A Traceable Block Cipher

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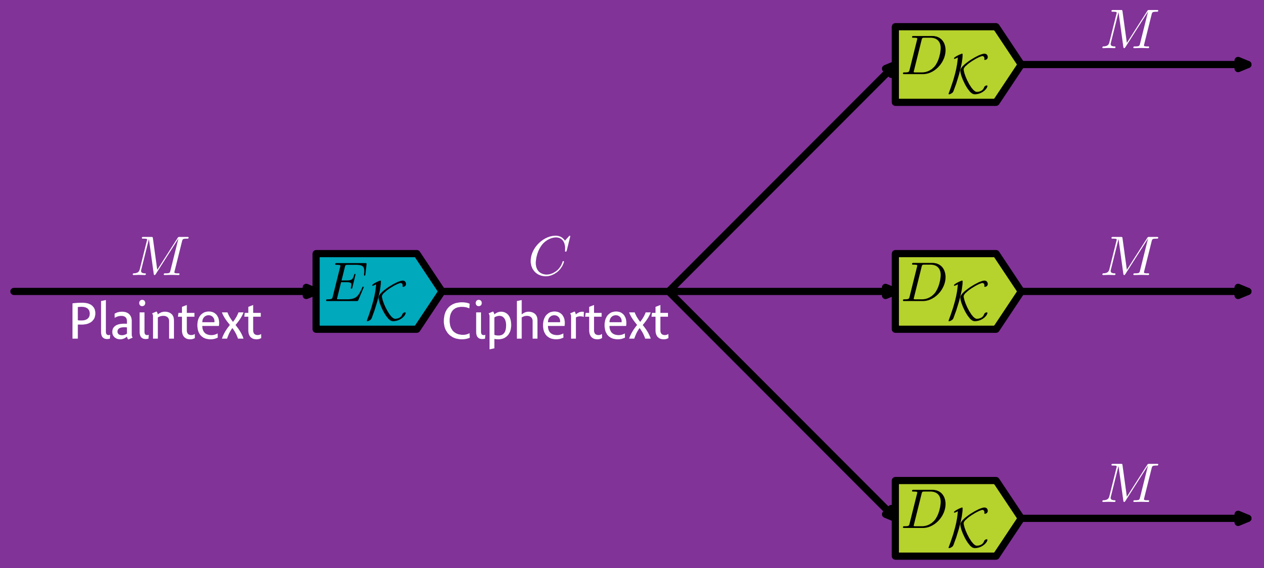
Content Distribution *Context*



Issues:

- Key Redistribution (by traitors to pirate users)
- Content Redistribution (not addressed here)

- Benny Chor, Amos Fiat, Moni Naor, 1994
- Each of the N users receives a personal key $\mathcal{K}_j$
  - » $\mathcal{K}_j$ enables user j to decrypt content
  - » $\mathcal{K}_j$ uniquely identifies user j
- No coalition of k traitors will produce an untraceable key
  - » allows a pirate to decrypt content
  - » conceals all traitors' identities

Traitor Tracing

- Four Procedures
  - Key Generation
  - Encrypt
  - Decrypt
  - Tracing

- Previous Constructions
  - Combinatorial Scheme [CFN 94, NP 98]
    headers $O(k \ln N)$
  - Asymmetric Algorithm [BF 99]
    expansion $O(k)$

Traceable Blockcipher

- $F_{\mathcal{K}}$ satisfies usual symmetric block cipher requirements
- generation from the meta-key $\mathcal{K}$ of keys $\mathcal{K}_j$ such that

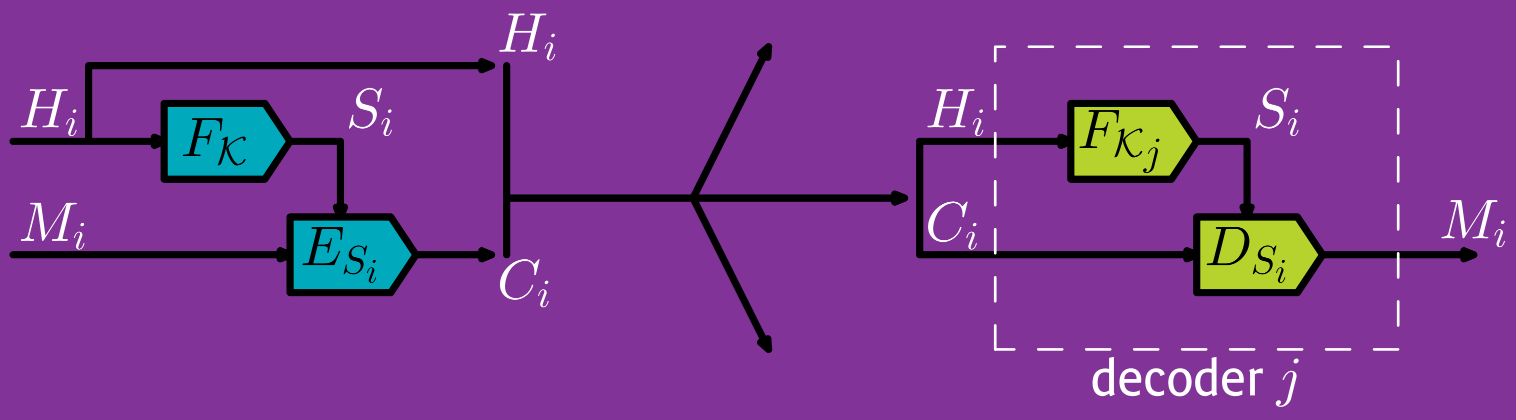
$$F_{\mathcal{K}} \equiv F_{\mathcal{K}_1} \equiv \cdots \equiv F_{\mathcal{K}_j} \equiv \cdots \equiv F_{\mathcal{K}_N}$$

- k-traceability requirement:

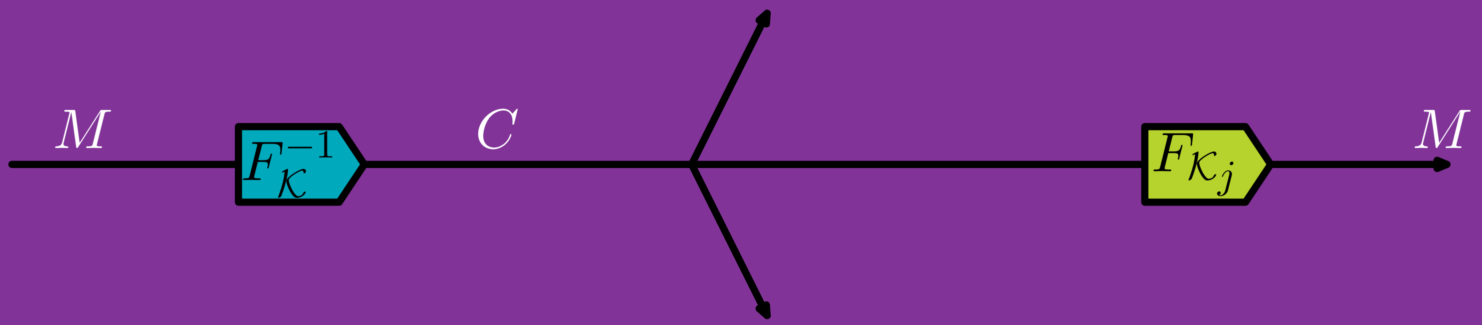
  an equivalent description produced from the knowledge of up to k equivalent descriptions $F_{\mathcal{K}_{j_1}}, \ldots, F_{\mathcal{K}_{j_k}}$ must reveal at least one of the identities $j_1, \ldots, j_k$

Operation Modes

- Mode with control words: $F_{\mathcal{K}} \equiv F_{\mathcal{K}_j}$



- Simple mode: $F_{\mathcal{K}}^{-1} \equiv F_{\mathcal{K}_j}$

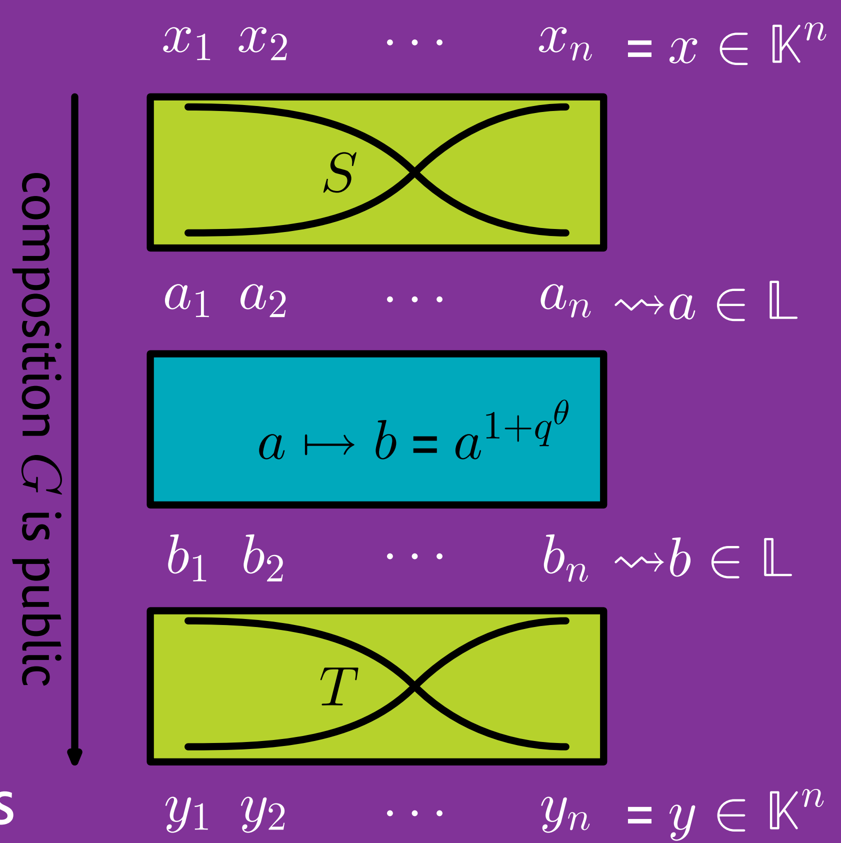


C^* Scheme *Matsumoto-Imai*

- parameters
  - » $\mathbb{K} = \mathrm{GF}(q) \quad q = 2^m$
  - » $\mathbb{L} \simeq \mathbb{K}^n$

    $\mathbb{L} = \mathbb{K}[X]/\pi_n(X)$
  - » $(1 + q^\theta) \perp (q^n - 1)$
- public key is a set of n quadratic equations in the variables x_i
- private key is (S, T)

  two invertible linear maps
- encrypt with G
- decrypt with $S^{-1} \circ g^{-1} \circ T^{-1}$



Underlying Problems

- Solving systems of multivariate equations
  - find one solution $(x_1, \ldots, x_n)$ over a finite field $\mathbb{K}$ of

$$\{y_i = P_i(x_1, \ldots, x_n)\}_{i \in [1,n]}$$

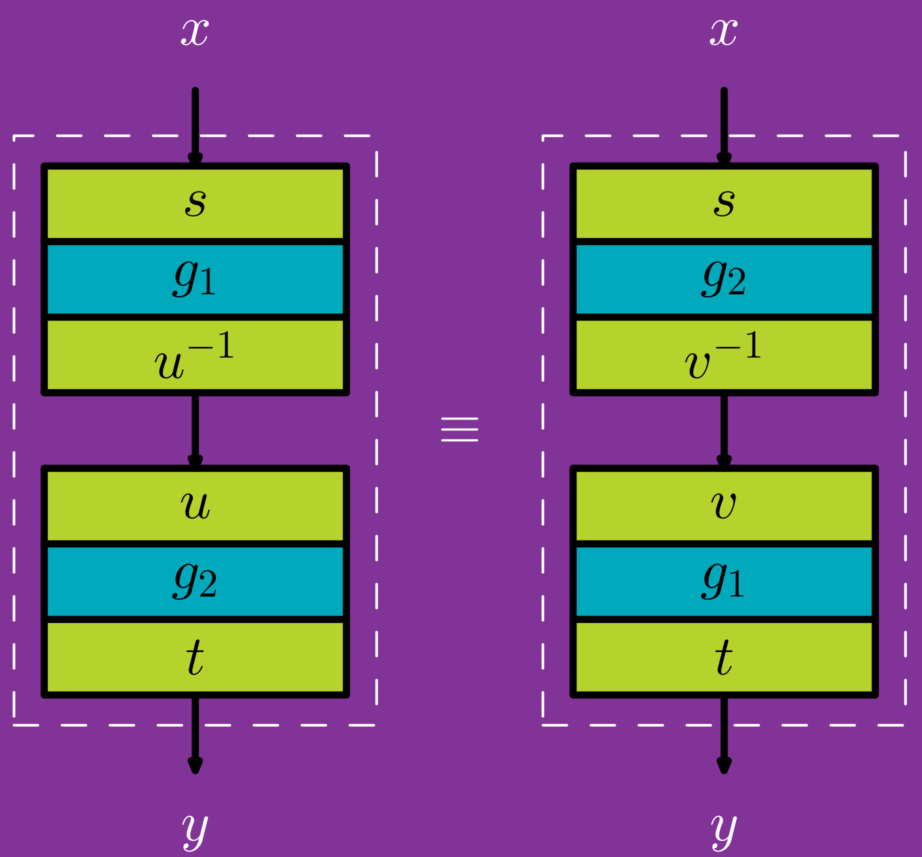
  - Decision problem is NP-complete, even over GF(2)
  - Patarin 1995 used structure of C^* to invert it
- IP: isomorphism of polynomials
  - given two sets of polynomials $\{P\}$ and $\{Q\}$
    find bijective linear maps A and B such that

$$B \circ (P_1, \ldots, P_n) \circ A = (Q_1, \ldots, Q_m)$$

  - IP is harder than IG
  - no polynomial algorithm is known [PGC, 1998]
  - relinearization attack for C^* degree 2 from [SK, 1999]

Commuting Blocks *Conducting Idea*

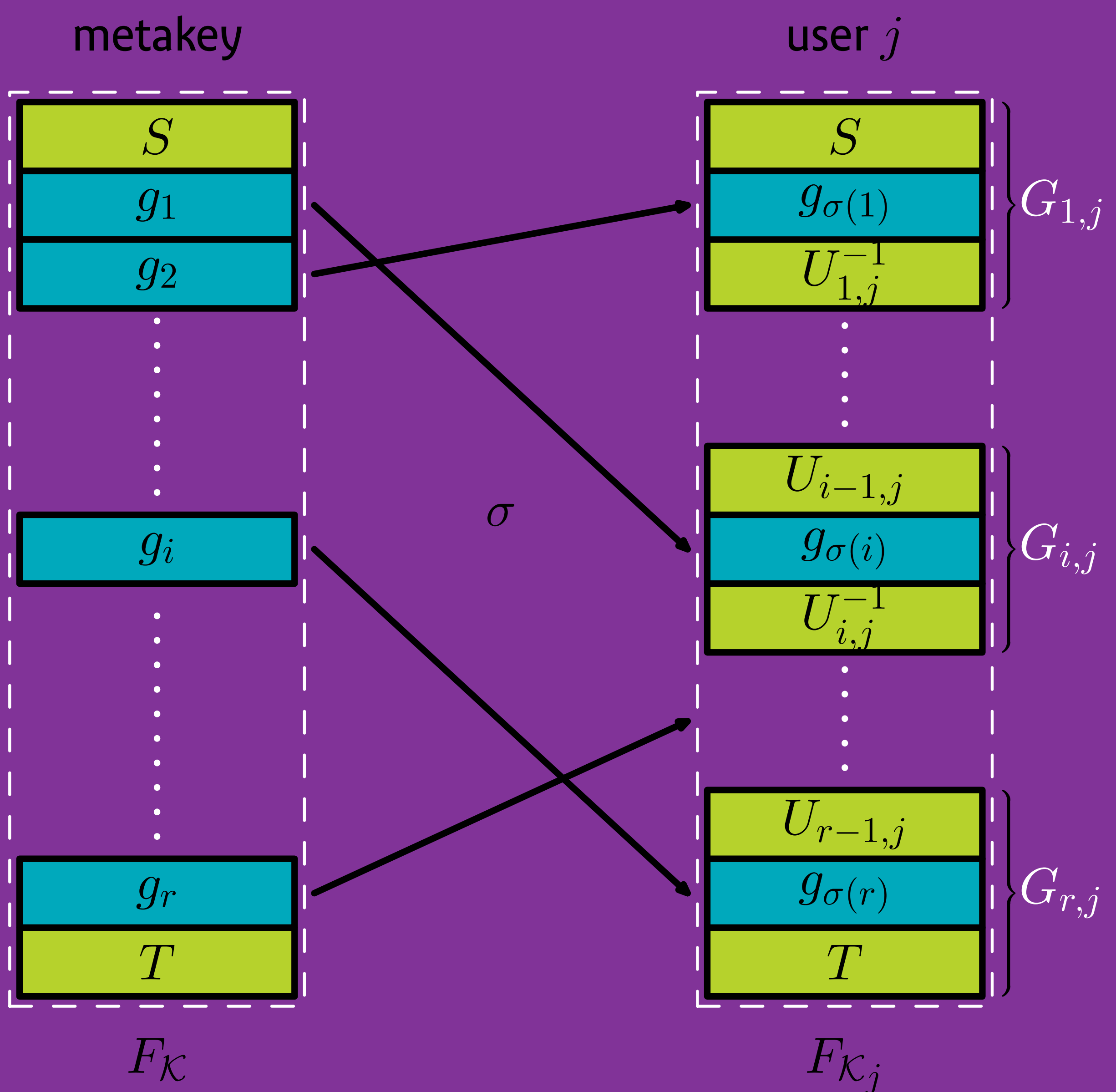
$g_1 \circ g_2 = g_2 \circ g_1$



- use a version of C^* with higher degree $d > 2$

$$g_i \; : \; a \mapsto b = a^{1+q^{\theta_1}+\ldots+q^{\theta_{d-1}}}$$

Commuting Blocks *Key Generation*



Parameters *Example*

- $q = 2^{16}$ $\mathbb{K} = \mathsf{GF}(q)$
- $n = 5$ block size is 80 bits
- $d = 4$

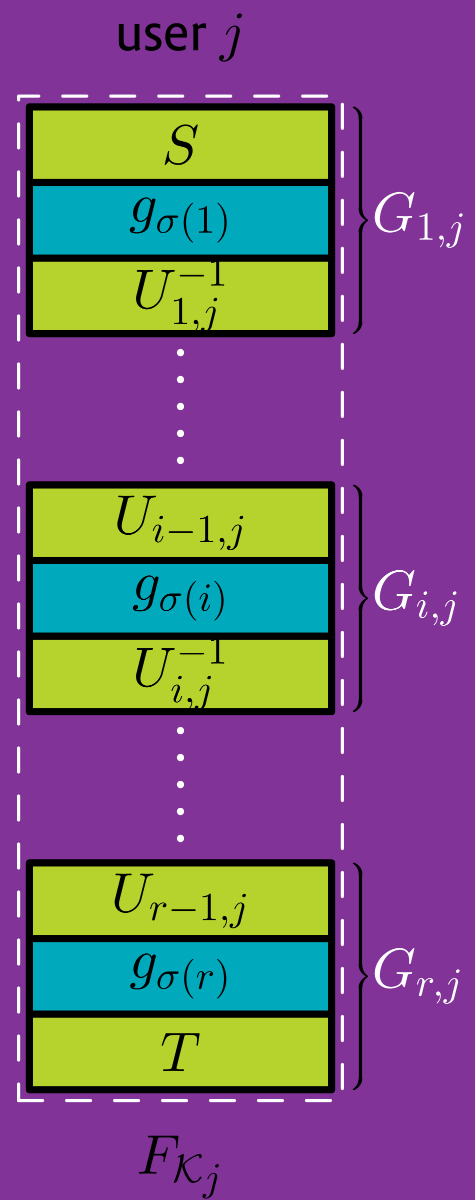
  equations for $G_{i,j}$ have degree 4

  about 70 monomials per equation

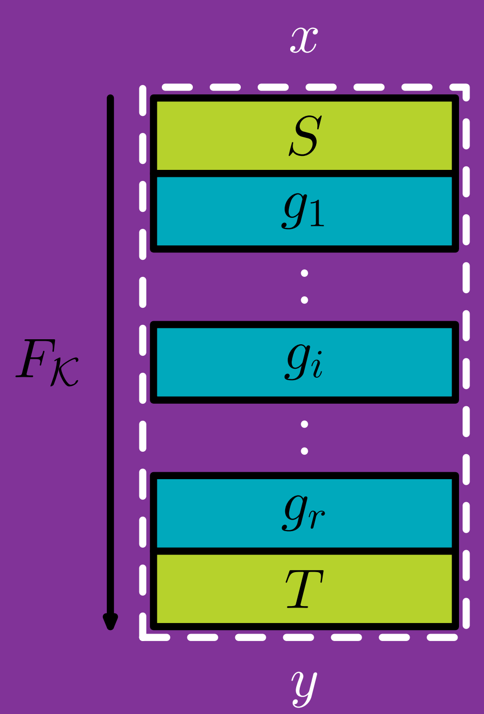
  computing $G_{i,j}$ is at most

  435 multiplications in $\mathbb{K}$
- $r = 32$ 32 rounds

  $F_{\mathcal{K}_j}$ is about 14000 mult. in $\mathbb{K}$
- size for $F_{\mathcal{K}_j}$ is 22 KB



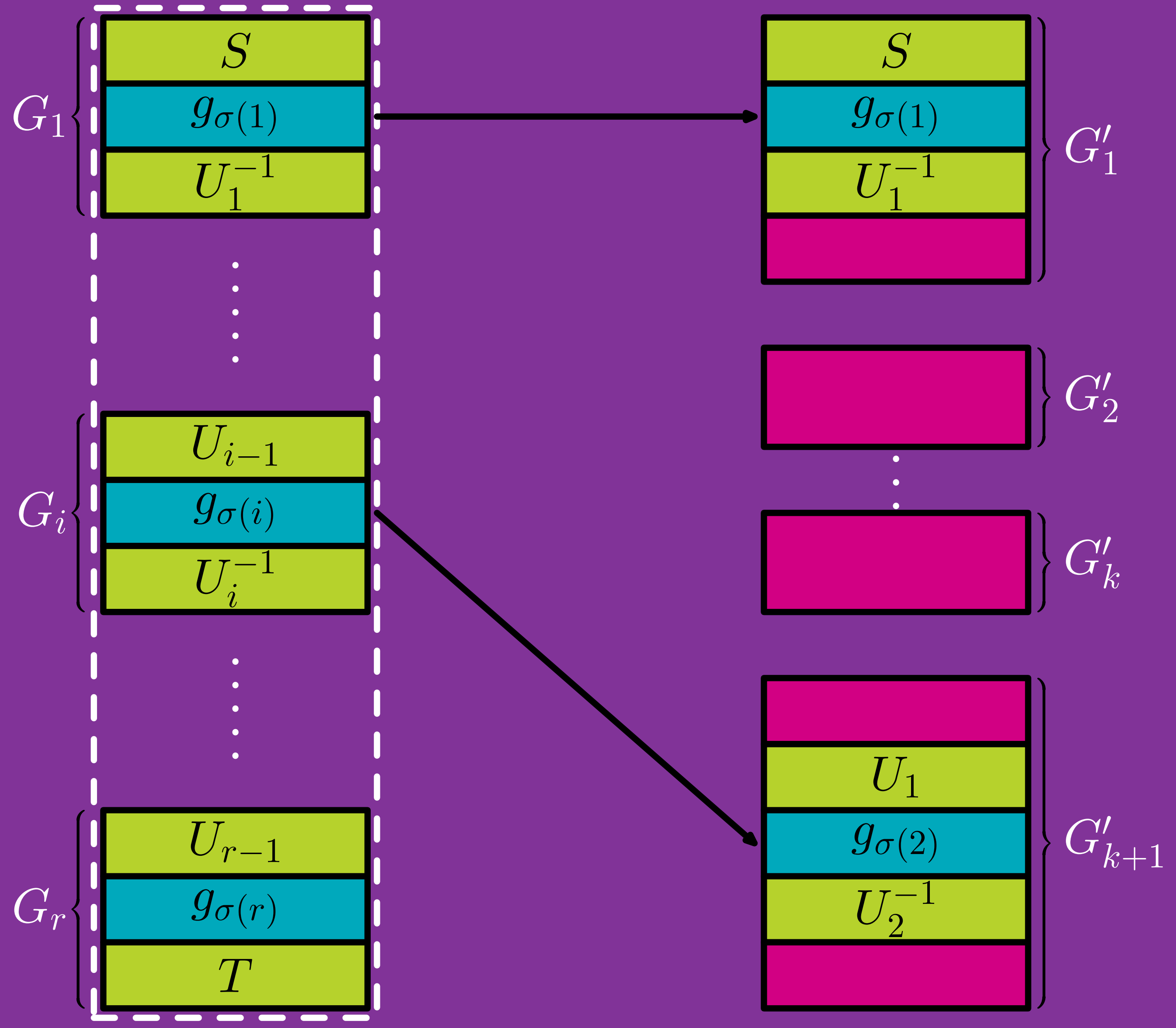
Security *as a Symmetric Cipher*



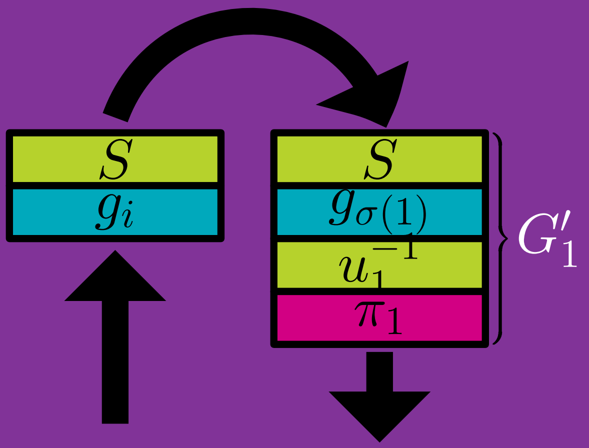
Input/Output observation must not allow

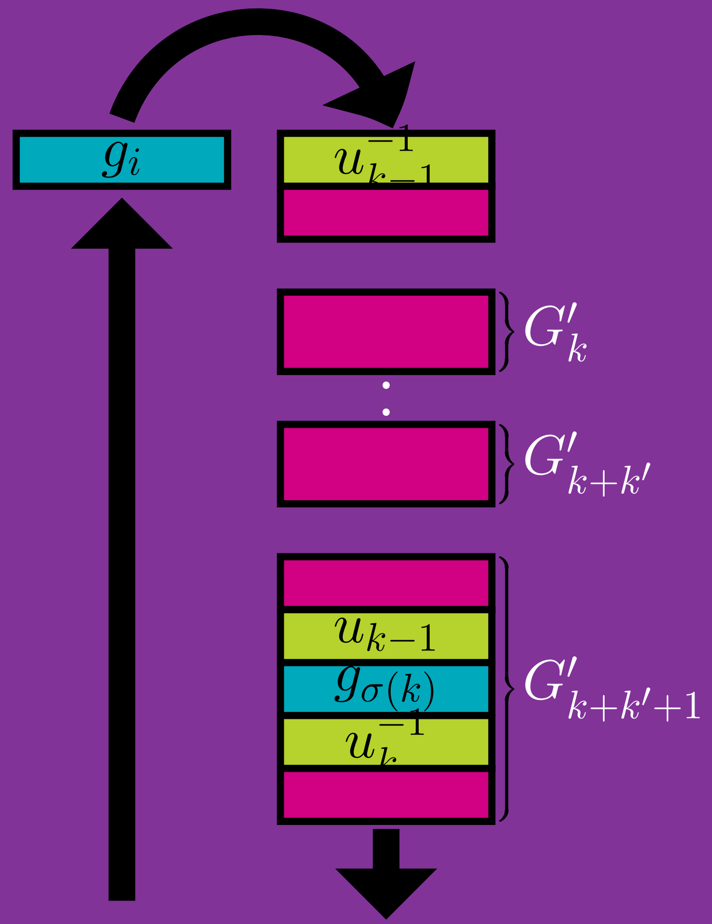
- to recover $F_{\mathcal{K}}$
- to interpolate $F_{\mathcal{K}}$
- to distinguish from a random permutation

Tracing One Traitor *Potential Strategy*



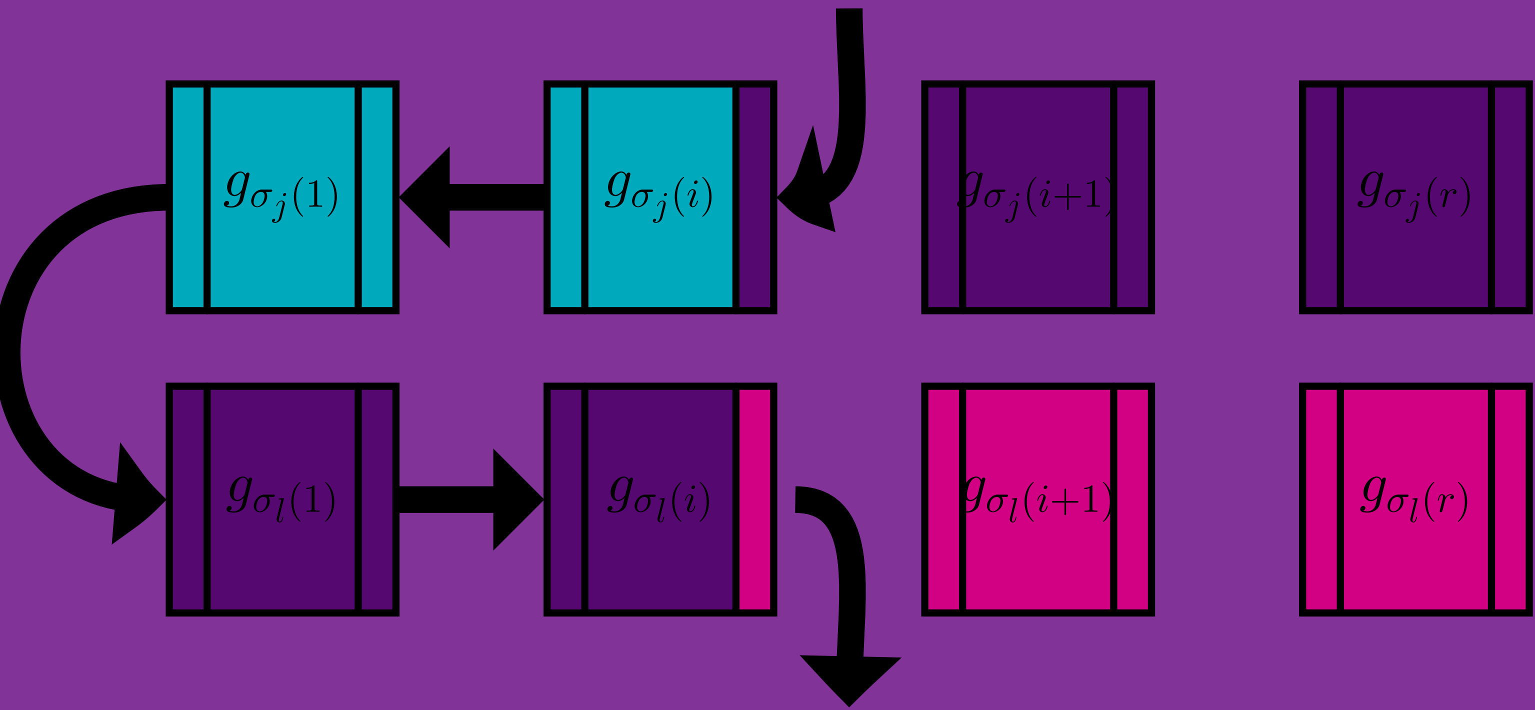
Tracing One Traitor





- step 1: guess $g_{\sigma(1)}$
- step i: guess $g_{\sigma(i)}$
- σ is known

Tracing *several Traitors*



- t-collision: $\{\sigma_j(i)\}_{i\in[1,t]} = \{\sigma_l(i)\}_{i\in[1,t]}$

- inner values reveal one identity

Conclusion

- Properties
  - » very low control word overhead: save bandwidth
  - » good behavior with high number of traitors
  - » good behavior with huge number of users: scalable
  - » speed of symmetric block cipher
  - » no black box yet
- Security
  - ? IP for extended C^* with degree higher than 2
- Applications
  - » White Box Cryptography
  - » Other instantiations