

THURSDAY, OCTOBER 26, 1893.

ANALYTICAL MECHANICS.

A Treatise on Analytical Statics. With numerous Examples. Vol. II. By Edward John Routh, Sc.D., LL.D., M.A., F.R.S. (Cambridge: at the University Press, 1892.)

THIS volume finishes Dr. Routh's work on the subject of analytical statics, the first volume of which was reviewed in NATURE, June 16, 1892. It contains, in three long Sections or Books, the subjects of Attraction, Bending of Rods, and Astatics, left over from Vol. I.

In Attraction a start is made with the Newtonian Law, and the Gravitation Constant is introduced.

The experimental redetermination of the numerical value of the Gravitation Constant is engaging the attention of Mr. Poynting (who has just been awarded the Adams Prize for his Essay on this subject) and of Mr. C. V. Boys. But we cannot hope to obtain, with the greatest refinements, an accuracy of determination within limits of error of less than one per cent.; the Astronomical Unit of Mass, defined in § 3, would be subject to the same limits of error, which are far beyond what is permissible in careful measurements with the Balance.

The only reason for the introduction of the Astronomical Unit of Mass is to save the trouble of writing down k , the Gravitation Constant, in our equations; but we agree with Prof. Minchin, in his Analytical Statics, in thinking that it tends to clearness if we take the trouble to write k in its proper place, so as always to measure m in such well-determined units as the gramme or kilogramme.

Nowadays the theorems of Attraction receive their most appropriate interpretation, analytical and experimental, from the subject of Electrostatics; the theorems on the Potential of Laplace, Poisson, and Gauss, on Tubes of Force, Green's Theorem, Inversion, Laplace's Functions, and on the Attraction of Ellipsoids of Chasles, all present themselves as fundamental in the Electrostatical chapters of Maxwell's "Electricity and Magnetism;" insomuch that Maxwell ventured to present a demonstration of some of the most abstruse analytical results of Laplace's Functions, founded on physical principles of Electrostatics, and thereby excite the ire of certain mathematicians of the purest proclivities.

For instance, the complicated theorems on Centrobaric Bodies, discussed in §§ 111, 116, become self-evident when interpreted as the analogues of the electricity induced on an uninsulated closed surface by an electrical point in the interior. The external electrical effect being zero, the potential of the induced electricity is equal and opposite to that of the point, and therefore the surface has an electrical coating which is centrobaric, the function which represents the superficial density being *Green's Function* for the surface and the point.

If the dielectric in the interior is stratified, an electrical concentration is distributed throughout the space, and thus the analogue of the centrobaric body is obtained; but incidentally the electric analogy shows that the strata of equal density in the centrobaric body are each separately centrobaric, so that the centrobaric

body is built up of centrobaric shells. The sphere is the homogeneous centrobaric body, as Newton showed in the "Principia"; and an application of Sir W. Thomson's powerful geometrical method of electrical inversion deduced the fact that a solid sphere whose density varies inversely as the fifth power of the distance from an external point O' is centrobaric with respect to the interior inverse point O . So also for a spherical shell, either this or composed of a series of concentric strata; and this by inversion leads to the theorem that a shell bounded by two excentric spheres of which the limiting points are O and O' is centrobaric if the density at any point P in it is

$$OP^{-5}\phi(OP/O'P/)$$

The discovery of Green's function for a given surface, or rather the discovery of surfaces for which Green's function can be assigned, is one of the most difficult and baffling of modern analysis; and it has so far only been effected for some few simple cases.

The British Association met recently at Nottingham, the birthplace of George Green in 1793. There must be people still living there who remember him, and could supply now, before it is too late, some interesting details of the causes which led to the development of his wonderful mathematical genius, at a time too when little encouragement was vouchsafed to such abnormal proclivities. In France a statue would long ago have arisen in his honour; but at least an interesting paper on the subject of Green's life could be communicated to Section A.

The theorems of Chasles and Maclaurin on the attraction of homœoids and focaloids are fully discussed in §182; the homœoids receive ample illustration in electrical phenomena; but Maclaurin's theorem on the attraction of confocal homogeneous solid ellipsoids is rendered more convincing by supposing the smaller confocal to be scooped out of the larger so as to form a thick focaloid, the matter which is scooped out being condensed homogeneously with the rest of the substance. The effect of this operation is to leave unaltered the external potential, and the original matter may thus ultimately be condensed into a thin focaloid, in which the thickness is inversely proportional to the perpendicular on the tangent plane; and this focaloid will have the same external equipotential surfaces as the solid ellipsoid.

Part ii., on the Bending of Rods, does not assume any new experimental knowledge beyond that of the proportionality of the curvature to the bending moment, an assumption which we know from Prof. Karl Pearson's "History of Elasticity" to be only a first rough approximation to the truth.

The analytical consequences of the hypothesis are, however, very elegant and instructive, and Dr. Routh has brought together an interesting collection of illustrative examples.

He does not, however, develop the elliptic function solution of the plane *Elastica* or associated *Lintearia*, curves which can now be drawn with great accuracy and rapidity by Mr. C. V. Boys's scale. He also restricts himself to the uniform helix in the tortuous *Elastica*; but the student who wishes to pursue this branch of the

subject to its fullest development must consult vol. ii. of Mr. Love's "Elasticity," which has recently appeared.

Kirchoff's Kinetic Analogue between this *Elastica* and the motion of a Top, makes the same analysis serve for both; thus, as pseudo-elliptic solutions, we may mention that tortuous *Elasticas* are given by:—

$$(i.) r^2 e^{2i(\psi + p\phi)} = \sqrt{\{r^2 - \frac{1}{4}c(1-4c)a^2\}} \sqrt{\{r^2 + \frac{1}{4}(1-2c)(1-4c)a^2\}} \\ + \frac{1}{2}i(1-4c)a \sqrt{\{\frac{1}{2}c(1-2c)a^2 - r^2\}};$$

$$(ii.) r^2 e^{3i(\psi + p\phi)} = \frac{1}{2}r^2 + (1-c)(2-3c)a^2 \sqrt{\{r^2 - (2c-3c^2)a^2\}} \\ + i(2-3c) \sqrt{\frac{1}{2} - r^4 - (1-c)(1-3c)a^2 r^2 + (1-c)^2(2c-3c^2)a^4};$$

corresponding to parameters $\omega_1 + \frac{1}{2}\omega_3$ and $\omega_1 + \frac{2}{3}\omega_3$ of the related Elliptic Integrals of the third kind.

Here a determines the scale of the figure, and c is an arbitrary parameter, upon which p depends; and it is curious that in case (ii.) the value $c = \frac{1}{3}$ makes p vanish, and then $\omega_3 = \sqrt{(-3)}\omega_1$.

Other interesting applications of the Theory of the Bending of Rods, requiring Bessel Functions, are the investigations of the greatest height consistent with stability to which a vertical wire or mast can be carried, or to which a tree can grow, without drooping over under its own weight; we can thus supply the analysis required in the old German proverb, quoted by Goethe, "Es ist dafür gesorgt, dass die Bäume nicht in den Himmel wachsen."

The third part, on Astatics, is intimately bound up with the distribution in space of Poinso's central axis for a system of forces; or with Sir Robert Ball's investigation on Screws. A great analogy exists with the analysis required in the distribution of principal axes in space. A problem which might well find a place here is, "The moment of inertia of a body of mass M about any generating line of the hyperboloid of one sheet

$$\frac{x^2}{a^2 + \mu} + \frac{y^2}{b^2 + \mu} + \frac{z^2}{c^2 + \mu} = 1,$$

confocal with the ellipsoid of gyration

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is constant, and equal to

$$M(a^2 + b^2 + c^2 + 2\mu).$$

Dr. Routh has now completed his work on "Analytical Statics," and the two volumes form an indispensable addition to the library of the mathematical student.

A. G. GREENHILL.

MOLESWORTH'S POCKET-BOOK.

Pocket-Book of Useful Formulæ and Memoranda for Civil and Mechanical Engineers. By Sir Guilford L. Molesworth, K.C.I.E., M.Inst.C.E., and Robert Bridges Molesworth, M.A., Assoc.M.Inst.C.E. 23rd edition. (London: E and F. N. Spon, 1893.)

OF all the many books published for the assistance of engineers generally there is none so well known to the profession as "Molesworth." This pocket-book is to be found in the possession of every engineer, and rightly so, because it is certainly the most useful and accurate of the many to be obtained.

Any work which has reached the 23rd edition requires no praise to verify its position. This edition is said to contain new and important information on recent

engineering and industrial developments, many of them entirely new, and much of the matter in previous editions has been revised. "Molesworth" treats with nearly all the various branches of engineering, and so extensively has this been carried out that it is impossible to notice but slightly its contents; besides the many purely technical formulæ there is to be found a collection of most useful tables applying generally to engineering. There are, however, a few mistakes in the mass of matter brought together in this book, and in some instances statements are made which would have been better omitted.

On page 254 we read that Vickers' straight steel axles should have an ultimate tensile strength of not more than 23 tons per square inch, and that this test can only be made by destroying the tested axle. Crank axles should also have a maximum tensile strength of 23 tons. There is either a typographical error or gross mistake in this statement. Had the maximum limit been given 33 tons it would have been nearer the mark; 23 tons is absurd. Probably the best tensile tests for steel axles would be 30 tons per square inch, 25 per cent. extension measured over a length of three inches, and 40 per cent. contraction of area at point of fracture. The author omits to state that straight axles are tested under the tup, and that it is this test which destroys them. It is usual to take the tensile sample from this axle. Again, in the tests for steel tyres given on page 255, we notice that a tensile strength of 47 tons is required, but no extension or contraction of area is specified. This is all very well, but tyres have been known to stand the tup test and give the proper tonnage in the tensile test, still the extension has only been 5 to 8 per cent. on three inches, when 16 per cent. is the lowest limit safety demands.

On p. 410, under the head of workshop recipes, there are several mixtures given for case-hardening of wrought iron. The first recipe is used by a few people, but the majority use ordinary charcoal mixed with about 2 per cent. of soda ash. This gives a very uniform and close-grained casing. The author gives no time-allowance for the articles to remain in the furnace; this is all-important, because the time governs the depth of the casing. Further on, at p. 418, we find some recipes to prevent the incrustation of boilers. One cannot help being amused to discover in "Molesworth" of 1893 that potatoes, $\frac{1}{3}$ th the weight of water in the boiler, when put in prevent adherence of scale. Twelve remedies are given, but the only one a man having any regard for his boiler would use is that of frequent blowing off.

When dealing with the question of the proportions of locomotive boilers on p. 453, the statement is made that—(1) no fixed rule can be established as to the best relative proportions of grate, fire-box, and tube surfaces. (2) Length of tube does not affect economic result. (3) Diameter of tube is a matter of indifference.

These conclusions are, to say the least of it, very dogmatical. Given the class of fuel to be consumed and the work to be done, then the question of the design of boiler is not very difficult, and the general practice in this respect may be said to be uniform. This practice is certainly approaching a fixed rule. Given the conditions, the design, or we should say the proportions, becomes an easy matter to designers worthy of the name.