A TOOL FOR THE COMPREHENSIVE ANALYSIS OF POWER SYSTEM DYNAMIC STABILITY

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Abstract - This is an applications paper reporting on the methodological basis and the development of a production-grade software package for use in the dynamic stability (or "small disturbance") analysis of large power systems. This package, known as EISEMAN (EIgenSystem Evaluation - Machine And Network), is capable of studying a wide range of dynamic stability phenomena, such as subsynchronous resonance, inter-machine rotor oscillations, and the effects of excitation and turbine-governor systems on stability. All of the eigenvalues of the system are calculated, as well as desired eigenvalue sensitivities. A noteworthy feature of this eigenvalue-based tool is its capability of modeling to various degrees of detail the dynamics of the power system components - the network, synchronous machines and control systems. This tool can also be used to develop root locus plots and the frequency response characteristics of the power system. The present version of EISEMAN can be used for the analysis of systems containing up to 250 machines, 1,500 buses, 2,000 lines, and 500 dynamic states. Several test cases demonstrate the application of this tool.

I. INTRODUCTION

The dynamic stability problem studies the dynamic behavior of a power system which has been subjected to small perturbations. As used in this paper, the term "dynamic" stability is synonymous with the recently adopted IEEE definition of "small disturbance" stability. Typical dynamic stability phenomena are self-excitation, network-torsional interactions, control system-related oscillations, inter-machine (rotor electromechanical) interactions, turbine-governor related oscillations, and monotonic instabilities associated with exceeding the (classical) steady-state power transmission limits of the system. The potential for the occurrence of dynamic instabilities has increased markedly due to recent trends in the design and operation of power systems, such as the operation of power systems closer to their (classical) steady-state stability limits, and the use of series compensation, fast-response excitation systems and machines with smaller H constants.

A strong need exists for analytical tools capable of studying the wide range of dynamic stability phenomena. The appropriate level of detail of models of the power system components - which include the transmission network, synchronous generators, and control equipment - is determined by the type of phenomena judged to be important in a particular stability study. The information in Table 1 points out the need for a wide range of models in general purpose dynamic stability analysis tools.

A second modeling issue focuses on the following question: How much of the system external to the study area needs to be represented in order to obtain meaningful results? This question and related ones, such as what transmission level marks the point below which network, machine and load dynamics can be neglected without significant effects, can at present only be dealt with on a trial and error basis. The questions themselves underscore the important need for tools which have the capability of analyzing very large power networks, and can also evaluate the sensitivity of stability predictions to the discarded portions of the power system.

Component Dynamics \ Phenomenon of Interest	Self-excitation	Network-torsional interactions (SSR)	Excitation system performance	Electromechanical (rotor) oscillations	Turbine-governor effects	Monotonic instability
Typical Frequency Range (Hz)	10-60	5-60	1-10	0.5-3	<1	0
Machine Stator Dynamics	R	R				
Machine Damper Dynamics	R	R	D	D		
Machine Field Dynamics	R	R	R	R	R	R
Machine Torsional Dynamics		R				
Exciter-Stabilizer Dynamics	R		R	R	R	R
Turbine-Governor Dynamics				D	R	
Network Dynamics	R	R				
Load Dynamics			D		D	

TABLE 1 - Component Modeling for Dynamic Stability Studies (R:required; D:desirable)

This paper reviews the statement, component modeling and analytical formulation of the dynamic stability problem. An efficient and original approach to the construction of the system matrix is presented. Significant computational aspects associated with the study of realistically-sized power systems are discussed. Finally, the application and versatility of EISEMAN, the production-grade software package developed at the Pacific Gas and Electric Company, are demonstrated by several test cases.

II. THE DYNAMIC STABILITY PROBLEM

The dynamic behavior of the power system operating in a balanced, three-phase mode may be described by a set of nonlinear differential equations:

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t)),\ \underline{x}(t_o) = \underline{x}_o \qquad (1)$$

Here, $\underline{x}(t)$ denotes the p-dimensional state of the system and $\underline{x}_o$ is the initial system operating point.

A dynamic stability study investigates the dynamic behavior of the power system in the "neighborhood" of a point $\underline{x}_s$ which lies on the trajectory of (1). Typically, the state $\underline{x}_s$ of interest is the operating state of the power system, $\underline{x}_o$. Because the focus of the study is on the small excursions of $\underline{x}$ about $\underline{x}_s$ in response to a small disturbance, it is only necessary to model the power system accurately in the neighborhood of $\underline{x}_s$. In order to investigate the so-called small-signal behavior of the power system, the system is "frozen" in the state $\underline{x}_s$ and the nonlinear differential equation (1) is linearized about $\underline{x}_s$. The dynamic stability of the state $\underline{x}_s$ is determined from the stability characteristics of the resulting set of linear equations

$$\dot{\underline{\Delta x}} = \frac{\partial \underline{f}}{\partial \underline{x}}\bigg|_{\underline{x}_s} \underline{\Delta x} \triangleq \underline{A}\ \underline{\Delta x} \qquad (2)$$

where $\underline{\Delta x}$ represents the incremental state vector about $\underline{x}_s$. The Jacobian in (2) is referred to as the system matrix. $\underline{A}$ is a function of all system parameters and the state $\underline{x}_s$ about which (1) is linearized. The eigenvalues of $\underline{A}$ ($\lambda_i = \sigma_i + j\omega_i$, $i = 1,2,\ldots p$) completely specify the nature of the modeled system's response (i.e., frequency and decay characteristics) to small disturbances. In the time domain, the components of the incremental state $\underline{\Delta x}$ are expressed as linear combinations of the modes $e^{\lambda_i t}$. The oscillatory frequencies of the system response are given by $\omega_i/2\pi$, and the σ_i define the decay rates of the corresponding modes. The system which is modeled by (2) will be stable if all σ_i are negative. It will be unstable if any σ_i is positive. If any of the σ_i are zero, the system will either be marginally stable or unstable, depending on the multiplicity of this eigenvalue.

Knowledge of the eigenvalues alone is not sufficient, however, for complete characterization of stability. The many uncertainties in the power system data, such as inaccuracy of data, variations in parameter values and variations in initial conditions, will cause the eigenvalues of $\underline{A}$ to differ from those of the actual power system. It is important to know how the excursions in parameter values affect the eigenvalues. For this reason, information about the sensitivity of the eigenvalues with respect to system parameters, such as excitation system gains, machine inertias and line reactances, is of importance. This information is obtained from the eigenvectors of $\underline{A}$ and $\underline{A}^T$ using the relationship

$$\frac{\partial \lambda_j}{\partial \alpha} = \frac{\underline{w}_j^T \left(\frac{\partial \underline{A}}{\partial \alpha}\right) \underline{u}_j}{\underline{w}_j^T \underline{u}_j} \qquad (3)$$

where $\underline{u}_j$ ($\underline{w}_j$) are the eigenvectors of $\underline{A}$ ($\underline{A}^T$) associated with λ_j, and α is a system parameter of interest [1].

Eigenvalue sensitivity can be used to ascertain which power system parameters have a major impact on the damping of particular modes. Once these parameters have been identified, the sensitivity information can also be used to estimate the changes in the parameters necessary to assure adequate damping. This approach can be used to estimate appropriate settings for tunable parameters, such as stabilizer gains. Moreover, knowledge of the eigensystem - the eigenvalues and the eigenvectors - suffices for the calculation of the frequency domain characteristics - gain and phase margins, Bode and Nyquist plots, etc. - of the power system.

Many approaches for dynamical system stability investigations are based on the determination of the spectrum (i.e., the set of eigenvalues) of the system matrix $\underline{A}$ [2] - [7]. A method for the automatic formulation of the system matrix for large, arbitrary linear systems was presented in [2]. The effects of different levels of modeling of power system components were investigated using a three machine power system in [3].

An imaginative frequency domain approach based on the multidimensional Nyquist criterion is reported in [8]. From the standpoint of computer resource requirements, this approach is more efficient than eigenvalue-based analysis techniques. However, it is difficult to relate the stability margin information obtained to particular system parameters when frequency response techniques are used. Eigensystem-based techniques can provide eigenvalue sensitivity information for any parameters of interest [2].

The dynamic stability tool reported in [5] utilizes a transient stability simulation program to construct the system matrix via numerical differentiation. This approach is attractive and relatively easy to implement; it may, however, be susceptible to problems of inaccuracy caused by the numerical instability which is associated in general with the numerical differentiation process [9].

A characteristic common to eigenvalue-based approaches is the requirement for large amounts of computer storage and time for the study of systems of high dimensionality. To overcome this problem, approaches that focus on evaluating a subset of the spectrum have been proposed [6], [7]. These techniques may, in certain cases, fail to indicate critical eigenvalues.

Our work in the dynamic stability area was undertaken with the aim of developing a tool to handle effectively the analysis of the wide range of dynamic stability phenomena for power systems of practical interest. We have implemented into a production-grade software package an eigenvalue-based dynamic stability analysis tool possessing a large degree of modeling flexibility and capable of studying large-scale systems. The EISEMAN package can be used to analyze systems with as many as 250 machines, 1500 buses, 2000 lines, and 500 dynamic states. The package has found practical application in the study of subsynchronous resonance problems and the planning of series compensation levels; the identification of possible torsional/water hammer interactions in hydro units; the study of rotor oscillation damping and inter-machine oscillations; and the study of the effects of excitation system and power system stabilizer parameters on system dynamic stability.

III. POWER SYSTEM MODELS

The constituent components of the power system - the synchronous machines and associated equipment, and the transmission network - are described in this section.

Synchronous Machine Electrical Dynamics

A three-phase two-pole synchronous machine consists of three identical, symmetrically placed, lumped armature windings (a,b,c) in the stator, and up to four lumped rotor windings (F,G,D and Q). The F coil represents the rotor field winding. The fictitious G coil, whose flux is in the quadrature axis, is used to represent the effects of eddy currents which circulate in the solid steel of the rotor. The damper windings are represented by the fictitious D and Q coils in the direct and quadrature axes, respectively [10].

The most detailed machine representation commonly used for the study of dynamic stability models the dynamics of the stator direct and quadrature axis windings plus the dynamics of the four rotor windings discussed above.

Simpler machine representations are often desirable or necessary for the following reasons:
(i) For the modeling of salient-pole machines, the representation of a fictitious "G" coil and its dynamics is not appropriate.
(ii) For studies in which only low-frequency phenomena are of interest, the modeling of stator dynamics may be neglected.
(iii) When modeling machines which are electrically "distant" from the study region, the detailed modeling of these machines may not be necessary. Some or all of the rotor and/or stator dynamics of these machines may be neglected.
(iv) When approximating the dynamics of "distant" portions of the power system by dynamic equivalents, a minimum amount of "equivalent synchronous machine" data may be available, necessitating the use of simpler machine representations.

Lower-order representations are derived by neglecting the dynamics of appropriate rotor and stator winding fluxes and currents. The number of dynamical states associated with the different synchronous machine models are given in Table 2.

Machine Model	Windings			State Variables	
	Stator	Rotor Direct Axis	Rotor Quadrature Axis	With Stator Dynamics	Without Stator Dynamics
Round Rotor with Damper Windings	d,q	F,D	G,Q	6	4
Round Rotor - No Damper Windings	d,q	F	G	4	2
Salient Pole with Damper Windings	d,q	F,D	Q	5	3
Salient Pole - No Damper Windings	d,q	F		3	1
Classical Transient Model	d,q	F with constant flux		2	0

TABLE 2 - Dynamic Modeling of Synchronous Machines

The effects of magnetic saturation in the generator may also be of interest in some studies. These nonlinear effects are typically accounted for by representing the mutual inductances of the machine as functions of the air-gap fluxes in the machine. Saturation of the leakage inductances is usually neglected [11].

Machine Shaft Dynamics

The machine shaft may be represented by a multi-mass - spring - dashpot system, using as many rotating masses as data availability permits [10]. The multi-mass model is typically used when subsynchronous resonance (SSR) and other torsional system-related oscillations are to be analyzed. For other studies, a single-mass mechanical system description is sufficient. The number of states associated with shaft dynamics is equal to twice the number of masses represented.

Auxiliary Equipment

The auxiliary equipment of interest in dynamic stability studies consists of excitation systems, power system stabilizers, and turbine-governor systems. Low frequency models of typical excitation systems are described in detail in [12]. For dynamic stability studies, linearized versions of these models are used. Power system stabilizers are commonly represented by a signal washout plus two lead-lag pairs, or by a signal washout plus a pair of complex poles and zeros. The input to the stabilizer can be the shaft slip of a machine mass, the machine terminal frequency deviation, or the machine accelerating power. Turbine-governor models are described in detail in [13].

Network Representation

Each transmission line is represented as a lumped RLC circuit. Lumped parameter representations are also used for series and shunt capacitors, shunt inductors, variable-tap transformers and phase-shifting transformers. When high-frequency dynamics may be of interest, the dynamics of the network's inductive and capacitive elements (i.e., the time derivatives of inductor currents and capacitor voltages) must be explicitly represented. This level of modeling is typically used in studies of network-machine interactions such as SSR and self-excitation. The dynamics of the elements in the *abc* coordinate system are transformed via Park's Transformation [14] to a synchronously rotating *dqo* reference frame, and the zero-sequence dynamics are dropped since balanced system operation is assumed. Each capacitor or inductor whose dynamics are explicitly represented gives rise to two dynamical states.

When the focus of a dynamic stability study is on the typically low-frequency inter-machine and inter-area electromechanical oscillations, the dynamics of the network elements may be neglected and the synchronous frequency, "steady-state" model of the network elements may be used. This component model is valid for oscillations at or near the synchronous frequency (ω_S). Since the "low-frequency" inter-machine modes of interest ($\omega_R/2\pi < 3$ Hz) are referred against a rotating *dqo* reference frame, then with respect to the stationary *abc* reference frame, these modes appear to be oscillations at frequencies of $(\omega_S \pm \omega_R) \cong \omega_S$. Thus, the approximation that the signals throughout the network are at the synchronous frequency is a reasonable one when only the "low-frequency" inter-machine phenomena are of interest.

Note that in both models of the network, "infinite buses" may be represented as direct connections to ground since, by definition, these buses have no incremental characteristics, and thus, their small-signal dynamic properties are identical to those of the ground node.

IV. CONSTRUCTION OF THE SYSTEM MATRIX

While conceptually straightforward, the actual automatic formulation of the system $\underline{A}$ matrix is quite complicated. The approach proposed below accomplishes the task in a computationally efficient manner. The network subsystem representation is assembled from the RLC element representations in one of two different ways, depending on whether network dynamics are modeled or neglected. When the dynamics of the network are represented, the capacitor voltages and inductor currents chosen as state variables must provide a minimal complete characterization of the network dynamics. A linearly independent set of variables is specified by choosing as state variables a set of capacitor voltages possessing the property that the associated capacitors form no capacitive looops, and a set of inductor currents possessing the property that the associated inductors form no inductive cutsets. The set of state equations which describe the network dynamics is then systematically formulated using the procedure developed by Kuh and Rohrer [15].

When the network dynamics are neglected, the synchronous steady-state description of the network is used:

$$\underline{0} = \underline{\tilde{i}} - \underline{Y}\,\underline{\tilde{v}} \qquad (4)$$

For a network with b nodes, $\underline{Y}$ is the 2b x 2b nodal admittance matrix, and $\underline{\tilde{i}}$ ($\underline{\tilde{v}}$) is the vector of nodal injections (voltages).

In both network representations, the *abc* network variables are transformed to the synchronously rotating reference frame through Park's Transformation.

The representations for each machine and its auxiliary equipment are coupled to the network representation by the machine stator current and voltage variables, which are related to the nodal injection and voltage variables at the machine buses. Because the machine stator currents and voltages are referenced against the rotating quadrature axes of the machines, these currents and voltages must be transformed to the synchronously rotating reference frame by a linear transformation of the form

$$\underline{T} = \begin{bmatrix} \cos\delta_h & -\sin\delta_h \\ \sin\delta_h & \cos\delta_h \end{bmatrix} \qquad (5)$$

where δ_h is the angle between the quadrature axis of machine h and the systemwide reference axis.

The equations which describe the network, machine and equipment subsystems are linearized about the system state $\underline{x}_s$. When network dynamics are represented, the linearized stator currents of machine h (Δi_{d_h} and Δi_{q_h}) are components of the machine/equipment state variable ($\underline{\Delta x}_{M_h}$), and the linearized terminal voltages of the machine (Δv_{d_h} and Δv_{q_h}) are components of the network state variable ($\underline{\Delta x}_N$).

The system representation is:

$$\begin{bmatrix} \dot{\underline{\Delta x}}_{M_1} \\ \dot{\underline{\Delta x}}_{M_2} \\ \vdots \\ \dot{\underline{\Delta x}}_{N} \end{bmatrix} = \begin{bmatrix} \underline{A}_{M_1,M_1} & \underline{0} & \underline{0} \cdots & \underline{A}_{M_1,N} \\ \underline{0} & \underline{A}_{M_2,M_2} & \underline{0} \cdots & \underline{A}_{M_2,N} \\ \vdots & \vdots & & \\ \underline{A}_{N,M_1} & \underline{A}_{N,M_2} & & \underline{A}_{N,N} \end{bmatrix} \begin{bmatrix} \underline{\Delta x}_{M_1} \\ \underline{\Delta x}_{M_2} \\ \vdots \\ \underline{\Delta x}_{N} \end{bmatrix} \qquad (6)$$

The matrix $\underline{A}_{N,N}$ is usually quite sparse, reflecting the fact that a node in a power network is directly connected to very few other nodes in the network. Moreover, the coupling matrices $\underline{A}_{M_h,N}$ and $\underline{A}_{N,M_h}$ are extremely sparse because their non-zero elements reflect the stator voltage and current inputs to the machine and network subsystems, respectively. Therefore the system matrix $\underline{A}$ has an "almost" block diagonal structure.

When network dynamics are not represented, consistency of modeling assumptions demands that the machine stator dynamics be neglected as well. In this case, the linearized equations for the machine/auxiliary equipment subsystems have the form of (7), in which the algebraic equations arise from the assumption that the incremental stator flux dynamics can be neglected.

$$\begin{bmatrix} \dot{\underline{\Delta x}}_{M} \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \underline{\tilde{Q}}_{M_h,M_h} & \underline{\tilde{R}}_{M_h,N} & \underline{\tilde{S}}_{M_h,N} \\ \underline{\tilde{T}}_{M_h,M_h} & \underline{\tilde{U}}_{M_h,N} & \underline{\tilde{V}}_{M_h,N} \end{bmatrix} \begin{bmatrix} \underline{\tilde{\Delta x}}_{M_h} \\ \underline{\tilde{\Delta v}}_{M_h} \\ \underline{\tilde{\Delta i}}_{M_h} \end{bmatrix} \qquad (7)$$

where $\underline{\tilde{\Delta i}}_{M_h} = \left[\Delta i_{d_h},\ \Delta i_{q_h}\right]^T$ and $\underline{\tilde{\Delta v}}_{M_h} = \left[\Delta v_{d_h},\ \Delta v_{q_h}\right]^T$.

For the network, the algebraic system

$$\underline{0} = \begin{bmatrix} \underline{\tilde{\Delta i}}_M \\ \underline{0} \end{bmatrix} - \begin{bmatrix} \underline{Y}_{MM} & \underline{Y}_{MN} \\ \underline{Y}_{NM} & \underline{Y}_{NN} \end{bmatrix} \begin{bmatrix} \underline{\tilde{\Delta v}}_M \\ \underline{\tilde{\Delta v}}_N \end{bmatrix} \qquad (8)$$

is used, where

$$\underline{\tilde{\Delta i}}_M = \left[\underline{\tilde{\Delta i}}_{M_1},\ \underline{\tilde{\Delta i}}_{M_2}, \ldots, \underline{\tilde{\Delta i}}_{M_k}\right]^T$$
$$\underline{\tilde{\Delta v}}_M = \left[\underline{\tilde{\Delta v}}_{M_1},\ \underline{\tilde{\Delta v}}_{M_2}, \ldots, \underline{\tilde{\Delta v}}_{M_k}\right]^T$$

and $\underline{\tilde{\Delta v}}_N$ is the vector of direct and quadrature axis nodal voltages at the non-machine nodes.

The equations in (7) and (8) are combined into a linear differential-algebraic system, and the algebraic equations are eliminated from the system formulation using sparsity-oriented Gaussian elimination. This results in the linear differential system

$$\frac{d}{dt}\begin{bmatrix} \underline{\tilde{\Delta x}}_{M_1} \\ \underline{\tilde{\Delta x}}_{M_2} \\ \vdots \\ \underline{\tilde{\Delta x}}_{M_k} \end{bmatrix} = \begin{bmatrix} \underline{\tilde{A}}_{M_1,M_1} & \underline{\tilde{A}}_{M_1,M_2} & \cdots & \underline{\tilde{A}}_{M_1,M_k} \\ \underline{\tilde{A}}_{M_2,M_1} & \underline{\tilde{A}}_{M_2,M_2} & \cdots & \underline{\tilde{A}}_{M_2,M_k} \\ \vdots & \vdots & & \\ \underline{\tilde{A}}_{M_k,M_1} & \underline{\tilde{A}}_{M_k,M_2} & & \underline{\tilde{A}}_{M_k,M_k} \end{bmatrix} \begin{bmatrix} \underline{\tilde{\Delta x}}_{M_1} \\ \underline{\tilde{\Delta x}}_{M_2} \\ \vdots \\ \underline{\tilde{\Delta x}}_{M_k} \end{bmatrix} \qquad (9)$$

The system matrix in (9) is typically of much lower dimension than the system matrix which represents the same system with network dynamics (6). This is so because no dynamical states are associated with the network elements, and because for each machine, the number of states is reduced by two. However, the system matrix (9) is usually much less sparse than the system matrix (6).

V. EIGENSYSTEM EVALUATION

Once the system matrix $\underline{A}$ is constructed, the eigensystem of $\underline{A}$ may be evaluated. Because $\underline{A}$ is non-symmetric and has no special exploitable structure, a general eigenvalue evaluation scheme, based on the conversion of the system matrix to upper Hessenberg form and Francis' QR algorithm, is used. This algorithm is noted for its great accuracy, efficiency, and numerical stability [9]. However, the iterative solution technique which it employs destroys the sparsity of $\underline{A}$ in the course of the solution process. (For this reason, the loss of $\underline{A}$ matrix sparsity due to the Gaussian elimination of the algebraic network equations in the "low-frequency" formulation (9) is of little consequence.) Since the algorithm cannot exploit matrix sparsity, it is necessary to allocate computer storage for all p^2 elements of the p-dimensional system matrix $\underline{A}$ for the calculation of the eigenvalues.

Constructing the $\underline{A}$ matrix from the differential equations on the subsystem-by-subsystem basis discussed earlier keeps the number of computational operations associated with this step - matrix inversions, multiplications, etc. - small, thereby reducing both the computational resource requirements and numerical accuracy problems associated with the $\underline{A}$ matrix formation. Because little numerical error is introduced during the construction of the system matrix, the accuracy of the eigenvalues is limited primarily by the numerical precision of the eigenvalue evaluation subroutines. The EISPACK general-purpose eigenanalysis package is used for the calculation of the eigenvalues. For systems with as many as 500 state variables, the eigenvalues calculated have been shown to be accurate to $\pm 10^{-6}\ \mathrm{sec}^{-1}$.

For sensitivity analysis of the eigenvalues with respect to system parameters, eigenvectors of $\underline{A}$ and $\underline{A}^T$ must be evaluated. Relationship (3) indicates that only those eigenvectors associated with eigenvalues of interest need be determined. For this reason, it is most appropriate to calculate the necessary eigenvectors individually. An effective technique is the modified inverse iteration algorithm [16]. Sparsity-oriented techniques can be employed to great advantage during these calculations.

VI. THE EISEMAN PACKAGE

The EISEMAN dynamic stability analysis package is a computer tool which implements the component modeling features and solution methodology discussed above. The package automatically formulates the system $\underline{A}$ matrix and calculates all of its eigenvalues and the sensitivity of the eigenvalues to power system parameters of interest. In addition, EISEMAN can evaluate the eigenvalues of the machine torsional subsystems and the eigenvalues of the network subsystem. The package can also be used to produce root loci of eigenvalues with respect to system parameters.

A wide variety of component models have been incorporated into the EISEMAN package thus far:

- The synchronous machine rotor can be modeled using from 0 to 4 rotor circuits, as outlined in Table 2. The dynamics of the stator windings may be modeled or neglected, at the user's discretion.
- Detailed models which account for generator magnetic saturation are provided.
- The machine shaft may be modeled using as many masses as the user desires.
- Ten turbine-governor models, with as many as eight state variables each; nine excitation systems models, with as many as seven state variables each; and power system stabilizer models have been implemented.
- The network model - transmission lines, RLC shunt elements, and transformers - may be represented with or without dynamics, depending on the user's needs.
- Power system loads can be represented as having the dynamics of equivalent shunt impedance elements, infinite buses, equivalent synchronous machines, or they can be neglected.

With the exception of the eigenvalue calculation subroutines, all of the software for the EISEMAN package was developed at the Pacific Gas and Electric Company. The package has been implemented in a modular manner. Consequently, additional component models can be easily incorporated.

A most significant feature of the EISEMAN package is its capability of analyzing large dynamic systems. Power systems with up to 250 machines, 1,500 buses, 2,000 lines, and 500 dynamic states can be studied using the present version of the program. Insofar as the authors are aware, no other dynamic stability analysis tool possesses the capability of providing the complete spectrum for power systems of this size. This FORTRAN package has been implemented using variable dimensioning of arrays, so the capabilities and size of the program can be easily increased or decreased to conform to the applications and computer resources at hand.

An innovative feature of EISEMAN is the implementation of the algorithm discussed in Section IV for the construction of the $\underline{A}$ matrix. EISEMAN's unique approach to the setup of the network equations and the $\underline{A}$ matrix is computationally straightforward, and the sparsity-oriented techniques used to form both the system $\underline{A}$ and $\partial \underline{A}/\partial \alpha$ matrices are highly beneficial from the standpoints of preserving accuracy and reducing computer CPU time requirements. For large systems in which network dynamics are explicitly represented, typically fewer than 5% of the elements of the system matrix are non-zero. Very little numerical error is introduced in the evaluation of the elements of $\underline{A}$ and $\partial \underline{A}/\partial \alpha$.

The amount of computer memory required by the package is determined primarily by the dimension p of the state space of the system to be analyzed. The double-precision arithmetic and the QR algorithm employed for the calculation of the eigenvalues require that the program size be at least $8p^2$ bytes. The use of overlaying techniques confines the computer memory requirements to approximately $8p^2$ + 450 K of memory.

The CPU time used in the calculation of the eigenvalues has been empiricially determined to be roughly proportional to $p^{2.83}$. The CPU time used during the formation of the system matrix is usually quite small in comparison to the eigenvalue calculation time requirements.

VII. APPLICATION EXAMPLES

The usefulness of a versatile stability analysis tool is illustrated in this section by examples of EISEMAN's application to several power systems.

Subsynchronous Resonance Benchmark Test System

This test system was developed as a standard test case for the purpose of facilitating comparisons of analytical tools capable of studying SSR. The linearized model of the system consists of a single machine connected to an infinite bus through a series-compensated transmission line. The machine is modeled as a round rotor machine with damper windings in the direct and quadrature axes. A six-mass model describes the machine's torsional dynamics. No auxiliary equipment is represented. Data for this test case can be found in [18].

In order to analyze the interaction of the network and the machine torsional subsystems, network dynamics, stator dynamics, and machine torsional dynamics must all be explicitly represented. The eigenvalues of this system and their sensitivities to the series capacitance are presented in Table 3.

Complete Model	Sensitivity ($\partial\lambda/\partial C$)	Damper Windings Neglected	Single-Mass Model	Origin of Eigenvalue
$-4.62 \pm j\ 593.97$ $-4.51 \pm j\ 159.97$	$+0.02 \mp j\ 31.1$ $-1.59 \pm j\ 25.6$	$-4.41 \pm j\ 590.16$ $-4.34 \pm j\ 163.14$	$-4.62 \pm j\ 593.94$ $-3.40 \pm j\ 159.90$	RLC mode transformed to dqo frame
$-0 \pm j\ 298.18$ $+0.007 \pm j\ 202.82$ $+1.07 \pm j\ 160.44$ $+0.005 \pm j\ 127.08$ $+0.009 \pm j\ 99.46$	≈ 0 $+0.01 \mp j\ 0.09$ $+1.30 \pm j\ 5.96$ $-0.01 \mp j\ 0.05$ $-0.02 \mp j\ 0.23$	$-0 \pm j\ 298.18$ $+0.010 \pm j\ 202.81$ $+0.823 \pm j\ 161.07$ $+0.004 \pm j\ 127.07$ $+0.004 \pm j\ 99.41$		Machine torsional system
-41.122 -25.404	+0.194 +0.009		-41.122 -25.405	Damper windings
-0.948	≈ 0	-1.215	-0.948	"G" winding
-0.710	+0.076	-0.880	-0.710	Field Winding
$-1.107 \pm j\ 10.05$	$+0.15 \mp j\ 0.66$	$-1.311 \pm j\ 9.76$	$-1.125 \pm j\ 10.13$	Swing Dynamics

TABLE 3 - Eigenvalues for SSR Benchmark Test System (in sec^{-1})

Although each mode of an eigensystem will appear to a greater or lesser extent in the response of each of the state variables of the system (as determined by the eigenvectors of $\underline{A}$), for "weakly coupled" subsystems, it is often possible to associate the origin of certain system modes with one or more subsystems. The sensitivities of the eigenvalues to various system parameters reveal these relationships. Column 5 of Table 3 notes the correspondence between the system eigenvalues and the various subsystems. Columns 3 and 4 present the eigenvalues for the cases in which the damper dynamics and the torsional dynamics are not modeled.

Figure 1 presents the loci of the real parts of some of the eigenvalues as a function of the level of capacitive compensation. Note that as the compensation increases, the subsynchronous electrical frequency decreases, and the eigenvalues associated with the different torsional modes are destabilized in turn.

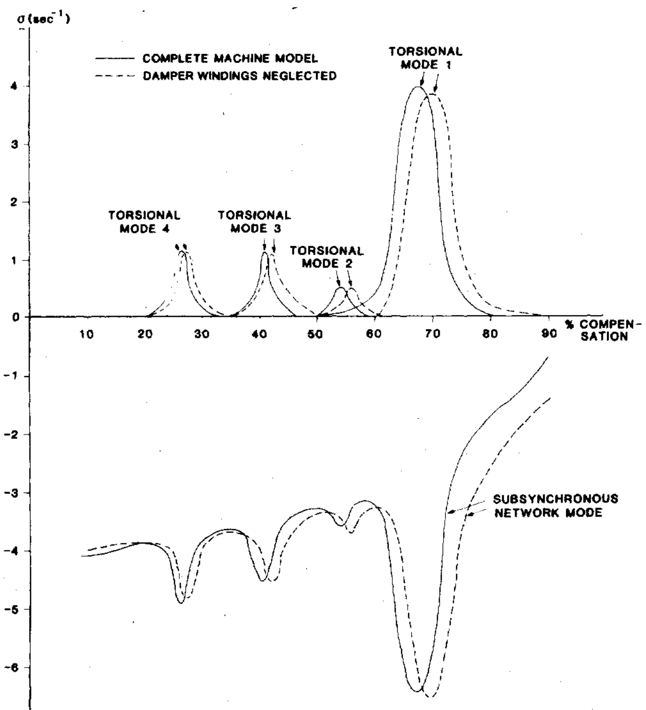

FIG. 1 - Damping of Eigenvalues as a Function of Compensation

<u>New England Test System</u>

The application of EISEMAN to the study of inter-machine oscillations is illustrated by this 10 machine, 39 bus, 46 line test case. This test system was developed as one representative of transmission systems in the Northeastern United States. The network diagram and data for this case can be found in [19].

The dynamic stability phenomena of primary interest in this study are the "low-frequency" 1 Hz - 2 Hz oscillations which are strongly related to the machine swing equations. In the base case studied here, network and machine stator dynamics, rotor transient dynamics (F and G windings), electromechanical dynamics (single-mass machine models are used), and excitation system dynamics are modeled. Saturation and turbine-governor dynamics are neglected. Loads throughout the network are modeled dynamically as equivalent RLC elements.

The eigenvalues calculated by EISEMAN for this case are shown in Table 4.

(i)	-1860.80 ±j 9214.40	(iv)
-65.79 ±j 376.92	-1860.50 ±j 8459.07	-0 467 ±j 8.96
-34.38 ±j 376.99	-1506.47 ±j 8309.14	-0.395 ±j 8.81
-30.43 ±j 376.94	-1807.80 ±j 7888.63	-0.368 ±j 8.61
-28.25 ±j 376.93	-1506.50 ±j 7554.60	-0.285 ±j 7.50
-27.59 ±j 376.91	-1807.86 ±j 7133.68	-0.113 ±j 7.09
-27.31 ±j 376.99	-735.03 ±j 6600.17	-0.295 ±j 6.94
-24.39 ±j 376.97	-1462.08 ±j 6557.92	-0.282 ±j 6.26
-22.71 ±j 376.99	-1176.09 ±j 6557.69	-0.302 ±j 5.80
-22.11 ±j 376.96	-1810.09 ±j 6043.49	-0.280 ±j 3.69
-15.37 ±j 376.99	-735.04 ±j 5846.19	(v)
-14.51 ±j 376.87	-1176.55 ±j 5804.38	-6.75
-12.35 ±j 376.93	-1462.64 ±j 5799.79	-6.05
-9.07 ±j 376.94	-935.82 ±j 5489.72	-5.30
-6.30 ±j 376.95	-1756.11 ±j 5132.24	-5.03
-6.02 ±j 376.95	-579.42 ±j 4931.06	-4.60
-3.75 ±j 376.95	-941.96 ±j 4707.17	-3.32
-2.29 ±j 376.99	-576.34 ±j 4172.11	-2.23
-0.76 ±j 376.98	-195.23 ±j 3770.10	-1.73
-0.46 ±j 376.99	-533.55 ±j 3599.51	-1.55
-0.38 ±j 376.99	-466.34 ±j 3284.82	-1.48
-0.35 ±j 376.99	-1234.50 ±j 3267.69	-1.40
-0.30 ±j 376.99	-194.94 ±j 3016.03	-1.32
-0.27 ±j 376.99	-535.33 ±j 2846.15	-1.29
-0.21 ±j 376.99	-482.92 ±j 2531.34	-1.08
-0.18 ±j 376.99	-1236.30 ±j 2486.23	-0.966
-0.13 ±j 376.99	-865.69 ±j 2134.70	-0.958
-0.11 ±j 376.99	-1037.42 ±j 1580.58	-0.906
-0.083 ±j 376.99	-864.39 ±j 1341.85	-0.195
-0.077 ±j 376.99	-1184.93 ±j 773.19	-0.802 ±j 1.70
-0.065 ±j 376.99	-1799.95 ±j 233.99	-0.826 ±j 0.859
-0.043 ±j 376.99	(iii)	-0.225 ±j 0.831
-0.036 ±j 376.99	-49.35	-0.412 ±j 0.632
-0.026 ±j 376.99	-49.35	-0.295 ±j 0.512
-0.004 ±j 376.99	-49.13	-0.657 ±j 0.503
-0.002 ±j 376.99	-48.30	-0.210 ±j 0.456
(ii)	-48.25	-0.125 ±j 0.424
-2715.21 ±j 9819.94	-17.71	-0.422 ±j 0.392
-992.84 ±j 9784.04	-15.85	-0.016 ±j 0.014
-1114.53 ±j 9416.02	-15.80	
	-15.74	

TABLE 4 - Eigenvalues for New England Test System (in sec^{-1})

The eigenvalues fall into five distinct groups:

(i) <u>Oscillatory modes at or very near the synchronous frequency</u>. These modes are closely associated with the R-L elements of the transmission network and loads. The purely real eigenvalues associated with R-L networks are transformed to oscillations at the synchronous frequency when viewed from the rotating reference frame. The neglect of load dynamics results in the elimination of all of the eigenvalues in this group for which $\sigma > -1$, reflecting the strong association between those system modes and the dynamics of the network elements used to model the loads.

(ii) <u>Other oscillatory modes at frequencies greater than 35 Hz</u>. Most of the eigenvalues in this group are closely associated with the representation of line-charging capacitors. The interaction of these capacitors with the inductive components of the transmission lines results in the creation of oscillatory modes. These modes occur at frequencies of 220 Hz and above, and reflect the relatively small effects of line-charging in tightly interconnected eastern U.S. power systems. The remaining eigenvalues in this group are associated with the capacitive elements which are used to models loads for which reactive power consumption is negative.

(iii) <u>Non-oscillatory modes with $-50 < \sigma < -15$</u>. The eigenvalues in this group are primarily associated with excitation system dynamics.

(iv) Oscillatory modes with $1 < \omega < 10$. The eigenvalues in this group correspond to the modes observed in the responses of the machine rotors to small disturbances on the power system. It is this group of eigenvalues which is the primary focus of this dynamic stability study.

(v) Lower frequency oscillatory modes and slowly decaying non-oscillatory modes. The eigenvalues in this group are related to the transient electrical dynamics of the machines (the field and "G" windings) and to excitation system dynamics.

The neglect of load dynamics and network dynamics has very little effect on the eigenvalues of primary interest, those in group (iv). It is important, however, to note the impacts of different degrees of modeling detail on the computational resources required for the analysis of system dynamic stability, as illustrated in Table 5.

Case	Dynamic States	Computer Memory (Bytes)	CPU Time (IBM 3033 OS/MVS)
New England Base Case	288	1110 K	55.2 sec
Line Charging Neglected	212	810 K	28.6 sec
Load Dynamics Neglected	248	940 K	40.4 sec
All Network Dynamics Neglected	66	400 K	2.2 sec

TABLE 5 - New England Test System - Comparisons of Computer Resource Requirements

Western United States Test System

This study demonstrates the application of EISEMAN to the study of potential subsynchronous oscillations on a realistically-sized power system. The system, illustrated in figure 2, is representative of those in the Western United States. The transmission network is composed of a 500 KV transmission system and an underlying 230 KV system. Series compensation is used throughout the 500 KV system in order to increase the power transmission capability of the transmission network.

Eight large synchronous machines connected to the 500 KV system are represented in this study using the most detailed synchronous machine model. Because SSR phenomena are the primary focus of the study, stator dynamics, network dynamics, and torsional dynamics must all be explicitly represented. No excitation systems are modeled. Four of the units are modeled with ten-mass torsional systems. The torsional systems of the four remaining machines are represented with six-mass models. Fifty-six oscillatory modes, with frequencies from 6 Hz to 600 Hz, are associated with these torsional subsystems.

As discussed earlier, several questions of major importance in the modeling of large power systems revolve around defining the system to be studied. How much of an interconnected power system should be modeled in order to obtain accurate results from a study? How should the portions of the system beyond the chosen study area be represented? In this case study, two possible models of the study network are examined. In the first (System A), only the 500 KV system is retained and the 230 KV system is modeled by equivalents at the 8 buses through which the 500 KV and 230 KV systems are interconnected. In the second (System B), both the 500 KV and 230 KV systems are

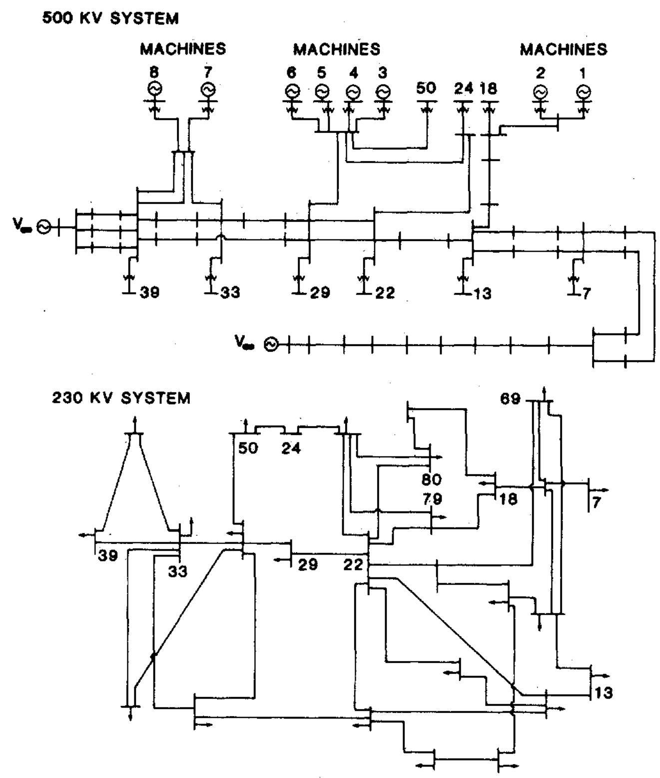

FIG. 2 - Western United States Test System

retained, with equivalents at the 15 buses which tie the 230 KV network to lower voltage networks and machines or to portions of the 230 KV system defined to be beyond the study area. The equivalents used here are calculated from the short circuit duty at the "boundary" buses using a short circuit analysis package. Each equivalent impedance developed in this manner is connected between the appropriate boundary bus and the infinite bus. The System A representation consists of 79 lines, 68 buses, and 370 state variables. The System B representation consists of 109 lines, 78 buses, and 428 state variables.

As in the New England test case discussed above, the eigenvalues of each of these systems can be classified into groups which are closely associated with the torsional, network, machine transient, machine subtransient, or electromechanical (rotor) dynamics. For either of the two system representations, all but two of the system modes appear to be adequately damped. The positive eigenvalues, which are closely related to the torsional subsystems of machines 1 and 2, are shown below:

System A: $+0.17 \pm j\ 126.9$
$+0.08 \pm j\ 114.9$
System B: $+0.12 \pm j\ 126.8$
$+0.11 \pm j\ 114.9$

The sensitivity analysis feature of the EISEMAN package can be used to determine which series capacitors have the greatest impact on the potential SSR problems, possibly leading to more appropriate choices for series compensation. This feature may also be used for studying the sensitivities of the eigenvalues to the system equivalents used. The sensitivities of all of the eigenvalues in the region of the two positively damped modes are presented in Table 6 for System A. For this system, the parameters of interest are the inductances of the equivalents which represent the 230 KV transmission system. The data in Table 6 shows that the unstable modes are most sensitive to the equivalent at bus 18.

Bus at Which Equivalent (L_{eq}) Is Located	Eigenvalues and Sensitivities ($\partial\lambda/\partial L_{eq}$) (in sec^{-1})					
	+0.17 +j 126.9	-0.09 +j 127.1	+0.09 +j 114.9	-0.08 +j 115.1	-8.9 +j 146.5	-9.2 +j 128.5
7	-0.27 -j 0.20	0	+0.15 -j 0.40	0	-0.01 +j 0.03	-2.3 +j 3.2
13	+0.53 -j 0.14	0	+0.24 -j 0.06	0	-0.23 +j 0.26	+5.9 +j 8.3
18	+1.3 +j 2.3	0	-0.40 +j 0.47	0	+6.2 +j 12.2	+24.6 +j 92.6
22	-0.08 +j 0.10	0	-0.07 +j 0.25	0	+17.1 +j 169.	+0.09 +j 0.10
24	-0.02 +j 0.01	0	-0.01 +j 0.03	0	+11.5 +j 40.2	+0.00 +j 0.00
29	0 -j 0.02	0	≈0	0	+5.9 +j 30.0	+0.19 +j 0.25
33	-0.01 +j 0.00	0	≈0	0	+7.7 +j 29.5	+0.02 +j 0.01
39	+0.06 -j 0.00	0	+0.03 +j 0.04	0	+0.67 +j 17.2	+0.90 +j 1.2

TABLE 6 - Sensitivities of Eigenvalues to Equivalents - System A

Bus at Which Equivalent (L_{eq}) Is Located	Eigenvalues and Sensitivities ($\partial\lambda/\partial L_{eq}$) (in sec^{-1})					
	+.12 +j 126.8	-.09 +j 127.1	+.11 +j 114.9	-.08 +j 115.1	-9.3 +j 142.8	-9.6 +j 125.4
69	-.02 +j .07	0	+.06 -j .05	0	+.01 +j .01	+.17 +j .04
79	+.22 +j .06	0	-.01 +j .02	0	+.09 +j 2.1	+.41 +j 8.8
80	+.19 +j .06	0	+.01 -j .04	0	-.37 +j 19.7	+1.80 +j 1.5

TABLE 7 - Sensitivities of Eigenvalues to Equivalents - System B

The sensitivities of the corresponding eigenvalues are displayed in Table 7 for System B. In this case, the system parameters of interest are the equivalents at buses 69, 79 and 80. (These are the "boundary" buses of the 230 kV system which are closest to bus 18, the bus at which the greatest sensitivity of the critical eigenvalues to the 230 KV equivalent was noted.) The sensitivity of the critical eigenvalues to the equivalents is markedly lower in this case. The use of the more detailed system representation decreases the sensitivity of the study results to the non-modeled portions of the power system. It must be kept in mind, however, that the more detailed representation increases the system dimension from 370 to 428 states, increasing the CPU time required for the computation of the system eigenvalues from 134 seconds to 211 seconds.

VIII. CONCLUSION

This paper has described in detail an effective eigenvalue-based tool, known as the EISEMAN package, which was developed for the comprehensive analysis of dynamic stability phenomena in power systems. The efficient manner in which EISEMAN handles the dynamic stability analysis of large power systems, the broad flexibility permitted in the modeling of power system components, and the original approach to the automated setup of the network equations make EISEMAN a unique tool. EISEMAN has been applied to study subsynchronous resonance phenomena, inter-machine oscillations, control system design and the influence of modeling detail in system component representation. Extensive use of EISEMAN bears out the effectiveness of this tool for the analysis of power system dynamic stability.

An important feature of the EISEMAN package is its modular nature. This permits the implementation of additional models in a rather straightforward manner. Extension of the work reported here is already underway. The scope of the system model is being expanded to incorporate DC transmission lines, static VAR sources and induction machine representations. Dynamic load models, as well as the application of coherency based dynamic equivalents [17] are being investigated. Results of work in these areas will be reported later.

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