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A Trade-Off Study of Rotor Tip Clearance Flow in a Turbine/Exhaust Diffuser System

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ABSTRACT

In a modern gas turbine power plant, the axial exhaust diffuser accounts for up to 10% of the generator power. An unshrouded rotor, due to its highly energetic tip clearance flow, improves the pressure recovery characteristic of the exhaust diffuser, while the power production within the blading suffers a loss as a result of the tip leakage flow. In this paper, these conflicting trends are thermodynamically investigated and nondimensional expressions are derived which facilitate the task of a gas turbine system designer. Conservatively, 1% thermal efficiency gain results from elimination of the last rotor tip clearance flow. The corresponding increase in thermal efficiency of a modern gas turbine power plant due to enhanced diffuser pressure recovery is less than one percent.

NOMENCLATURE

Latin

A = area
AR = diffuser area ratio
a = speed of sound
$\underset{\sim}{C}$ = absolute velocity vector
C_D = discharge coefficient $\equiv \dot{m}_{\varepsilon\ \mathrm{actual}}/\dot{m}_{\varepsilon\ \mathrm{ideal}}$
C_p = diffuser pressure recovery coefficient
h = specific enthalpy, channel height
m = $(\rho u)_j/(\rho u)_o$, mass flux ratio
$\dot{m}$ = mass flow rate
M = Mach number
p = pressure
P = power
q = $\frac{1}{2}\rho u^2$, dynamic pressure
r = radius
$°R$ = degree of reaction
s = specific entropy
u = fluid velocity
U = ωr, blade rotational speed
w = rotor specific work
$\underset{\sim}{W}$ = relative velocity vector
x = diffuser axis in Fig. 3
Z = coordinate in axial direction

Greek

α = absolute flow angle w.r.t axial direction
β = relative flow angle w.r.t axial direction
γ = specific heat ratio
ε = tip clearance height
λ = ε/h, relative clearance height
δ = increment of
Δ = change of
π = p_5/p_4, turbine pressure ratio
π' = $p_{5'}/p_4$
ρ = fluid density
η = efficiency
ϕ = C_z/U, flow coefficient
ψ = $2\Delta h_o/U^2$, stage loading coefficient
τ = T_5/T_4, turbine temperature ratio
θ = tangential direction, diffuser divergence half-angle
ω = shaft angular velocity

Subscripts

o = zero clearance, stagnation state, core flow
ε = with clearance
1 = rotor inlet
2 = rotor exit
4 = turbine inlet
5 = turbine outlet with higher diffuser pressure recovery
5' = turbine outlet with lower diffuser pressure recovery
6 = diffuser outlet with higher diffuser pressure recovery
6' = diffuser outlet with lower diffuser pressure recovery
h = hub
j = jet
t = turbine, tip
m = mean
s = isentropic
θ = tangential component
z = axial component

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1. INTRODUCTION

Today gas turbine power plants employ axial exhaust diffusers to increase both the power output and the cycle thermal efficiency. The power production within the turbine blading suffers a loss as a result of rotor tip clearance flow. To avoid the radial tip leakage, shrouded tip rotor blades are employed, which increase rotor blade centrifugal stresses as well as manufacturer's costs. It is to be noted that rotor tip shrouding introduces a new type of leakage, i.e. loss, in the form of axial leakage through the labyrinth seals. However, as the second form of leakage, i.e. the axial, does not roll into a vortex near the blade tip, the tip unloading effect of free-tip rotors is not encountered. Since the tip leakage flow emerges as a highly energetic wall jet through unshrouded turbine rotors, the axial exhaust diffuser enjoys an enhancement in its pressure recovery characteristics (Kruse et al. 1983). As a result, a trade-off study should be conducted by a gas turbine system designer/optimizer to answer the necessity of last-stage rotor tip shrouding in a gas turbine power plant. Figure 1 shows the schematics of a turbine/axial exhaust diffuser system.

In this paper, the turbine power enhancement, δP_t, due to higher diffuser pressure recovery, δC_p, is thermodynamically analyzed (section 2.1). For simplicity of analysis, the gas expansion process is assumed to be adiabatic. The analysis shows that $\delta P_t/P_t$ is a function of turbine temperature ratio τ_t, among other parameters. The effect of cooling may also be accounted for through a modified turbine temperature ratio, τ_t', where the turbine inlet stagnation temperature, T_{o4}, is replaced by an effective, "mixed" temperature, $T_{o4,mixed}$. Although turbine inlet temperature correction for the effect of coolant is customary and straight forward, the present analysis does not differentiate between τ_t and τ_t'. It is interesting to note however that a cooled turbine, i.e. one with higher effective τ_t, produces a higher δP_t for a given δC_p than an uncooled turbine, all other parameters being equal (see Fig. 6). The thermodynamic analysis of the turbine in this paper further assumes equal flow Mach numbers at the turbine inlet and exit. However, this is not a restrictive assumption as the condition $M_4 \approx M_5$ holds in most axial-flow gas turbines. Utilizing equal-Mach-number assumption, turbine temperature and pressure ratios, i.e. τ_t and π_t, represent both static and total conditions.

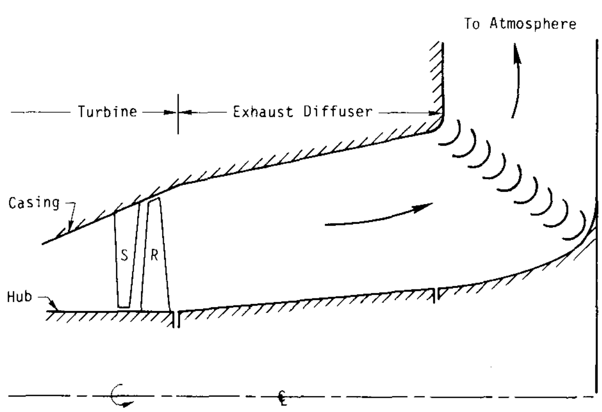

Fig. 1 Schematics of turbine/axial-exhaust diffuser system

The emergence of tip clearance flow through a turbine rotor blade as a wall jet has been experimentally shown by Bammert et al. (1968) (see Fig. 2). A parameter characterizing the strength of the wall jet is the mass flux ratio, $m \equiv (\rho u)_j/(\rho u)_o$. Back and Cuffel (1982) have experimentally investigated the effect of m on the pressure recovery characteristics of rectangular diffusers. A summary of their results is shown in Fig. (3). This confirms the experience in industry, (Rappard von, 1984), that a free-tip rotor discharging in an axial exhaust diffuser allows a greater diffuser divergence angle than a shrouded-tip rotor. However, as the diffuser performance is a strong function of its exact inlet conditions, the data obtained by Back and Cuffel (1982) in an isolated diffuser must be viewed with caution when applied to actual gas turbine power plants. The limited results obtained by Kruse et al. (1983) (see Fig. 4), where axial exhaust diffusers

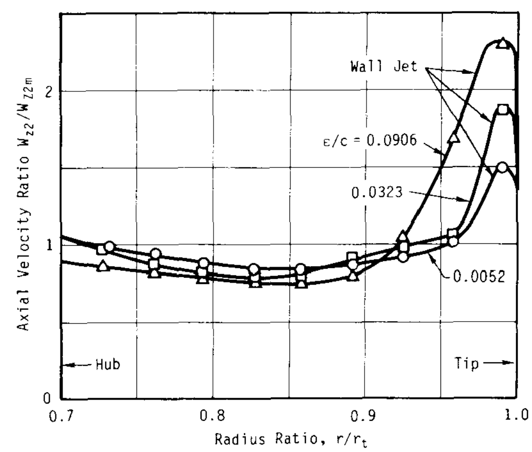

Fig. 2 Axial velocity distribution at the rotor exit in a single-stage axial-flow turbine (at design point) (data from Bammert et al. 1968)

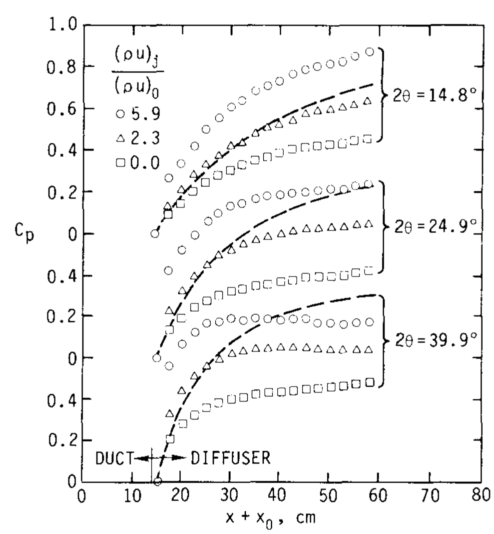

Fig. 3 Pressure recovery enhancement in a rectangular diffuser with wall jet (data from Back and Cuffel 1982) (dashed lines represent ideal diffuser pressure recovery)

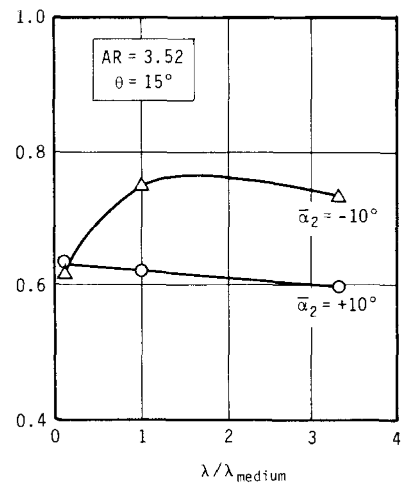

Fig. 4 Annular exhaust diffuser pressure recovery downstream of a turbine with relative rotor tip clearance (data from Kruse et al. 1983)

downstream of turbine stages are tested, can be directly applied to power plant analysis. Figure 4 shows that the diffuser pressure recovery improvement, downstream of a turbine, is a function of clearance height as well as the net swirl at the diffuser inlet.

2. ANALYSIS

2.1 Turbine Power Increase Due to Higher Diffuser Pressure Recovery

The effect of higher C_p on the increased turbine power output is schematically shown in Fig. 5. Flow expansion in the turbine is approximated by a straight line from 4 to 5; and the compression from p_5 to the ambient pressure, p_6, depicts the diffuser flow. The slope of the expansion line from 4 to 5 is indicative of the extent of the boundary layer and shock losses within an adiabatic turbine. Now, a reduced pressure recovery diffuser, i.e. one without the rotor tip clearance discharge, will increase the turbine back

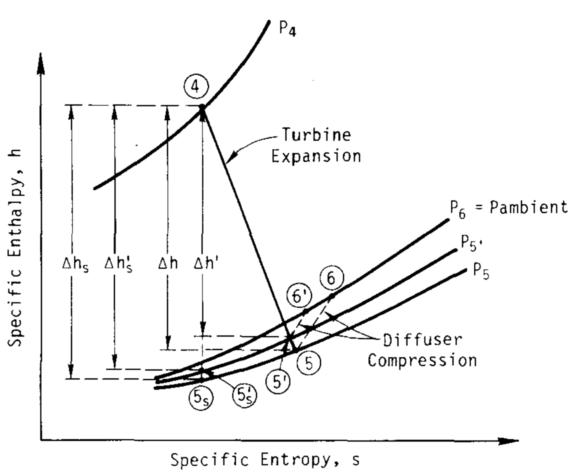

Fig. 5 Mollier diagram for the flow through an adiabatic turbine and exhaust diffuser

pressure from p_5 to $p_{5'}$. In general the expansion $p_{5'}/p_5 << p_4/p_5$ (see Appendix A) and thus the turbine power enhancement $\delta P_t << P_t$. Consequently the turbine adiabatic efficiency η_t remains, to the first order, unaffected by the diffuser pressure recovery increase. Therefore,

$$\eta_t = \Delta h/\Delta h_s \approx \Delta h'/\Delta h'_s \tag{1}$$

where subscript s stands for isentropic expansion process. The increment of turbine specific work, δw_t, is

$$\delta w_t = \Delta h - \Delta h' = \Delta h - \eta_t \Delta h'_s \tag{2}$$

From isentropic relations we can write

$$\Delta h_{s'} = h_4 [1 - \pi_t'^{\frac{\gamma-1}{\gamma}}] \tag{3}$$

where γ is the ratio of specific heats in the turbine, typically 1.33 for hydrocarbon fuels and air mixture. In (3), the working fluid is a calorically perfect gas. From (1) and (3) we get

$$\Delta h'/h_4 = \eta_t [1 - \pi_t'^{\frac{\gamma-1}{\gamma}}] \tag{4}$$

and similarly,

$$\Delta h/h_4 = \eta_t [1 - \pi_t^{\frac{\gamma-1}{\gamma}}] \tag{5}$$

Substituting (4) and (5) in (2), the nondimensional turbine work increment becomes

$$\delta w_t/h_4 = \eta_t [\pi_t'^{\frac{\gamma-1}{\gamma}} - \pi_t^{\frac{\gamma-1}{\gamma}}] \tag{6}$$

Now, by defining the diffuser pressure recovery

$$C_p = (p_6 - p_5)/q_5 \tag{7}$$

and the diffuser reduced pressure recovery, i.e. without the rotor tip clearance discharge as

$$C_{p'} = (p_6 - p_{5'})/q_{5'} \tag{8}$$

We can write the increment of diffuser recovery, δC_p as

$$\delta C_p \equiv C_p - C_{p'} \approx (p_{5'} - p_5)/q_5 \tag{9}$$

(Note that the dynamic pressure at the turbine exit, q_5 is, to the first order, equal to $q_{5'}$ since additional expansion $p_{5'}/p_5 << p_4/p_5$, see Appendix A.)

Writing $p_{5'}$ in terms of δC_p from equation (9) and factoring the turbine pressure ratio term from the bracket in (6) we get

$$\delta w_t/h_4 = \eta_t \pi_t^{\frac{\gamma-1}{\gamma}} [(1 + \frac{\delta C_p q_5}{p_5})^{\frac{\gamma-1}{\gamma}} - 1] \tag{10}$$

As will be shown in Appendix A, $\delta C_p q_5/p_5 << 1$; thus (10) via binomial expansion simplifies to

$$\delta w_t/h_4 \approx \frac{\gamma-1}{\gamma} \eta_t \frac{\delta C_p q_5}{p_5} \pi_t^{\frac{\gamma-1}{\gamma}} \tag{11}$$

Now, the turbine specific work, w_t, can be written in terms of turbine adiabatic efficiency and pressure ratio via (5)

$$w_t = h_4 - h_5 = \eta_t h_4 [1 - \pi_t^{\frac{\gamma-1}{\gamma}}] \tag{12}$$

From (12),

$$\pi_t^{\frac{\gamma-1}{\gamma}} = 1 - w_t/\eta_t h_4 \tag{13}$$

which, upon substitution in (11) and some simplification, yields

$$\frac{\delta w_t}{w_t} = \frac{\gamma-1}{\gamma} \cdot \frac{\delta C_p}{(p_5/q_5)} \cdot [\frac{\eta_t h_4}{w_t} - 1] \tag{14}$$

The ratio p_5/q_5 can be further simplified to

$$p_5/q_5 = 2p_5/\rho_5 u_5^2 = 2/\gamma M_5^2 \tag{15}$$

via the speed of sound definition, $a^2 \equiv \gamma p/\rho$, where M_5 is the flow Mach number at the turbine exit; and

$$\frac{h_4}{w_t} = \frac{h_4}{h_4 - h_5} = \frac{1}{1 - h_5/h_4} \approx \frac{1}{1 - T_5/T_4} = \frac{1}{1 - \tau_t} \tag{16}$$

Substituting (15) and (16) into (14) yields the final nondimensional result as

$$\frac{\delta P_t}{P_t} = \frac{\delta w_t}{w_t} \approx (\frac{\gamma-1}{2}) \cdot \delta C_p \cdot M_5^2 (\frac{\eta_t}{1 - \tau_t} - 1) \tag{17}$$

From equation (17) we note that any small decrease in turbine adiabatic efficiency, η_t, due to less pressure recovery in the diffuser results in a second-order correction to $\delta P_t/P_t$. Hence, our assumption of constant adiabatic efficiency, to the first order, in section 2.1 is consistent with our analysis (for further discussions see Appendix A). Graphical representations of equation (17) are plotted in Figures 6 and 7.

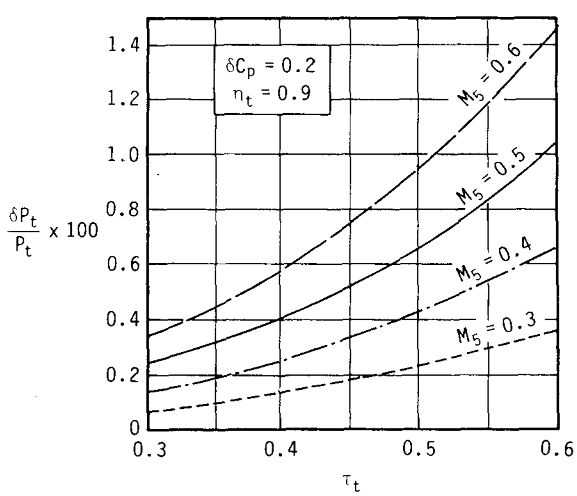

Fig. 6 Percentage of turbine power increase with turbine temperature ratio ($\gamma = 1.33$)

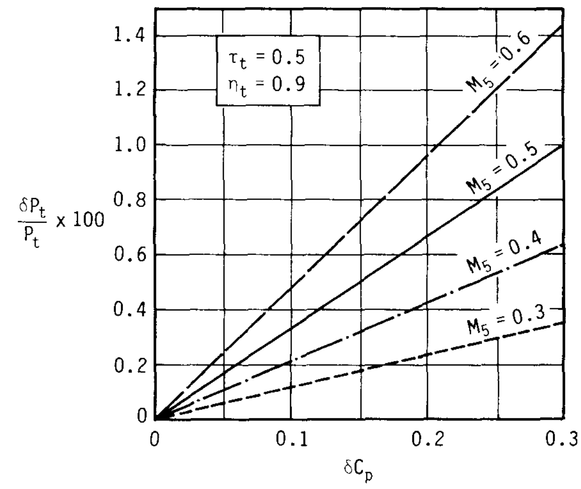

Fig. 7 Percentage of turbine power increase with increment of diffuser pressure recovery ($\gamma = 1.33$)

2.2 Turbine Power Increase Due to Rotor Tip Shrouding

Many researchers have investigated the rotor tip clearance loss in turbomachinery since the appearance of steam turbines. Some prominent loss models are discussed and compared by Farokhi (1987), and Lakshminarayana's model (1970) is found to be the most versatile of the reviewed models. The model presented in this section is developed by Farokhi (1987) based on the hypotheses proposed by Bammert et al. (1968). The experimental results obtained by Booth et al. (1982) and Wadia and Booth (1982) for the tip discharge coefficient, C_D, are used as guidelines in the proposed model. When applied to the efficiency measurements of Kofskey and Nusbaum (1968), in a two-stage axial-flow turbine, a linearized approximation of the model reproduced the experimental data with reasonable accuracy (Farokhi, 1987). Geometry of a free-tip turbine rotor is schematically shown in Fig. 8.

The fraction of turbine power loss, $\delta P_t/P_{t_o}$, due to rotor tip clearance is assumed to be proportional to the fraction of tip leakage flow, i.e.

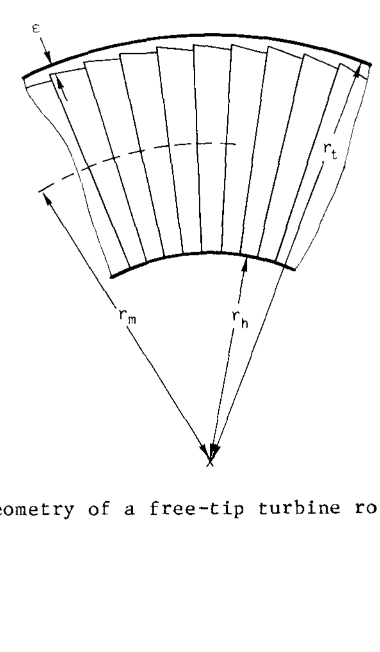

Fig. 8 Geometry of a free-tip turbine rotor

$$\delta P_t/P_{t_o} \propto \dot{m}_\varepsilon/\dot{m}_o \tag{18}$$

The zero-clearance turbine power is defined as

$$P_{t_o} = \eta_o P_{t\ ideal} \tag{19}$$

where η_o is turbine adiabatic efficiency with zero clearance and turbine power output with clearance is

$$P_{t_\varepsilon} = \eta_\varepsilon P_{t\ ideal} \tag{20}$$

where η_ε is turbine adiabatic efficiency with rotor tip clearance. Now, defining

$$\delta P_t \equiv P_{t_o} - P_{t_\varepsilon} \tag{21}$$

and

$$\Delta\eta \equiv \eta_o - \eta_\varepsilon \tag{22}$$

we get

$$\frac{\delta P_t}{P_t} = \frac{\Delta\eta}{\eta_o} \propto \dot{m}_\varepsilon/\dot{m}_o \tag{23}$$

With similar assumptions as Bammert et al. (1968), namely a) conservation of angular momentum for the clearance flow, i.e. $W_{\theta 2_\varepsilon} = W_{\theta 1}$ and b) equal expansion of the tip and core flow, i.e. $|\underset{\sim}{W}_{2\varepsilon}| = |\underset{\sim}{W}_2|$ (see Fig. 9) the mass flow ratio can be expressed as

$$\frac{\dot{m}_\varepsilon}{\dot{m}_o} = \frac{A_\varepsilon}{A_o}\ [1 + \tan^2\beta_2 - (\frac{W_{z1}}{W_{z2}} \tan\beta_1)^2]^{0.5} \tag{24}$$

which can be easily deduced from Fig. 7.

Now, the proportionality factor in (23) which should be a function of a) tip shape, b) relative wall-motion, c) clearance gap Reynolds number and d) clearance-to-blade height ratio, λ, among other parameters is shown to be the tip discharge coefficient, C_D by Farokhi (1987). Hence

$$\frac{\delta P_t}{P_t} = C_D \frac{A_\varepsilon}{A_o}\ [1 + \tan^2\beta_2 - (\frac{W_{z1}}{W_{z2}} \tan\beta_1)^2]^{0.5} \tag{25}$$

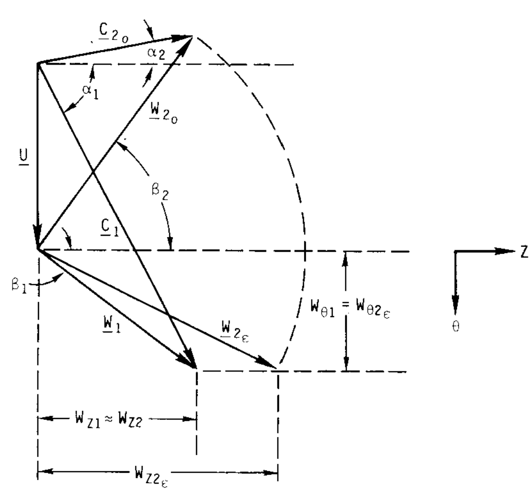

Fig. 9 Velocity triangles at the tip section of a free-tip rotor

In terms of stage loading factor, ψ and flow coefficient ϕ at the blade tip, equation (25) can be rewritten in the following form,

$$\frac{\delta P_t}{P_t} = C_D \frac{A_\varepsilon}{A_o}\ [1 - \frac{\psi}{\phi} \tan\beta_m]^{0.5} \tag{26}$$

where $W_{z1} \approx W_{z2}$.

Also

$$\frac{A_\varepsilon}{A_o} \approx (\frac{r_t}{r_m})\ (\frac{\varepsilon}{h}) = \frac{r_t}{r_m} \cdot \lambda \tag{27}$$

neglecting λ^2 term, the decrease in turbine power, δP_t, can be written in the final form

$$\frac{\delta P_t}{P_t} \approx C_D\ (\frac{r_t}{r_m})\ \lambda\ \ [1 - \frac{\psi}{\phi} \tan\beta_m]^{0.5} \tag{28}$$

Note that the mean flow angle in a turbine rotor, β_m, is either zero or negative, i.e.

$$\beta_m \leq 0 \tag{29}$$

where zero value holds for an impulse stage and negative values for reaction stages.

From the above model (eq. 28), an accurate estimation of tip clearance power loss, δP_t, is contingent upon the knowledge of tip discharge coefficient, C_D. However this parameter is extremely difficult to measure experimentally in an actual operating turbine, and to author's knowledge, no such data has been reported to date. Hence, turbine model tests are needed to establish some insight into the magnitude of C_D with/without relative wall motion effect. To this end, experimental results of Booth et al. (1982) and Wadia and Booth (1982) provide valuable quantitative information on the tip discharge coefficient. Water rig flow visualization experiments of Graham (1986) also assist in qualitative understanding of rotor tip discharge coefficient with relative wall motion. However, for the purpose of discussion, a new nondimensional turbine power loss parameter, $(1/C_D)\delta P_t/P_t$, is considered, where $1/C_D$ is the scaling factor allowing the influence of new tip geometries and wall treatments be efficiently incorporated in the model.

3. RESULTS AND DISCUSSION

Nondimensional expressions are derived which relate the turbine power increase to enhanced diffuser pressure recovery and the last turbine rotor tip shrouding. Some typical turbine parameters are chosen and the results are presented in Figs. 6-7 and 10-12. From experimental results of Back and Cuffel (1982), an increased diffuser pressure recovery of $\delta C_p \approx 0.2$ is realized in a rectangular diffuser with wall jet of strength $m \approx 2.3$. A mass flux ratio of nearly two, i.e. $m \approx 2$, is also measured at the rotor exit in a single- (and seven) stage turbine by Bammert et al. (1968) (see Fig. 2). Annular exhaust diffuser runs downstream of turbine stages performed by Kruse et al. (1983) point to similar (i.e. $\delta C_p \approx 0.2$) diffuser pressure recovery improvements as well (see Fig. 4). Therefore, it may be reasonable to expect a δC_p of ≈ 0.2 in an exhaust diffuser, as a result of rotor tip clearance flow. Fig. 6 examines the percentage of turbine power increase with turbine temperature ratio,

τ_t, at different turbine exit flow Mach numbers, for an enhanced diffuser pressure recovery of $\delta C_p = 0.2$. For a fixed turbine temperature ratio τ_t, δP_t increases with exit Mach number, M_5. This is due to higher dynamic pressure, q_5, which is partly converted to diffuser static pressure rise. Now, for a fixed exit Mach number, M_5, δP_t increases as turbine temperature ratio is increased. Noting that $q_5 = (\gamma/2)p_5 M_5^2$, the higher τ_t corresponds to a higher p_5, hence q_5, taking turbine inlet conditions as constant. The percentage of turbine power increase with diffuser pressure recovery is shown in Fig. 7. As expected, δP_t increases linearly with δC_p, and exhibits similar trends as in Fig. 6, as exit flow Mach number, M_5, increases.

Results of parametric investigation of the turbine power enhancement with the rotor tip shrouding are shown in Figs. 10-12. Nondimensional turbine

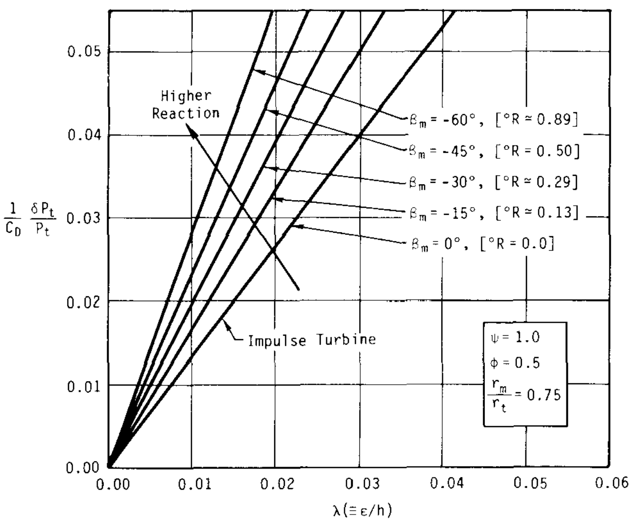

Fig. 10 Nondimensional turbine power increase due to shrouding vs. clearance

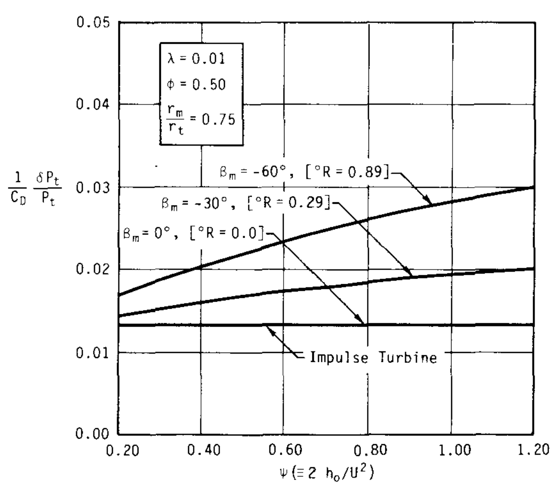

Fig. 11 Effect of stage loading on the nondimensional turbine power increase

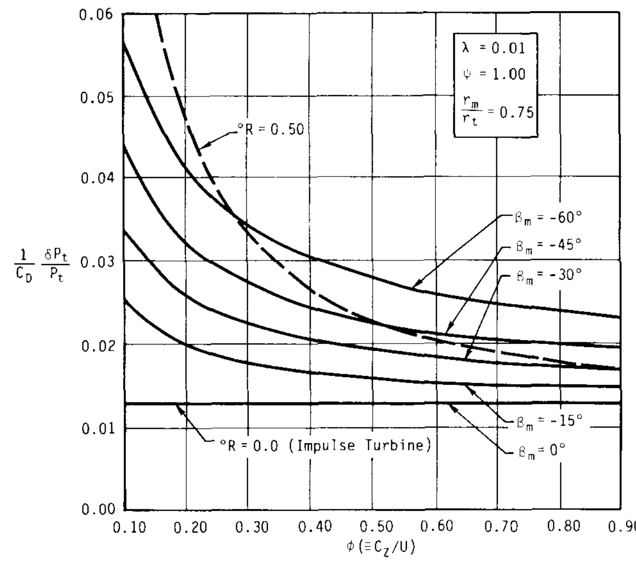

Fig. 12 Effect of flow coefficient and stage reaction on the nondimensional turbine power increase

power increase, $(1/C_D)(\delta P_t/P_t)$, is shown (Fig. 10) to vary linearly with tip clearance λ and increase as stage reaction increases. The fan-like distribution of Fig. 10 is similar to the tip clearance loss model presented by Cordes (1964). The effects of stage loading and flow coefficient on the turbine power enhancement are separately examined in Figs. 11 and 12. It should be noted that the magnitude of the power increase due to tip shrouding is linearly proportional to the tip discharge coefficient, C_D. Based on the limited available experimental results, the value of C_D lies between 0.5 to 0.9, where lower values correspond to new tip geometries (e.g. winglet) and the upper values belong to flat-tip, high-reaction rotor blades. As clearly shown in Figs. 10-12 the magnitude of $\delta P_t/P_t$ strongly depends on stage loading ψ, flow coefficient ϕ, clearance height λ, mean relative flow angle β_m and blade tip discharge coefficient, C_D. Especially note that the values read from Figures 10-12 need to be multiplied by the rotor tip discharge coefficient, C_D, before concluding any turbine power enhancement percentage. Hence, the gas turbine system designer/optimizer is provided only with the tools to estimate the corresponding performance gains achieved through tip shrouding and enhanced diffuser pressure recovery.

Now, in terms of generator output, which is a fraction of turbine power, say 1/2, the results shown in Figs. 6-7 and 10-12 should be amplified by the inverse of power fraction. Similarly, the cycle thermal efficiency defined as the ratio of generator output and the heating power of the fuel takes on the amplified values of the results shown in Figs. 6-7 and 10-12. Thus, for a 0.5% increase in P_t, the generator power and thermal efficiency will both rise by nearly 1%. In modern gas turbines, where new tip geometries and casing treatments are employed, it is possible to reduce the tip clearance losses significantly, and thus 0.5% increase in P_t or nearly 1% in thermal efficiency gain is achievable. Enhanced pressure recovery in an exhaust diffuser due to highly energetic tip leakage flow provides somewhat lower performance gains in thermal efficiency (see Figs. 6 and 7).

Finally, it should be noted that rotor tip shrouding does more than eliminating/reducing the tip clearance flow: namely, it adds rigidity to the blades and increases structural damping in cases of forced- and self-excited vibrations. In this respect, aeroelastic considerations clearly dominate the thermodynamic cycle performance aspects in shrouding the last-stage rotor blades.

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Appendix A: Some Pertinent Derivations and Orders of Magnitude

To derive an expression for the additional expansion $p_{5'}/p_5$ in terms of known turbine/diffuser parameters, we write

$$\delta C_p = \frac{p_{5'} - p_5}{q_5} = \frac{(p_{5'}/p_5) - 1}{(q_5/p_5)} \tag{A.1}$$

and upon simplification, we get

$$p_{5'}/p_5 = 1 + \delta C_p \, (q_5/p_5) \tag{A.2}$$

Substitute from equation (15) for q_5/p_5 in (A.2) and get

$$p_{5'}/p_5 = 1 + \frac{\gamma}{2} \, \delta C_p \, M_5^2 \tag{A.3}$$

Now, for a turbine with exit flow Mach number of 0.5, specific heat ratio of 1.33 and diffuser pressure recovery improvement of $\delta C_p \approx 0.2$, we get

$$p_{5'}/p_5 = 1 + \frac{1.33}{2} (0.20)(0.5)^2 \approx 1.033 \tag{A.4}$$

Typical turbine expansion ratios, p_4/p_5, for modern gas turbine power plants are ~ 14-18. Hence,

$$p_{5'}/p_5 \ll p_4/p_5 \quad (\text{or } \delta P_t/P_t \ll 1) \tag{A.5}$$

Also note from (A.2) and (A.3) that

$$\delta C_p (q_5/p_5) = \frac{\gamma}{2} \, \delta C_p \, M_5^2 \tag{A.6}$$

and for typical turbine parameters, as described above,

$$\delta C_p \, (q_5/p_5) \approx 0.033 \ll 1 \tag{A.7}$$

Therefore, maintaining only the linear term in the binomial expansion of equation (10) is admissible.

Finally, note that isobars p_5 and $p_{5'}$ are in practice much closer to each other than depicted in Fig. 5, as shown in (A.4). Hence, divergence of the constant-pressure lines on the h-s diagram would not affect the approximation $\eta_t \approx \eta_{t'}$ in section 2.1.