

# A Turbo FDE Technique for Reduced-CP SC-Based Block Transmission Systems

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**Abstract**—For conventional cyclic-prefix (CP)-assisted block transmission systems, the CP length is selected on the basis of the expected maximum delay spread. With regard to single-carrier (SC)-based block transmission implementations, a full-length CP is recommendable, since it allows good performances through the use of simple frequency-domain equalization (FDE) techniques. In this letter, a soft-decision-directed correction (SDDC)-aided turbo FDE technique is presented for reduced-CP SC-based block transmission systems using conventional frame structures. The relations with some already known iterative FDE techniques are established, and a set of performance results is reported and discussed. The advantages of the proposed approach are emphasized, namely, the possibility of approximately achieving (besides the obvious bandwidth efficiency gain) the maximum power efficiency gain that a strong CP reduction allows.

**Index Terms**—Block transmission, cyclic prefix (CP), frequency-domain equalization (FDE), single-carrier (SC) modulations.

## I. INTRODUCTION

CONVENTIONAL single-carrier (SC) modulations have been shown to be suitable for cyclic-prefix (CP)-assisted block transmission within broadband wireless systems, similar to the usually proposed orthogonal frequency-division multiplexing (OFDM) modulations [1]. With a CP long enough to cope with the maximum relative channel delay, a low-complexity frequency-domain equalization (FDE) technique, involving simple fast Fourier transform (FFT) computations, can be employed to solve the severe intersymbol interference (ISI) problem: this is due to the fact that under full-length CP conditions, any interblock interference (IBI) is avoided; moreover, the linear convolutions which are inherent to the time-dispersive channels become equivalent to circular convolutions, corresponding to frequency-domain multiplications. In conventionally designed block transmission systems, after selecting a full-length CP according to the channel memory order, the data block size is chosen to be small enough to ensure a negligible channel variation over the block, but large enough to avoid a significant degradation of both bandwidth and power efficiencies.

In recent years, the possibility of achieving improved FDE performances in SC-based systems, under full-length CP conditions, was considered by several authors. One approach,

Paper approved by G. M. Vitetta, the Editor for Equalization and Fading Channels of the IEEE Communications Society. Manuscript received October 31, 2005; revised March 28, 2006. This paper was presented in part at the IEEE International Symposium on Turbo Codes and Related Topics, Munich, Germany, May 2006.

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Digital Object Identifier 10.1109/TCOMM.2006.887482

as presented in [2] and [3], is turbo equalization in the frequency domain (turbo FDE), where the linear FDE procedures and the decoding procedures (assuming a coded data transmission) are jointly performed, in an iterative way. Another FDE approach, with lower complexity, is the so-called iterative block decision-feedback equalization (IB-DFE), which does not use decoding within the iterative process. This approach, introduced in [4], was later extended and shown to be easily compatible with space diversity and multiple-input multiple-output (MIMO) systems [5], [6], as well as selected CP-assisted orthogonal frequency-division multiple-access (OFDMA)-type and code-division multiple-access (CDMA) schemes [7]–[9].

Since a full-length CP reduces the block transmission efficiency, the possibility of adopting a reduced CP (below the channel memory order), while keeping an essentially FFT-based implementation, deserves to be considered. In [10], a basic algorithm for a decision-directed correction (DDC) of the FDE inputs under reduced-CP conditions was shown to provide good performances, without significant error propagation, when using low-complexity, noniterative receiver techniques for especially designed SC-based frame structures. Selected time-domain computations, similar to those behind the DDC algorithm, have already been proposed by several authors, within iterative algorithms which provide IBI suppression and CP reconstruction in more complex receivers, for SC-based and OFDM-based block transmission systems (see, for example, [11] and [12]). In this letter, in the reduced-CP context, we consider a soft-decision version of the DDC algorithm (Section II), in an iterative way, as an aid to turbo-type FDE techniques similar to those reported above (Section III). Section IV (performance results) and Section V (conclusions and final remarks) complete the letter.

## II. SDDC ALGORITHM FOR REDUCED-CP BLOCK TRANSMISSION

For a length- $L$  channel impulse response (CIR), let us consider the transmission of length- $N$  time-domain symbol blocks  $\mathbf{s}(m) = [s_0(m), s_1(m), \dots, s_{N-1}(m)]^T$ , with  $N > L$ , which are related to data. Whenever a length- $L$  CP is appended to each data block, the length- $m$ th useful received block can be represented by  $\mathbf{y}_{\text{CP}}(m) = \mathbf{H}\mathbf{s}(m) + \mathbf{n}(m)$ , where  $\mathbf{n}(m) = [n_0(m), \dots, n_{N-1}(m)]^T$  is the  $m$ th received noise vector, and  $\mathbf{H}$  is the  $N \times N$  circulant matrix which describes the channel effects. The entries of this square matrix, given by  $h_{j,k} = h_{(j-k) \bmod N}$ , are related to the length- $L$  CIR ( $h_n = 0$  for  $n = L + 1, \dots, N - 1$ ).

By assuming the transmission of length- $N$  blocks with a length- $L_R$  CP ( $0 \leq L_R < L, N + L_R \geq 2L$ ), the initial

$\Delta L = L - L_R$  samples of each received block will differ from the corresponding samples under full-length CP, unless

$$\Delta s_p(m) = s_p(m) - s_{p+L_R}(m-1) \quad (1)$$

is equal to zero with  $p = N - L, \dots, N - L_R - 1$ . The insufficient CP leads to some IBI, and also to an imperfect circular convolution regarding the channel impact on the data block contents. When using  $\mathbf{y}(m)$  to denote the new length- $N$  received block, it can be shown that

$$\mathbf{y}_{\text{CP}}(m) - \mathbf{y}(m) = \mathbf{I}'_{\Delta L} \mathbf{H} \Delta(m) \quad (2)$$

with  $\mathbf{I}'_{\Delta L}$  and  $\Delta(m)$  as follows:

$$\mathbf{I}'_{\Delta L} = \text{diag}[\underbrace{1, \dots, 1}_{\Delta L}, \underbrace{0, \dots, 0}_{N-\Delta L}] \quad (3)$$

$$\Delta(m) = [\underbrace{0, \dots, 0}_{N-L}, \underbrace{\Delta s_{N-L}(m), \dots, \Delta s_{N-L_R-1}(m)}_{\Delta L}, \underbrace{0, \dots, 0}_{L_R}]^T. \quad (4)$$

When an estimate  $\hat{\Delta}(m)$  of  $\Delta(m)$  is available, a DDC of  $\mathbf{y}(m)$  can be carried out to obtain a suitable approximation to  $\mathbf{y}_{\text{CP}}(m)$

$$\begin{aligned} \hat{\mathbf{y}}_{\text{CP}}(m) &= \mathbf{y}(m) + \mathbf{I}'_{\Delta L} \mathbf{H} \hat{\Delta}(m) \\ &= \mathbf{y}(m) + \mathbf{I}'_{\Delta L} \mathbf{F}^{-1} \text{diag}[H_0, \dots, H_{N-1}] \mathbf{F} \hat{\Delta}(m) \end{aligned} \quad (5)$$

where  $\mathbf{F}$  and  $\mathbf{F}^{-1}$  denote a discrete Fourier transform (DFT) matrix and an inverse (IDFT) matrix, respectively, and  $[H_0, H_1, \dots, H_{N-1}]^T$  is the DFT of  $[h_0, h_1, \dots, h_{N-1}]^T$ . This is the basis for the DDC algorithm, as presented in [10].

In the following, we assume a conventional block transmission, with length- $N$  useful symbol blocks (corresponding to blocks of coded data), and a length- $L_R$  CP for every block; however, we will consider that the CP length ( $L_R$ ) can be chosen to be smaller than  $L$ . In this context, a very simple iterative FDE technique using a DDC aid could easily be devised. For a given block  $m$  and a given FDE iteration  $i \in \{1, 2, \dots, I\}$ , the length- $N$  time-domain input block  $\hat{\mathbf{y}}_{\text{CP}}^{(i)}(m)$  for FDE purposes could be obtained from the received vector  $\mathbf{y}(m)$  simply by using the DDC algorithm described here, with

$$\hat{\Delta s}_p^{(i)}(m) = \hat{s}_p^{(i-1)}(m) - \hat{s}_{p+L_R}^{(I)}(m-1) \quad (6)$$

taking advantage of the appropriate  $\Delta L$  current decisions on symbols of block  $m$  (iteration  $i-1$ ), and the final decisions regarding the last  $\Delta L$  symbols of block  $m-1$  (iteration  $I$ ). Obviously, for  $i=1$ , no previous decisions regarding block  $m$  are available; therefore, we should assume  $\hat{s}_p^{(0)}(m) = 0$  when using (6) for  $i=1$ . Certainly, there is a more efficient way of using the DDC aid; it consists of replacing the hard decisions by some kind of soft decisions, derived from the soft information that a soft-in, soft-out (SISO) channel decoder can provide. An appropriate choice is a soft-decision-directed correction (SDDC)

scheme, as an alternative to the DDC scheme described above, where the  $\hat{\Delta}^{(i)}(m)$  vector is replaced by

$$\begin{aligned} \bar{\Delta}^{(i)}(m) &= [\underbrace{0, \dots, 0}_{N-L}, \underbrace{\bar{\Delta s}_{N-L}^{(i)}(m), \dots, \bar{\Delta s}_{N-L_R-1}^{(i)}(m)}_{\Delta L}, \underbrace{0, \dots, 0}_{L_R}]^T. \end{aligned} \quad (7)$$

In this new vector, for  $p = N - L, \dots, N - L_R - 1$

$$\bar{\Delta s}_p^{(i)}(m) = \bar{s}_p^{(i-1)}(m) - \bar{s}_{p+L_R}^{(I)}(m-1) \quad (8)$$

with  $\bar{s}_p^{(i-1)}(m)$  and  $\bar{s}_{p+L_R}^{(I)}(m-1)$  being mean symbol values (in the statistical sense) rather than hard decisions. In Appendix A, we indicate a way to compute these values, based on the log-likelihood ratios (LLRs) of the coded bits, provided by the channel decoder, for a quaternary phase-shift keying (QPSK) symbol constellation.

### III. TURBO FDE TECHNIQUES FOR SC-BASED BLOCK TRANSMISSION

When a full-length CP is employed, turbo FDE techniques such as those proposed in [2] and [3] can provide a strongly improved FDE performance, while avoiding a high complexity of implementation. Such techniques, which resort to a conventional, linear, single-tap FDE scheme, take advantage of the SISO decoder outputs in order to carry out a turbo soft cancellation of residual ISI through the use of the soft information on the coded bits. According to the switched approximate linear equalization/matched filtering (APPLE/MF) approach [3], the FDE coefficients for each iteration are selected on the basis of an average signal-to-noise ratio (SNR) concerning the length- $N$  block of equalizer outputs. This block provides an extrinsic information on the coded data block, delivered by the equalizer to the channel SISO decoder. For each iteration, the receiver uses the algorithm (APPLE or MF) leading to the largest estimated SNR.

In the following, we describe a new conventional (for full-length CP conditions) turbo FDE technique which is strongly related to that proposed in [3], but replaces the selection principle (APPLE/MF) regarding the linear FDE parameters by an appropriate compromise choice. In fact, the values of the linear FDE parameters are adaptively adjusted, iteration by iteration, according to the available block of SISO decoder outputs. The proposed receiver structure is depicted in Fig. 1 ( $\Pi$  and  $\Pi^{-1}$  stand for ‘‘interleaver’’ and ‘‘deinterleaver,’’ respectively, and  $\odot$  denotes ‘‘element-by-element multiplication’’). At the equalizer output, the time-domain vector  $\check{\mathbf{s}}^{(i)}(m)$  is the IDFT of

$$\check{\mathbf{S}}^{(i)}(m) = \mathbf{F}^{(i)}(m) \odot \mathbf{Y}(m) + \mathbf{G}^{(i)}(m). \quad (9)$$

Therefore, the  $N$  entries of  $\check{\mathbf{S}}^{(i)}(m)$  can be written as

$$\check{S}_k^{(i)}(m) = F_k^{(i)}(m) Y_k(m) + G_k^{(i)}(m) \quad (10)$$

where  $F_k^{(i)}(m), k = 0, 1, \dots, N-1$  are the multiplicative FDE parameters for iteration  $i$ , and

$$G_k^{(i)}(m) = (\gamma^{(i)}(m) - F_k^{(i)}(m) \hat{H}_k) \bar{S}_k^{(i-1)}(m) \quad (11)$$

with  $\gamma^{(i)}(m) = (1/N) \sum_{k=0}^{N-1} F_k^{(i)}(m) \hat{H}_k$  are complementary FDE parameters for ISI soft-cancellation purposes.  $\bar{\mathbf{S}}^{(i-1)}(m) = [\bar{S}_0^{(i-1)}(m), \bar{S}_1^{(i-1)}(m), \dots, \bar{S}_{N-1}^{(i-1)}(m)]^T$  is the DFT of  $\bar{\mathbf{s}}^{(i-1)}(m) = [\bar{s}_0^{(i-1)}(m), \bar{s}_1^{(i-1)}(m), \dots, \bar{s}_{N-1}^{(i-1)}(m)]^T$ ,

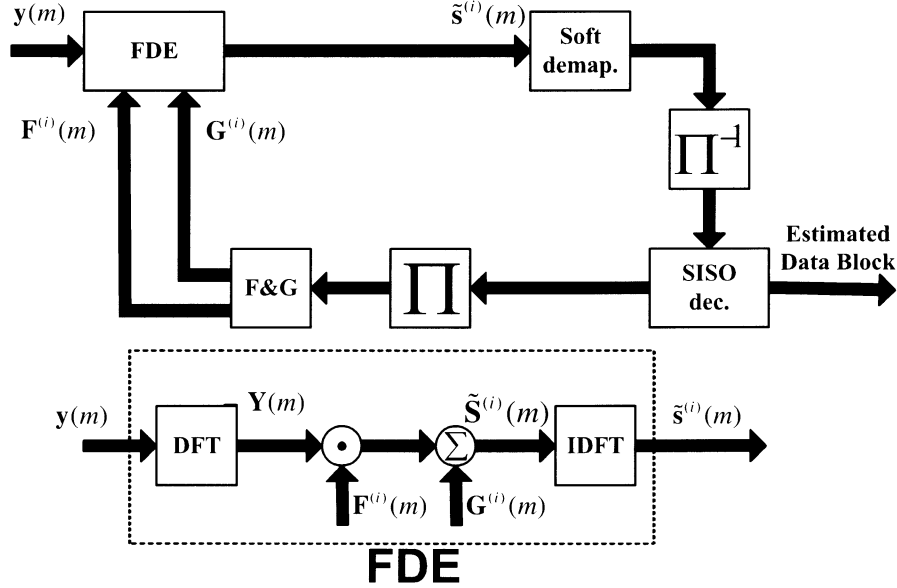


Fig. 1. Turbo FDE receiver structure and characterization of the FDE unit for SC-based block transmission.

resulting from the soft information provided by the SISO decoder, as shown in Appendix A, when assuming a QPSK modulation. It should be noted that, in this case

$$\bar{s}_p^{(i-1)}(m) = \frac{\sigma_s}{\sqrt{2}} \cdot \left( \tanh \left( \frac{L_{p,I}^{(i-1)}(m)}{2} \right) + j \tanh \left( \frac{L_{p,Q}^{(i-1)}(m)}{2} \right) \right) \quad (12)$$

where  $L_{p,I}^{(i-1)}(m)$  and  $L_{p,Q}^{(i-1)}(m)$  are the LLRs of the in-phase and the quadrature coded bits, respectively.

The soft demapper in Fig. 1 provides the inputs to the SISO decoder (LLRs of the several coded bits). The decoder outputs must correspond to the full soft information, not the extrinsic one. As to the FDE parameters, instead of  $F_k^{(i)}(m) = \hat{H}_k^*/(\hat{\alpha} + |\hat{H}_k|^2)$  (APPLE) or  $F_k^{(i)}(m) = \hat{H}_k^*$  (MF) [3], with  $\alpha = (\sigma_n^2/\sigma_s^2)$  ( $\sigma_n^2$  denoting the variance of the Gaussian noise components), we adopt

$$F_k^{(i)}(m) = \frac{K_F^{(i)}(m)\hat{H}_k^*}{\hat{\alpha} + (1 - (\hat{\rho}^{(i-1)}(m))^2)|\hat{H}_k|^2} \quad (13)$$

where  $\hat{\rho}^{(i-1)}(m) = (1/N) \sum_{n=0}^{N-1} (E[s_n^*(m)\hat{s}_n^{(i-1)}(m)]/E[|s_n(m)|^2])$  is an overall correlation coefficient. It can be obtained as an average value of the  $2N$  correlation coefficients per bit  $(\rho_{n,I}^{i-1}(m), \rho_{n,Q}^{i-1}(m))$ , derived from the SISO decoder outputs.  $K_F^{(i)}(m)$  is a normalization factor. Using  $\gamma^{(i)}(m) = 1$ ,  $\tilde{s}_n^{(i)}(m) = s_n(m) + \xi_n^{(i)}(m)$ , where  $\xi_n^{(i)}(m)$  is the zero-mean error (assumed to be approximately complex Gaussian) concerning symbol  $s_n(m)$  at the FDE output. Under the Gaussian assumption, the LLRs of the in-phase and quadrature bits at the SISO decoder input, are given by  $L_{n,I}^{(i)}(m) = (\sqrt{8}/\sigma_{\text{eq}}^{2(i)}(m))\sigma_s \Re\{\tilde{s}_n^{(i)}(m)\}$

and  $L_{n,Q}^{(i)}(m) = (\sqrt{8}/\sigma_{\text{eq}}^{2(i)}(m))\sigma_s \Im\{\tilde{s}_n^{(i)}(m)\}$ , respectively, where  $\sigma_{\text{eq}}^{2(i)}(m)$  is the mean-squared error in the time-domain samples  $\tilde{s}_n^{(i)}(m)$ . It can easily be estimated as  $\hat{\sigma}_{\text{eq}}^{(i)}(m) = (1/N) \sum_{n=0}^{N-1} |\tilde{s}_n^{(i)}(m) - s_n^{(i)}(m)|^2$ , with  $s_n^{(i)}(m) = (\sigma_s/\sqrt{2}) (\text{sgn}(\Re\{\tilde{s}_n^{(i)}(m)\}) + j\text{sgn}(\Im\{\tilde{s}_n^{(i)}(m)\}))$ .

We point out that for  $i = 1$ , the  $F_k^{(i)}(m)$  parameters meet the minimum mean-squared error (MMSE) criterion, since  $\hat{\rho}^{(i-1)}(m) = 0$  in (13). After a number of iterations and/or for high SNRs, typically  $\hat{\rho}_{n,I}^{(i-1)}(m) \approx 1$  and  $\hat{\rho}_{n,Q}^{(i-1)}(m) \approx 1$ , leading to  $\hat{\rho}^{(i-1)}(m) \approx 1$  in (13), and, therefore, to  $F_k^{(i)}(m)$  parameters approximately in accordance with the MF criterion. When replacing the APPLE/MF selection [3] of the  $F_k^{(i)}(m)$  parameters by the proposed compromise choice, according to (13), a small performance gain is achieved [13].

When the CP length ( $L_R$ ) is smaller than the channel memory order ( $L$ ), good performances through the iterative receiver technique of Fig. 1 cannot be ensured. In this case, Fig. 2 shows a suitable receiver technique, which actually uses an SDDC aid, as proposed in Section II, to the turbo FDE technique described above. The  $\Delta$  unit computes the vector  $\bar{\Delta}^{(i)}(m)$ , which is related to  $\bar{s}^{(i-1)}(m)$  and  $\bar{s}^{(I)}(m-1)$ , and has entries according to (7) and (8); for each iteration, the time-domain input to the FDE unit is updated, not only the FDE parameters.

Simplified iterative FDE implementations, based on the ideas above, can be considered, with no decoding effort really involved in the FDE process [13]. The corresponding receiver derives from that shown in Figs. 1 and 2 by suppressing the SISO decoder. Therefore, no extrinsic information is provided to help the iterative FDE process, and the soft FDE outputs in a given iteration ( $i-1$ ) are directly used to compute  $F_k^{(i)}(m)$  and  $G_k^{(i)}(m)$  for the next iteration. An additional simplification is to replace the correlation coefficient concerning every bit by  $\hat{\rho}^{(i-1)}(m)$  in the computation of  $\bar{s}_n^{(i-1)}(m)$ . Therefore,  $\bar{s}_k^{(i-1)}(m) = \hat{\rho}^{(i-1)}(m)\hat{S}_k^{(i-1)}(m)$

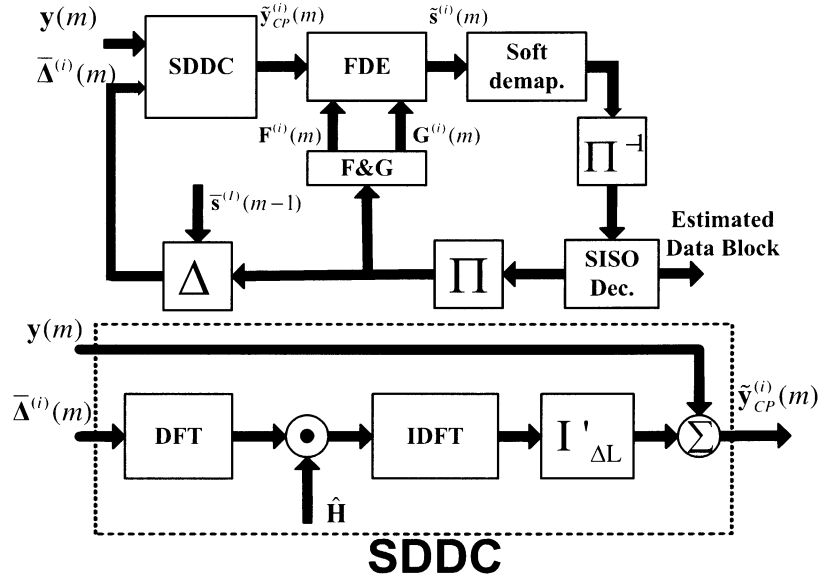


Fig. 2. Turbo SDDC-FDE receiver structure for SC-based block transmission, with characterization of the SDDC unit.

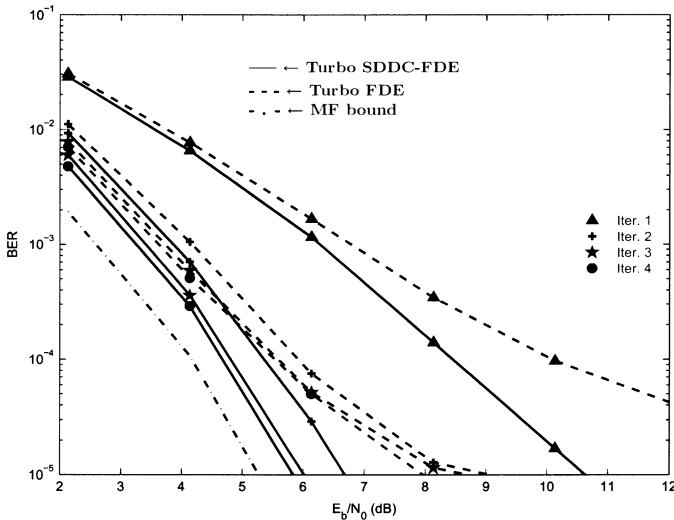


Fig. 3. Coded BER performances when  $L_R < L$ , showing the advantages of the SDDC-aided turbo FDE technique over the conventional turbo FDE technique.

(with  $[\hat{s}_0^{(i-1)}(m), \hat{s}_0^{(i-1)}(m), \dots, \hat{s}_{N-1}^{(i-1)}(m)]^T$  as the DFT of  $[\hat{s}_0^{(i-1)}(m), \hat{s}_0^{(i-1)}(m), \dots, \hat{s}_{N-1}^{(i-1)}(m)]^T$ ), leading to an iterative block (IB)-DFE receiver [4], [13].

#### IV. PERFORMANCE RESULTS

A set of numerical results is presented in the following, with regard to broadband transmission over a strongly frequency-selective Rayleigh fading channel, when the iterative techniques of Sections II and III are employed under perfect channel estimation. We adopt the power delay profile type C within HIPERLAN/2, with uncorrelated Rayleigh fading on the different paths. A conventional CP-assisted block transmission scheme is assumed, with  $N = 256$  QPSK data symbols per block. The duration of the useful part of each block is  $5 \mu\text{s}$ , and we consider either a full-length CP ( $L_R = L = 64$ ) or a reduced CP ( $L_R = L/8 = 8$ ); with the latter alternative,

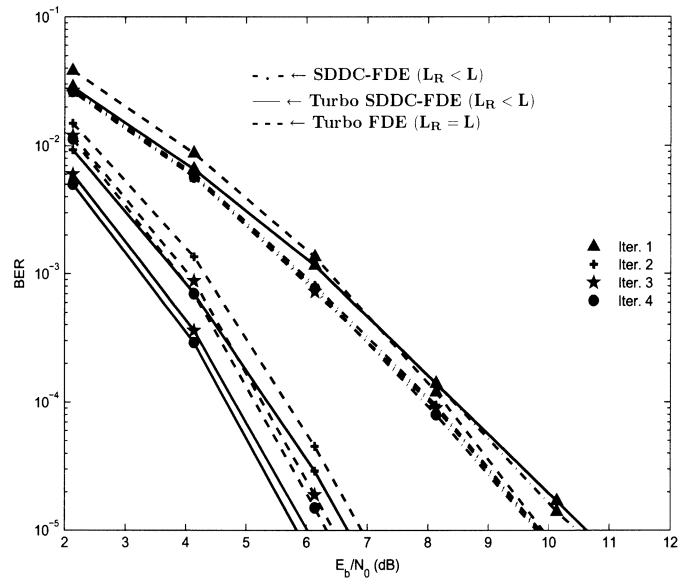


Fig. 4. Coded BER performances for the SDDC-aided techniques ( $L_R < L$ ) and the conventional turbo FDE technique ( $L_R = L$ ).

about 1/3 of the CIR energy falls outside the time interval of the reduced CP.

For the performance results of Figs. 3 and 4, we assumed a rate-1/2 convolutional code with  $G(D) = [1 (1+D^2)/(1+D+D^2)]$ , a low-complexity SISO decoding through the use of the Max-Log-MAP algorithm [14], and the close equalization/decoding cooperation which is allowed by the receivers shown in Figs. 1 and 2, as described in Section III. Fig. 3 shows coded bit-error rate (BER) performances when  $L_R = L/8 = 8$ , for both the conventional turbo FDE technique (Fig. 1) and the turbo SDDC-FDE technique (Fig. 2). The advantage of the SDDC-aided technique is very clear, in spite of the moderate amount of additional complexity, and it should be noted that it allows a close approximation to the new MF bound (with  $L_R = L/8 = 8$  and  $N = 256$ , for the coded SC transmission

under consideration). Moreover, the conventional turbo FDE technique (designed for full-length CP conditions) is not able to avoid a significant performance degradation, characterized by a clear error floor. To conclude, Fig. 4 compares coded BER results in the following cases: the turbo FDE technique of Fig. 1, with  $L_R = L = 64$ ; and the turbo SDDC-FDE technique of Fig. 2, with  $L_R = L/8 = 8$ . For the sake of comparison, we also include performance results for an iterative SDDC-aided FDE technique which does not upgrade the FDE procedures during the iterative process (it corresponds to using  $\mathbf{G}^{(i)}(m) = 0$  and  $\mathbf{F}^{(i)}(m) = \mathbf{F}^{(1)}(m)$  in the receiver of Fig. 2), similar to the iterative equalization/decoding technique in [12]. The proposed turbo SDDC-FDE technique clearly provides a much better performance, under a moderate amount of additional complexity.

Special attention should be paid to the following pair of performance curves: the best solid line, as compared with the best dashed line (turbo FDE performances). We must keep in mind that when reducing the CP length from  $L_R = L = 64$  to  $L_R = L/8 = 8$  (e.g., to increase the bandwidth efficiency by about  $(L - L_R/N + L_R) \times 100\%$ , which gives 21.2% in our case), the maximum achievable power efficiency gain is  $10 \log_{10}((N + L)/(N + L_R)) \approx 0.84$  dB. Therefore, it is clear that the SDDC-aided approach can practically ensure that there is no degradation of the power efficiency as a downside of that reduction, and, on the contrary, there is a gain close to the maximum.

## V. CONCLUSIONS AND FINAL REMARKS

An SDDC-aided turbo FDE technique, based on the DDC algorithm [10], was presented for reduced-CP, SC-based block transmission systems using conventional frame structures.

The relations with some already known iterative FDE techniques [2]–[4], [12] were established. The advantages of the SDDC-aided approach were emphasized, namely, the possibility of approximately achieving the maximum power efficiency gain that a strong CP reduction allows. The SDDC-aided techniques seem to be especially well-suited for the uplink of future broadband wireless systems, following the recommendation of a hybrid air interface for those systems (SC-based uplink and OFDM-based downlink) [15], [16]. In this context, an SC-based transmitter at the mobile terminal (MT) and a turbo SDDC-FDE receiver at the base station (BS) turn out to be realistic, complementary choices for system implementation. Therefore, power amplifiers at the MTs do not need to be highly linear, and, by an appropriate selection of block and CP durations (according to  $L_R \ll N$ ), it is possible to achieve excellent uplink performances even for channels which are both strongly time-dispersive and strongly time-varying, under a moderate complexity charge at the BSs.

To conclude, we point out that conventional turbo FDE techniques in accordance with Fig. 1, designed for full-length CP conditions, exhibit a strong complexity advantage over turbo equalization techniques in the time domain [3], but suffer from the power/bandwidth drawback which is inherent to the full-length CP. By allowing a reduced-CP system choice, the proposed turbo SDDC-FDE technique (Fig. 2) essentially preserves the above-mentioned advantage, while avoiding the above-mentioned drawback.

## APPENDIX A

### QPSK SYMBOL STATISTICS USING SOFT DECODER OUTPUTS

Let us assume QPSK symbol coefficients  $s_n = s_{n,I} + js_{n,Q}$ , with  $s_{n,I} = \pm\sigma_s/\sqrt{2}$  and  $s_{n,Q} = \pm\sigma_s/\sqrt{2}$  ( $n = 0, 1, \dots, N - 1$ ), according to the coded data block. When the LLRs concerning the  $n$ th in-phase bit and the  $n$ th quadrature bit, as provided by the channel decoder, are  $L_{n,I}$  and  $L_{n,Q}$ , respectively, the resulting expected value  $\bar{s}_n$  can be expressed as  $\bar{s}_n = \bar{s}_{n,I} + j\bar{s}_{n,Q}$  with  $\bar{s}_{n,I} = (\sigma_s/\sqrt{2}) \tanh(L_{n,I}/2)$  and  $\bar{s}_{n,Q} = (\sigma_s/\sqrt{2}) \tanh(L_{n,Q}/2)$ .

Let us define the coded bit decisions  $\hat{s}_{n,I} = \pm\sigma_s/\sqrt{2}$  and  $\hat{s}_{n,Q} = \pm\sigma_s/\sqrt{2}$ , according to the signs of  $L_{n,I}$  and  $L_{n,Q}$ , respectively, and the following correlation coefficients:  $\rho_{n,I} = (E[s_{n,I}\hat{s}_{n,I}]/E[|s_{n,I}|^2])$ ;  $\rho_{n,Q} = (E[s_{n,Q}\hat{s}_{n,Q}]/E[|s_{n,Q}|^2])$ . Since  $\rho_{n,I} = 1 - 2\text{Prob}(\hat{s}_{n,I} = -s_{n,I}|L_{n,I}) = \tanh(|L_{n,I}|/2)$  and  $\rho_{n,Q} = 1 - 2\text{Prob}(\hat{s}_{n,Q} = -s_{n,Q}|L_{n,Q}) = \tanh(|L_{n,Q}|/2)$  (leading to  $0 \leq \rho_{n,I} \leq 1$  and  $0 \leq \rho_{n,Q} \leq 1$ ), the average values  $\bar{s}_{n,I}$  and  $\bar{s}_{n,Q}$  can be written as  $\bar{s}_{n,I} = \rho_{n,I}\hat{s}_{n,I}$  and  $\bar{s}_{n,Q} = \rho_{n,Q}\hat{s}_{n,Q}$ .

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