Filomat 32:5 (2018), 1523–1533
https://doi.org/10.2298/FIL1805523L

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

A time-preference and VIKOR-based dynamic intuitionistic fuzzy decision making method

Jin-qiu Li[a], Wei Chen[a], Zao-li Yang[b], Chuan-yun Li[a]

[a]*School of Economy and Management, Harbin Engineering University, Harbin 150001, China*
[b]*School of Economics and Management, Beijing University of Technology, Beijing 100124, China*

Abstract. According to the decision information of multi-attribute decision-making problem with fuzzy and temporal characteristics, a dynamic intuitionistic fuzzy decision making method based on time preference and VIKOR is proposed. First, we determined the attribute weights under different time sequence based on intuitionistic fuzzy entropy minimization; secondly, we introduced the time degree function reflecting the decision makers' subjective time preference, and established a multi-objective programming model to obtain time weights; then we used dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator to integrate different time periods of the intuitionistic fuzzy decision matrices; the VIKOR method is used in ranking solutions that takes account of group effectiveness maximization and individual regret minimization, and obtained the optimal scheme that is closet to ideal solution; finally, the feasibility and effectiveness of the proposed method is verified by the example of a technology innovation alliance partner selection.

1. Introduction

Since Zadeh [1] first proposed the concept of fuzzy sets, fuzzy theory and method has attracted wide attention in academia. On the base of the fuzzy set, some scholars put forward the fuzzy number to describe the characteristics of fuzzy of objective objects, because it is only using the membership to describe fuzziness, and cannot accurately portray the fuzziness in reality of the world. Therefore, Atanassov (1986,1989) [2, 3] further proposed the concept of intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets, can also express membership, non-membership degree and hesitancy degree information, so as to fully describe the fuzziness and uncertainty. On this base, many scholars carry out the research by intuitionistic fuzzy theories and methods.

Most of the current scholars mainly study on single time period of intuitionistic fuzzy multiple attribute decision making problems [4, 5]. But only considering single time period of decision information to make comparison of alternatives, ignore the impact of timing characteristics on decision results, and lead to the decision results are not comprehensive and scientific. Based on this idea, some scholars began to study on dynamic intuitionistic fuzzy multiple attribute decision making problems embedded characteristics of

2010 *Mathematics Subject Classification*. Primary 03E72.
Keywords. Intuitionistic fuzzy entropy; time degrees; TOPSIS method; VIKOR method; dynamic decision making.
Received: 5 September 2017; Accepted: 1 November 2017
Communicated by Hari M. Srivastava
This work was supported by the National Fund Support Project on Natural Science of China (No. 71704007).
Email addresses: hanying911@126.com (Jin-qiu Li), 598742058@qq.com (Wei Chen), yangprofessor@163.com (Zao-li Yang), yundanfengqing812@126.com (Chuan-yun Li)

sequential information at different time periods. Such as Xu (2008), Yang (2011) aggregated different time periods of interval decision information by using uncertain dynamic weighted average (UDWA) operator, uncertain dynamic weighted geometric averaging (UDWGA) operator [6, 7]. Research of these scholars has laid a theoretical foundation for the dynamic interval multi-attribute decision-making problem. Among them, scientifically and reasonably determine the time weight and rank of alternatives are the key to solve dynamic multiple attribute decision making problems. In determining of time weight, there are abundant achievements: such as Park (2013) [8] established a time dimension-based dynamic intuitionistic fuzzy multiple attribute decision making model, but the time weight is directly affected by subjective of decision makers, which is not scientific. In order to overcome the subjective of determining time weight, some scholars used objective weighting method to improve the calculation accuracy of time weight, such as normal distribution method [9], the entropy principle [10] and exponential decay model [11], the damping coefficient model [12] to determine the time weight. Most of these methods are on the base of mathematical probability theory and physics to determine time weight, although they are able to fully exploit the objective decision information, did not consider the characteristics of time preference and subjective, ignore the effectiveness of recent information in decision-making, fewer scholars both considered the effect of objective decision information and subjective of decision makers, so they cannot fully reflect the influence of time series on the result of decision.

In the research of intuitionistic fuzzy multiple attribute decision making method, has made some achievements. Some scholars proposed a fuzzy multi attribute decision making method based on intuitionistic TOPSIS method [13–15], but because the TOPSIS method only considers the distance between schemes and the ideal solution, ignored correlation distance between the scheme and the positive and negative ideal solution, so it cannot fully reflect the advantages and disadvantages of the schemes; Wei (2011), Wu (2011), Dymova (2014) [16–18] put forward fuzzy multi-attribute decision making method based on gray correlation model, ELECTRE method, D-S evidence theory. However, these methods based on expected utility theory did not take into account the psychological factors of decision makers influencing on the decision results.

In view of the above analysis, according to the current research deficiencies, a dynamic intuitionistic fuzzy MADM method is proposed. The contents of this paper are mainly as follows: Section 2 included prerequisite knowledge, introduced the definition of intuitionistic fuzzy number, and the related algorithm and concepts; Section 3 constructed a novel method of solving time weights combining the subjective factor of decision makers and the sequential objective information via a bi-objective programming model based on the variance minimization and TOPSIS method; Section 4 sorted schemes using the VIKOR method taking account of group effectiveness maximization and the individual regret minimization; Section 5 verified the feasibility and validity of the method proposed in this paper via a numerical example analysis; and Section 6 provided the conclusion and future work of the paper.

2. Preliminaries

Definition 1. [2] *Let a set $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, then define the set $A = \{< x, u_A(x), v_A(x) | x \in X >\}$ as an intuitionistic fuzzy set. Where, $u_A(x)$ and $v_A(x)$ denote the membership degree and non-membership degree of the element x in X to A, respectively, and $\mu_A(x) : X \to [0, 1]$, $\upsilon_A(x) : X \to [0, 1]$, with the condition $0 \le \mu_A(x) + \upsilon_A(x) \le 1, x \in X$, $\pi_A = 1 - \mu_A(x) - \upsilon_A(x)$ is called the degree of indeterminacy of x in X to A.*

Definition 2. [19] *Let $\alpha_1 =< \mu_{\alpha_1}, \nu_{\alpha_1} >$ and $\alpha_2 =< \mu_{\alpha_2}, \nu_{\alpha_2} >$ be two IFNs, λ be a real number, and $\lambda > 0$, then*

(1) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1} \cdot \mu_{\alpha_2}, \upsilon_{\alpha_1} \cdot \upsilon_{\alpha_2})$;

(2) $\alpha_1 \otimes \alpha_2 = (\mu_{\alpha_1} \cdot \mu_{\alpha_2}, \mu_{\alpha_1} + \mu_{\alpha_2} - \upsilon_{\alpha_1} \cdot \upsilon_{\alpha_2})$;

(3) $\lambda\alpha_1 = (1 - (1 - \mu_{\alpha_1})^\lambda, \upsilon_{\alpha_1}^\lambda)$;

(4) $\alpha_1^\lambda = (\mu_{\alpha_1}^\lambda, (1 - \mu_{\alpha_1})^\lambda)$.

Definition 3. [9] *Let t be a timing variable, then we call $\alpha(t) = \left(\mu_{\alpha(t)}(x), \nu_{\alpha(t)}(x)\right)$ an IFN, where $\mu_{\alpha(t)}(x) \in [0, 1], \upsilon_{\alpha(t)} \in [0, 1]$, $\mu_{\alpha(t)}(x) + \nu_{\alpha(t)}(x) \le 1$, if $t_1, t_2, \ldots, t_n$, then $\alpha(t_1), \alpha(t_2), ..., \alpha(t_p)$ denotes an IFS of p different periods.*

Definition 4. [20] *Let* $\alpha_j =< \mu_{A_j}(x), \nu_{A_j}(x) >, j = 1, 2, ..., n$*, be an IFN, then we call*

$$IFWA_\omega(\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{j=1}^{n} \omega_j \alpha_j =< 1 - \prod_{j=1}^{n} \left(1 - \mu_{A_j}(x)\right)^{\omega_j}, \prod_{j=1}^{n} \nu_{A_j}(x)^{\omega_j} > \tag{1}$$

as an intuitionistic fuzzy weighted averaging (IFWA) operator. Where, $IFWA : Q^n \to Q, \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T, \omega_j \geq 0, j = 1, 2, \ldots, n, \sum_{j=1}^{n} \omega_j = 1.$

Definition 5. [9] *Let* $\alpha_{t_k} =< \mu_{t_k}, \nu_{t_k} > (k = 1, 2, \ldots, p)$ *be an IFN at period* t_k*, and* $\eta(t_k) = (\eta(t_1), \eta(t_2), \ldots, \eta(t_p))^T$ *be the weight vector of the periods* t_k *then we call*

$$DIFWG_{\eta(t)}(\alpha_{t_1}, \alpha_{t_2}, ..., \alpha_{t_p}) = \prod_{k=1}^{p} \alpha_{t_k}^{\eta(t_k)} =< \prod_{k=1}^{p} \mu_{t_k}^{\eta(t_k)}, 1 - \prod_{k=1}^{p} (1 - \nu_{t_k})^{\eta(t_k)} > \tag{2}$$

as a dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator. Where, $\eta(t_k) \in [0, 1], \sum_{k=1}^{p} \eta(t_k) = 1(k = 1, 2, \ldots, p)$

Definition 6. [21] *Let A and B be two IFS in a fixed set* $X = \{x_1, x_2, ..., x_n\}$*, the difference of the IFS can be characterized by Euclidean distance:*

$$d(A, B) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} \left[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2\right]} \tag{3}$$

Definition 7. [22] *Let* $\alpha =< \mu_A(x), \nu_A(x) >$ *be an IFN, and define* $S(\alpha) = \mu_A(x) - \nu_A(x)$ *is the score function of the IFN, wherein,* $S(\alpha) \in [-1, 1]$ *and the bigger* $S(\alpha)$ *is*$\alpha =< \mu_A(x), \nu_A(x) >$ *is*

Definition 8. [23] *Let* $\alpha =< \mu_A(x), \nu_A(x) >$ *be a IFN, and define* $H(\alpha) = \mu_A(x) + \nu_A(x)$ *is the accuracy function of the IFN, wherein,* $H(\alpha) \in [0, 1]$ *and the bigger* $H(\alpha)$ *is , the higher of accuracy* $\alpha =< \mu_A(x), \nu_A(x) >$ *is*

3. Determination of weights

3.1. Attribute weights based on IFE

In different periods of decision making, the difference of preference to attribute of decision makers at different periods is obscure, so we need to determine the attribute weights in different time series using the intuitionistic fuzzy entropy (IFE), which combined the difference of decision-making information and preference to attribute of decision makers at different periods. Based on the literature [24], we calculate the attribute weights as follows:

Let that $X(t_k) = \left(x_{ij}^{t_k}\right)_{m \times n}$ be an intuitionistic fuzzy decision matrix of the period t_k where $x_{ij}^{t_k} =< \mu_{ij}^{t_k}, \nu_{ij}^{t_k} >$ is an attribute value, denoted by an IFN, $\mu_{ij}^{t_k}$ indicates the degree that the alternative i should satisfy the attribute $j$at period t_k, $\nu_{ij}^{t_k}$ indicates the degree that the alternative i should not satisfy the attribute j at period t_k, $\pi_{ij}^{t_k}$ indicates the degree of indeterminacy of the alternative i to the attribute j, such that $\pi_{ij}^{t_k} = 1 - \mu_{ij}^{t_k} - \nu_{ij}^{t_k}$.

Based on the principle of maximize the uptake ability of objective information, we use the entropy theory and apply these method in intuitionistic fuzzy sets proposed by former scholars, first, we can compute the intuitionistic fuzzy entropy $E_j(t_k)$ of the attribute$j$at period t_k:

$$E_j(t_k) = \frac{1}{m} \sum_{i=1}^{m} \left\{1 - \sqrt{\left(1 - \pi_{ij}^{t_k}\right)^2 - \mu_{ij}^{t_k} \nu_{ij}^{t_k}}\right\} \tag{4}$$

Let $\omega_j(t_k)$ be the weight of the attribute j at period t_k, and the optimization model $(M-1)$ of attribute weights at period t_k is established as follows:

$$(M-1) = \begin{cases} \min \sum_{j=1}^{n} (\omega_j(t_k))^2 E_j(t_k) \\ s.t \sum_{j=1}^{n} \omega_j(t_k) = 1 \end{cases}$$

To solve the above model, we construct the Lagrange function of the constrained optimization model $(M-1)$, and we can have the weight of attribute j at period t_k:

$$\omega_j(t_k) = \frac{\left(E_j(t_k)\right)^{-1}}{\sum_{j=1}^{n} \left(E_j(t_k)\right)^{-1}} \tag{5}$$

3.2. Time weights via a multi-objective programming model based on time preference

In this paper, on the basis of research [24, 25], the function of time degree is applied in MADM problems of dynamic intuitionistic fuzzy numbers, to determine decision makers preference towards decision-making information at different time periods. A nonlinear multi-objective programming model is established to determine time weights by combining minimization of variance and ideal solution, the optimization model aiming at improving accuracy of the solved time weight, enabling more reasonable time weight results.

Definition 9. *[25] Suppose $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$ indicates a timing weight vector. Wherein, $\eta(t_k)$ is weight of k-th time period, $0 \le \eta(t_k) \le 1$, $\sum_{k=1}^{P} \eta(t_k) = 1$. The timing weight reflects the importance of different time periods in decision-making process. Let $\theta = \sum_{k=1}^{p} \frac{p-k}{p-1} \eta(t_k)$, $\theta \in [0, 1]$ then θ is referred to as time degree of timing weight vector $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$. Time degree reflects degree of preference of decision makers for time series. Decision makers generally provide θ value based on experience and preference. The closer to 0 θ is, the greater preference of decision makers for recent time series information is; the closer to 1 θ is, the greater preference of decision makers for forward time series information is.*

3.2.1. Determination of time weight based on time degree and ideal solution

According to definition 9, when $\eta_{t_k} = (0, 0, ..., 1)^T, \theta = 0$, indicating that decision makers have total preference for recent time series information, and denote $\eta_{t_k}^{+} = (0, 0, \ldots, 1)^T$ as positive ideal time weight vector; when $\eta_{t_k} = (1, 0, ..., 0)^T, \theta = 1$, indicating that decision makers have total preference for forward time series information and denote $\eta_{t_k}{}^{-} = (0, 0, ..., 1)^T$ as negative ideal time weight vector.

Let distance between the two time weight vectors $\underline{\eta}_{t_k} = (\underline{\eta}(t_1), \underline{\eta}(t_2), ..., \underline{\eta}(t_p))^T$ and $\bar{\eta}_{t_k} = (\bar{\eta}(t_1), \bar{\eta}(t_2), ..., \bar{\eta}(t_p))^T$ as:

$$d(\underline{\eta}_{t_k}, \bar{\eta}_{t_k}) = \sqrt{\sum_{k=1}^{p} (\underline{\eta}(t_k) - \bar{\eta}(t_k))^2} \tag{6}$$

Then respective distance of a certain time weight vector $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$ from positive, negative time weight vector is:

$$d(\eta_{t_k}, \eta_{t_k}^{+}) = \sqrt{\sum_{k=1}^{p-1} \eta(t_k)^2 + (1 - \eta(t_p))^2} \tag{7}$$

$$d(\eta_{t_k}, \eta_{t_k}^-) = \sqrt{(1-\eta(t_1))^2 + \sum_{k=2}^{p} \eta(t_k)^2} \tag{8}$$

Then relative close degree of time weight vector η_{t_k} and ideal time weight vector $\eta_{t_k}^+$ can be obtained as follows:

$$c(\eta_{t_k}, \eta_{t_k}^+) = \frac{d(\eta_{t_k}, \eta_{t_k}^-)}{d(\eta_{t_k}, \eta_{t_k}^+) + d(\eta_{t_k}, \eta_{t_k}^-)} \tag{9}$$

According to the thought of "stressing the present rather than the past", when recent information can more fully reflect characteristics of decision attributes, it is more effective to obtain decision evaluation result. Thus, time weight vector is solved based on time degree and TOPSIS method, and nonlinear programming $(M-2)$ is constructed as follows:

$$(M-2)\begin{cases} \max \quad c(\eta_{t_k}, \eta_{t_k}^+) = \dfrac{\sqrt{(1-\eta(t_1))^2 + \sum\limits_{k=2}^{p} \eta(t_k)^2}}{\sqrt{(1-\eta(t_1))^2 + \sum\limits_{k=2}^{p} \eta(t_k)^2} + \sqrt{\sum\limits_{k=1}^{p-1} \eta(t_k)^2 + (1-\eta(t_p))^2}} \\ s.t.\ \theta = \sum\limits_{k=1}^{p} \frac{p-k}{p-1}\eta(t_k),\ \sum\limits_{k=1}^{p} \eta(t_k) = 1, \eta(t_k) \in [0,1], k = 1,2,..,p \end{cases}$$

In this $(M-2)$ model, the objective is to maximize the relative close degree of time weight vector η_{t_k} and ideal time weight vector $\eta_{t_k}^+$, $\eta(t_k)$ is the unknown variable, and the value of θ is given by the decision makers according their experience, then we can solve $(M-2)$ model with Lingo11.0 software, and then timing weight vector $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$ can be obtained.

The Model $(M-2)$ is considering the subjective preferences of decision makers in time sequence. However, only considering the recent efficacy of decision information and time preference of decision makers, it easily leads to the obscure volatility of information with time weight aggregated attribute values of the samples, and the instability of the time weights. Therefore, it is critical to determine stable time weights, which can aggregate each period of decision information, and minimum the fluctuation of decision information between the samples.

3.2.2. *Determination of time weight based on time degree and minimization of variance*

At the period of t_k, considering the feature of the difference and uncertainty of the alternatives in the time series , the distance between each alternative is represented as:

$$d_{t_k} = \sum_{i=1}^{m} \sum_{r=i+1}^{m} \sum_{j=1}^{n} d(x_{ij}^{t_k}, x_{rj}^{t_k}), k = 1,2,...,p \tag{10}$$

Because of variance can reflect the fluctuation of time weight aggregating each period of decision information, and thus we can obtain the formula as follows.

$$D^2(d_{t_k}, \eta(t_k)) = \sum_{k=1}^{p} [d_{t_k}\eta(t_k) - E(d_{t_k}\eta(t_k))]^2 = \sum_{k=1}^{p} \frac{1}{p}(d_{t_k}\eta(t_k))^2 - \frac{1}{p^2}\left(\sum_{k=1}^{p} d_{t_k}\eta(t_k)\right)^2 \tag{11}$$

Wherein, $E(d_{t_k}\eta(t_k)) = \frac{1}{p}\sum\limits_{k=1}^{p} d_{t_k}\eta(t_k)$,$(k = 1,2,...,p)$

In order to make best use of the objective information in time sequence , and determine stable time weights aggregating each period of decision information, which can minimum the fluctuation of decision information between the samples. Thus, we construct the nonlinear programming model $(M-3)$ is as follows:

$$(M-3)\begin{cases} \min D^2(d_{t_k},\eta(t_k)) = \min \sum\limits_{k=1}^{p} \frac{1}{p}(d_{t_k}\eta(t_k))^2 - \frac{1}{p^2}\left(\sum\limits_{k=1}^{p} d_{t_k}\eta(t_k)\right)^2 \\ s.t.\ \theta = \sum\limits_{k=1}^{p} \frac{p-k}{p-1}\eta(t_k),\ \sum\limits_{k=1}^{p}\eta(t_k) = 1, \eta(t_k)\in[0,1], k = 1,2,..,p \end{cases}$$

In this $(M-3)$ model, the objective is to minimize the variance, which can represent the fluctuation of time weights aggregating each period of decision information, and the value of θ is given by the decision makers according their experience, then we can also solve $(M-3)$ model with Lingo11.0 software, and then timing weight vector $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$ can be obtained.

3.2.3. Comprehensive time weight

Based on models $(M-2)$ and $(M-3)$, we combined the subjective factor of decision makers and the sequential objective information, ,considering time preference and the effectiveness of recent decision information, according to the principle of minimum variance and TOPSIS method we build a bi-objective nonlinear programming model $(M-4)$ is as follows:

$$(M-4)\begin{cases} \min \quad D^2(d_{t_k},\eta(t_k)) = \sum\limits_{k=1}^{p} \frac{1}{p}(d_{t_k}\eta(t_k))^2 - \frac{1}{p^2}\left(\sum\limits_{k=1}^{p} d_{t_k}\eta(t_k)\right)^2 \\ \max \quad c(\eta_{t_k},\eta^+_{t_k}) = \dfrac{\sqrt{(1-\eta(t_1))^2+\sum\limits_{k=2}^{p}\eta(t_k)^2}}{\sqrt{(1-\eta(t_1))^2+\sum\limits_{k=2}^{p}\eta(t_k)^2}+\sqrt{\sum\limits_{k=1}^{p-1}\eta(t_k)^2+(1-\eta(t_p))^2}} \\ s.t.\ \theta = \sum\limits_{k=1}^{p} \frac{p-k}{p-1}\eta(t_k),\ \sum\limits_{k=1}^{p}\eta(t_k) = 1, \eta(t_k)\in[0,1], k = 1,2,..,p \end{cases}$$

In this $(M-4)$ model, there are dual objectives, so it is quite difficult to solve this model, to simplify calculation, we the convert this bi-objective optimization model into a single-objective optimization model $(M-5)$ as follows:

$$(M-5)\begin{cases} \max \quad g = c\dfrac{\sqrt{(1-\eta(t_1))^2+\sum\limits_{k=2}^{p}\eta(t_k)^2}}{\sqrt{(1-\eta(t_1))^2+\sum\limits_{k=2}^{p}\eta(t_k)^2}+\sqrt{\sum\limits_{k=1}^{p-1}\eta(t_k)^2+(1-\eta(t_p))^2}} - (1-c)\left(\sum\limits_{k=1}^{p} \frac{1}{p}(d_{t_k}\eta(t_k))^2 - \frac{1}{p^2}\left(\sum\limits_{k=1}^{p} d_{t_k}\eta(t_k)\right)^2\right) \\ s.t.\ \theta = \sum\limits_{k=1}^{p} \frac{p-k}{p-1}\eta(t_k),\ \sum\limits_{k=1}^{p}\eta(t_k) = 1, \eta(t_k)\in[0,1], k = 1,2,..,p \end{cases}$$

In the $(M-5)$ model, the objective is single, and $\eta(t_k)$ is the unknown variable, the value of θ is given by the decision makers according their experience. Wherein, c is a balance coefficient, $c \in [0,1]$, generally it is 0.5. Similarly, Lingo11.0 software can be used to solve $(M-5)$ model and obtain timing weight vector $\eta_{t_k} = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$.

4. Decision-making method based on VIKOR method

4.1. Description of dynamic intuitionistic fuzzy MADM problems

In a dynamic MADM problem of intuitionistic fuzzy numbers, supposing $S = \{S_1, S_2, ..., S_m\}$ consists of m alternative solution sets, $G = \{G_1, G_2, ..., G_n\}$ is a set consists of n alternative solutions, $T = \{t_1, t_2, ..., t_k\}$ is a time periods set composed by k decision-making time periods. The attribute value of the i^{th} solution S_i under attribute G_j at different decision-making time period t_k is represented by intuitionistic fuzzy numbers, denoted by $X_{ij}^{t_k} =< \mu_{ij}^{t_k}, \upsilon_{ij}^{t_k} >$. Therefore, the decision-making matrix at time period t_k can be represented as:

$$X(t_k) = (X_{ij}^{t_k})_{m\times n} = (< \mu_{ij}^{t_k}, \upsilon_{ij}^{t_k} >)_{m\times n} =$$
$$\begin{bmatrix} < \mu_{11}^{t_k}, \upsilon_{11}^{t_k} > & < \mu_{12}^{t_k}, \upsilon_{12}^{t_k} > & \cdots & < \mu_{1n}^{t_k}, \upsilon_{1n}^{t_k} > \\ < \mu_{21}^{t_k}, \upsilon_{21}^{t_k} > & < \mu_{22}^{t_k}, \upsilon_{22}^{t_k} > & \cdots & < \mu_{2n}^{t_k}, \upsilon_{2n}^{t_k} > \\ \vdots & \vdots & \ddots & \vdots \\ < \mu_{m1}^{t_k}, \upsilon_{m1}^{t_k} > & < \mu_{m2}^{t_k}, \upsilon_{m2}^{t_k} > & \cdots & < \mu_{mn}^{t_k}, \upsilon_{mn}^{t_k} > \end{bmatrix}$$

Supposing attribute weight vector $\omega_j(t_k) = (\omega_1(t_k), \omega_2(t_k), ..., \omega_n(t_k))^T$at period t_k is unknown, and $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$, time weight vector $\eta(t_k) = (\eta(t_1), \eta(t_2), ..., \eta(t_k))^T$ is unknown, and $\eta(t_k) \in [0,1]$, $\sum_{k=1}^{p} \eta(t_k) = 1$. This paper conducts dynamic decision-making on fuzzy MADM problems with unknown attribute weight and unknown time weight.

4.2. *Steps of dynamic intuitionistic fuzzy MADM based on VIKOR method*

This paper introduces VIKOR method which takes account of group effectiveness maximization and the individual regret minimization into intuitionistic fuzzy MADM problem. This method can effectively overcome the shortcomings of traditional TOPSIS method, it considers the distances between decision alternatives and the positive, negative ideal solution, and obtains the optimal solution to meet with the closest distance from ideal solution and the farthest distance from negative ideal solution [26]. By using dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator to aggregate all time periods of decision information, we obtain a dynamic intuitionistic fuzzy comprehensive decision matrix, and then we use VIKOR method to rank the decision solutions in this type of MADM problem. The calculation steps of this method are as follows:

Step 1. Let that $X(t_k) = (x_{ij}^{t_k})_{m\times n} = (< \mu_{ij}^{t_k}, \upsilon_{ij}^{t_k} >)_{m\times n}$ be an intuitionistic fuzzy decision matrix at t_k time period, and calculate the attribute weights $\omega_j(t_k)$ at t_k time periods according to the Eq. (5);

Step 2. Construct a nonlinear multi-objective programming model $(M-4)$ and select an appropriate balance coefficient c , transform model $(M-4)$ into a single-objective programming model $(M-5)$, calculating the time weight vector $\eta(t) = (\eta(t_1), \eta(t_2), ..., \eta(t_p))^T$ by using Lingo 11.0 software;

Step 3. Calculate the weighted intuitionistic fuzzy decision matrix $R(t_k) = \left(r_{ij}^{t_k}\right)_{m\times n}$ at period t_k based on the intuitionistic fuzzy algorithm, wherein

$$r_{ij}^{t_k} = (1 - (1 - \mu_{ij}^{t_k})^{\omega_j(t_k)}, (\upsilon_{ij}^{t_k})^{\omega_j(t_k)}) \tag{12}$$

Step 4. Aggregate intuitionistic fuzzy decision information at multiple time periods by DIFWG operator, acquiring a dynamic comprehensive intuitionistic fuzzy decision matrix $X = (x_{ij})_{m\times n}$;

Step 5. Determine the intuitionistic fuzzy positive and negative ideal solution $X^+ = (x_1^+, x_2^+, ..., x_n^+)$ and $X^- = (x_1^-, x_2^-, ..., x_n^-)$ of decision matrix $X = (x_{ij})_{m\times n}$, where

$$x_j^+ =< \mu_j^+, \upsilon_j^+ > =< \max_{1\le i\le m} \mu_{ij}, \min_{1\le i\le m} \upsilon_{ij} >, (j = 1, 2, ..., n) \tag{13}$$

$$x_j^- =< \mu_j^-, \upsilon_j^- > =< \min_{1\le i\le m} \mu_{ij}, \max_{1\le i\le m} \upsilon_{ij} >, (j = 1, 2, ..., n) \tag{14}$$

Step 6. Calculate the Euclidean distance $d(x_{ij}, x_j^+)$ from each IFS x_{ij} to the intuitionistic fuzzy positive ideal solution, and the Euclidean distance $d(x_j^+, x_j^-)$ from the intuitionistic fuzzy positive ideal solution to the intuitionistic fuzzy negative ideal solution by Eq. (3);

Step 7. Calculate the group effectiveness value, individual regret value and compromise value of each alternative [26].

$$U_i = \sum_{j=1}^{n} \left(\omega_j \frac{d(x_{ij}, x_j^+)}{d(x_j^+, x_j^-)} \right), (i = 1, 2, ..., m) \tag{15}$$

$$K_i = \max_{1\le j\le n}\left(\omega_j \frac{d(x_{ij}, x_j^+)}{d(x_j^+, x_j^-)}\right), (i = 1, 2, ..., m) \tag{16}$$

$$Z_i = \lambda\left(\frac{U_i - \min\limits_{1\le i\le m} U_i}{\max\limits_{1\le i\le m} U_i - \min\limits_{1\le i\le m} U_i}\right) + (1-\lambda)\left(\frac{K_i - \min\limits_{1\le i\le m} K_i}{\max\limits_{1\le i\le m} K_i - \min\limits_{1\le i\le m} K_i}\right), (i = 1, 2, ..., m) \tag{17}$$

Where ω_j represents the attribute weight of intuitionistic fuzzy decision matrix, λ is referred to as compromise coefficient, and $\lambda \in [0, 1]$. If $\lambda > 0.5$, indicating decision makers prefer to make decision with group effectiveness maximization; if $\lambda < 0.5$, indicating decision makers prefer to make decision with individual regret maximization; if $\lambda = 0.5$, indicating decision makers prefer to take a balanced and compromised way to make decisions.

Step 8. Determine the ordering of alternatives according to the compromise value Z_i, and the greater Z_i, the worse alternative S_i; Conversely, the smaller Z_i, the better alternative S_i.

5. Analysis of Examples

A high-tech enterprise is now seeking for a R&D partner; considering five indicators: R&D capability (G_1); level of resource sharing (G_2); ability of risk monitoring (G_3); level of technology update(G_4); level of cooperation (G_5), the attribute set is denoted as $G = \{G_1, G_2, G_3, G_4, G_5\}$. There are four development partners as the alternatives, denoted as $S = \{S_1, S_2, S_3, S_4\}$. Experts evaluate all the indicators denoted by IFNs for each alternative in three different time periods ($t_1 < t_2 < t_3$) , to obtain the relevant intuitionistic fuzzy decision matrix. As illustrated in tables 1 to 3.

Table 1: Intuitionistic fuzzy decision matrix $X(t_1)$ at period t_1

t_1	G_1	G_2	G_3	G_4	G_5
S_1	$< 0.6, 0.3 >$	$< 0.5, 0.4 >$	$< 0.4, 0.5 >$	$< 0.4, 0.4 >$	$< 0.4, 0.5 >$
S_2	$< 0.5, 0.4 >$	$< 0.4, 0.3 >$	$< 0.5, 0.2 >$	$< 0.5, 0.4 >$	$< 0.6, 0.3 >$
S_3	$< 0.7, 0.2 >$	$< 0.5, 0.3 >$	$< 0.8, 0.1 >$	$< 0.5, 0.3 >$	$< 0.5, 0.2 >$
S_4	$< 0.4, 0.5 >$	$< 0.6, 0.3 >$	$< 0.3, 0.6 >$	$< 0.7, 0.1 >$	$< 0.6, 0.1 >$

Table 2: Intuitionistic fuzzy decision matrix $X(t_2)$ at period t_2

t_2	G_1	G_2	G_3	G_4	G_5
S_1	$< 0.4, 0.5 >$	$< 0.7, 0.2 >$	$< 0.6, 0.2 >$	$< 0.7, 0.2 >$	$< 0.4, 0.5 >$
S_2	$< 0.5, 0.4 >$	$< 0.6, 0.1 >$	$< 0.4, 0.3 >$	$< 0.5, 0.4 >$	$< 0.6, 0.3 >$
S_3	$< 0.7, 0.1 >$	$< 0.5, 0.3 >$	$< 0.5, 0.2 >$	$< 0.6, 0.1 >$	$< 0.5, 0.4 >$
S_4	$< 0.8, 0.1 >$	$< 0.4, 0.4 >$	$< 0.7, 0.2 >$	$< 0.5, 0.3 >$	$< 0.3, 0.5 >$

Table 3: Intuitionistic fuzzy decision matrix $X(t_3)$ at period t_3

t_3	G_1	G_2	G_3	G_4	G_5
S_1	$< 0.3, 0.5 >$	$< 0.5, 0.2 >$	$< 0.5, 0.3 >$	$< 0.5, 0.4 >$	$< 0.6, 0.1 >$
S_2	$< 0.3, 0.6 >$	$< 0.6, 0.3 >$	$< 0.7, 0.2 >$	$< 0.8, 0.1 >$	$< 0.4, 0.5 >$
S_3	$< 0.1, 0.7 >$	$< 0.4, 0.4 >$	$< 0.6, 0.3 >$	$< 0.2, 0.7 >$	$< 0.1, 0.4 >$
S_4	$< 0.2, 0.7 >$	$< 0.3, 0.4 >$	$< 0.5, 0.1 >$	$< 0.5, 0.3 >$	$< 0.2, 0.5 >$

According to model $(M-1)$ and model $(M-5)$ in steps 1 and 2, using Lingo11.0 software to solve the attribute weights $\omega_j(t_k)$ and time weights $\eta(t_k)$ in different time periods. As shown in Table 4.

Table 4: Time weights and attribute weights

	$\eta(t_k)$	$\omega_1(t_k)$	$\omega_2(t_k)$	$\omega_3(t_k)$	$\omega_4(t_k)$	$\omega_5(t_k)$
t_1	0.121	0.193	0.183	0.217	0.207	0.199
t_2	0.357	0.225	0.202	0.196	0.205	0.173
t_3	0.522	0.195	0.204	0.184	0.210	0.207

According to step 3, using $\omega_j(t_k)$ and $X(t_k)$ in table 4, calculate weighted intuitionistic fuzzy decision matrices in different periods, and then according to step 4, using the DIFWG operator to gather the weighted intuitionistic fuzzy decision matrices at different periods, and obtain dynamic comprehensive intuitionistic fuzzy decision matrix, as shown in table 5.

Table 5: Dynamic comprehensive intuitionistic fuzzy decision matrix

	G_1	G_2	G_3	G_4	G_5
S_1	$<0.183, 0.757>$	$<0.155, 0.740>$	$<0.132, 0.787>$	$<0.155, 0.793>$	$<0.125, 0.784>$
S_2	$<0.114, 0.851>$	$<0.157, 0.740>$	$<0.146, 0.757>$	$<0.218, 0.715>$	$<0.122, 0.840>$
S_3	$<0.168, 0.728>$	$<0.112, 0.811>$	$<0.156, 0.759>$	$<0.168, 0.694>$	$<0.163, 0.740>$
S_4	$<0.143, 0.809>$	$<0.179, 0.739>$	$<0.138, 0.726>$	$<0.143, 0.763>$	$<0.103, 0.789>$

According to step 5, ideal positive solution X^+ and negative ideal solution X^- of dynamic comprehensive intuitionistic fuzzy matrix can be obtained.

$$X^+ = (<0.183, 0.728>, <0.179, 0.739>, <0.156, 0.726>, <0.218, 0.694>, <0.163, 0.740>)$$

$$X^- = (<0.114, 0.851>, <0.112, 0.811>, <0.132, 0.787>, <0.143, 0.793>, <0.103, 0.840>)$$

According to step 6 and step 7, we suppose that compromise coefficient is $\lambda = 0.5$, calculating by Eq. (15) ~ Eq. (17), we can obtain the group effectiveness value U_i, individual regret value K_i and compromise value Z_i of each alternative.

Table 6: The value of U_i, K_i and Z_i of each alternative

	U_i	K_i	Z_i	Ranking
S_1	0.6070	0.2015	0.947	3
S_2	0.6086	0.2050	1.000	4
S_3	0.4657	0.2010	0.445	2
S_4	0.4911	0.1687	0.089	1

As showed in Table 6, the ranking of compromise value is $Z_4 < Z_3 < Z_1 < Z_2$. So according to step 8, we can determine the corresponding descending ordering of R&D partners is $S_4 > S_3 > S_1 > S_2$, so S_4 is the best partner.

In addition, by using the data of dynamic comprehensive intuitionistic fuzzy matrix in Table 5, and the IFWA operator and the score function of IFNs, we can obtain the ranking of alternatives $S_4 > S_3 > S_1 > S_2$, which is in accord with results of the proposed method. But this calculation process is more comprehensive than VIKOR method, and the difference of evaluation results between alternatives is lower, so VIKOR method has higher precision of identification, can be showed more apparent ranking results. Thus, the proposed method of dynamic MADM problem is reasonable and feasible.

We compare the proposed method in this paper with TOPSIS method presented in literature [13] and literature [14], and the ranking results of alternatives is $S_3 > S_1 > S_4 > S_2$, which is not in line with the proposed algorithm in this paper and the method based on IFWA operator and score function. This is because the TOPSIS method has a drawback, the optimal solution maybe the one is not only the closest to the positive ideal solution (PIS), but also the closest to the negative ideal solution(NIS), but VIKOR method can effectively overcome the shortcomings of traditional TOPSIS method, in which the aggregation function takes account of group effectiveness maximization and the individual regret minimization, and we can obtain the optimal solution to meet with the closest distance from ideal solution and the farthest distance from negative ideal solution, thus this method is more reasonable and scientific.

6. Conclusion

This paper presents a dynamic intuitionistic fuzzy multi-attribute decision making method based on time preference. In this method, time degree function is introduced to denote subjective preference, and a multi-objective nonlinear programming model is established to determine time weights, which takes a full consideration of the subjective characteristics of decision makers in time preference and objective preference toward time sequence information, enhancing accuracy of the solved time weight. We use VIKOR method to sort schemes taking account of group effectiveness maximization and the individual regret minimization, and obtain the optimal scheme that is the closest to ideal solution, which considers the psychological state of decision-makers, and can fully excavate the subjective preference characteristics of decision makers, and the objective information of sample in difference time series. So this method can overcome the possible drawback of reversed order that may occur in ordering result by using traditional TOPSIS method.

In the future work, we will further extend this method in solving dynamic MADM problem involving attribute information of interval valued intuitionistic fuzzy numbers, triangular and trapezoidal intuitionistic fuzzy numbers.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control, 8(1965) 338–353.
[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986) 87–96.
[3] K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31(1989) 343–349.
[4] Z. S. Xu, R. R. Yager, Power-geometric operators and their use in group decision making, IEEE Transactions on Fuzzy Systems, 8(2010) 94–105.
[5] Z. S. Xu, Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators, Knowledge-Based Systems, 24(2011) 749 –760.
[6] Z. S. Xu, On multi-period multi-attribute decision making, Knowledge-Based Systems, 21(2008) 164–171.
[7] W. Yang, Y. F. Pang, An uncertain dynamic weighted averaging geometric operator and its application multiple attribute decision making, Mathematics in practice and theory, 41(2011) 14–18.
[8] J. H. Park, H. J. Cho, Y. C. Kwun, Extension of the VIKOR method to dynamic intuitionistic fuzzy multiple attribute decision making, Computers and Mathematics with Applications, 65(2013) 731–744.
[9] Z. S. Xu, R. R. Yager, Dynamic intuitionistic fuzzy multi-attribute decision making, International Journal of Approximate Reasoning, 48(2008) 246–262.
[10] C. Tan, X. Chen, Dynamic similarity measures between intuitionistic fuzzy sets and its application, International Journal of Fuzzy System, 16(2014) 511–519.
[11] J. J. Mao, D. B. Yao, C. C. Wang, H. Y. Chen, Group decision-making method based on time-series fuzzy soft sets, systems Engineering-Theory & Practice, 34(2014) 182–189.
[12] B. Sun, X. F. Xu. A dynamic stochastic decision-making method based on discrete time sequences, Knowledge-Based Systems, 105(2016) 23–28.
[13] D. Joshi, S. Kumar, Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making, Egyptian Informatics Journal, 15(2014) 97–104.
[14] T. Y. Chen, The inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making, Applied Soft Computing, 26(2015) 57–73.
[15] S. M. Chen, S. H. Cheng, T. C. Lan, Multi-criteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values, Information Sciences, s 367–368(2016) 279–295.
[16] G. W. Wei, Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making, Expert Systems with Applications, 38(2011) 11671–11677.
[17] M. C. Wu, T. Y. Chen, The ELECTRE multi criteria analysis approach based on Atanassov's intuitionistic fuzzy sets, Expert Systems with Applications, 38(2011) 12318–12327.
[18] L. Dymova, P. Sevastjanov, A new approach to the rule-base evidential reasoning in the intuitionistic fuzzy setting, Knowledge-Based Systems, 61(2014) 109–117.
[19] Z. S. Xu, Intuitionistic preference relations and their application in group decision making, Information Science,177(2007) 2363–2379.

[20] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Transactions on Fuzzy Systems, 15(2007) 1179–1187.
[21] J. X. Nan, D. F. Li, M. J. Zhang, TOPSIS for multi-attribute decision making in IF-set setting, Operations Research and Management Science, 17(2008) 34–37.
[22] S. M. Chen, J. M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems, 67(1994) 163–172.
[23] D. H. Hong, C. H. Choi, Multicriteria fuzzy problems based on vague set theory, Fuzzy Sets and Systems, 114(2000) 103–113.
[24] X. Z. Zhang, C. X. Zhu, L. Zhu, Generalized precedence ordering method with ranking preference for time series-based multi-attribute decision making, Chinese Journal of Management Science, 22(2014) 105–111.
[25] R. R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Transactions on Systems, Man, and Cybernetics, 18(1988) 183–190.
[26] S. Opricovic, G. H. Tzeng, Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research, 156(2004) 445–455.