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A TRACKING FILTER FOR MANEUVERING SOURCES

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ABSTRACT

It is well known that the extended Kalman filtering methodology works well in situations characterized by a high signal-to-noise ratio, good observability and a valid state trajectory for linearization. This paper considers a problem not characterized by these favorable conditions. A large number of ad hoc modifications are required to prevent divergence, resulting in a rather complex filter. However, performance is quite good as judged by comparison of Monte-Carlo simulations with the Cramer-Rao lower bound, and by the filter's ability to track maneuvering targets.

I.   INTRODUCTION

Much work has been done in the past few years on the problems of tracking using active tracking systems [1,3,11-16]. The high signal to noise ratios and good observability properties of such problems have allowed the consideration of maneuvering as well as steady state trajectories [3,11,13-16]. The passive tracking problem, which uses signals radiated from the source itself as the source of information, has not received as much attention [7]. This is particularly true of the case where only one receiver is available to supply observations, due to the low signal to noise ratios and limited observability typically encountered. A solution to this problem based on the extended Kalman filter [2,8] and a simple maneuver detection and compensation scheme is presented here.

The specific problem treated is that of estimating the path of a source of CW energy. A single sensor is available to detect the center frequency of the signal and the direction from which it is arriving. The relative motion of the target and sensor produce Doppler shifts and source bearings that change through time. The sensor will be fairly modeled as observing these quantities in the presence of uncorrelated, zero mean Gaussian noise.

Since the frequency and bearing observations obtained at a single instant of time are insufficient to uniquely determine the position of the target, a model of the source behavior is unavoidable. The simplest model assumes that the source is moving at constant speed along a straight line path. Deviations from this path (maneuvers) can take many forms.

The simplest type, considered here, is a sudden change in speed or heading. After a maneuver, it is assumed that the target will return to a constant speed, straight line course. The center frequency of the signal is assumed to remain constant throughout the maneuver.

A previous solution to this problem was reviewed in [7]. A Cartesian state

$$x' = [x(t),y(t),v_x,v_y,f] \ ,$$

where the components of velocity v_x, v_y and the center frequency of the source are constant, was used as the basis for a standard extended Kalman filter. The current state estimate was used as the reference track, and mechanisms were included for both local and global iteration over the observations to reduce the effect of the nonlinearities [5], [8]. A class of gentle maneuvers could be modeled by adding fictitious system noise on the velocity states.

Subsequent work revealed three drawbacks to this approach. The most important was the requirement for a good path estimate early in the track about which linearization could take place so the filter would not diverge. In most cases, an estimate of the required quality is simply not available. Also, the filter tended to become conceited in the sense that its estimate of the state error covariance matrix was consistently smaller than the statistics of the actual errors, leading to an underweighting of high quality observations. Finally, the method of adding system noise to describe maneuvers posed problems. Too little noise restricted the class of maneuvers that could

be tracked; too much led to filter divergence as perfectly good velocity

information was exponentially forgotten. The present paper describes a

new approach to the problem that eliminates these drawbacks.

II. FILTER DESIGN

Design of a filter for this problem was to a large extent a trial

and error procedure. A number of less successful designs are described

in [7]; the present paper simply summarizes the final design.

An important innovation was the introduction of an alternative

to Cartesian coordinates, termed relative coordinates (Fig. 1).

The coordinates base the description of the track on the position and

speed of the target when it is at its closest point of approach (CPA)

to the sensor. The observations are then

$$y_\theta(t) = \tan^{-1}(\frac{v}{r}(t-t_o)) + \beta + w_\theta \tag{1}$$

$$y_f(t) = f(1 - \frac{v}{c} \sin \alpha(t)) + w_f \tag{2}$$

$$\sin \alpha(t) = \frac{v(t-t_0)}{\sqrt{r^2+v^2(t-t_0)^2}} , \tag{3}$$

where c is the speed of signal propagation. The problem is thus to

estimate the parameters. $X' = [-\frac{v}{r}t_o, \frac{v}{r}, \beta, f, -\frac{fv}{c}]$ of the track

from the time-varying non-linear observations (1)-(3).

To apply the extended Kalman filtering methodology to the problem,

linearization of the measurement equations is required. Consider first
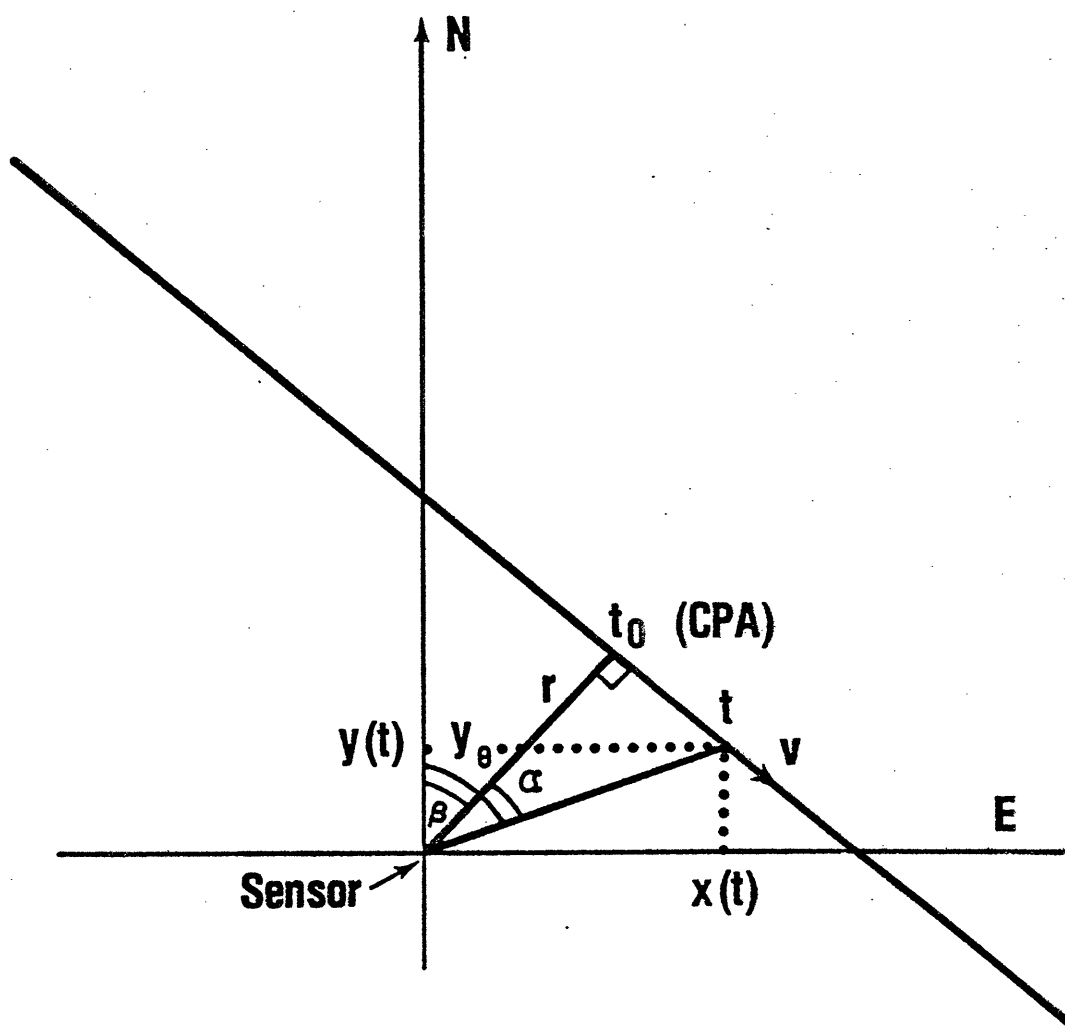
the bearing measurement,

Figure 1 Relative coordinates

$$y_\theta(t) = \tan^{-1}[(x_1 + x_2 t)] + x_3 + w_\theta \tag{4}$$

This equation can be transformed by application of the tangent function and linearizing about $x_3 + w_\theta = 0$ to give

$$\tilde{y}_\theta(t) = \tan(y_\theta(t)) \cong x_1 + x_2 t + (1 + (x_1 + x_2 t)^2)x_3 + (1 + (x_1 + x_2 t)^2)w_\theta$$

$$\tag{5}$$

If $|x_3 + w_\theta|$ is sufficiently small, one can approximate

$$x_1 + x_2 t \cong \tilde{y}_\theta(t) \tag{6}$$

yielding an approximate measurement equation

$$\tilde{y}_\theta(t) \cong x_1 + x_2 t + (1 + \tilde{y}_\theta^2)x_3 + (1 + \tilde{y}_\theta^2)w_\theta \tag{7}$$

The advantage of this form is that the linearized equation is linear in the states, rather than state deviations, and the coefficients can be evaluated without a reference trajectory. Thus an initial state estimate is unnecessary for the linearization to take place. The requirement that $|\beta + w_\theta|$ be small can be satisfied by having several trackers operating in parallel, each using a different reference value of β, namely γ_i, and each tracking the difference $\beta - \gamma_i$. The key to the

success of this approach is that the linearizations differ only by one parameter, which is hard to estimate early. The filter using the value of γ_i nearest the true value of β can be selected later in the path by a straightforward residual test on incoming observations.

Now consider the doppler measurement. Since

$$\sin \alpha(t) = \frac{v(t-t_0)}{\sqrt{r^2+v^2(t-t_0)^2}} \tag{8}$$

$$\approx \frac{\tilde{y}_\theta}{\sqrt{1+\tilde{y}_\theta^2}} \tag{9}$$

the observation becomes

$$y_f = x_4 + x_5 \frac{\tilde{y}_\theta}{\sqrt{1+\tilde{y}_\theta^2}} + w_f . \tag{10}$$

Again, these equations are linear in the states and require no reference information other than that provided directly by the measurements. However, writing the measurement in this form ignores the information on x_1 and x_2 contained in the frequency data.

The frequency observation equation can also be written as

$$\tilde{y}_f = x_4 + \frac{x_5}{\sqrt{1+(x_1+x_2t)^2}} x_1 + \frac{x_5 t}{\sqrt{1+(x_1+x_2t)^2}} x_2 + w_f \tag{11}$$

which brings out the dependency on x_1 and x_2. This equation is still linear in the states if an estimate of

$$\frac{x_5}{\sqrt{1+(x_1+x_2 t)^2}}$$

is available. If one regards the frequency observation as two observations, with perfectly correlated measurement noise, one can take advantage of both ways of writing the observation equation, thus gaining information on all states possible. This strategy results in a measurement matrix

$$\underline{M} = \begin{bmatrix} 1 & t & 1+\xi_1^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dfrac{\xi_1}{\sqrt{1+\xi_1^2}} \\ \dfrac{\xi_2}{\sqrt{1+\xi_1^2}} & \dfrac{\xi_2 t}{\sqrt{1+\xi_1^2}} & 0 & 1 & 0 \end{bmatrix} \tag{12}$$

and measurement noise covariance

$$\underline{R} = \begin{bmatrix} (1+\xi_1^2)^2 R_\theta & 0 & 0 \\ 0 & R_f & R_f \\ 0 & R_f & R_f \end{bmatrix} \tag{13}$$

where ξ_1 is the estimate of $x_1 + x_2 t$ used in the linearization (here $\xi_1 = \tilde{y}_\theta$) and ξ_2 the estimate of x_5 ($= \hat{x}_5$).

Two quantities need to be estimated for the measurement matrix $\underline{M}$ to be computed. The ξ_1 can be found using either the measurements as described earlier, or from the predicted state estimate as in a classical extended Kalman filter. Both of these approaches result in an estimate with known mean and variance. Since observation noise is independent of the error on the predicted state $\hat{x}(t|t-1)$, these approximations can be combined into one estimate which is likely to be better than either alone. If

$$\xi_1' = \tilde{y}_\theta \qquad \xi_1' \sim N(\xi_1, R_1') \qquad R_1' \stackrel{\sim}{=} R_{\theta i} \qquad (14)$$

$$\xi_1'' = \hat{x}_1 + \hat{x}_2 t \qquad \xi_1'' \sim N(\xi_1, R_1'') \qquad R_1'' = P_{11}(t|t-1)$$
$$+ 2tP_{12}(t|t-1) + t^2 P_{22}(t|t-1) \qquad (15)$$

the minimum variance linear combination is

$$\hat{\xi}_1 = \frac{R_1''}{R_1' + R_1''} \xi_1' + \frac{R_1'}{R_1' + R_1''} \xi_1''$$

$$\text{var} (\hat{\xi}_1) = \frac{R_1'' R_1'}{R_1' + R_1''} \qquad (16)$$

Combining these this way produces a considerable improvement in the estimation of $\underline{M}$. Early in the path, $\hat{\xi}_1$ is based mainly on the observations. After CPA, the observation noise increases, but the state estimate has become very good. Thus it is better to use it as a basis for computing $\underline{M}$.

A similar procedure can be used in the determination of an estimate of $\xi_2 = -\frac{fv}{c}$. One source of information is the state estimate

$$\xi_2' = \hat{x}_5 \qquad \xi_2' \sim N(\xi_2, R_2') \qquad R_2' = P_{55} \tag{17}$$

Since the state x_5 is difficult to observe early in the path, an a priori estimate of the velocity v_a is assumed to be available, along with a variance R_a. Typically, $R_a \overset{\sim}{=} v_a^2$, indicating a very crude guess.

$$\xi_2'' = -\frac{y_f v_a}{c} \qquad \xi_2'' \sim N(\xi_2, R_2'') \qquad R_2'' = \frac{y_f^2 R_a}{c^2} \tag{18}$$

and these two estimates can be combined to form $\hat{\xi}_2$.

The effect of this procedure is to base the approximation of ξ_2 on the a priori estimate early in the run. When the filter acquires $\hat{x}_5$, near CPA, the estimate switches over to it.

Thus the combination of information from several sources can be used to improve the estimation of the quantities necessary to evaluate $\underline{M}$. This was found in result in a decrease in the RMS errors incurred in the Monte Carlo simulations to be described later.

One fault that early versions of this tracker shared with many other extended Kalman filters is that the covariances computed by the filter were much smaller than the actual statistics of the state errors. This suggested that errors were being introduced by uncertainties not accounted for in the calculation of the covariance matrix. An important source of error was found to be the inaccurate estimation of $\underline{M}$.

If the errors on the $\hat{\xi}_i$ are made explicit by writing

$$\xi_i = \hat{\xi}_i + \delta\xi_i \qquad \delta\xi_i \sim N(0, R_{\xi i}) \tag{19}$$

the measurement matrix M becomes, to first order,

$$\underline{M} = \underline{M}(\hat{\xi}_1, \hat{\xi}_2) + \frac{\partial\underline{M}}{\partial\xi_1}\delta\xi_1 + \frac{\partial\underline{M}}{\partial\xi_2}\delta\xi_2 \tag{20}$$

and the observation equation

$$\underline{y}(t) \overset{\sim}{\sim} \underline{M}(\hat{\xi}_1, \hat{\xi}_2)\underline{x} + \frac{\partial\underline{M}}{\partial\xi_1}\underline{x}\delta\xi_1 + \frac{\partial\underline{M}}{\partial\xi_2}\underline{x}\delta\xi_2 + \underline{w} \tag{21}$$

Letting

$$\underline{\Lambda} = \left[\frac{\partial\underline{M}}{\partial\xi_1}\underline{x} \;\middle|\; \frac{\partial\underline{M}}{\partial\xi_2}\underline{x} \right] \tag{22}$$

this becomes

$$\underline{y}(t) = \underline{M}(\hat{\xi}_1, \hat{\xi}_2)\underline{x} + [\underline{I} \quad \underline{\Lambda}] \begin{bmatrix} w \\ \cdots \\ \delta\xi_1 \\ \cdots \\ \delta\xi_2 \end{bmatrix} \tag{23}$$

Thus the inaccuracies in the esimtates of the ξ_i have, to first order, the same effect as an additional noise process. The vector $w_c = [w' \quad \delta\xi_1 \quad \delta\xi_2]'$ is zero mean and has an easily determined covariance. This covariance is at least as large as the observation covariance R_s and reflects additional uncertainty due to the approximate evaluation of $\underline{M}$. Thus, the replacement of $\underline{R}$ by $\underline{R}_c = \text{cov}(\underline{w}_c)$ in the filter equations reflects the error process. This results in a larger, more accurate state error covariance estimate than that produced using $\underline{R}$ only. Note also that R_c, unlike R, is nonsingular.

Since the system model incorporates no dynamics and no system noise, the inverse form of the Kalman filter equations can be used, which in the case of a static state with no system noise reduce to

$$\underline{Z}(t+1) = \underline{Z}(t) + \underline{M}'(t) \; \underline{R}_c^{-1}(t) \; \underline{M}(t) \qquad \underline{Z}(t) = \underline{0} \qquad (24)$$

$$\underline{Zx}(t+1) = \underline{Zx}(t) + \underline{M}'(t) \; \underline{R}_c^{-1}(t) \; \underline{y}(t) \qquad \underline{Zx}(0) = \underline{0} \qquad (25)$$

$$\underline{P}(t) = \underline{Z}^{-1}(t) \qquad (26)$$

$$\hat{\underline{x}}(t) = \underline{P}(t) \; \underline{Zx}(t) \qquad (27)$$

where $\underline{M}(t)$ is the observation matrix, $R_c(t)$ the compensated observation noise covariance, $\underline{P}(t)$ the state error covariance, and $\hat{\underline{x}}(t)$ the state estimate. The computational simplicity of these equations offsets the need for several filters in parallel. The great advantage, however, lies in the fact that numerical errors caused by the matrix inversion when it is nearly singular are not propagated through time. This contrasts with the standard form in which the accumulation of numerical errors can lead to divergence. Also, no initial state and covariance are required to start the filter.

Extraction of the state estimate $\hat{x}$ from $\underline{Z}$ and $\underline{Zx}$ via (26) and (27)
is fraught with numerical error when $\underline{Z}$ is nearly singular. This is particularly
a problem here since the limited observability of the path causes $\underline{Z}$ to be
singular until near CPA. Two methods for avoiding these errors were included
in the tracker. The first simply suppresses the retrieval process if certain
tests on $\underline{Z}$ indicate that it is nearly singular. The state $\hat{x}$ is still set to
$\underline{P}^{-1} \cdot \underline{Zx}$, but $\underline{P}$ is set to a very large value. This allows a state estimate
to be produced even though it is known to be unreliable.

The second merely involves estimating the error introduced into $\hat{x}$ when
the retrieval is performed, and increasing the estimate of $\underline{P}$ to account
for it. Note that neither of these methods introduce compensations that
are directly propagated through time. The recursively computed quantities,
$\underline{Z}$ and $\underline{Zx}$, are left untouched. Of course, the choice of the optimal linear-
ization is affected, as intended.

III. MANEUVERS

The filter design of the previous section suffices to produce both an
accurate state estimate and a covariance estimate that closely matches the
actual error statistics. This latter point is extremely useful in detecting
maneuvers. Examination of the residuals

$$\underline{\gamma}(t) = y(t) - \hat{y}(t) \tag{28}$$

with respect to their covariance

$$\underline{R}_{\gamma} = \underline{R}_c(t) + \underline{M}(t)\underline{P}(t)\underline{M}'(t) \tag{29}$$

provides an indicator of the consistency of the observed measurement residuals with their expected magnitudes. The validity of this test is highly dependent on the existence of an accurate P, which is not available in many approximate nonlinear filtering algorithms.

Of course, the idea of examining filter residuals is common in the literature. The approach outlined in [18-19] suggested averaging these chi-square indicators over time to reduce the susceptibility of the tracker to short, unexpected measurement noise bursts. A long time window results in delay between the maneuver and its detection, making filter adjustment difficult. In this problem, the indicators were averaged over a time window of two observations with an extra term included to make use of the whiteness property of the residuals as well as their size.

$$\ell(t) = \underline{\gamma}'(t)\underline{R}_\gamma^{-1}(t)\underline{\gamma}(t) + [\underline{\gamma}(t) + \underline{\gamma}(t-1)]'[R_\gamma(t) + \underline{R}_\gamma(t-1)]^{-1}[\underline{\gamma}(t) + \underline{\gamma}(t-1)]$$

$$+ \underline{\gamma}'(t-1) \underline{R}_\gamma^{-1}(t-1)\underline{\gamma}(t-1) \tag{31}$$

The motivation for this type of residual test was the effect of changes in speed and heading on the observations. These appear almost instantly as a jump on the Doppler shift which remans biased for some time. The bearings exhibit a ramp type deviation, again remaining biased. Thus maneuvers are characterized by residuals that are not only large, but highly correlated through time.

When the $\ell(t)$ for the filter that is tracking the target is computed, it is compared against an empirically determined threshold. If it is larger than the threshold, the maneuver hypothesis is accepted and a compensation process is invoked. Otherwise, the system tracks normally.

The detection of a maneuver signals an increase in the uncertainty of only part of the state, that associated with v and β. If the maneuver occurs after the tracker has acquired a good estimate of the path, the x and y position of the target at the time of the maneuver is known. If the new heading β' and velocity v' could be estimated, the filters could be reset to incorporate an initial path estimate and covariance consistent with this information.

Since each of the parallel filters assumes that the target path has a bearing at CPA near its γ_i, it can be initialized to a path departing from the known x y location with bearing at CPA γ_i. The filter selection mechanism will use the filter which tracks the new path best as the output, so the heading change can be handled easily.

The new velocity can also be estimated, directly from the post maneuver Doppler shift. Since the center frequency of the transmitted signal is known, and does not change, the Doppler shift yields a velocity estimate. When the maneuver results in a CPA on the new path, this velocity estimate has a large variance. It can be combined with the a priori velocity estimate used in the optimal linearization procedure to limit the uncertainty.

Thus the geometry of the old path, the characteristic angle of each filter, and the postmaneuver observations are used to compute a new state for the filter. Once each of the filters are reset, the tracker operates in normal mode until the next maneuver detection.

IV. EVALUATION

The following figures illustrate the effectiveness of the tracking system for a typical trajectory. Estimated and actual RMS errors, averaged over 25 runs and normalized, are plotted on logarithmic scales. The Cramer-Rao lower bound for the problem was computed ignoring any a priori information
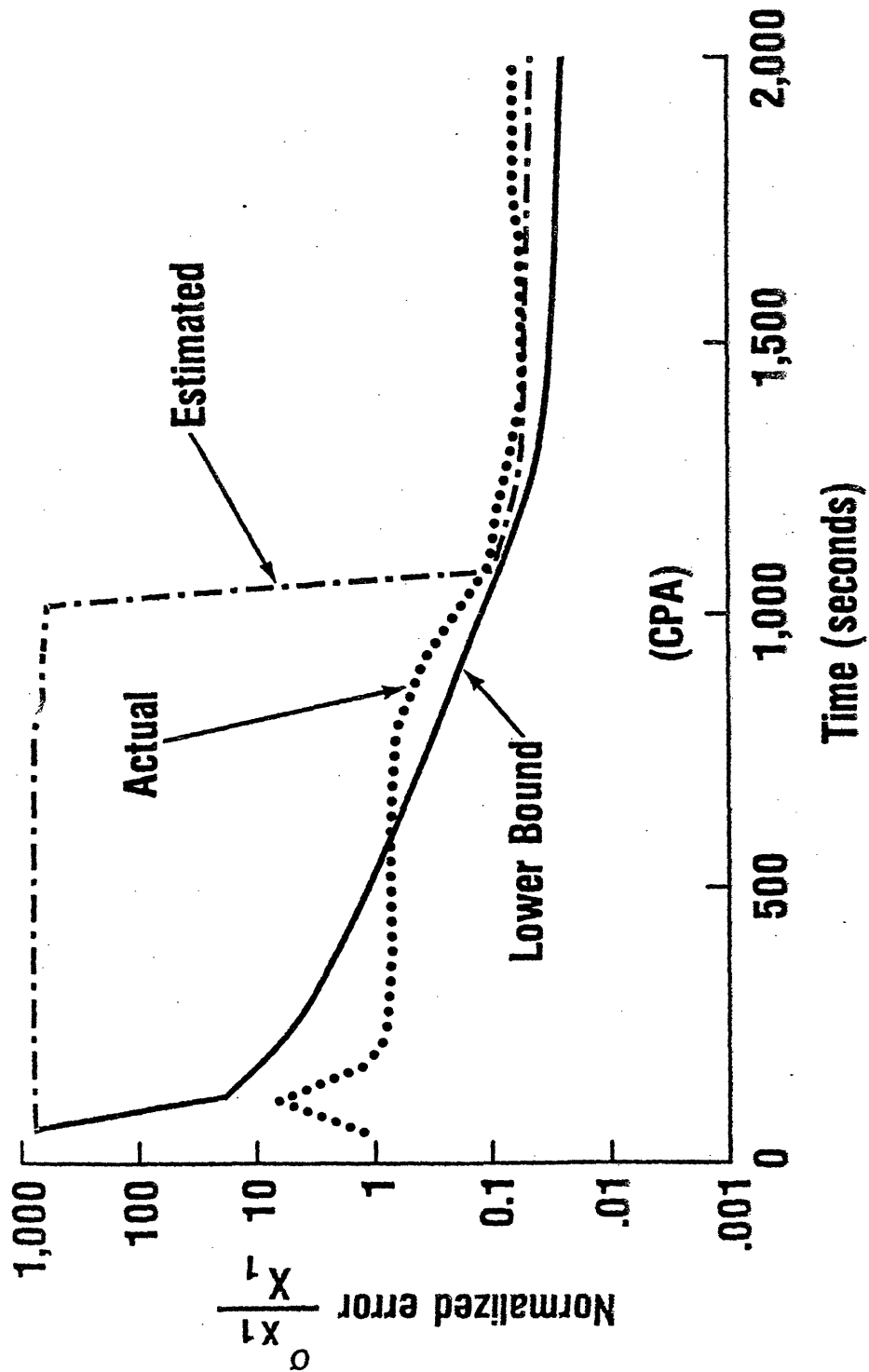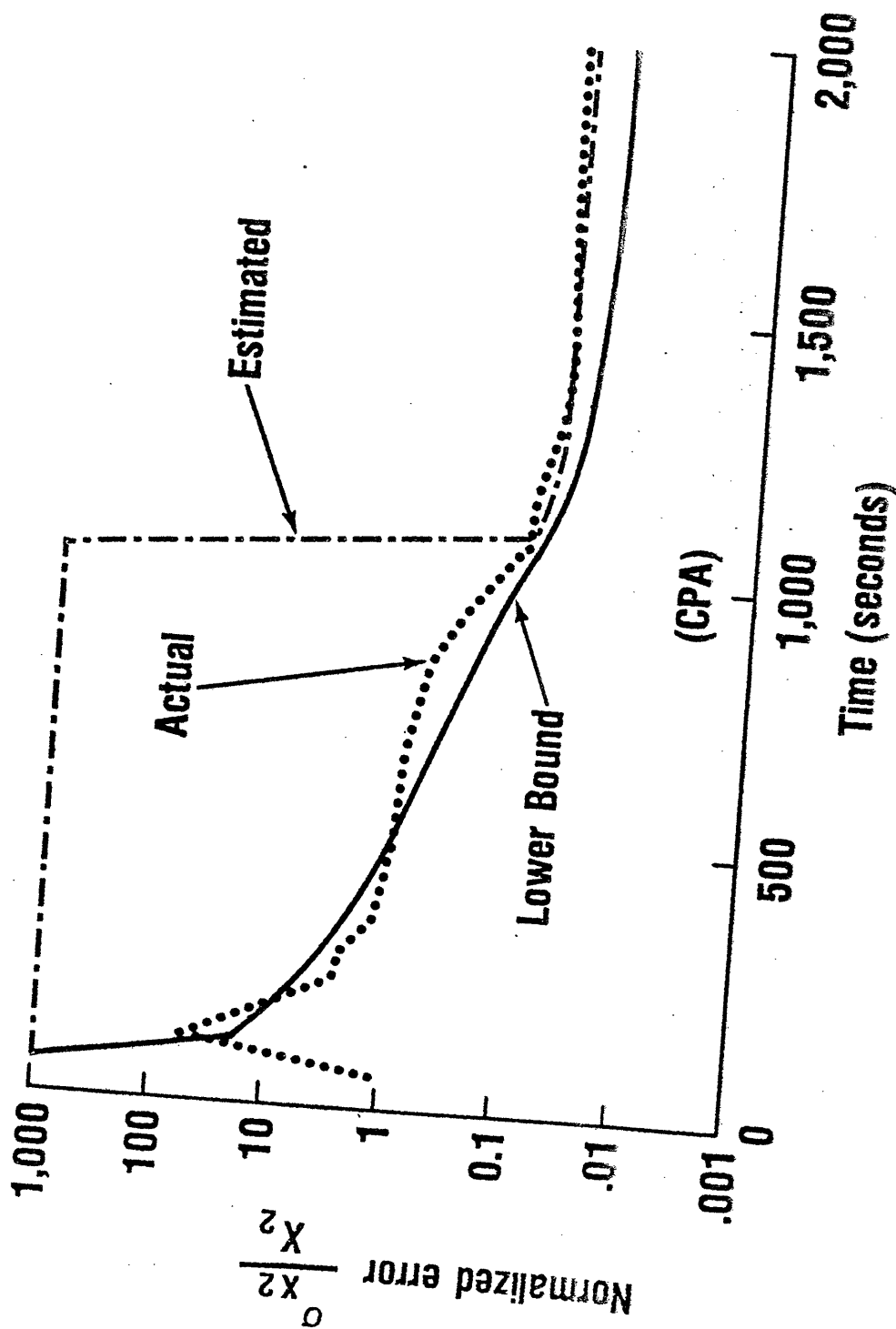
Figure 2   Error in Estimate of $\frac{-V_{to}}{r}$

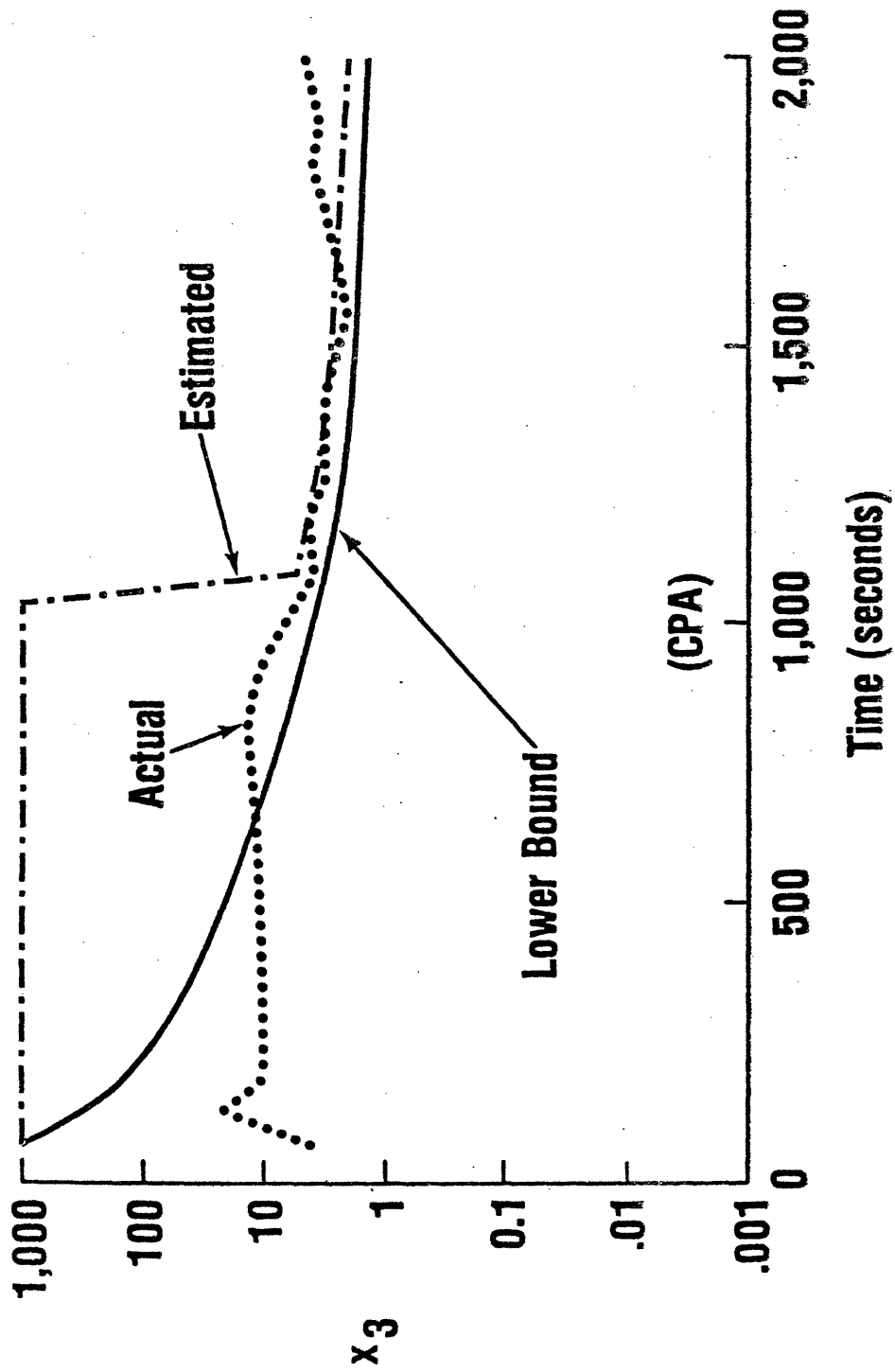Figure 3   Error in Estimate of $\dfrac{V}{r}$
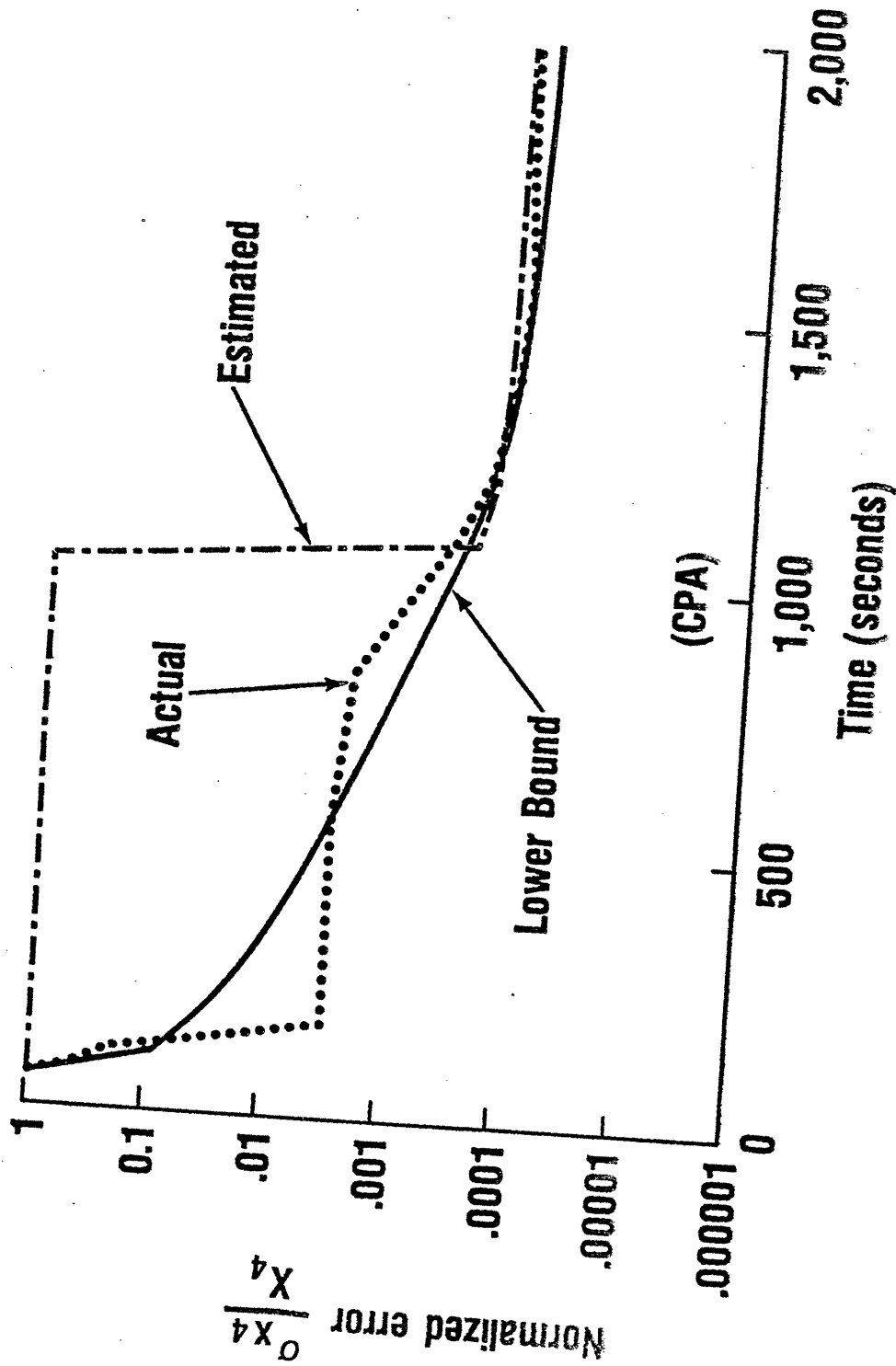
Figure 4   Error in Estimate in β
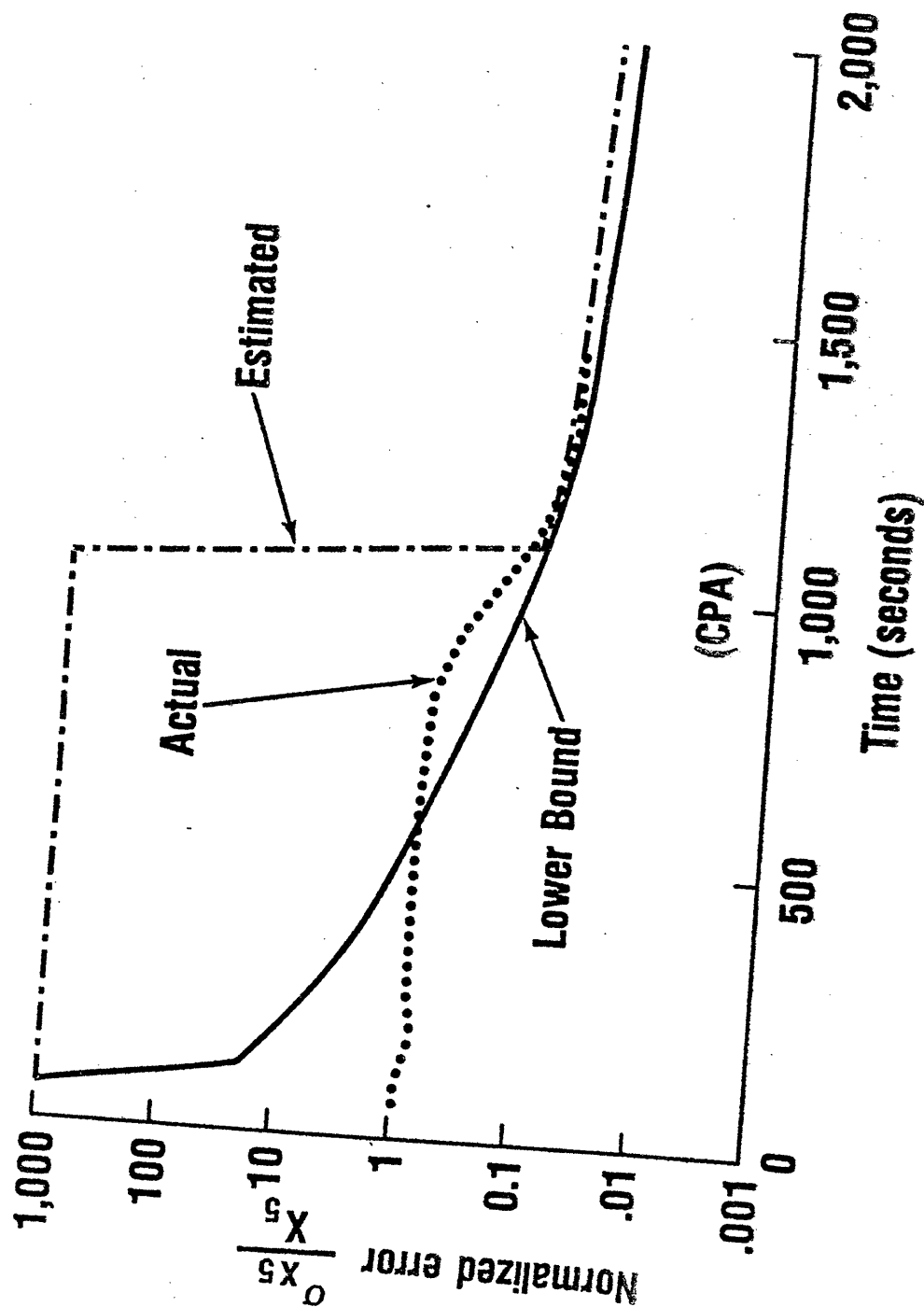
Figure 5   Error in Estimate of f

Figure 6 Error in Estimate of $\frac{-fv}{c}$

and is also shown. The results shown correspond to the parameters

$$\frac{v}{r} = .004 \text{ seconds}^{-1}$$

$$t_0 = 1000 \text{ seconds}$$

$$\beta = 0°$$

$$f = f \text{ Hertz}$$

$$R_\theta = [3° \cdot \frac{r(t)}{r(t_0)}]^2 \times \frac{1}{60} \quad \text{i.e. degenerating with range}$$

$$R_f = \frac{10^{-5} \cdot f^2}{60} \text{ Hertz}^2$$

$$0 \leq t \leq 2000$$

with observations every 60 seconds. The results are plotted as ratios of the standard deviation of each state error to the actual value of that state (with the exception of state 3). This is the same trajectory and format used in the Monte Carlo evaluations to be presented later.

Notice that the average actual estimation errors are quite close to the average errors estimated by the filter, and both are reasonably close to the Cramer-Rao lower bound values. The lack of observability of the problem is reflected by the poor trajectory estimates before CPA in Figure 7. In Figures 2-6, the filter is unable to accurately invert the $\underline{Z}$ matrix before CPA to produce an estimate of $\underline{P}$.

The determination of the number of subfilters is crucial to the computational efficiency of this tracker. The errors encountered as a function of $|\beta - \gamma_i|$ are shown in Table 1. For $|\beta - \gamma_i| \leq 4°$, the tracking errors are dominated by sources other than the approximations that assume that it is small. At 8° tracking errors increase noticeably, indicating that these latter errors are dominating performance. Since the number of subfilters must be kept as small as possible for efficient computation, the γ_i were
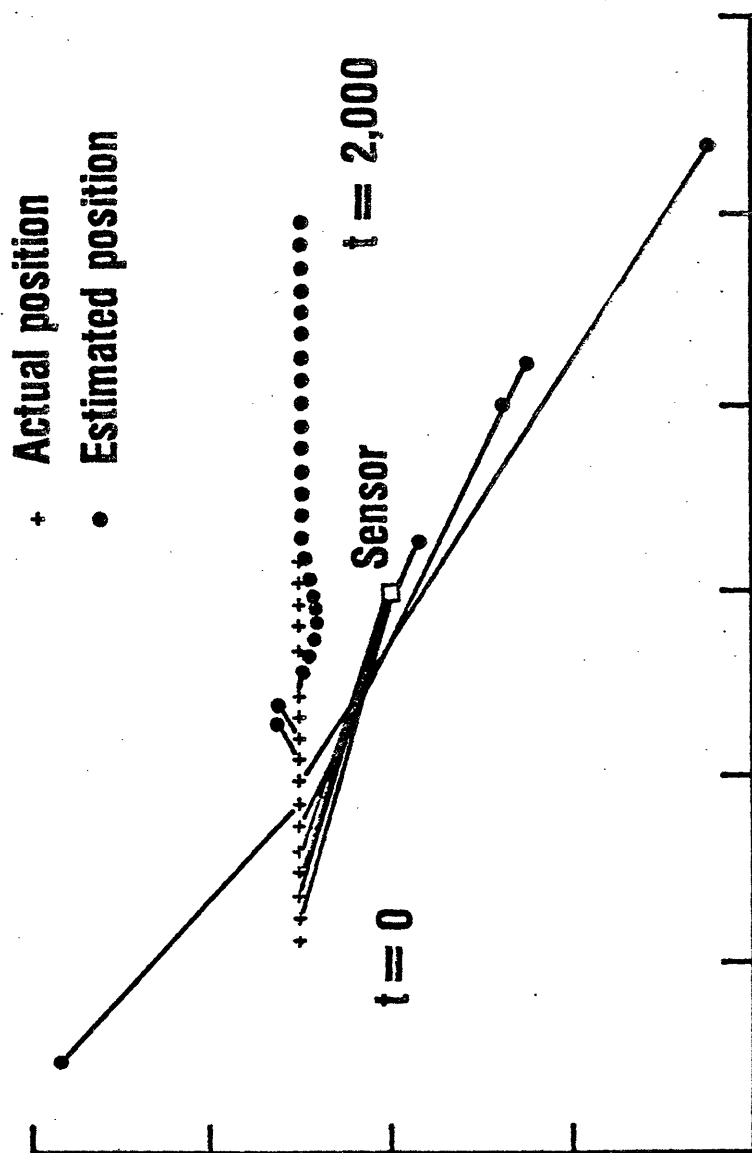
+ Actual position

● Estimated position

Sensor

t = 2,000

t = 0

Figure 7 Typical Trajectory and Estimate

$$\beta$$

	0⁰	4⁰	8⁰	12⁰
State 1	8%	8%	9%	10%
State 2	8%	8%	8%	9%
State 3	2⁰	3⁰	6⁰	10⁰
State 4	.01%	.02%	.03%	.04%
State 5	5%	5%	10%	12%

Table 1   State error as function of β

chosen so that the actual value of β never differed from one of them by more than 7 1/2°. It was felt that the few tracks that approached this limit, if β were uniformly distributed, would not degrade overall performance enough to justify a smaller spacing. The Monte Carlo simulations included a representative sample of these marginal tracks.

The ability of the tracker to operate on various values of measurement noise near the limits of good performance is demonstrated in Table 2. It can be seen that bearing noise near the 7 1/2° limit degrades filter performance just as much as $|\beta - \gamma_i|$ does, since both must be relatively small for the linearization approximations to hold. Frequency noise does not have quite as significant an effect, for the values shown. The center values used in the Monte Carlo simulations shown in Figures 2-8 lie at the edge of the region of good filter performance.

Figure 8 illustrates the behavior of the filter for a path with several maneuvers. The initial path and measurement noise were the same as for the Monte Carlo runs. The errors at the beginning of the second leg were due to the selection of the wrong subfilter for three observations. The loss of information after the third maneuver seems to have caused divergence, but the errors before CPA on the fourth leg are much smaller than those encountered for a similar path with no initial information. In all cases, the position estimate is best near CPA, when the target path is most observable. Beyond, the tracker is merely extrapolating the path obtained from earlier data.

V. SUMMARY

A. Review

The basic approach adopted for this passive tracking system was that of the extended Kalman filter. The final tracking system incorporated

Bearing noise at CPA

	1°	3°	6°

3×10^{-6} — 6%

10^{-5} — 1.5% 6% 11% X_1

3×10^{-5} — 8%

3×10^{-6} — 6%

10^{-5} — 1.5% 6% 11% X_2

3×10^{-5} — 8%

Frequency noise

$$\frac{\sigma_f^2}{f^2}$$

3×10^{-6} — 1.5°

10^{-5} — 1° 1.5° 3° X_3

3×10^{-5} — 3°

3×10^{-6} — .001%

10^{-5} — .01% .01% .02% X_4

3×10^{-5} — .02%

3×10^{-6} — 2%

10^{-5} — 3% 5% 8% X_5

3×10^{-5} — 8%

Table 2   RMS State Estimation Error as a Function of Measurement Noise Intensity
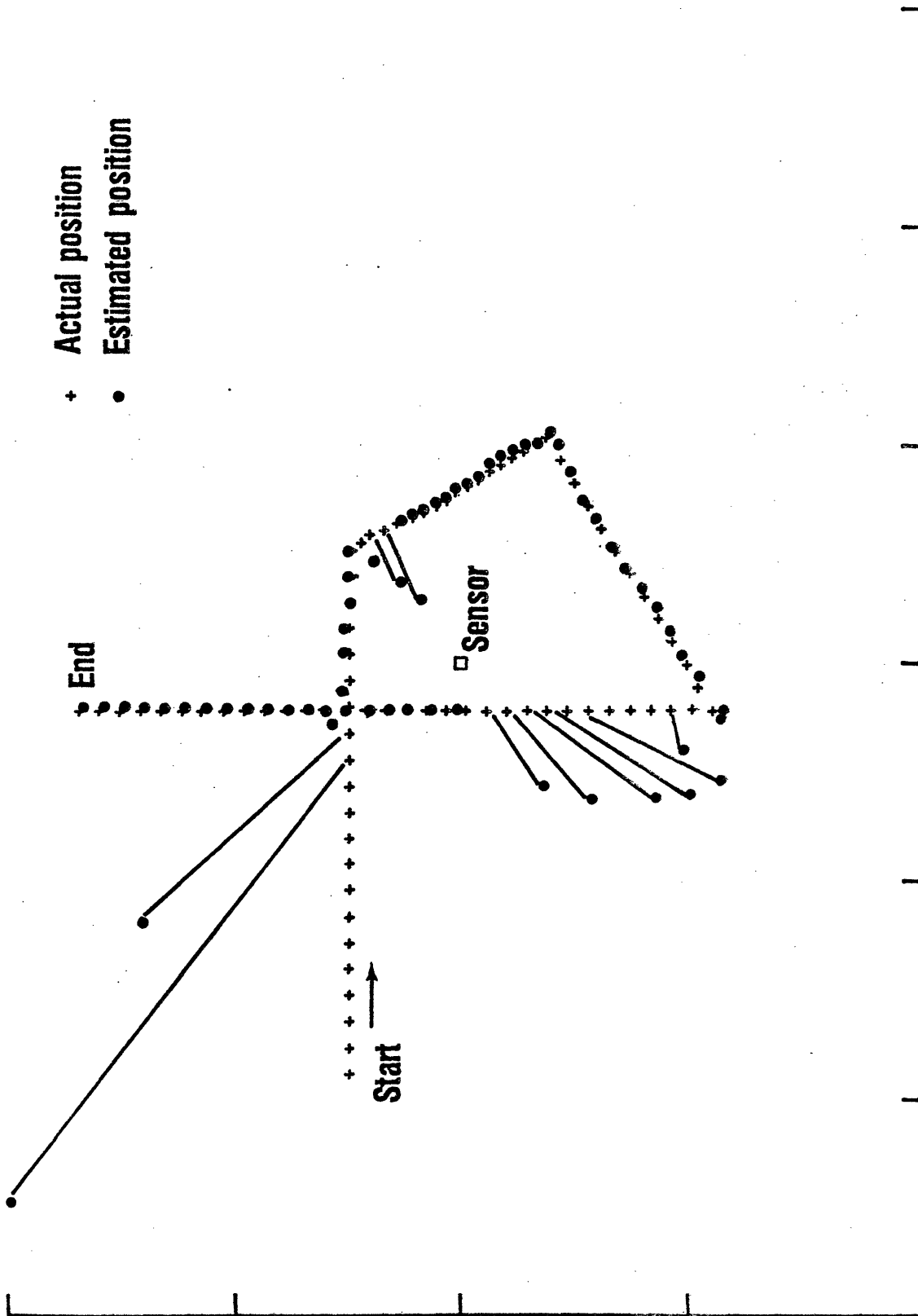
Figure 8   Multimaneuver path

several techniques that led to its success. The most notable of these were :

1. The choice of a state with several useful properties:

a) The measurements, by use of a nonlinear transformation, can be expressed as a linear combination of the states with uncertain parameters.

b) The quantities that are needed to compute these parameters are few, namely $x_1 + x_2 t$ and x_5.

c) The more useful Cartesian state is extractable by a very nonlinear function. This, combined with (a), produces a formulation in which the estimation is linear, with nonlinear transformations acting only on quantities with small variance (the filter state). Thus the estimate is obtained with minimal nonlinear effects, these being reserved for the conversion process.

2. The use of several filters based on various reference values of one state, permitting the above, and of a selection process that chooses the subfilter that best matches the observations over a period of time.

3. The combination of all sources of information, (observations, state estimates, and a priori data,) to obtain a minimum variance estimate of the uncertain parameters.

4. The adjustment of the measurement noise covariance matrix to reflect the remaining uncertainties.

5. The suppression of a state estimate when conditions guaranteeing large biases exist.

6. The use of an inverse form of the Kalman filter equations which, in a case of parameter estimation with no plant noise, are very simple and not prone to the propagation of numerical errors. They also remove the need for an initial state estimate and covariance.

7. Increasing the covariance matrix to account for state retrieval errors.

8. Detection of maneuvers based on a hypothesis test using the subfilter that exhibits the best behavior. This preserves as much information about a straight line course as possible.

9. The preservation of information during maneuver compensation in the form of an initial state estimate for each of the subfilters.

10. The increase of the variance of those states affected by a maneuver to allow the filter to adjust itself to the new trajectory.

B. Extensions.

The tracking system presented here can easily be extended along two different avenues. The first is the use of batch processing, and the second is the availability of multiple sensors.

Global iteration over time would be just one example of nonrecursive approaches in which several observations would be stored in a batch and some nonlinear processing performed on them. This type of processing is more suitable to this problem, characterized by few observations, than it might be to other estimation problems. Certainly global iteration would not degrade the performance of this tracker, since information contained in the early observations could be used to a greater extent.

The most tractable problem would be an extension to a multiple sensor situation. The added observability would allow a much more general class of maneuvers to be considered, as well as providing a better straight line path estimate.

C. Conclusions

It is well known that the extended Kalman filtering methodology works well for situations characterized by a high signal-to-noise ratio, good

observability, and a valid state trajectory for linearization. This paper illustrates the considerable additional filter complexity required in situations not characterized by these favorable conditions. A number of the ideas of the paper are quite specific to the particular problem considered, although a few may be of more general interest - all are rather ad hoc. However, the filter performs quite well as evidenced by comparison of the results of Monte-Carlo simulation and the Cramer-Rao lower bound, and by the filter's ability to track maneuvering vehicles.

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