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A Threshold-Parameter-Dependent Approach to Designing Distributed Event-Triggered H_∞ Consensus Filters Over Sensor Networks

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***Abstract*—This paper is concerned with distributed event-triggered H_∞ consensus filtering for a discrete-time linear system over a sensor network. Different from some existing event-triggered communication schemes (ETCSs), a new distributed ETCS is first developed to reduce the communication frequency of neighboring sensors, where the threshold parameter in an event triggering condition is time-varying with attainable upper and lower bounds. Then a threshold-parameter-dependent approach is proposed to derive criteria for designing the desired H_∞ consensus filters and the ETCS such that the resultant filtering error system is asymptotically stable with the prescribed H_∞ performance while maintaining satisfactory resource efficiency. Furthermore, a polytope-like transformation with regard to time-varying threshold parameters is performed and a recursive algorithm is presented to determine the threshold-parameter-dependent filter matrix sequences and event triggering weighting matrix sequence. Two illustrative examples are employed to show the effectiveness of the developed approach.**

***Index Terms*—Distributed H_∞ filtering, event-triggered communication scheme, sensor network, threshold-parameter-dependent approach.**

I. Introduction

RECENT developments in sensing, computing and wireless communication technologies have led to numerous applications over sensor networks, such as battlefield surveillance, target tracking, cooperative detection of toxic chemicals in contaminated environments, search and rescue operations after disasters, forest fire monitoring [1]. In a sensor network, a basic idea is to employ a large number of spatially distributed sensor nodes to cooperatively monitor physical/environmental conditions or collaboratively gather scientific data from moving targets through wireless channels. In practice, noise inevitably occurs when the signal of interest is sensed and/or communicated by sensors through wireless channels, which often causes the degradation of the system performance. Thus, it is desired to settle the problem of how to develop a distributed algorithm to estimate an unavailable state signal through the noisy measurement and/or a disturbed plant [2]–[5]. Some existing results along this line of research can be arguably classified into three categories: 1) distributed Kalman filtering [6]–[8]; 2) distributed H_∞ filtering [4], [5], [9]–[15]; and 3) distributed set-membership estimation [16]–[19].

Note that the majority of the distributed filtering or estimation algorithms aforementioned require consecutive measurement transmissions or periodical updates of the signal of interest on each sensor node. This may lead to the inefficient use of limited communication resources, such as sensor power and bandwidth in sensor networks [12], [20], [21]. Therefore, from a resource conservation perspective, it is wasteful for each sensor node to communicate with its neighboring nodes at every instant of sampling time especially when there is a little fluctuation between two consecutive sampled-data.

In order to reduce the frequency of occupying scarce communication resources, event-triggered communication schemes (ETCSs) have been emerging to mitigate the unnecessary use of resources while preserving certain system performance. We refer to survey papers [22]–[25] regarding some recent advances in event-triggered control and estimation. Under ETCSs, an objective control or estimation task is only executed after the occurrence of a specified "event" [26]–[32]. For example, the event is usually generated by an event processer by checking whether or not the following triggering condition is satisfied: $\|e\|^2 - \sigma > 0$ or $\|e\|^2 - \delta\|x\|^2 > 0$, where e denotes the error between the current sampled-data and the latest transmitted data; $\sigma > 0$ is a predesigned threshold; x represents either the current sampled-data or the latest transmitted data; and $\delta > 0$ stands for a constant threshold parameter. Once the event triggering condition above holds, the current sampled-data will be broadcast and transmitted to a remote controller or an estimator. In this sense, communication resources are occupied only when "necessary" and the number of transmitted task executions may be reduced significantly.

It should be pointed out that compared with many results on event-triggered control in the literature, there are some results available on distributed event-triggered filtering or estimation over sensor networks. For example, in [12], a co-design problem of distributed H_∞ filters and ETCSs was addressed

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for continuous-time linear systems over sensor networks subject to communication delays. In [16]–[19], ETCSs were implemented to tackle the distributed set-membership filtering problem for discrete-time systems subject to unknown-but-bounded noises. In [33], a distributed event-based H_∞ state estimation problem was considered for discrete-time stochastic nonlinear systems in lossy sensor networks. By developing an ETCS, a recursive distributed filtering approach was developed in [34] for a class of discrete time-varying linear systems such that the filtering error covariance was minimized locally.

It is noteworthy that the distributed event-triggered filtering or estimation algorithms aforementioned are based on an explicit assumption of "static" event triggering conditions, i.e., a threshold or a threshold parameter therein is fixed *a priori* during the entire implementation of an ETCS. Recently, an adaptive algorithm for determining a threshold parameter was developed in [35] to tackle distributed event-based filtering for continuous-time nonlinear stochastic systems over wireless sensor networks. However, one limitation of this algorithm is that the time-varying nature of the threshold parameter is not considered during the performance analysis and filter design. In practice, motivations of considering a time-varying threshold parameter in an ETCS may arise from the following three aspects, either explicitly or implicitly. *First*, it is well recognized that a threshold parameter determines how frequently data transmission occurs or how many data packets are transmitted [26]. For example, lowering the threshold parameter causes a small interevent time (the time interval between two consecutive events), thus leading to more data packets being transmitted through communication networks [12], [36]–[38]. Therefore, the threshold parameter is closely related to the data transmission rate through a communication channel. However, in practical communication networks, such as IEEE 802.11 wireless local area network, a transmission rate may vary over time due to time-varying interference and random wireless fading. Specifically, an IEEE 802.11b standard can provide different raw transmission rates at 1, 2, 5.5, and 11 Mb/s. In this situation, the threshold parameter should essentially vary with time to reflect such an engineering practice of time-varying wireless transmission channels [26]. *Second*, event processers are usually embedded in mobile devices which in most cases are powered by small battery. It is possible that the threshold parameter varies from time to time due to power allocation decisions, limited chipset's processing capacity and parameter variations. *Third*, it has been shown in the literature that, under ETCSs, various system performance can be achieved by choosing different threshold parameters, such as selecting a larger threshold parameter can lead to a deteriorative H_∞ filtering performance index [12], [36]. Hence, threshold parameter variations have an impact on the system performance, and it is essential to incorporate the time-varying nature of a threshold parameter into a distributed event-triggered filtering framework. To the best of the authors' knowledge, there have been few results investigating the effect of a time-varying threshold parameter in the context of distributed event-triggered H_∞ filtering, which motivates this paper.

In this paper, we will consider the problem of distributed event-triggered H_∞ consensus filtering for a discrete-time linear-invariant system over a wireless sensor network. Sensor nodes are spatially distributed and intercommunicated through a wireless network medium. We summarize the main contributions as follows.

1) A new distributed ETCS will be developed to determine when each sensor's communication actions with its neighboring sensors should be executed so as to alleviate the continual occupancy of communication resources. Different from some existing ETCSs, the threshold parameter in the event triggering condition is time-varying with accessible bound information. In this sense, each sensor's inter-event time is dynamically adjusted in accordance with both the time-varying threshold parameter and its current state estimation information.
2) Delicate distributed event-triggered H_∞ consensus filters will be constructed, where filters' gain matrices are dependent on the time-varying threshold parameter. Each sensor implements a threshold-parameter-dependent consensus filter by following two steps. First, each sensor receives an actual measurement output from the system and takes it as an input of the filter. Second, each sensor computes a state estimation based on consensus strategies and locally triggered state estimations of neighboring sensors, and uses it to form an output of the filter.
3) A specific threshold-parameter-dependent filter design approach will be proposed to derive criteria on the existence of the desired filters and ETCS. By constructing a threshold-parameter-dependent Lyapunov functional, sufficient conditions are derived to guarantee the asymptotic stability of the resultant filtering error system while preserving the prescribed H_∞ performance and reducing the frequency of sensors' communication.
4) A recursive optimization algorithm will be put forward such that the threshold-parameter-dependent filter gain matrix sequences and event triggering weighting matrix sequence can be efficiently solved out. Due to the time-varying characteristics of the threshold parameter, a polytope-like transformation is first performed to convert the infinite matrix inequalities into finite and numerically tractable linear matrix inequalities (LMIs). Then, the desired filter gain matrix sequences and weighting matrix sequences are obtained by solving a recursive convex optimization algorithm in terms of recursive LMIs.

Notation: $\text{diag}_{\mathcal{N}}\{Z\}$ denotes a block-diagonal matrix with $\mathcal{N}$ blocks $Z, \ldots, Z$ and $\text{diag}^i_{\mathcal{N}}\{Z_i\}$ denotes a block-diagonal matrix with $\mathcal{N}$ blocks $Z_1, \ldots, Z_{\mathcal{N}}$. $\text{col}_{\mathcal{N}}\{Z\}$ stands for an $\mathcal{N}$-block column vector $[Z^T \ \cdots \ Z^T]^T$ and $\text{col}^i_{\mathcal{N}}\{Z_i\}$ denotes an $\mathcal{N}$-block column vector $[Z_1^T \ \ldots \ Z_{\mathcal{N}}^T]^T$. The space of square-summable vector functions over $[0, \infty)$ is denoted as $l_2[0, \infty)$ and for any $w(k) \in l_2[0, \infty)$, its norm is given by $\|w(k)\|_2 = \sqrt{\sum_{k=0}^{\infty} w^T(k)w(k)}$. $\otimes$ represents the Kronecker product for matrices. Other notations are standard.

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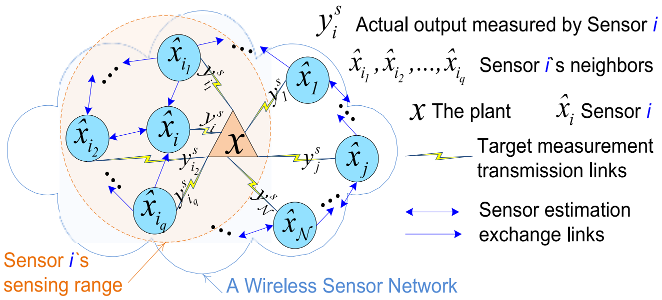

Fig. 1. Schematic of distributed filtering over a wireless sensor network.

II. Problem Formulation

A. Plant

Consider the plant described by a discrete-time linear-invariant system of the following form:

$$\begin{cases} x(k+1) = Ax(k) + Bw(k), \;\; x(0) = x_0 \\ z(k) = Ex(k) + Fw(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector; $w(k) \in \mathbb{R}^{n_w}$ is the exogenous disturbance input and belongs to $l_2[0, \infty)$; $z(k) \in \mathbb{R}^{n_z}$ is the objective output signal to be estimated; x_0 is the initial state of the plant; and A, B, E, and F are known constant matrices with appropriate dimensions.

B. Sensor Network Topology

$\mathcal{N}$ cooperative sensors spatially distributed over a wireless communication network are deployed to monitor the plant and estimate the state of the plant, which is illustrated in Fig. 1. It is natural to represent the sensors' network topology, which models information exchange among sensors, by means of directed graphs, see [2], [5], [9], [12], [13], [15], [20] and references therein.

Let $\mathcal{V} = \{1, 2, \ldots, \mathcal{N}\}$ denote an index set of $\mathcal{N}$ sensor nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denote the edge set of paired sensor nodes and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{\mathcal{N} \times \mathcal{N}}$ denote the weighted adjacency matrix. The directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is used to model the network topology of interacting sensors. An edge of $\mathcal{G}$ is denoted by (i, j). The adjacency element $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$ represents a positive weighting of the edge between two adjacent sensors, which implies that sensor i receives information from sensor j or sensor j transmits information to sensor i, otherwise, $a_{ij} = 0$ if no information link exists from sensor j to sensor i. Self-loops are excluded in the graph, i.e., $a_{ii} = 0$, $i \in \mathcal{V}$. The Laplacian matrix of the digraph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{W} - \mathcal{A}$, where $\mathcal{W} = \mathrm{diag}^i_{\mathcal{N}}\{w_i\}$ with the diagonal element $w_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. Node j is considered as a neighbor of node i if $(i, j) \in \mathcal{E}$. The set of neighbors of node i excluding the node itself is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

C. Sensor Measurement Output Model

In practice, sensors are often deployed either inside a monitoring region of the plant or very close to the plant. Furthermore, sensors are equipped with on-board sensing units, which renders that sensors can observe the plant and sense the plant's state signal with noisy measurement [7]. Thus, it is assumed that sensor i, $i \in \mathcal{V}$, takes the actual measurement in the form of

$$y_i^s(k) = C_i x(k) + D_i v_i(k), \;\; \forall\, i \in \mathcal{V} \tag{2}$$

where $y_i^s(k) \in \mathbb{R}^{n_y}$ is the measurement output vector received by sensor node i from the plant; $v_i(k) \in \mathbb{R}^{n_v}$ belonging to $l_2[0, \infty)$ is the measurement noise through measurement transmission link i; C_i and D_i, $i \in \mathcal{V}$, are known constant matrices with appropriate dimensions. As can be seen from (2), the measurement $y_i^s(k)$ is assumed to be taken at every instant of time by sensor i. Although in some cases, this assumption may be unfavorable as it may result in the inefficient use of resources on sensor measuring, we consider in this paper that the main cause for resource consumption comes from the data transmission and receiving between sensors. This enables us to focus exclusively on reducing the frequency of communication among sensors so as to achieve better resource efficiency.

D. New Distributed Event-Triggered Communication Scheme

The purpose of this section is to develop a new distributed ETCS to determine how frequently each sensor's data should be broadcast and transmitted over the wireless network.

A conceptual framework of the distributed ETCS on sensor node i is shown in Fig. 2. An event processor (EP) which consists of an *event generator* and a *buffer* is embedded within each sensor node to determine when each sensor's current information of a state estimation should be broadcast to its neighboring nodes for state update. Denote by $\{t_k^i |\; t_k^i \in \mathbb{N}\}$ the broadcasting time sequence and $\{s_k^i |\; s_k^i = t_k^i + m_i, m_i = 0, 1, \ldots, M_i, M_i = t_{k+1}^i - t_k^i - 1\}$ the current time sequence between two consecutive broadcasting instants. At each time $k \in \mathbb{N}$, sensor i's computes its state update $\hat{x}_i(k)$ as an estimation of the state $x(k)$ of plant. This state estimation $\hat{x}_i(k)$ and its time-stamp k are encapsulated into a data packet $(k, \hat{x}_i(k))$. We assume that t_k^i also denotes the time when sensor node i successfully transmits its state estimation $\hat{x}_i(t_k^i)$ to its neighboring nodes. Therefore, all transmitted packets $(t_k^i, \hat{x}_i(t_k^i))$ are time-stamped. The broadcasting and transmitting time instants are determined recursively by an event generator according to the following scheme:

$$t_{k+1}^i = t_k^i + \inf_{m_i \geq 0} \left\{ m_i + 1 \mid f\left(s_k^i\right) > 0 \right\} \tag{3}$$

where

$$\begin{aligned} f\left(s_k^i\right) &= \mathcal{X}_i^T\left(s_k^i\right) \Phi_i\left(\delta_i\left(s_k^i\right)\right) \mathcal{X}_i\left(s_k^i\right) \\ &\quad - \delta_i\left(s_k^i\right) \mathcal{Y}_i^T\left(s_k^i\right) \Phi_i\left(\delta_i\left(s_k^i\right)\right) \mathcal{Y}_i\left(s_k^i\right) \\ \mathcal{X}_i\left(s_k^i\right) &= \hat{x}_i\left(s_k^i\right) - \hat{x}_i\left(t_k^i\right) \\ \mathcal{Y}_i\left(s_k^i\right) &= \sum_{j \in \mathcal{N}_i} a_{ij} \left(\hat{x}_i\left(t_k^i\right) - \hat{x}_j\left(t_{\tilde{k}_j}^j\right) \right) \end{aligned}$$

with $\tilde{k}_j \triangleq \arg\min_{\tilde{k}} \{s_k^i - t_{\tilde{k}}^j \mid s_k^i > t_{\tilde{k}}^j, \tilde{k} \in \mathbb{N}\}$. For each index i, $\delta_i(k)$ is a time-varying threshold parameter satisfying

$$0 \leq \bar{\delta}_{1,i} \leq \delta_i(k) \leq \bar{\delta}_{2,i} < 1, \;\; \forall\, i \in \mathcal{V} \tag{4}$$

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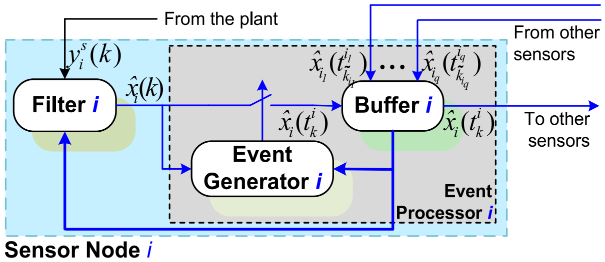

Fig. 2. Conceptual framework of distributed event-triggered communication on sensor i, $i \in \mathcal{V}$.

with constants $\bar{\delta}_{1,i}$ and $\bar{\delta}_{2,i}$ denoting the lower bound and the upper bound of $\delta_i(k)$, respectively. Under the communication scheme (3), once the current sampled-data $\hat{x}_i(s^i_k)$ satisfies the event triggering condition $f(s^i_k) > 0$, EP i immediately broadcasts sensor i's current state estimation to its underlying neighbors. It is assumed that the time-varying threshold parameter can be estimated or measured via statistical tests in real time. $\Phi_i(\delta_i(k))$ denotes a symmetric positive definite weighting matrix sequence that depends on the time-varying threshold parameter. The buffer in EP i is assumed to be event-driven and has the logical capability of checking the time stamps of the newly arrived data packets and choosing the newest one to actuate filter i, as shown in Fig. 2. Even though transmitted data packets from neighboring nodes may arrive with a different temporal order, buffer i is configured to keep the arrived packet only when its time stamp is greater than that of currently stored packet. In other words, buffer i will reserve the latest data packets and keep sensor i's state update unchanged until new data packets are arrived. For simplicity, it is assumed that data transmission over the network is performed in a single packet manner.

Remark 1: Different from some existing distributed/decentralized ETCSs [12], [17], [19], [29], [33]–[35], [39]–[41], the distributed ETCS (3) is dependent on the time-varying threshold parameter $\delta_i(k)$. Generally, changing the value of the threshold parameter leads to a variation of the average inter-event time. Consequently, the above scheme is capable to dynamically adjust the released event time interval according to both the current sampled-data and the time-varying threshold parameter. In (4), we consider that $\delta_i(k)$ takes values on the interval $[\bar{\delta}_{1,i}, \bar{\delta}_{2,i}]$ mainly based on the three aspects mentioned in the introduction of this paper.

E. Distributed Event-Triggered H_∞ Consensus Filters

To estimate the state of the plant, sensor i is assumed to run a threshold-parameter-dependent consensus filter of the form

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + K_i(\delta_i(k))\big(y^s_i(k) - C_i\hat{x}_i(k)\big) \\ \qquad\qquad + G_i(\delta_i(k)) \sum\limits_{j\in\mathcal{N}_i} a_{ij}\left(\hat{x}_i\left(t^i_k\right) - \hat{x}_j\left(t^j_{\tilde{k}_j(k)}\right)\right) \\ \hat{z}_i(k) \qquad = E\hat{x}_i(k) \end{cases} \tag{5}$$

where $k \in \Upsilon \triangleq \{t^i_k, t^i_k + 1, \ldots, t^i_{k+1} - 1\}$ with Υ representing the holding time sequence of filter i's state update; $\tilde{k}_j(k) \triangleq \arg\min_{\tilde{k}}\{k - t^j_{\tilde{k}} \mid k > t^j_{\tilde{k}}, \tilde{k} \in \mathbb{N}\}$; $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the state estimation computed by sensor node i; $\hat{z}_i(k) \in \mathbb{R}^{n_z}$ is the output of sensor node i and represents an estimation of the objective output signal $z(k)$; the initial condition of filter i is given by $\hat{x}^i_0$; $G_i(\delta_i(k))$ and $K_i(\delta_i(k))$, $\forall\ i \in \mathcal{V}$, are the filter gain matrix sequences to be determined.

Remark 2: From Fig. 1, one can see that state estimations of neighboring nodes $i_1, i_2, \ldots, i_q \in \mathcal{N}_i$ are exchanged with sensor node i to provide some freedom and flexibility in choosing information sent through the network. Furthermore, information exchanged among neighboring nodes plays an important role in collaborative information processing over wireless sensor networks. Similar to [6], [13], and [42], the consensus-based distributed filters are constructed in (5), where filter i's dynamics consists of two parts: one is derived from a local Luenberger-like observer weighted with the matrix sequence $K_i(\delta_i(k))$; and the other arises from consensus and takes into account state estimation information collected from sensor node i's all underlying neighbors weighted with the matrix sequence $G_i(\delta_i(k))$. However, the proposed filters (5) are inherently distinct with the ones in [6], [13], and [42] from the following three aspects.

1) The latter rely on a periodical or continuous communication scheme while the proposed filters are based on an event-triggered communication paradigm.
2) The filters in [6] and [13] are constructed only for continuous-time systems and the filter proposed in [42] does not take the effects of external disturbance and measurement noise into account.
3) The estimator gain matrices in [6], [13], and [42] are fixed permanently while the filter gain matrices in (5) are time-varying and dependent on the threshold parameter $\delta_i(k)$.

F. Filtering Error Dynamics

For sensor node i, $\forall\ i \in \mathcal{V}$, define a state estimation error vector $e_i(k) = x(k) - \hat{x}_i(k)$, an output estimation error vector $\tilde{z}_i(k) = z(k) - \hat{z}_i(k)$ and a state update error vector $\hat{h}_i(k) = \hat{x}_i(k) - \hat{x}_i(t^i_k)$, $k \in \Upsilon$. Combining (1), (2), and (5), we have the following the filtering error system on node i:

$$\begin{cases} e_i(k+1) = (A - K_i(\delta_i(k))C_i)e_i(k) \\ \qquad\qquad + G_i(\delta_i(k)) \sum\limits_{j\in\mathcal{N}_i} a_{ij}\left(\hat{h}_i(k) - \hat{h}_j(k)\right) \\ \qquad\qquad + G_i(\delta_i(k)) \sum\limits_{j\in\mathcal{N}_i} a_{ij}\big(e_i(k) - e_j(k)\big) \\ \qquad\qquad - K_i(\delta_i(k))D_i v_i(k) + Bw(k) \\ \tilde{z}_i(k) = Ee_i(k) + Fw(k). \end{cases} \tag{6}$$

For notational brevity, we denote $e(k) = \mathrm{col}^i_{\mathcal{N}}\{e_i(k)\}$, $h(k) = \mathrm{col}^i_{\mathcal{N}}\{\hat{h}_i(k)\}$, $\tilde{z}(k) = \mathrm{col}^i_{\mathcal{N}}\{\tilde{z}_i(k)\}$, $v(k) = \mathrm{col}^i_{\mathcal{N}}\{v_i(k)\}$, $\bar{A} = \mathrm{diag}_{\mathcal{N}}\{A\}$, $\bar{B} = \mathrm{col}_{\mathcal{N}}\{B\}$, $\bar{C} = \mathrm{diag}^i_{\mathcal{N}}\{C_i\}$, $\bar{D} = \mathrm{diag}^i_{\mathcal{N}}\{D_i\}$, $\bar{E} = \mathrm{diag}_{\mathcal{N}}\{E\}$, $\bar{F} = \mathrm{col}_{\mathcal{N}}\{F\}$, $\Xi(k) = \mathrm{diag}^i_{\mathcal{N}}\{\delta_i(k)\}$, $\bar{\Phi}(\Xi(k)) = \mathrm{diag}^i_{\mathcal{N}}\{\Phi_i(\delta_i(k))\}$, $\bar{G}(\Xi(k)) = \mathrm{diag}^i_{\mathcal{N}}\{G_i(\delta_i(k))\}$, and $\bar{K}(\Xi(k)) = \mathrm{diag}^i_{\mathcal{N}}\{K_i(\delta_i(k))\}$.

From (6), the filtering error system can be rewritten in a compact form

$$\begin{cases} e(k+1) = \tilde{A}e(k) + \bar{B}w(k) + \tilde{C}h(k) + \tilde{D}v(k) \\ \tilde{z}(k) = \bar{E}e(k) + \bar{F}w(k) \end{cases} \tag{7}$$

where $\tilde{A} = \bar{A} - \bar{K}(\Xi(k))\bar{C} + \bar{G}(\Xi(k))\tilde{\mathcal{L}}$, $\tilde{C} = \bar{G}(\Xi(k))\tilde{\mathcal{L}}$, $\tilde{D} = -\bar{K}(\Xi(k))\bar{D}$, and $\tilde{\mathcal{L}} = \mathcal{L} \otimes I$.

G. Problem to Be Addressed

For a prescribed level of disturbance attenuation performance $\gamma > 0$ and a measurable threshold parameter $\delta_i(k) \in [\bar{\delta}_{1,i}, \bar{\delta}_{2,i}] \subseteq [0, 1)$, the objective of the distributed event-triggered H_∞ consensus filtering problem is to design desired distributed event-triggered H_∞ consensus filters in the form of (5) and event triggering weighting matrix sequences $\Phi_i(\delta_i(k))$ for all $i \in \mathcal{V}$, such that:

1) The resultant estimation error system (7) with $w(k) = 0$ and $v_i(k) = 0$ is asymptotically stable; and
2) The following H_∞ performance index:

$$\frac{1}{\mathcal{N}}\sum_{i=1}^{\mathcal{N}}\sum_{k=0}^{\infty}\tilde{z}_i^T(k)\tilde{z}_i(k) < \gamma^2\sum_{k=0}^{\infty}w^T(k)w(k) + \gamma^2\frac{1}{\mathcal{N}}\sum_{i=1}^{\mathcal{N}}\sum_{k=0}^{\infty}v_i^T(k)v_i(k) \tag{8}$$

is satisfied under nonzero $w(k), v_i(k) \in l_2[0, \infty)$ and zero initial condition.

III. Criteria for Designing Desired Distributed Event-Triggered H_∞ Consensus Filters

In this section, criteria on the existence of desired distributed event-triggered H_∞ consensus filters of the form (5) and distributed ETCS in the form of (3) are derived to guarantee the asymptotic stability of the resultant filtering error system (7) with a prescribed H_∞ performance index while reducing the communication frequency of interacting sensors.

A. Criteria in the Case of Time-Varying Threshold Parameters

Choose a new *threshold-parameter-dependent* Lyapunov functional candidate as

$$V(k) = e^T(k)\bar{P}(\Xi(k))e(k) \tag{9}$$

where $\bar{P}(\Xi(k)) = \text{diag}_{\mathcal{N}}^i\{P_i(\delta_i(k))\}$, which depends on the threshold parameter $\delta_i(k)$, $i \in \mathcal{V}$, is a time-varying matrix sequence to be determined. We now state and establish the following result.

Theorem 1: Given scalars $\gamma > 0$, $\alpha > 0$, and a measurable threshold parameter $\delta_i(k) \in [\bar{\delta}_{1,i}, \bar{\delta}_{2,i}] \subseteq [0, 1)$, $i \in \mathcal{V}$, the proposed distributed event-triggered H_∞ consensus filtering problem is solvable by the distributed filters (5), if there exist a constant diagonal matrix R, diagonal matrix sequences $\bar{P}(\Xi(k)) > 0$, $\bar{\Phi}(\Xi(k)) > 0$, $\tilde{G}(\Xi(k))$ and $\tilde{K}(\Xi(k))$ of appropriate dimensions such that

$$\Gamma < 0 \tag{10}$$

where $\Gamma = [\Gamma^{(pq)}]_{6\times 6}$ is a symmetric block matrix with its nonzero entries given by $\Gamma^{(11)} = -\bar{P}(\Xi(k)) + \Theta(\Xi(k))$, $\Gamma^{(12)} = \Theta(\Xi(k))$, $\Gamma^{(22)} = \Theta(\Xi(k)) - \bar{\Phi}(\Xi(k))$, $\Gamma^{(15)} = \bar{E}^T$, $\Gamma^{(35)} = \bar{F}^T$, $\Gamma^{(16)} = \bar{A}^T R^T - \bar{C}^T\tilde{K}^T(\Xi(k)) + \tilde{\mathcal{L}}^T\tilde{G}^T(\Xi(k))$, $\Gamma^{(26)} = \tilde{\mathcal{L}}^T\tilde{G}^T(\Xi(k))$, $\Gamma^{(36)} = \bar{B}^T R^T$, $\Gamma^{(46)} = -\bar{D}^T\tilde{K}^T(\Xi(k))$, $\Gamma^{(33)} = -\gamma^2 I$, $\Gamma^{(44)} = -(\gamma^2/\mathcal{N})I$, $\Gamma^{(55)} = -\mathcal{N}I$, $\Gamma^{(66)} = \alpha^2\bar{P}(\Xi(k+1)) - \text{sym}(\alpha R)$, and $\Theta(\Xi(k)) = \tilde{\mathcal{L}}^T(\Xi(k) \otimes I)\bar{\Phi}(\Xi(k))\tilde{\mathcal{L}}$. Furthermore, the filter gain matrix sequences are given by

$$\bar{G}(\Xi(k)) = R^{-1}\tilde{G}(\Xi(k)),\ \bar{K}(\Xi(k)) = R^{-1}\tilde{K}(\Xi(k)). \tag{11}$$

Proof: See the Appendix. ■

Remark 3: From Theorem 1, it is clear to see that in order to ensure $\Gamma < 0$, one has to guarantee $\Gamma^{(22)} = \tilde{\mathcal{L}}^T(\Xi(k) \otimes I)\bar{\Phi}(\Xi(k))\tilde{\mathcal{L}} - \bar{\Phi}(\Xi(k)) < 0$. On the other hand, we have

$$\begin{aligned}&\tilde{\mathcal{L}}^T(\Xi(k)\otimes I)\bar{\Phi}(\Xi(k))\tilde{\mathcal{L}} - \bar{\Phi}(\Xi(k))\\ &\quad\le \lambda_{\max}^2(\Xi(k)\otimes I)\bar{\Phi}(\Xi(k)) - \bar{\Phi}(\Xi(k))\\ &\quad= \left(\left(\lambda_{\max}^2\Xi(k) - I\right)\otimes I\right)\bar{\Phi}(\Xi(k))\end{aligned}$$

where $\lambda_{\max}$ is the largest eigenvalue of the Laplacian matrix $\mathcal{L}$ of the graph $\mathcal{G}$. Since $\bar{\Phi}(\Xi(k)) > 0$, it can be concluded that if

$$0 \le \bar{\delta}_{1,i} \le \delta_i(k) \le \bar{\delta}_{2,i} < \frac{1}{\lambda_{\max}^2} \tag{12}$$

then $\Gamma^{(22)} < 0$ holds. As a consequence, the above constraint on the time-varying threshold parameter establishes the relationship between the upper bound of the time-varying threshold parameter and the largest eigenvalue of the Laplacian matrix. However, the Laplacian matrix relies on the entire sensor network topology. In other words, Theorem 1 identifies a way to build the relationship between how to bound the threshold parameter and how to select a suitable network topology.

Note that the number of matrix inequalities in Theorem 1 tends to be *infinite* due to the time-varying parameters $\Xi(k)$ and $\Xi(k+1)$. Therefore, it is difficult to directly solve the inequalities to obtain the filter gain parameters. In the following, we will convert the matrix inequalities (10) into *finite* LMIs by resorting to a *polytope-like transformation*.

Choose the matrix sequences $P_i(\delta_i(k+1))$ and $\nabla_i(\delta_i(k))$ of the following structure:

$$\begin{aligned}P_i(\delta_i(k+1)) &= P_{i,0} + \delta_i(k+1)P_i\\ \nabla_i(\delta_i(k)) &= \nabla_{i,0} + \delta_i(k)\nabla_i\end{aligned} \tag{13}$$

where the symbol ∇ denotes P, Φ, G, and K, respectively. Consequently, it can be easily observed from (5) that the proposed distributed event-triggered filters comprise two kinds of estimator gain parameters: 1) the constant (or fixed) parameters $G_{i,0}$, $K_{i,0}$, G_i, and K_i and 2) the time-varying parameter $\delta_i(k)$. In this sense, the distributed event-triggered filters can dynamically schedule their gain parameters according to the measured threshold parameter $\delta_i(k)$.

We define some new variables

$$\beta_{1,i}(k) = \frac{\bar{\delta}_{2,i} - \delta_i(k+1)}{\bar{\delta}_{2,i} - \bar{\delta}_{1,i}},\ \beta_{2,i}(k) = \frac{\delta_i(k+1) - \bar{\delta}_{1,i}}{\bar{\delta}_{2,i} - \bar{\delta}_{1,i}}$$

$$\alpha_{1,i}(k) = \frac{\bar{\delta}_{2,i} - \delta_i(k)}{\bar{\delta}_{2,i} - \bar{\delta}_{1,i}},\ \alpha_{2,i}(k) = \frac{\delta_i(k) - \bar{\delta}_{1,i}}{\bar{\delta}_{2,i} - \bar{\delta}_{1,i}}.$$

It can be readily derived that

$$\delta_i(k+1) = \sum_{l=1}^{2}\beta_{l,i}(k)\bar{\delta}_{l,i},\ \delta_i(k) = \sum_{m=1}^{2}\alpha_{m,i}(k)\bar{\delta}_{m,i}. \tag{14}$$

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Then from (13) we have that the matrix sequences $P_i(\delta_i(k+1))$ and $\nabla_i(\delta_i(k))$ belong to the following convex compact sets or polytopes:

$$\check{P}_i \triangleq \left\{P_i : P_i(\delta_i^+) = \sum_{l=1}^{2}\beta_{l,i}(k)(P_{i,0} + \bar{\delta}_{l,i}P_i),\ \beta_i \in \Omega_i^\beta\right\}$$

$$\Omega_i^\beta \triangleq \left\{\beta_i : \beta_{l,i}(k) \geq 0,\ l = 1, 2;\ \sum_{l=1}^{2}\beta_{l,i}(k) = 1\right\}$$

$$\check{\nabla}_i \triangleq \left\{\nabla_i : \nabla_i(\delta_i) = \sum_{m=1}^{2}\alpha_{m,i}(k)\big(\nabla_{i,0} + \bar{\delta}_{m,i}\nabla_i\big),\ \alpha_i \in \Omega_i^\alpha\right\}$$

$$\Omega_i^\alpha \triangleq \left\{\alpha_i : \alpha_{m,i}(k) \geq 0,\ m = 1, 2;\ \sum_{m=1}^{2}\alpha_{m,i}(k) = 1\right\}$$

where $P_{i,0} + \bar{\delta}_{l,i}P_i$, $l = 1, 2$, are two vertices of the polytope $\check{P}_i$ and $\beta_i = \mathrm{col}_2\{\beta_{1,i}(k), \beta_{2,i}(k)\}$ is the polytope coordinate; $\nabla_{i,0} + \bar{\delta}_{m,i}\nabla_i$, $m = 1, 2$, are two vertices of the polytope $\check{\nabla}_i$ and $\alpha_i = \mathrm{col}_2\{\alpha_{1,i}(k), \alpha_{2,i}(k)\}$ is the polytope coordinate; the symbol ∇ denotes P, Φ, G, and K, respectively.

In the sequel, $\forall\ m, l = 1, 2$, we denote $\bar{\Xi}_m = \mathrm{diag}^i_{\mathscr{N}}\{\bar{\delta}_{m,i}\}$, $\Lambda_m(k) = \mathrm{diag}^i_{\mathscr{N}}\{\alpha_{m,i}(k)\}$, $\Omega_l(k) = \mathrm{diag}^i_{\mathscr{N}}\{\beta_{l,i}(k)\}$, $\bar{P}_0 = \mathrm{diag}^i_{\mathscr{N}}\{P_{0,i}\}$, $\bar{P} = \mathrm{diag}^i_{\mathscr{N}}\{P_i\}$, $\bar{\Phi}_0 = \mathrm{diag}^i_{\mathscr{N}}\{\Phi_{0,i}\}$, $\bar{\Phi} = \mathrm{diag}^i_{\mathscr{N}}\{\Phi_i\}$, $\bar{G}_0 = \mathrm{diag}^i_{\mathscr{N}}\{G_{0,i}\}$, $\bar{G} = \mathrm{diag}^i_{\mathscr{N}}\{G_i\}$, $\bar{K}_0 = \mathrm{diag}^i_{\mathscr{N}}\{K_{0,i}\}$, $\bar{K} = \mathrm{diag}^i_{\mathscr{N}}\{K_i\}$, $\check{P}_m = \bar{P}_0 + \bar{\Xi}_m\bar{P}$, $\check{\Phi}_m = \bar{\Phi}_0 + \bar{\Xi}_m\bar{\Phi}$, $\check{G}_m = \tilde{G}_0 + \bar{\Xi}_m\tilde{G}$, and $\check{K}_m = \tilde{K}_0 + \bar{\Xi}_m\tilde{K}$.

Applying the polytope-like transformation formulated above, it follows from (10) that:

$$\Gamma = \sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{l=1}^{2}\Lambda_m(k)\Lambda_n(k)\Omega_l(k)\Gamma_{m,n,l} < 0 \tag{15}$$

where $\Gamma_{m,n,l} = [\Gamma^{(pq)}_{m,n,l}]_{6\times6}$ is a symmetric block matrix with its nonzero entries given by $\Gamma^{(11)}_{m,n,l} = -\check{P}_m + \tilde{\mathcal{L}}^T(\bar{\Xi}_n \otimes I)\check{\Phi}_m\tilde{\mathcal{L}}$, $\Gamma^{(12)}_{m,n,l} = \tilde{\mathcal{L}}^T(\bar{\Xi}_n \otimes I)\check{\Phi}_m\tilde{\mathcal{L}}$, $\Gamma^{(15)}_{m,n,l} = \bar{E}^T$, $\Gamma^{(35)}_{m,n,l} = \bar{F}^T$, $\Gamma^{(22)}_{m,n,l} = \tilde{\mathcal{L}}^T(\bar{\Xi}_n \otimes I)\check{\Phi}_m\tilde{\mathcal{L}} - \check{\Phi}_m$, $\Gamma^{(16)}_{m,n,l} = \bar{A}^TR^T - \bar{C}^T\check{K}_m^T + \tilde{\mathcal{L}}^T\check{G}_m^T$, $\Gamma^{(26)}_{m,n,l} = \tilde{\mathcal{L}}^T\check{G}_m^T$, $\Gamma^{(36)}_{m,n,l} = \bar{B}^TR^T$, $\Gamma^{(46)}_{m,n,l} = -\bar{D}^T\check{K}_m^T$, $\Gamma^{(33)}_{m,n,l} = -\gamma^2 I$, $\Gamma^{(44)}_{m,n,l} = -(\gamma^2/\mathscr{N})I$, $\Gamma^{(55)}_{m,n,l} = -\mathscr{N}I$, and $\Gamma^{(66)}_{m,n,l} = \alpha^2\check{P}_l - \mathrm{sym}(\alpha R)$, and other zero entries.

Now, we are in a position to present the following result.

Theorem 2: Given scalars $\gamma > 0$, $\alpha > 0$, $\bar{\delta}_{1,i}$ and $\bar{\delta}_{2,i}$ satisfying $0 \leq \bar{\delta}_{1,i} \leq \bar{\delta}_{2,i} < 1$, $i \in \mathcal{V}$, the proposed distributed event-triggered H_∞ consensus filtering problem is solvable by the distributed filters (5) if there exist constant diagonal matrices $\bar{P}_0 > 0$, $\bar{P} > 0$, R, $\bar{\Phi}_0 > 0$, $\bar{\Phi} > 0$, $\tilde{G}_0$, $\tilde{G}$, $\tilde{K}_0$, and $\tilde{K}$ of appropriate dimensions such that

$$\Gamma_{m,n,l} < 0, \quad m, n, l = 1, 2. \tag{16}$$

Furthermore, the constant filter gain matrices are given by

$$\bar{G}_0 = R^{-1}\tilde{G}_0, \bar{G} = R^{-1}\tilde{G}, \bar{K}_0 = R^{-1}\tilde{K}_0, \bar{K} = R^{-1}\tilde{K}. \tag{17}$$

With Theorem 2, the distributed event-triggered H_∞ consensus filtering problem can be transformed into the following optimization problem (OP):

$$\min_{\mathcal{F}}\ \lambda \qquad \text{subject to (16)}$$

where $\lambda = \gamma^2$ and $\mathcal{F}$ is the set of all feasible solutions from LMIs (16) in Theorem 2. The optimal H_∞ performance level $\gamma = \sqrt{\lambda}$ can be obtained by solving the minimization problem formulated above.

In the following, to obtain the threshold-parameter-dependent event triggering weighting matrix sequence $\Phi_i(\delta_i(k))$ and filter gain matrix sequences $G_i(\delta_i(k))$ and $K_i(\delta_i(k))$, $\forall\ i \in \mathcal{V}$, a recursive algorithm which outlines the design procedure is presented.

Algorithm 1: To determine the sequences of $\Phi_i(\delta_i(k))$, $G_i(\delta_i(k))$, and $K_i(\delta_i(k))$, $\forall\ i \in \mathcal{V}$.

Step 1: Given the system matrices A, B, C_i, D_i, E, and F, scalars $\bar{\delta}_{1,i}$ and $\bar{\delta}_{2,i}$ satisfying $0 \leq \bar{\delta}_{1,i} \leq \bar{\delta}_{2,i} < 1$. Choose a fixed network topology $\mathcal{G}$, an initial time $k = 0$ and a simulation time T_{sim}.

Step 2: Solve the OP to obtain $\bar{\Phi} = \mathrm{diag}^i_{\mathscr{N}}\{\Phi_i\}$, R, $\tilde{G}_0$, $\tilde{G}$, $\tilde{K}_0$, and $\tilde{K}$. Calculate the constant filter gain matrices $\bar{G}_0 = \mathrm{diag}^i_{\mathscr{N}}\{G_{0,i}\}$, $\bar{G} = \mathrm{diag}^i_{\mathscr{N}}\{G_i\}$, $\bar{K}_0 = \mathrm{diag}^i_{\mathscr{N}}\{K_{0,i}\}$, and $\bar{K} = \mathrm{diag}^i_{\mathscr{N}}\{K_i\}$ according to (17).

Step 3: For measurable threshold parameters $\delta_i(k)$, compute the threshold-parameter-dependent event triggering weighting matrix sequence and filter gain matrix sequences as follows. $\Phi_i(\delta_i(k)) = \Phi_{0,i} + \delta_i(k)\Phi_i$, $G_i(\delta_i(k)) = G_{0,i} + \delta_i(k)G_i$ and $K_i(\delta_i(k)) = K_{0,i} + \delta_i(k)K_i$, $\forall\ i \in \mathcal{V}$ and set $k = k + 1$.

Step 4: If $k < T_{\mathrm{sim}}$, go to step 3. Otherwise go to step 5.

Step 5: Output $\Phi_i(\delta_i(k))$, $G_i(\delta_i(k))$ and $K_i(\delta_i(k))$, $\forall\ i \in \mathcal{V}$. Exit.

Remark 4: Note from Theorem 2 that the infinite time-varying LMIs established in Theorem 1 are converted into a set of finite LMIs by reformulating the time-varying threshold parameter $\delta_i(k)$ into a polytopic form (14). As a result, desired threshold-parameter-dependent filter gain matrix sequences $G_i(\delta_i(k))$ and $K_i(\delta_i(k))$ in (5) and event triggering weighting matrix sequences $\Phi_i(\delta_i(k))$ in (3) can be recursively calculated by Algorithm 1 at each time k. Recall that $G_i(\delta_i(k))$, $K_i(\delta_i(k))$ and $\Phi_i(\delta_i(k))$ depend on the threshold parameter $\delta_i(k)$. In this paper, we refer to this filter design approach as a *threshold-parameter-dependent* approach.

B. Criterion in the Case of Constant Threshold Parameters

When the threshold parameter $\delta_i(k)$, $i \in \mathcal{V}$, is fixed *a priori*, i.e., $\delta_i(k) \equiv \delta_i$, the ETCS (3) can be written as

$$t^i_{k+1} = t^i_k + \inf_{m_i \geq 0}\left\{m_i + 1 \mid \tilde{f}_i(s^i_k) > 0\right\} \tag{18}$$

where $\tilde{f}_i(s^i_k) = \mathcal{X}_i^T(s^i_k)\Phi_i\mathcal{X}_i(s^i_k) - \delta_i\mathcal{Y}_i^T(s^i_k)\Phi_i\mathcal{Y}_i(s^i_k)$. The objective is thus to design constant event triggering weighting matrix Φ_i and desired estimators in the form of (5) with constant filter gain matrices G_i and K_i, $i \in \mathcal{V}$, such that the resultant filtering error system is asymptotically stable with a prescribed H_∞ performance index γ. As a by-product, the following theorem is straightforward from Theorem 2.

Theorem 3: Given scalars $\gamma > 0$, $\alpha > 0$ and $\delta_i \in [0, 1)$, $i \in \mathcal{V}$, the distributed event-triggered H_∞ consensus filtering problem in the case of constant threshold parameters is solvable by the distributed filters if there exist constant

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diagonal matrices R, $\bar{P} > 0$, $\bar{\Phi} > 0$, $\tilde{G}$ and $\tilde{K}$ of appropriate dimensions such that $\hat{\Gamma} < 0$, where $\hat{\Gamma} = [\hat{\Gamma}^{(pq)}]_{6\times 6}$ is a symmetric block matrix with its nonzero entries given by $\hat{\Gamma}^{(11)} = -\bar{P} + \tilde{\mathcal{L}}^T(\Xi \otimes I)\bar{\Phi}\tilde{\mathcal{L}}$, $\hat{\Gamma}^{(12)} = \tilde{\mathcal{L}}^T(\Xi \otimes I)\bar{\Phi}\tilde{\mathcal{L}}$, $\hat{\Gamma}^{(15)} = \bar{E}^T$, $\hat{\Gamma}^{(35)} = \bar{F}^T$, $\hat{\Gamma}^{(22)} = \tilde{\mathcal{L}}^T(\Xi \otimes I)\bar{\Phi}\tilde{\mathcal{L}} - \bar{\Phi}$, $\hat{\Gamma}^{(33)} = -\gamma^2 I$, $\hat{\Gamma}^{(16)} = \bar{A}^T R^T - \bar{C}^T\tilde{K}^T + \tilde{\mathcal{L}}^T\tilde{G}^T$, $\hat{\Gamma}^{(26)} = \tilde{\mathcal{L}}^T\tilde{G}^T$, $\hat{\Gamma}^{(36)} = \bar{B}^T R^T$, $\hat{\Gamma}^{(46)} = -\bar{D}^T\tilde{K}^T$, $\hat{\Gamma}^{(44)} = -(\gamma^2/\mathcal{N})I$, $\hat{\Gamma}^{(55)} = -\mathcal{N}I$, and $\hat{\Gamma}^{(66)} = \alpha^2\bar{P} - \text{sym}(\alpha R)$, and other zero entries. Furthermore, the filter gain matrices are given by $\bar{G} = R^{-1}\tilde{G}$ and $\bar{K} = R^{-1}\tilde{K}$ with $\bar{G} = \text{diag}^i_{\mathcal{N}}\{G_i\}$ and $\bar{K} = \text{diag}^i_{\mathcal{N}}\{K_i\}$.

Remark 5: The problem of distributed event-triggered H_∞ filtering for continuous-time linear systems over sensor networks with constant threshold parameters was investigated in [12]. A co-design algorithm was proposed to design both the filter parameter matrices and the event-triggered parameters including the threshold parameters and weighting matrices so as to simultaneously preserve the desired system performance and expected network resource occupancy. By following similar analysis and design procedures in [12], interested readers may extend Theorem 3 in the case of constant threshold parameters to relevant co-design results for discrete-time linear systems, which is omitted due to page limitations.

IV. Criterion for Designing Distributed Time-Triggered H_∞ Consensus Filters

Note that if one sets $\delta_i = 0$, $i \in \mathcal{V}$, it can be clearly seen from (18) that $t^i_{k+1} = t^i_k + 1$ because $\mathcal{X}_i^T(s^i_k)\Phi_i\mathcal{X}_i(s^i_k) > 0$ holds for every instant k, which means that sensor node i periodically updates its state estimation. As a result, the proposed ETCS (3) reduces to a time-triggered communication scheme (TTCS) and the corresponding distributed event-triggered H_∞ consensus filters (5) can be reconstructed as

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + K_i\big(y^s_i(k) - C_i\hat{x}_i(k)\big) \\ \qquad\qquad + G_i \sum_{j\in\mathcal{N}_i} a_{ij}\big(\hat{x}_i(k) - \hat{x}_j(k)\big) \\ \hat{z}_i(k) = E\hat{x}_i(k) \end{cases} \quad (19)$$

where G_i and K_i, $i \in \mathcal{V}$, are constant filter gain matrices to be determined. In this case, similar to the proof of Theorem 1, one can derive the following result to design desired distributed time-triggered H_∞ consensus filters (19) such that the resultant filtering error system is asymptotically stable with the prescribed H_∞ performance.

Theorem 4: For prescribed scalars $\gamma > 0$ and $\alpha > 0$, the distributed time-triggered H_∞ consensus filtering problem is solvable by the distributed filters (19) if there exist constant diagonal matrices R, $\bar{P} > 0$, $\bar{\Phi} > 0$, $\tilde{G}$ and $\tilde{K}$ of appropriate dimensions such that $\check{\Gamma} < 0$, where $\check{\Gamma} = [\check{\Gamma}^{(pq)}]_{5\times 5}$ is a symmetric block matrix with its nonzero entries given by $\check{\Gamma}^{(11)} = -\bar{P}$, $\check{\Gamma}^{(14)} = \bar{E}^T$, $\check{\Gamma}^{(15)} = \bar{A}^T R^T - \bar{C}^T\tilde{K}^T + \tilde{\mathcal{L}}^T\tilde{G}^T$, $\check{\Gamma}^{(22)} = -\gamma^2 I$, $\check{\Gamma}^{(24)} = \bar{F}^T$, $\check{\Gamma}^{(25)} = \bar{B}^T R^T$, $\check{\Gamma}^{(35)} = -\bar{D}^T\tilde{K}^T$, $\check{\Gamma}^{(33)} = -(\gamma^2/\mathcal{N})I$, $\check{\Gamma}^{(44)} = -\mathcal{N}I$, and $\check{\Gamma}^{(55)} = \alpha^2\bar{P} - \text{sym}(\alpha R)$, and other zero entries. Furthermore, the filter gain matrices are given by $\bar{G} = R^{-1}\tilde{G}$ and $\bar{K} = R^{-1}\tilde{K}$ with $\bar{G} = \text{diag}^i_{\mathcal{N}}\{G_i\}$ and $\bar{K} = \text{diag}^i_{\mathcal{N}}\{K_i\}$.

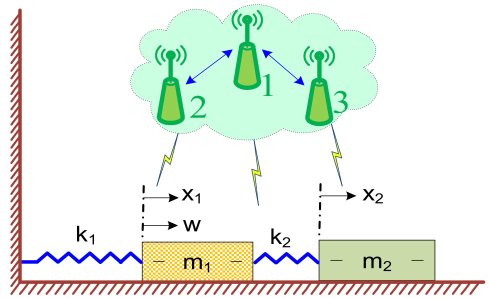

Fig. 3. Mechanical system with two masses and two springs over a wireless sensor network.

V. Illustrative Examples

In this section, two examples, which have been commonly used in the filtering/estimation literature, are given to illustrate the effectiveness of the proposed design approach.

A. Two-Mass-Spring Mechanical System

Consider a mechanical system with two masses and two springs [43], illustrated in Fig. 3. The positions of these two masses m_1 and m_2 are denoted as x_1 and x_2, respectively. The parameters k_1 and k_2 stand for the spring constants. The parameter c represents the viscous friction coefficient between the masses and the horizontal surface. The system is disturbed by a noise input w. Denoting $x = \text{col}_4\{x_1, x_2, \dot{x}_1, \dot{x}_2\}$, a state-space realization of this two-mass-spring system is given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c}{m_1} & 0 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & -\frac{c}{m_2} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} w(t).$$

The parameters are chosen as $m_1 = 1$ kg, $m_2 = 0.5$ kg, $k_1 = k_2 = 1$ N/m, and $c = 0.5$ N·s/m. It is assumed that three sensor nodes ($\mathcal{N} = 3$) are deployed to cooperatively estimate the position and the velocity of two masses over a wireless communication network. The network topology is characterized by a directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the nodes $\mathcal{V} = \{1, 2, 3\}$, sets of edges $\mathcal{E} = \{(1, 2), (1, 3), (2, 1), (3, 1)\}$ and the adjacency elements $a_{ij} = 1/\mathcal{N}$, $\forall\ (i, j) \in \mathcal{E}$. Similar to [15], we choose the sampling period $T = 0.3s$ and obtain by discretization the discrete-time model in the form of the first equation in (1) with system parameter matrices given by

$$A = \begin{bmatrix} 0.9612 & 0.0416 & 0.2703 & 0.0040 \\ 0.0792 & 0.9202 & 0.0079 & 0.2515 \\ -0.5328 & 0.2624 & 0.7810 & 0.0376 \\ 0.4872 & -0.4951 & 0.0753 & 0.6686 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.0422 & 0.0006 & 0.2703 & 0.0079 \end{bmatrix}.$$

The measurement output parameter matrices on sensors are given by $C_1 = [2\ \ 2\ \ -1\ \ -2]$, $C_2 = [1\ \ 2\ \ 1\ \ -2]$, $C_3 = [2\ \ 1\ \ -2\ \ 3]$ and $D_1 = D_2 = D_3 = 0.1$. We are interested in estimating the position of mass 1, thus one has $z(k) = x_1(k)$, i.e., $E = [1\ 0\ 0\ 0]$ and $F = 0$.

The objective is twofold. *First*, to build up a multisensor estimation framework for estimating the position of mass 1

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TABLE I
COMPARISONS OF DIFFERENT COMMUNICATION SCHEMES

	γ_{min}	NoTDP 1*	NoTDP 2	NoTDP 3
Total Packets	-	80	80	80
TTCS	0.2476	80	80	80
ETCS-C	0.3749	71	75	76
ETCS-TV	0.4240	48	61	65

* NoTDP i denotes the number of transmitted data packets on sensor node i, $\forall\ i \in \mathcal{V}$

such that an optimal estimation performance index can be achieved. *Second*, to reduce the communication frequency among neighboring sensors in a quantitative manner so as to achieve better resource efficiency.

For this purpose, we apply the obtained theoretical results in three different cases. The time-triggered communication scheme (TTCS) case, see [5], [9], [15], [20], [42]. In this case, each sensor broadcasts and transmits its current state estimation $\hat{x}_i(k)$ to its neighboring sensors at every time step $k \in \mathbb{N}$. The ETCS with constant threshold parameters (ETCS-C) case, see [17], [19], [33]–[35], [41]. In this case, the decision of whether each sensor should broadcast and transmit its current state estimation $\hat{x}_i(k)$ to its neighbors is made by (18) in terms of a constant threshold parameter δ_i. To solve Theorem 3, we further choose $\delta_1 = 0.75$, $\delta_2 = 0.8$, and $\delta_3 = 0.85$. The newly proposed ETCS with time-varying threshold parameters (ETCS-TV) case. In this case, Theorem 2 is implemented and the time-varying threshold parameter sequence in (3) on each node is simulated as $\delta_1(k) = 0.65 + 0.2|\cos(k)|$, $\delta_2(k) = 0.7 + 0.2|\sin(k)|$ and $\delta_3(k) = 0.75 + 0.2|\sin(k)|$, respectively, as shown in Fig. 4. It can be checked that (12) holds since $\lambda_{\max} = 1$.

Applying Theorems 2–4, it is found that desired consensus-based distributed event-triggered filters can be designed such that the resultant filtering error system (7) is asymptotically stable in each case while preserving the optimal H_∞ performance index $\gamma_{\min}$, as given in Table I, respectively. The designed filter gain matrices and event triggering weighting matrices in difference cases are omitted for space limitations.

To provide the quantitative analysis on saving communication resources, we perform the simulation for 80 s and calculate the number of transmitted data packets (NoTDP) on each sensor node in the three cases aforementioned. The comparison result is presented in Table I. The specific sensors' event release instants in different cases are depicted in Fig. 5. This example reveals the following two facts. *First*, the TTCS leads to the smallest H_∞ performance index and the ETCS-C achieves the less H_∞ performance index than the ETCS-TV. *Second*, the ETCS-TV results in the fewest transmitted data packets through the communication network. In other words, for this example, the proposed ETCS-TV outperforms the TTCS and ETCS-C in light of reducing the communication frequency among sensors thus saving a certain amount of communication resources, with slight degradation of the H_∞ performance. Generally, when an ETCS is employed to design H_∞ filters, there is a tradeoff between achieving better resource efficiency and preserving less H_∞ performance. However, it is necessary to point out that the above comparison

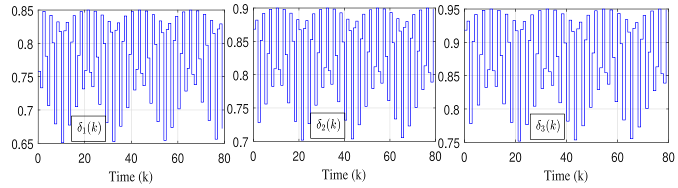

Fig. 4. Time-varying threshold parameter sequence $\delta_i(k)$ on sensor i ($i = 1, 2, 3$) over [0 s, 80 s).

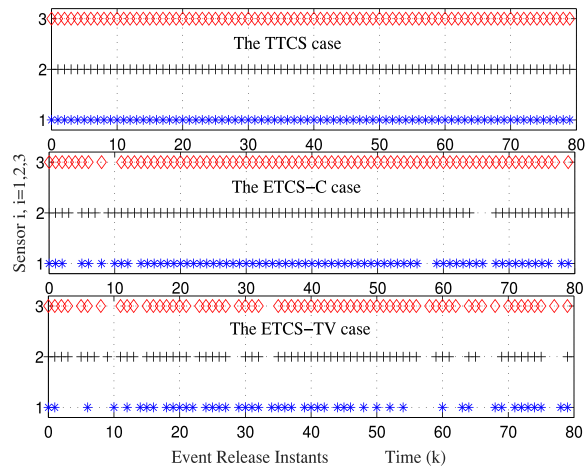

Fig. 5. Event release instants on sensor i ($i = 1, 2, 3$) over [0 s, 80 s) under TTCS, ETCS-C, and ETCS-TV.

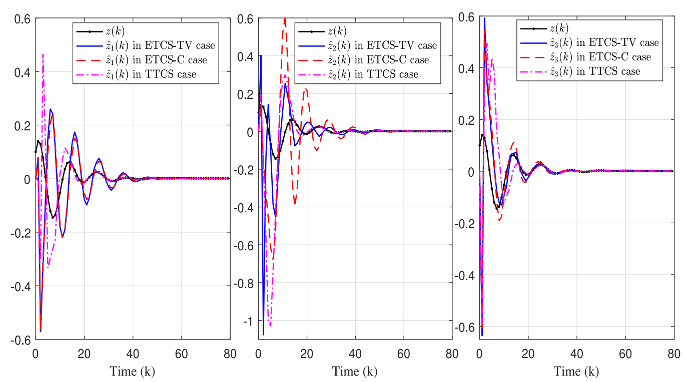

Fig. 6. Objective output $z(k)$ and its estimation $\hat{z}_i(k)$ on sensor i ($i = 1, 2, 3$) under TTCS, ETCS-C, and ETCS-TV.

does not mean that the ETCS-TV always leads to less transmitted data packets than the ETCS-C as the later depends on the choice of constant threshold parameters.

In the sequel, choose the initial conditions as $x_0 = \text{col}_4\{0.1, -0.3, 0.2, -0.3\}$ and $\hat{x}_0^i = \text{col}_4\{0\}$, $\forall\ i \in \mathcal{V}$. The external disturbance and the measurement noise are taken as $w(k) = 0.5\text{rand}(1)/(2 + 5k)$, $v_1(k) = \exp(-0.4k)$, $v_2(k) = \exp(-0.2k)$ and $v_3(k) = \exp(-0.3k)$, respectively. We connect the obtained filters with the system under consideration and apply the determined ETCS, Fig. 6 depicts the objective output $z(k)$ and its estimations $\hat{z}_i(k)$, $\forall\ i \in \mathcal{V}$, under different communication schemes. It can be seen from Fig. 6 that the designed filters well estimate the masses' positions as time goes by.

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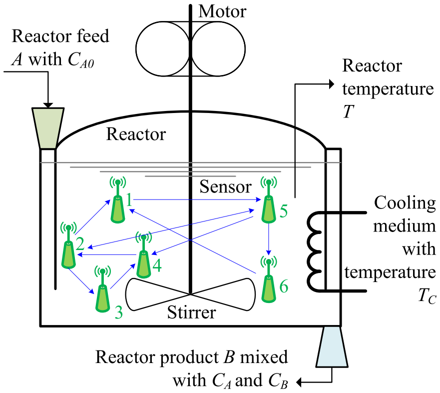

Fig. 7. CSTR monitored by a wireless sensor network consisting of six cooperative sensors.

B. Continuous Stirred Tank Reactor

In this section, we consider an industrial nonisothermal continuous stirred tank reactor (CSTR), where chemical species A reacts to form species B [44]. Fig. 7 demonstrates a simple physical structure of the CSTR. The reactor inflow contains only the educt A in low concentration C_{A0} and the reactor outflow contains the desired product B mixed with A. C_A represents the output concentration of the educt A; C_B stands for the output concentration of the desired product B within the reactor; T denotes the reactor temperature; and T_c is the cooling medium temperature. The CSTR model has been widely adopted in the literature to address H_∞ filtering problems, see [45], [46]. Considering that it may be costly and practically difficult to implement a direct measure of the concentration by using a traditional chemical approach, an alternative way is to apply a signal processing technique to estimate the concentration based on local measurement of the system. In the following, the state matrix of the discretized and linearized state-space model of the CSTR is borrowed from [45], [46] and given by

$$\begin{bmatrix} x^{(1)}(k+1) \\ x^{(2)}(k+1) \end{bmatrix} = \begin{bmatrix} 0.9719 & -0.0013 \\ -0.0340 & 0.8628 \end{bmatrix} \begin{bmatrix} x^{(1)}(k) \\ x^{(2)}(k) \end{bmatrix} \quad (20)$$

where $x^{(1)}(k)$ denotes the output concentration of the educt A; and $x^{(2)}(k)$ represents the reactor temperature. In practice, the process noise may stem from poisoning of the reaction, fouling of the cooling coils or temperature fluctuation, which can be regarded as an exogenous disturbance input and can be mathematically described by adding one term $Bw(k)$ in (20) with $B = [0.1 \ \ 0.3]^T$. The actual measurement output on each sensor and the objective output are given by

$$\begin{cases} y_i(k) = [-0.2 \ \ 0.1 + 0.1i]x(k) + \frac{1}{i}v_i(k) \\ z(k) = [0 \ \ 1]x(k) + 0.1w(k). \end{cases}$$

The initial condition is $x(0) = \mathrm{col}_2\{2, -1\}$. The external disturbance and the measurement noise are taken as $w(k) = 3\mathrm{rand}(1)/(0.5+0.1k^2)$, $v_1(k) = \exp(-0.05k)$, $v_2(k) = \exp(-0.06k)$, $v_3(k) = \exp(-0.08k)$, $v_4(k) = \exp(-0.07k)$, $v_5(k) = \exp(-0.04k)$, and $v_6(k) = \exp(-0.09k)$, respectively.

In what follows, we apply the developed distributed event-triggered H_∞ filtering approach to estimate the reactor temperature. To enhance the reliability and to improve the estimation

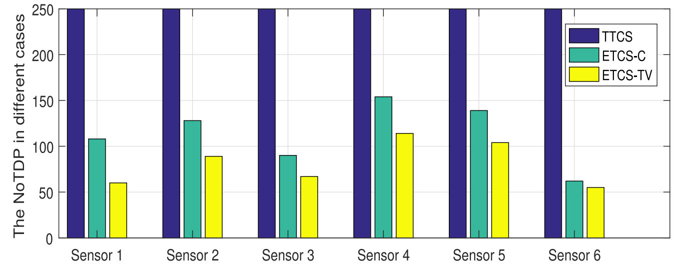

Fig. 8. Comparisons of the NoTDP on sensor i ($i = 1, 2, \ldots, 6$) over [0 s, 250 s) under TTCS, ETCS-C, and ETCS-TV.

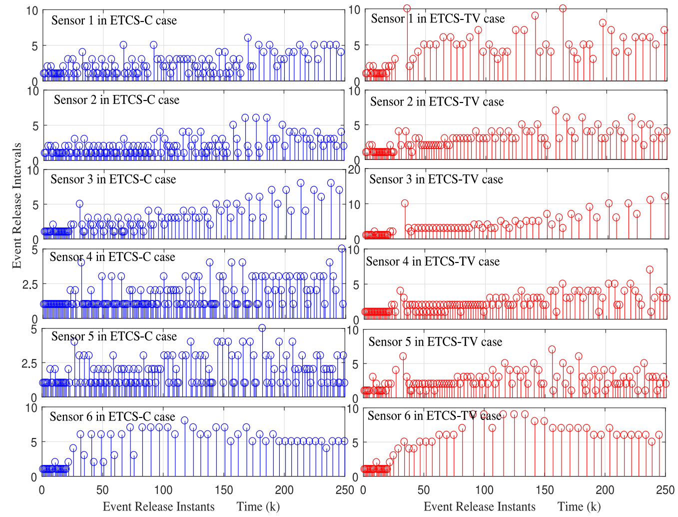

Fig. 9. Event release instants and event release intervals on sensor i ($i = 1, 2, \ldots, 6$) over [0 s, 250 s) under ETCS-C and ETCS-TV.

performance, six cooperative sensors, i.e., $\mathcal{N} = 6$, are deployed to monitor the reactor. The network topology characterized by a directed weighted graph is illustrated in Fig. 7. When each sensor should broadcast and share its local estimation to its neighbors is determined by the proposed ETCS-TV. For comparison purposes, the following simulation is carried out under three different communication schemes, i.e., TTCS, ETCS-C, and ETCS-TV. It is assumed that the time-varying threshold parameter sequence on each node satisfies $\delta_1(k) = 0.50 + 0.33|\sin(k)|$, $\delta_2(k) = 0.53 + 0.28|\cos(k)|$, $\delta_3(k) = 0.59 + 0.25|\sin(k)|$, $\delta_4(k) = 0.52 + 0.32|\cos(k)|$, $\delta_5(k) = 0.55 + 0.27|\cos(k)|$, and $\delta_6(k) = 0.56 + 0.29|\sin(k)|$, respectively. In the ETCS-C case, the constant threshold parameters are chosen as $\delta_1 = 0.665$, $\delta_2 = 0.670$, $\delta_3 = 0.715$, $\delta_4 = 0.680$, $\delta_5 = 0.685$, and $\delta_6 = 0.705$.

Applying Theorems 2–4, we conclude that the proposed distributed H_∞ consensus filtering problem is solvable under either TTCS, ETCS-C, or ETCS-TV with the minimal H_∞ performance index $\gamma_{\min} = 1.1675$, $\gamma_{\min} = 1.1890$, and $\gamma_{\min} = 1.1964$, respectively. The filter parameter matrices are omitted for space limitations. Comparisons of the NoTDP on each sensor node between TTCS, ETCS-C, and ETCS-TV are provided in Fig. 8. The specific event release instants and event release intervals under ETCS-C and ETCS-TV are depicted in Fig. 9. Connecting the designed threshold-parameter-dependent filters with the CSTR and performing

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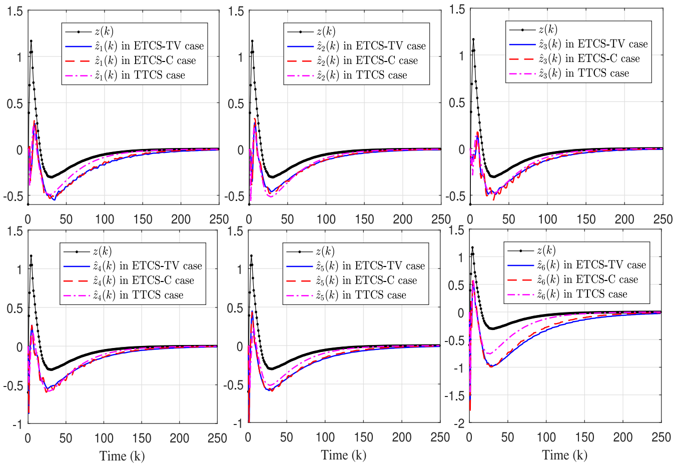

Fig. 10. Objective output $z(k)$ and its estimation $\hat{z}_i(k)$ on sensor i ($i = 1, 2, \ldots, 6$) under TTCS, ETCS-C, and ETCS-TV.

the determined communication schemes, the evolution of the objective output $z(k)$ and its estimation $\hat{z}_i(k)$ on sensor i, $\forall\, i \in \mathcal{V}$ is shown in Fig. 10.

Based on the simulation above, it can be seen that the estimation performance under the TTCS is better than the ETCS-C and ETCS-TV, however, at the expense of sacrificing the communication resources since more data packets need to be broadcast and transmitted among sensors to perform the cooperative estimation task, and compared with the TTCS and ETCS-C, the ETCS-TV releases the fewest data packets through the wireless communication network, thus has more potential to save certain communication resources in practice. However, the overall estimation performance is compromised. This is due to the tradeoff between system performance and communication cost.

VI. Conclusion

The problem of distributed event-triggered H_∞ consensus filtering for a discrete-time linear system over a sensor network has been addressed. In order to reduce sensor update frequency, a new distributed ETCS which depends on the time-varying threshold parameter has been proposed. To solve the distributed event-triggered H_∞ consensus filtering problem, a new threshold-parameter-dependent filter design approach has been developed. Sufficient conditions on the existence of desired distributed H_∞ consensus filters and ETCS have been presented such that the resultant filtering error system is asymptotically stable with the prescribed H_∞ performance. To deal with the time-varying threshold parameter residing in the proposed design criteria, a polytope-like transformation has been introduced to convert the time-varying LMIs into a set of finite LMIs, which are numerically tractable. Then, a recursive algorithm has been presented to compute the time-varying threshold-parameter-dependent filter gain matrix sequences and event triggering weighting matrix sequence. Two illustrative examples have been provided to show the effectiveness and merits of the threshold-parameter-dependent filter design approach. One future topic that deserves deep investigation is to study the resource efficiency problem under the simultaneous presence of quantized and event-triggered data communication. This is significant because in general data quantization determines the transmission bit rate of data packets thus influences the usage of communication resources. Another future topic may lie in applying the developed threshold-parameter-dependent filter design approach to deal with heterogenous sensor networks, where different types of sensors possess different sensing/processing capabilities and/or random sensor networks [14], [47], [48], where the network topology randomly varies over time.

Appendix

Proof of Theorem 1

We first prove the asymptotic stability of the resultant filtering error system (7) with the prescribed H_∞ performance. Define the forward difference of $V(k)$ in (9) as $\triangle V(k) \triangleq V(k+1) - V(k)$. Calculating the forward difference along the system (7) yields

$$\begin{aligned}\triangle V(k) = {} & e^T(k+1)\bar{P}(\Xi(k+1))e(k+1) \\ & - e^T(k)\bar{P}(\Xi(k))e(k). \end{aligned} \tag{21}$$

On the other hand, for all $k \in \Upsilon = [t_k^i, t_{k+1}^i)$, there is no event released. According to the proposed ETCS (3), we have that

$$\hat{h}_i^T(k)\Phi(\delta_i(k))\hat{h}_i(k) \le \delta_i(k)\tilde{\mathcal{Y}}_i^T(k)\Phi(\delta_i(k))\tilde{\mathcal{Y}}_i(k) \tag{22}$$

where $\tilde{\mathcal{Y}}_i(k) = \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_i(t_{\tilde{k}_i}^i) - \hat{x}_j(t_{\tilde{k}_j}^j))$ with $\tilde{k}_i \triangleq \arg\min_{\tilde{k}}\{k - t_{\tilde{k}}^i \mid k \ge t_{\tilde{k}}^i, \tilde{k} \in \mathbb{N}\}$. Thus, it leads to

$$h^T(k)\bar{\Phi}(\Xi(k))h(k) \le \tilde{h}^T(k)\Theta(\Xi(k))\tilde{h}(k) \tag{23}$$

where $\tilde{h}(k) = e(k) + h(k)$ and $\Theta(\Xi(k)) = \tilde{\mathcal{L}}^T(\Xi(k) \otimes I)\bar{\Phi}(\Xi(k))\tilde{\mathcal{L}}$, which can be rewritten as

$$\begin{bmatrix} e(k) \\ h(k) \end{bmatrix}^T \mathcal{R} \begin{bmatrix} e(k) \\ h(k) \end{bmatrix} \ge 0 \tag{24}$$

where

$$\mathcal{R} = \begin{bmatrix} \Theta(\Xi(k)) & \Theta(\Xi(k)) \\ \Theta^T(\Xi(k)) & \Theta(\Xi(k)) - \bar{\Phi}(\Xi(k)) \end{bmatrix}.$$

Combining (21) and the inequality (24) yields

$$\begin{aligned}\triangle V(k) \le {} & \psi^T(k)\left\{\tilde{\Gamma} + \frac{1}{\mathcal{N}}\rho^T\rho + \varrho^T\bar{P}(\Xi(k+1))\varrho\right\}\psi(k) \\ & + \gamma^2 w^T(k)w(k) + \frac{\gamma^2}{\mathcal{N}}v^T(k)v(k) - \frac{1}{\mathcal{N}}\tilde{z}^T(k)\tilde{z}(k) \end{aligned} \tag{25}$$

where $\psi(k) = \text{col}_4\{e(k), h(k), w(k), v(k)\}$, $\rho = [\bar{E} \ \ 0 \ \ \bar{F} \ \ 0]$, $\varrho = [\tilde{A} \ \ \tilde{C} \ \ \bar{B} \ \ \tilde{D}]$, and $\tilde{\Gamma} = [\tilde{\Gamma}^{(pq)}]_{4\times 4}$ is a symmetric block matrix with its nonzero entries given by $\tilde{\Gamma}^{(11)} = -\bar{P}(\Xi(k)) + \Theta(\Xi(k))$, $\tilde{\Gamma}^{(12)} = \Theta(\Xi(k))$, $\tilde{\Gamma}^{(22)} = \Theta(\Xi(k)) - \bar{\Phi}(\Xi(k))$, $\tilde{\Gamma}^{(33)} = -\gamma^2 I$, and $\tilde{\Gamma}^{(44)} = -(\gamma^2/\mathcal{N})I$.

By Schur complement, $\tilde{\Gamma} + (1/\mathcal{N})\rho^T\rho + \varrho^T\bar{P}(\Xi(k+1))\varrho < 0$ is equivalent to

$$\begin{bmatrix} \tilde{\Gamma} & \rho^T & \varrho^T \\ * & -\mathcal{N}I & 0 \\ * & * & -\bar{P}^{-1}(\Xi(k+1)) \end{bmatrix} < 0. \tag{26}$$

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Performing a congruence transformation to (26) by $\text{diag}_7\{I, I, I, I, I, R\}$, applying the inequality $-R\bar{P}^{-1}(\Xi(k+1))R^T \leq \alpha^2\bar{P}(\Xi(k+1)) - \text{sym}(\alpha R)$ and introducing new matrix sequences $\tilde{G}(\Xi(k)) = R\bar{G}(\Xi(k))$ and $\tilde{K}(\Xi(k)) = R\bar{K}(\Xi(k))$ yield $\Gamma < 0$. Therefore, if (10) holds, we have

$$\triangle V(k) \leq \gamma^2 w^T(k)w(k) + \frac{\gamma^2}{\mathcal{N}} v^T(k)v(k) - \frac{1}{\mathcal{N}}\tilde{z}^T(k)\tilde{z}(k). \tag{27}$$

Summing both sides of (27) from 0 to ∞ on k, one obtains

$$\frac{1}{\mathcal{N}}\sum_{k=0}^{\infty}\tilde{z}^T(k)\tilde{z}(k) \leq \gamma^2\sum_{k=0}^{\infty} w^T(k)w(k) + \frac{\gamma^2}{\mathcal{N}}\sum_{k=0}^{\infty} v^T(k)v(k) + V(0) - V(\infty). \tag{28}$$

Under zero initial condition $V(0) = 0$, we have

$$\frac{1}{\mathcal{N}}\sum_{k=0}^{\infty}\tilde{z}^T(k)\tilde{z}(k) \leq \sum_{k=0}^{\infty}\left(\gamma^2 w^T(k)w(k) + \frac{\gamma^2}{\mathcal{N}} v^T(k)v(k)\right) \tag{29}$$

from which the H_∞ performance (8) is guaranteed.

Assume that $w(k) = 0$ and $v(k) = 0$, combining (21) and (24) yields

$$\triangle V(k) \leq \psi^T(k)\left\{\tilde{\Gamma} + \varrho^T\bar{P}(\Xi(k+1))\varrho\right\}\psi(k). \tag{30}$$

Following similar line of analysis, one has that $\tilde{\Gamma} + \varrho^T\bar{P}(\Xi(k+1))\varrho < 0$ is deduced from (10). Therefore, there exists a scalar $\lambda > 0$ such that $\triangle V(k) \leq -\lambda||e(k)||_2^2 < 0$, from which it is concluded that the resultant filtering error system (7) is asymptotically stable.

We next solve out the filter gain parameters. Observe from $\Gamma < 0$ in (10) that $\Gamma^{(66)} = \alpha^2\bar{P}(\Xi(k+1)) - \text{sym}(\alpha R) < 0$, thereby leading to a nonsingular constant matrix R. As a consequence, the filter gain matrix sequences in (5) can be derived by (11). This completes the proof.

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