal{L}} E_i^O l_i + \sum_{(i,j) \in \mathcal{A}} \sum_{b \in \mathcal{B}} E_b^C y_{ijb}^I + \sum_{(i,j) \in \mathcal{A}} E_{ij}^I x_{ij}^I + \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn}} E_{ijvn}^L x_{ijvn}^L + w_n = H^B \qquad n \in \mathcal{E} \backslash \mathcal{E}_1 \qquad (39)$$

$$\sum_{(i,j) \in \mathcal{A}} T_{ij}^I z_{ij}^I + \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn}'} T_{ijvn}^L z_{ijvn}^L \leq H^T \qquad n \in \mathcal{E}_3 \qquad (40)$$

Constraints (39) ensure that the budget is honored, and calculate the residual budget. They apply to the last two stages separately, but both take 1st stage costs into account. Constraints (40) account for the time spent serving the different PODs by keeping convoy time within a given bound. However, seeing how the binary vehicle flow variables merely indicate whether or not one or more vehicles traverse an arc, the calculated time value should not be taken as the total delivery time.

Non-Negativity Constraints for All Variables

$$l_i \in \{0,1\} \qquad\qquad i \in \mathcal{L} \qquad (41)$$
$$x_{ij}^I \geq 0 \qquad\qquad \text{integer}, (i,j) \in \mathcal{A} \qquad (42)$$
$$y_{ijb}^I \geq 0 \qquad\qquad (i,j) \in \mathcal{A}, b \in \mathcal{B} \qquad (43)$$
$$z_{ij}^I \in \{0,1\} \qquad\qquad (i,j) \in \mathcal{A} \qquad (44)$$
$$d_{jbkn} \geq 0 \qquad\qquad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, n \in \mathcal{E}_2 \cup \mathcal{E}_3 \qquad (45)$$
$$x_{ijvn}^L \geq 0 \qquad\qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \qquad (46)$$
$$y_{ijbvn}^L \geq 0 \qquad\qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn}, b \in \mathcal{B} \qquad (47)$$
$$z_{ijvn}^L \in \{0,1\} \qquad\qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \qquad (48)$$
$$x_{ijvn}^L \geq 0 \qquad\qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}_{vn}' \qquad (49)$$
$$y_{ijbvn}^L \geq 0 \qquad\qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}_{vn}', b \in \mathcal{B} \qquad (50)$$
$$z_{ijvn}^L \in \{0,1\} \qquad\qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}_{vn}' \qquad (51)$$
$$w_n \geq 0 \qquad\qquad n \in \mathcal{E} \backslash \mathcal{E}_1 \qquad (52)$$

4 Computational Study

In this section, the applicability, quality and value of the proposed model will be evaluated by conducting a series of computational tests. The tests are divided in two parts. In the first part, the ability for commercial solvers to find optimal solutions to the model is tested, while varying the size of the instances. In the second part, the value of using a stochastic model is analyzed using a realistic case study.

Computational efficiency will be assessed by changing 1) the configurations of the network in terms of number of LDCs, 2) the number of PODs, 3) the number of possible outcomes in stage 2, 4) the number of possible outcomes per parent node in stage 3, and 5) the size of the budget. At least 30 instances are considered for each of the aspects subject to testing.

All tests are conducted for at least 15 different values for the parameter of interest, to enable identification of possible trends. All results relating to computational efficiency were obtained using Xpress-IVE on PCs connected to a computational cluster, equipped with 2x AMD Opteron 2431 2,4 GHz processors and 24 GB RAM.

The second part of tests makes use of a different set of test instances, based on a realistic base case study, to illustrate the value of using a stochastic model. The results for these tests were obtained using Xpress IVE on a PC equipped with Intel(R) Core(TM) i7-2600 3.40GHz processor and 16.0 GB RAM.

All instances are either stopped as run time exceeds 12 hours, or when a MIP solution within 0.10 % of the optimal solution has been found, unless otherwise stated.

4.1 Computational Efficiency

Limitations of the proposed model in relation to problem size will be discussed in this section, and questions regarding the applicability of the model as a decision support tool will be addressed. Unless otherwise stated, the results presented here will be for instances with 5 LDCs, 15 PODs, 2 vehicle types, and 9 scenarios.

The first tests consider varying the number of LDCs. Two different set of instances were tested, one with 15 PODs and another with 40 PODs. The number of LDCs in the instances is varied up to a maximum of 90. Of these tests, only for one instance (with 90 LDCs) was the solver unable to find an optimal solution within 12 hours. With less than 70 LDCs, the average solution times are less than 5,000 seconds. The computational tests also reveals that the optimal objective function value, other parameters being held constant, does not increase substantially when the number of LDCs are increased beyond 25 % of the number of PODs. This should indicate that in practice, the number of LDCs would not be critical for the model.

The second tests look at the number of PODs. Only a small number of demand points for distribution is usually considered in the literature. Vitoriano et al. (2010) treat 9 demand points (PODs), while 6 and 30 PODs are treated by Barbarosoğlu and Arda (2004) and Rawls and Turnquist (2010) respectively. The solution time of the model increases with the number of PODs, but in a relatively mild manner. With up to 250 PODs, the model can be solved to optimality within 45 minutes. This means that the model can be solved for realistic networks, in terms of the number of PODs.

In the third experiment we test both the budget size as well as the scenario tree size. By varying the number of successors individually, inspection of the complexity of the model is enabled by study of the effect of varying the number of scenario tree nodes in stage 2, or stage 3, whilst maintaining the number of scenarios. Figure 1 illustrates two possible instances of this type. The scenario tree at the left has three successor nodes per parent node in stage 2 and two in stage 3, whereas the right instance has two successor nodes per parent node in stage 2 and three in stage 3.
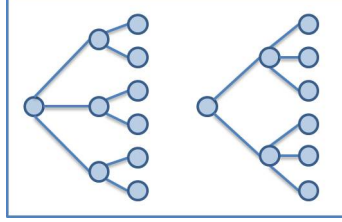

Figure 1: Illustration of two instances subject to varying numbers of successor nodes per parent node - Both consist of six scenarios

An increased number of scenarios obviously leads to longer solution times. Increasing the number of nodes per parent node in stage 2 leads to more scenario nodes overall than increasing the number of nodes per parent node in stage 3. However, computational results indicate that solution times are lower for the former, despite having more scenario nodes. A possible explanation is that even though the real size of the problem is bigger in terms of variables and constraints, the complexity and the amount of symmetric solutions may be greater for a problem with the highest number of successor nodes in stage 3. In general, the average time to find a solution that is within 1 % of optimality is less than 10 hours for up to over 300 scenarios, and the solution times behave rougly linearly in the number of scenarios.

It has also been observed that the solution times are smaller when the budget level is increased, and this may have an effect on how large scenario trees may be handled. Table 1 presents some illustrating results.

Table 1: Ability to find optimal solutions when varying the number of successor nodes per parent node in stage 2 and 3

Budget	# of nodes per parent node		# of scenarios	Solved to optimality?
	stage 2	stage 3		
100	10	3	30	yes
	20	3	60	no
	3	6	18	yes
	3	7	21	no
500	40	3	120	yes
	50	3	150	no
	3	20	60	yes
	3	30	90	no

4.2 Value of Using a Stochastic Model

This section will attempt to determine the value of using a stochastic model based on two measures: the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). These measures will be applied to a range of different test cases of interest, which serves to illustrate how the model performs under different conditions.

To calculate the EVPI one takes the difference between the wait-and-see solution (WS) and the solution obtained by solving the stochastic model referred to as the recourse problem (RP). The VSS on the other hand is calculated as the difference between RP and the expected result of using an expected value solution (EEV). While the RP and the WS are straighforward to find, the EEV needs to be defined more clearly.

WS is obtained by solving the model for each scenario separately, and then taking the expected objective function value, and RP is obtained by solving the full model directly. EEV is based on using the expected values for the uncertain parameters of the model and then solving a deterministic model. This does not work well for the parameters governing the infrastructure, as these as binary parameters, the average of which does not make sense. The expected network will therefore be represented by the initial network of stage 1. Escudero et al. (2007) describe a method for computing EEV that fits multi-stage problems which we will follow: A deterministic expected value scenario problem is solved in the first step of the procedure. The resulting optimal first stage variables are saved, and for the following stage in the scenario tree, an expected value scenario problem is solved for each scenario tree node leading up to the final stage. For each of the nodes, the random parameters of subsequent stages are estimated by their expected values, and all the variables of the preceding stages are fixed at the optimal solution values obtained in the chain leading up to the node in question. The optimal values of the variables of the current stage are still to be saved. The same procedure applies to the final stage, except that use of expected values is no longer relevant. This leads to a dynamic EEV value that is calculated based on the optimal values obtained for each scenario tree node of the final stage.

As the objective function of the proposed model sums the utility based on both stage 2 and stage 3, an alternative objective function will be used when calculating VSS and EVPI here. All EEV and RP values will be given in terms of optimal 3rd stage values exclusively, as opposed to the objective function value produced by the original model which include both 2nd stage and 3rd stage optimal variable values. This enables a comparison of these values with the WS value produced by the associated deterministic version of the model. The values given in terms of average third stage utility are denoted by WS', EEV' and RP' to indicate that they represent adjusted values compared to the objective function used in the stochastic model.

4.2.1 Introduction of the Base Case

The base case introduced in this section will serve as a platform upon which a range of test cases will be built. It will also act as a point of reference in evaluation of the valuation measures produced for the different test cases. The characteristics of the base case are inspired by real data from the earthquake which hit Haiti on January 12, 2010. Some modifications and assumptions regarding the data set are made when adequate information is lacking. The adopted data are provided via Logistics Cluster (2012).
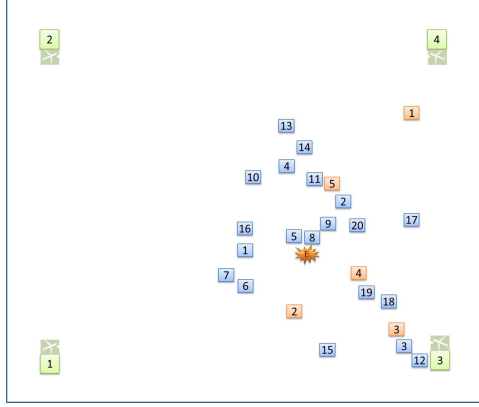
Figure 2: Configuration of the distribution network applied during validation of the model

The size of the base network replicates the distribution network utilized during the disaster in Haiti. It consists of $|\mathcal{P}| = 20$ PODs and $|\mathcal{L}| = 5$ LDCs, as depicted by red and blue boxes respectively in Figure 2, and $|\mathcal{I}| = 4$ IDCs. The reciprocal location of the nodes will also remain unaltered throughout the range of instances. The operativeness of the arcs connecting these nodes will vary between base cases in order to ensure representativeness of the valuation measures presented. They will however be identical for all test cases relating to the same base case. In addition to the parameters listed in Table 2, those omitted from Table 3 are common for all cases, irrespective of the originating base case.

Table 2: Static problem characteristics - common for all cases

# of IDCs	# of LDCs	# of PODs	# of vehicle types	Successor Nodes	
				stage 2	stage 3
4	5	20	2	3	3

Table 3: Characteristics of the base case - subject to change in the descendant test cases

Bud-get	Demand per commodity type				LDC cap						V-cap per vehicle type		
	1	2	3	Tot dem	1	2	3	4	5	Tot cap	1	2	Tot cap
0.5M	1020	2360	4380	30000	674	3014	2095	9796	9021	30000	88	45	30000

Besides, $|\mathcal{N}| = 9$ scenarios, $|\mathcal{V}| = 2$ vehicle types, $|\mathcal{B}| = 3$ commodity types have been considered, together with varying quantities of PODs. Complete demand for each of the commodity types across all demand nodes is given in Table 3, as is the capacity of the LDCs and the available distribution budget. The vehicle capacity of each vehicle type is also given, along with the

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accumulated capacity across the total number of available vehicles. Total demand, total LDC capacity and total vehicle capacity are converted into standard units, as opposed to number of items, to allow for comparison.

4.2.2 Presentation of the Test Cases

In order to evaluate the performance of the model in realistic test cases with different input parameters, the available level of LDC capacity, vehicle capacity and budget, together with the demand at the PODs, will be varied. The intention is to demonstrate efficiency in distribution when demand exceeds the level of resources needed, and also when the level of resources is sufficient relative to the level of demand.

An overview of the cases we have chosen to consider is given in Table 9. It should be noted that the levels of demand and capacity subject to change are stated in terms of *anticipated* quantities as given in the initial phase of distribution. This is due to the fact that realized levels of demand and capacity are randomly generated, thus exempt from manual manipulation. Four different versions of the base case and descendant test cases are created, and an average value to represent the valuation measures are calculated based on corresponding cases.

Table 4: Overview of the test cases

Case	Variable parameter	Value
1	Total demand	40 000
2	Total demand	1 000
3	Total LDC capacity	40 000
4	Total LDC capacity	1 000
5	Total vehicle capacity	40 000
6	Total vehicle capacity	1 000
7	Budget	1.0M
8	Budget	0.3M

4.2.3 Results and Discussion

The overall results from the test cases are given in Table 5, where EVPI' and VSS' are also given in terms of average third stage utility and in percent. The majority of instances have been solved to a gap of 0.1 %. If a solution has not been found within one hour, the current solution has been accepted. Nevertheless, for all test cases, WS'$\leq$SP'$\leq$EEV', and the average run time to optimality for the SP solution is 1132.24 sec. The run times to optimality for the EEV and WS models are negligible for the cases under consideration.

Case 1 and 2 test the effect of increasing and decreasing total demand respectively, relative to the base case. Comparing the VSS' of Case 1 and 2 with the base case show that the value of the stochastic solution decreases when there is a shortage in resources. In this case, stochasticity is not the critical restraining factor which depreciate the solution, but the fact that supply is scarce. When, on the contrary, resources are in excess, the value of the VSS' experiences a

Table 5: Resulting valuation measures in terms of average values

				EVPI'		VSS'	
Case	*WS'*	*EEV'*	*RP'*	*Value*	*%*	*Value*	*%*
Base case	802 028.0	564 314.5	743 843.8	58 184.3	8.1	179 529.3	31.4
1	770 623.0	552 018.0	712 042.8	58 580.3	8.6	160 024.8	28.5
2	217 182.5	106 212.4	201 895.5	15 287.0	7.8	95 683.1	92.6
3	802 176.8	566 538.5	743 765.5	58 411.3	8.2	177 227.0	30.8
4	371 568.0	242 460.8	339 980.0	31 588.0	9.1	97 519.3	39.7
5	1 023 402.5	653 499.5	919 238.3	104 164.3	11.7	265 738.8	40.8
6	144 814.5	118 339.2	132 709.3	12 105.2	9.6	14 370.1	12.4
7	852 053.3	619 224.0	795 126.7	56 926.5	7.4	175 902.7	27.8
8	707 662.3	445 005.3	595 680.0	111 982.3	18.7	150 674.8	33.8

substantial increase of 2 times the VSS' of the base case. This implies that evaluating the distribution policy and making decisions using a SP model is essential when resources exceed demand.

Considering the VSS' for Case 5 and 6, similar results are obtained by altering total vehicle capacity. The same reasoning as for Case 1 and 2 is applied. The adjusted value of the stochastic solution declines as vehicle capacity becomes insufficient. Consequently, toleration of the additional effort and complexity that the SP model entails is less convincingly justified. As vehicle capacity exceeds demand and total LDC capacity, VSS' increases, and the SP model gives a solution with higher levels of utility accumulated over every scenario than the expected result of using an EV solution.

Varying total LDC capacity in Case 3 and 4 indicates that further increase in capacity as compared to the base case has no effect on neither the EVPI' nor the VSS' value. Increasing LDC capacity beyond this level will not contribute to more efficient distribution, nor to increase the value of the stochastic solution. Still, the VSS' is high, and can justify the use of the SP model. However, insufficiency in total LDC capacity now yields an increase in the value of perfect information, as opposed to previous cases. Total LDC capacity is dispersed across several LDCs. Hence, selecting the LDCs which will provide the highest possible level of demand fulfillment, is of the utmost significance. When LDC capacity is restricted, the value of the stochastic solution shows an upward trend as expected value solutions are more likely to suggest suboptimal choices of LDCs to initialize.

An increase or decrease in budget (Case 7 and 8) will only have a slight impact on the VSS' and EVPI' as compared to the base case example. The budget is sufficient for supplying the demand nodes in the base case, and thus a further increase will not necessarily improve the solution in terms of an increase in the level of demand fulfillment. A reduction in budget such that it is not sufficient to supply all demand nodes increases EVPI' and VSS' as compared with when budget is in excess. The effect however, is smaller than in the previous cases, as the differences in VSS' and EVPI' are far less than in the other cases. The reason for this is that the budget has a small impact on the optimal solution as compared to the level of demand

fulfillment. This is because of the diminutive value given to the utility factor for residual budget in comparison with that of demand fulfillment, in agreement with the spirit of humanitarian logistics.

Average VSS' values of the test cases ranging from 12.4% to 92.6% indicate that the SP model yields considerable improvements and, to a greater or lesser extent, this applies to all the test cases. Furthermore, large VSS values indicate that uncertainty is of great importance to the optimal solution. An average EVPI' ranging from 7.4% to 18.7% demonstrates that obtaining higher quality forecasts of the state of the infrastructure, the level of demand and level of resources in the events of a disaster can prove somewhat beneficial for the cases under consideration.

Comparing the percentual EVPI' of the different test cases with that of the base case, the overall trend is an increase in its value in events of shortage of resources, and decline in situations of excess. We expect this effect to become even more evident as the level of excess or deficiency is further increased. There is less value of perfect information when the decision maker has adequate amounts of resources available. Excessive amounts of available transportation resources will increase the feasible area and make it easier for the SP model to produce sound solutions. Constraint (12) and (31) will in these cases be non-binding, and are in effect redundant. Because of the excessive amount of transportation resources, the MSP model potentially has a larger number of alternative solutions, and can be more certain of being able to satisfy demand. In consequence, when supply and capacity exceed demand perfect information is not a necessary prerequisite for producing a good solution. On the other hand, when demand exceeds available resources, we would be interested in investing in techniques to achieve better forecasts. However, acquiring forecasts of higher quality can more often than not prove too arduous in the event of an earthquake, causing the VSS to be of greater pertinency to the decision maker than the EVPI.

5   Concluding Remarks

This paper has presented a three-stage stochastic programming model for a disaster response problem, comprising facility location and last mile distribution decisions. The model handles uncertainty in vehicle availability, demand and infrastructure, as well as the distribution of multiple commodities and the utilization of different transportation modes. Fairness of distribution is also considered by the model by assigning different utilities to different groups of recipients, so that the most needed ones can receive a greater attention.

An extensive computational study to test the applicability, quality and value of the proposed model has been performed on a variety of realistic instances, some of which use data from the earthquake that hit Haiti on January 12, 2010. These experiments reveal that the use of a stochastic model is actually valuable, and that the proposed model can be solved for realistically sized test instances.

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