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A 'TWIN-EXHAUST' MODEL FOR DOUBLE RADIO SOURCES

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SUMMARY

A mechanism is proposed for the formation of radio components in strong double sources such as Cygnus A. Relativistic plasma generated in an active galactic nucleus cannot escape isotropically if the nucleus is surrounded by too much dense thermal gas. There is, however, a possible equilibrium flow in which the plasma escapes along two oppositely-directed channels or 'exhausts'. At all points on the boundary of these channels, the pressure of the relativistic (possibly magnetized) plasma must balance the pressure of the static thermal gas cloud. The outflow velocity becomes sonic (implying, for ultra-relativistic plasma, a velocity $c/\sqrt{3}$) where the external pressure is $\approx \frac{1}{2}$ its central value. The channel cross-section reaches a minimum value at this point. The channel then widens again as the external pressure drops still further, and, as in a de Laval nozzle, the flow becomes supersonic. Relativistic plasma can thus be collimated into two relativistic beams.

Hargrave & Ryle's high resolution maps of Cygnus A reveal 'hot spots' $\lesssim 2$ kpc in size at the outer edge of each individual component into which (it is believed) energy is being continuously supplied. We identify these hot spots with the regions where the beams impinge on the intergalactic medium. The dimensions and radio luminosity of Cygnus A imply that the central galactic nucleus must have maintained a power output $\sim 10^{46}$ erg s^{-1} for 10^6–10^7 yr. This outflow could have been collimated into two sufficiently narrow beams if the galactic nucleus were surrounded by gas with $T \simeq 10^8$ K, density $\sim 10^3$ particles cm^{-3} and scale height ~ 200 pc. Some aspects of the detailed morphology of Cygnus A are also interpreted on the basis of this general model.

The possible role of instabilities in the flow pattern, and the influence of magnetic fields, is discussed. Applications to other sources (which generally require less extreme parameters than Cygnus A) are briefly considered.

1. INTRODUCTION

Recent high-resolution observations of the powerful extragalactic double radio sources indicate that the energy for such objects is probably supplied continuously by the nuclear regions of large galaxies (usually giant ellipticals) or quasars. General theoretical reviews of alternative radio source models have been given by van der Laan & Perola (1969), Rees (1972) and Longair, Ryle & Scheuer (1973). In this paper we discuss further the possibility, previously suggested in a specialized form by one of us (Rees 1971), that the energy is supplied by a light fluid—composed of fast (possibly relativistic) particles, perhaps pervaded by electromagnetic fields—which is generated in the nuclear region and collimates into two oppositely directed beams or 'exhausts'. The bulk velocity of this fluid becomes very large (possibly $\sim c$). Each beam terminates, and reconverts much of its bulk energy into relativistic particles and fields, at a 'working surface' which advances outwards at a speed determined by a balance between the momentum flux in the beam and the 'ram pressure' of the intergalactic medium. This speed is sub-relativistic, but probably highly supersonic with respect to the external gas (though a subsonic variant has been proposed by Gull & Northover 1973). Scheuer (1974) has discussed some specific fluid dynamical aspects of beam models. An important feature of these models is that the energy content of extended sources can be assumed to accumulate gradually over the whole lifetime, at rates $\lesssim 10^{45}$ erg s^{-1}: there is no need (as there is in some other theories) to hypothesize a short-lived initial outburst of vastly higher power.

The recent high-resolution radio maps of Cygnus A (Hargrave & Ryle 1974) present in many respects the most stringent tests for any model. These maps reveal that there is a relatively compact (~ 2 kpc) feature of high surface-brightness near the outer edge of each component. The synchrotron lifetimes of the relevant electrons in these 'hot spots' are estimated to be shorter than the light travel time from the central galaxy, suggesting that the electron supply is being continuously replenished; and the permitted thermal gas density in the 'hot spots' seems to be inadequate for a De Young–Axford (1967) or inertial confinement model. On these grounds, Hargrave & Ryle argue that their data are best explained by a 'beam' model, the 'hot spots' being associated with the 'working surface'. A continuous power output $\gtrsim 3 \times 10^{45}$ erg s^{-1} would be necessary for such a model to be viable. This power is not necessarily excessive by comparison with the observed integrated luminosities of active galactic nuclei, or even the optical line and infrared emission of the nuclear region of Cygnus A (Mitton & Mitton 1972; Rieke & Low 1972); if it continued for 10^6–10^7 yr, the power supply would be more than sufficient to build up the presently-inferred energy content of the radio-emitting regions.

The pressure associated with this power supply within the nucleus would greatly exceed the likely pressure of the ambient extragalactic medium. The observed bifurcation suggests that, if the energy is released in a fluid form, there must be some material around the nucleus capable of balancing the large pressure and preventing a spherical outflow. We suppose that this material takes the form of relatively dense (possibly magnetized) gas, confined by the gravitational potential well at the centre of the galaxy. The scale height is assumed to be comparable with the dimensions of the nuclear region (i.e. much less than the overall size of the galaxy) so that the pressure in the cloud can greatly exceed that of the extragalactic medium. This is in contrast to the model investigated by Gull & Northover (1973) in which the energy is generated within a bubble, with an internal pressure comparable to that in the extragalactic medium, which attains galactic dimensions (~ 8-kpc radius for Cygnus A) before fission into two similar bubbles occurs. However, there is a similarity between the two models in that both essentially rely on buoyancy forces to raise a light relativistic fluid above a denser surrounding medium. We aim to show in Sections 2 and 3 that a (sufficiently stable) equilibrium flow pattern can establish itself in which the relativistic plasma emerges in two oppositely-directed beams. The collimation is achieved because the pressure of the thermal gas in the nucleus concentrates the relativistic outflow into two channels whose shapes adjust so that they act like de Laval nozzles.

The precise natures of both the central active region and the light fluid are for many purposes unimportant. The fluid is anticipated to be an uncertain mixture of static magnetic fields, hot (possibly relativistic) plasma, and perhaps a variety of low frequency wave modes.† This energy is widely assumed to be gravitational in origin, and possible constituents of the central active region are: clusters of pulsars, or stars collapsing under rotational support (Rees 1971; Blandford & Rees 1972; Arons, Kulsrud & Ostriker 1974); black holes of stellar mass, surrounded by accretion discs in which the dominant viscosity near the Schwarzschild radius is caused by electromagnetic fields (these could radiate similar *energies* to pulsars but with much smaller *power* for longer times); massive 'objects' (Hoyle & Fowler 1963; Salpeter 1971); massive black holes (Lynden-Bell 1969; Pringle, Rees & Pacholczyk 1973). A firm choice between these (or other) alternatives must await future studies of active nuclei. The only general prerequisites for our present discussion are that the central active region should be capable of maintaining its power output for at least 10^6–10^7 yr, and should be $\lesssim 10$ pc in size.

2. THE BASIC FLOW

We now consider the possible steady outflow of a light fluid continuously generated in the nucleus of a galaxy whose central region contains a 'cool' thermal gas cloud in which gravitational and pressure forces are in equilibrium. For illustrative purposes, we assume that the light fluid is a fully-relativistic gas with a specific heat ratio of 4/3. For the moment, we assume the flow is isentropic (ignoring dissipative processes perhaps resulting from instabilities) and, except very close to the nucleus, we approximate the flow as one-dimensional. We search for stationary flows in which the cross-sectional area of the channel (assumed circular of radius r) has adjusted so as to maintain a constant power L. A consequence of the relatively low ($\ll c$) external sound speed is that the direct influence of gravity on the flow of the light fluid can be ignored. The indirect influence is of course considerable as it is responsible for maintaining the external pressure distribution.‡

In the Appendix we describe the nature of one-dimensional isentropic flow of an ultra-relativistic fluid. The pressure, p, (assumed isotropic) measured in the co-moving frame is identical to the pressure exerted transversely on the walls of the channel by the relativistic fluid. It can thus be equated to the external pressure which falls monotonically with distance from the nucleus. If the central (stagnation) pressure is p_0, the Lorentz factor corresponding to the bulk fluid velocity v is

† In Rees (1971) it was assumed that this outflow was primarily in the form of low frequency waves, whose 'self-focusing' properties were invoked to achieve the collimation into beams. Our present view, however, is that *any* form of relativistic outflow can be collimated into beams, and can efficiently accelerate relativistic particles at the 'working surface'.

‡ Some justification is perhaps needed for treating the relativistic plasma as a fluid, even though the collisional mean-free-paths are very long. If a magnetic field B gauss were present, the Larmor radius for a proton of Lorentz factor γ would be $\sim 10^{-12}\gamma B^{-1}$ pc. The smallest length-scales of the flow patterns we should consider are 1–10 pc. Thus even a magnetic field whose dynamical effects were completely negligible could guarantee fluid-like behaviour. Collective processes in the plasma would also decrease the effective mean-free-path. (Similar arguments justify a fluid dynamical approach in discussing some aspects of—for instance—the solar wind.) The nature of the 'hot' fluid is discussed further in Sections 3 and 4. If the mean-free-paths are small, the path of a typical particle closely follows a streamline of the bulk flow; also we are justified in assuming a sharp boundary between the two fluids—except in so far as instabilities occur.

given by $\gamma = (p_0/p)^{1/4}$; and when $p < p^* = 4/9p_0$, the flow becomes supersonic (i.e. $v > c/\sqrt{3}$). At the sonic point ($R = R^*$) the cross-sectional area is minimized and a nozzle should therefore form. An expression for the minimum channel radius, r^*, at the nozzle is:

$$r^* \sim 0{\cdot}5(L/cp_0)^{1/2}. \tag{2.1}$$

If $L = 10^{46}L_{46}$ erg s^{-1} and $p_0 = 10^{-4}p_{0-4}$ dyne cm^{-2}, then

$$r^* \sim 9L_{46}{}^{1/2}p_{0-4}{}^{-1/2} \text{ pc}. \tag{2.2}$$

We then obtain

$$r = 0{\cdot}6[(p/p_0)(1-(p/p_0)^{1/2})]^{-1/4}\, r^*. \tag{2.3}$$

In laboratory or engineering applications of the Bernoulli equation, one customarily considers situations where gas flows through a pipe of *given* cross-section. The pressure and velocity can then be calculated. In the present application, however, it is the *pressure* $p(R)$ whose variation along the pipe is given, and the Bernoulli equation then tells us how the cross-section must adjust in order that the internal pressure balances the given external pressure. If p decreases by a factor $\gg 2$ along the pipe, the *only possible steady situation* is one in which there is a transition to supersonic flow.

If the relativistic plasma escapes along two cylindrical ' channels ', whose cross-sectional radii are $r(R)$, the ' nozzle ' forms at a radius comparable with the pressure scale-height for the gas, irrespective of the form of $p(R)$. The only general requirement is that the external pressure should be high enough to make the nozzle radius r^* (equation (2.2)) much less than R^*. This guarantees that the equilibrium shape of the channel along which the relativistic fluid escapes is more or less as shown in Fig. 1. Within R^*, the channel widens (its width being given in terms of $p(R)$ by (2.3)) until it forms a quasi-spherical ' bulb ' of radius R_0. Near R_0, the flow cannot be approximated as one-dimensional, but since the velocities are $\ll c$ the pressure can be assumed to be $\sim p_0$. For consistency, the relativistic energy must all originate in a region of dimensions $\lesssim R_0$. (If $r^* \ll R^*$, then in general R_0 is substantially smaller than R^*. The precise shape of the channel —i.e. the function $r(R)$—depends on the form of the gravitational potential well and the temperature distribution in the gas cloud.)

If the pressure distribution in the gas cloud is axisymmetric rather than strictly spherically symmetric (as would be expected if the gravitational equipotentials were oblate spheroids, or the cloud were rotating) then the jets would be expected to emerge along the axis of symmetry—any oblique orientation would plainly not be an equilibrium, let alone a stable one.

If for some reason a jet emerges only in *one* direction along the symmetry axis, application of the Bernoulli equation shows that the internal pressure would be maximal at the stagnation point opposite the jet. The *external* pressure at this point, however, is less than in the equatorial plane. Thus the flow pattern with a single jet is not an equilibrium situation—a second jet would develop antiparallel to the first. (We would indeed conjecture that a similar statement holds even if the gas cloud is strictly spherically symmetric; and that if one jet develops, another automatically tends to form in the opposite direction, so that the net momentum discharge is zero.)

The results quoted so far apply to an ultra-relativistic fluid, but the important qualitative features are unchanged if the internal sound speed is non-relativistic, provided that it is still much higher than the external sound speed and the escape velocity. The nozzle should still establish itself, the trans-sonic point coinciding with minimum cross-section. For a non-relativistic gas with specific heat ratio equal to

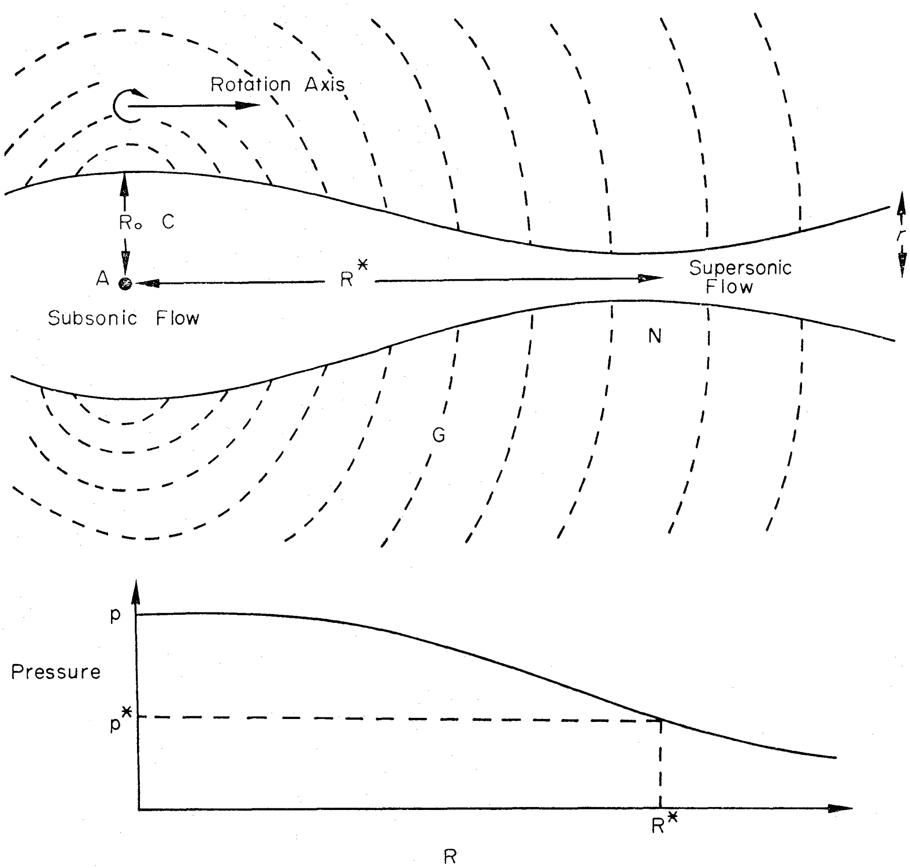

FIG. 1. *Hot (possibly relativistic) plasma is assumed to be generated continuously by an 'active' region (A) within a central cavity (C) confined by the thermal pressure of a gravitationally-confined rotating gas cloud (G) whose isobars are shown as dashed lines. The plasma flows along two oppositely-directed channels whose cross-sections reach a minimum radius r^* at the nozzle (N). An equilibrium flow is possible only if the channel adjusts so that the nozzles form when the external pressure has dropped to a value $p^* \simeq 4/9$ of the central pressure p_0. At the nozzles ($R = R^*$) the bulk velocity is sonic (i.e. $v = c/\sqrt{3}$ if the plasma is ultra-relativistic); and beyond R^*, where the external pressure may be $\ll p_0$, the outflowing material is collimated into beams. For this type of flow pattern to occur, p_0 must be high enough for r^* (given by equation (2.1)) to be much smaller than R^* (the 'scale height' of the gas cloud).*

$5/3$, $p^* = 0{\cdot}49 p_0$. In general the flow becomes supersonic when the pressure has approximately halved—a fact that follows immediately from the Bernoulli equation.

Once the flow velocity has become supersonic, the transverse pressure (given by (2.3)) becomes a decreasing (and eventually very small) fraction of the momentum flux density $L/\pi r^2 c$. This means that a very modest external pressure can collimate the outflow from each nozzle into a narrow beam. We discuss the properties of such beams, and their relationship to the radio components, in Section 4.

3. THE NUCLEAR REGION: PARAMETERS FOR CYGNUS A, STABILITY OF FLOW, ETC

We now discuss in rather more detail the physical conditions in the central region where the collimation into two beams is established. As already mentioned, we must postulate the existence of a dense gas cloud localized in a central gravitational potential well. The main contribution to the potential will probably come from the stars. Stellar densities of $\rho_s \simeq 10^4\,M_\odot\,\mathrm{pc}^{-3}$ are not ruled out within the central ~ 100 pc of massive ellipticals, permitting therefore a total mass of $\sim 10^{10}\,M_\odot$ within 100 pc radius (Wolfe & Burbidge 1970). The mass of the central active region, which is required to supply a power L for $10^7 Y_7$ yr, must exceed $1{\cdot}5\times 10^6 L_{46} Y_7 \Sigma^{-1}\,M_\odot$, where Σ is the efficiency with which rest mass is converted into relativistic plasma. This mass will typically be $\ll 10^{10}\,M_\odot$ provided that $\Sigma \gtrsim 10^{-2}$. The required mass of the gas itself is also (as we shall show) typically $\ll 10^{10}\,M_\odot$.

3.1 *Illustrative example of possible gas cloud parameters*

Consider, as an illustration, the simplest possible case where the stars are uniformly distributed with density ρ_s, producing a spherically symmetric potential well, and the gas is supported by thermal pressure and is isothermal with temperature T. (Using an adiabatic temperature gradient makes little difference.) Then,

$$p(R) = p_0\, e^{-R^2/K^2}, \tag{3.1}$$

where $K = (3kT/\pi G\rho_s m_p)^{1/2}$ and p_0 is the central pressure.

We regard K and p_0 (which follows from the total amount of gas in the cloud) as given. The assumption that ρ_s is constant is of course a reasonable approximation only if the random velocities of the stars exceed $(kT/m_p)^{1/2}$, i.e. the 'scale height' is larger for the stars than for the gas. If the cloud is uniformly rotating, (3.1) still applies along the rotation axis, though the scale height would be larger in the equatorial plane by a factor $f \simeq (1+3\Omega^2/4\pi G\rho_s)$. Unless we solve the two-dimensional flow equations, it is necessary to stipulate a somewhat arbitrary inner boundary condition on the one-dimensional equations. We use the condition $r(2R_0) = \frac{1}{2}R$, for convenience, but what follows is not particularly sensitive to this. We then obtain

$$\begin{aligned} R_0 &\sim \left(\frac{1{\cdot}5 L^2 kT}{\pi^3 G\rho_s m_p c^3 p_0^2}\right)^{1/6} \\ &\sim 20 L_{46}^{1/3} T_8^{1/6} \left(\frac{\rho_s}{10^4\,M_\odot\,\mathrm{pc}^{-3}}\right)^{-1/6} P_{0-4}^{-1/3}\ \mathrm{pc}. \end{aligned} \tag{3.2}$$

For $R_0 \lesssim R \lesssim R^*$ we have (approximately)

$$r \sim r^*(R/R^*)^{-1/2}$$

where

$$R^* = [\log_e (9/4)]^{1/2}\,\mathrm{K} \sim 120 T_8^{1/2} \left(\frac{\rho_s}{10^4\,M_\odot\,\mathrm{pc}^{-3}}\right)^{-1/2} \mathrm{pc}.$$

and

$$r^* \sim 6 L_{46}^{1/2} p_{0-4}^{-1/2}\ \mathrm{pc}. \tag{3.3}$$

The total free-free power P_{ff} produced by the cloud is

$$P_{ff} \sim 2 \times 10^{46} p_{0-4}{}^2 \left(\frac{\rho_g}{10^4\, M_\odot\, \mathrm{pc}^{-3}}\right)^{-3/2} f^2 \,\mathrm{erg\,s^{-1}} \qquad (3.4)$$

independent of T_8.

We can continue to use the observations of Cygnus A as a convenient but severe test of the model. Longair & Willmore (1974) report an X-ray luminosity from a 10′ arc region around Cygnus A of $\sim 2 \times 10^{45}$ erg s^{-1} up to an energy of 7·5 keV. The most likely origin of this is the free-free emission from a hot intra-cluster medium, but it can also be taken as an upper limit on the X-ray luminosity of the nucleus. If we take L_{46} as 0·5, the X-ray observations then give an upper limit on the nuclear pressure, $10^{-4}\, p_{0-4}$ dyne cm^{-2}, of $p_{0-4} \lesssim 2f^{-4} R^*{}_2{}^{-3/2} T_8{}^{3/4}$, where $R^*{}_2 = R^*/(100 \text{ pc})$, (as long as $T_8 \gtrsim 0{\cdot}7$; the limit is less stringent for lower temperatures).

The maximum gas density in the cloud is $\sim 70 p_{0-4} T_8{}^{-1}\, M_\odot$ pc^{-3}, confirming that (for p_{0-4}, $T_8 \sim 1$) the gas mass is indeed negligible compared with the likely density in stars. Now a necessary condition for the self-consistency of the model is that the radius of the nozzle should be much smaller than the radius of the central gas cloud. If we stipulate that $R^* \gtrsim 3R_0 \gtrsim 10r^*$, we obtain the inequality $p_{0-4} \lesssim 3f^{-1} T_8{}^3$. Improved resolution and higher energy observations could clearly tighten these limits, but at present the data permit a wide range of acceptable parameters. A consistent (but by no means obligatory) choice of parameters for Cygnus A is as follows:

$$\begin{aligned} L &\simeq 5 \times 10^{45} \,\mathrm{erg\,s^{-1}} \\ p_0 &\simeq 3 \times 10^{-5} \,\mathrm{dyne\,cm^{-2}} \\ T &\simeq 3 \times 10^{8}\,\mathrm{K} \\ R_0 &\simeq 30\,\mathrm{pc} \\ R^* &\simeq 220\,\mathrm{pc} \\ r^* &\simeq 10\,\mathrm{pc}. \end{aligned}$$

A central star density $\sim 10^4\, M_\odot$ pc^{-3} would be required to retain the gas cloud (whose central electron density would be ~ 300 cm^{-3}). Cygnus A is, of course, among the most powerful sources, and more 'typical' doubles could be explained with a wider range of less extreme parameters. We mention in Section 5 some reasons why gas might accumulate near the centres of elliptical galaxies.

(Calculations similar to the above can be carried out for different assumptions concerning the form of the potential well. If the potential is primarily due to a point mass, the solutions generally have $(R^* - R_0) \lesssim R$, and so a one-dimensional approximation to the flow is inadequate, and the stability of the cloud to secular changes in L questionable.)

3.2 *Instabilities, etc.*

When a dense gas cloud is supported in a gravitational field by a lighter fluid, the configuration is Rayleigh–Taylor unstable and we might expect the linear growth time of serious large-scale perturbations to be approximately the free-fall time across the nuclear region. For a gas cloud in an approximately parabolic potential well, this is $\sim 10^5 R^*{}_2 T_8{}^{-1/2}$ yr. The time that a fluid element takes to travel from $R \sim 0$ to $R \sim R^*$ is, however, $\sim 2\pi R_0{}^3 p_0/L$. This is the characteristic time scale associated with the relativistic flow, and is typically much shorter than the Rayleigh–Taylor time scale

$$(\text{by a factor } \sim LR^*(4\pi p_0 R_0{}^3 c_s)^{-1} \sim L_{46} p_{0-4}{}^{-1} R^*{}_2 R_{02}{}^{-3} T_8{}^{-1/2}).$$

There is thus no necessary inconsistency in treating this flow as steady, despite the possible development of Rayleigh–Taylor instabilities. The non-linear development of such an instability cannot even be guessed without specifying how the interior energy supply responds to perturbations. (The associated time scales are many orders of magnitude larger than those associated with compact variable sources, which presumably require an explanation in terms of processes occurring on length scales ~ 1 pc within the active region.)

Potentially more serious, as emphasized by Scheuer (1974), are Kelvin–Helmholtz instabilities that result from trying to make one fluid flow past a second fluid (see, for example, Gerwin 1968). Indeed, *if* the non-linear development of such an instability in the vicinity of the nozzle (where the growth rate would presumably be most rapid) caused entrainment of the surrounding gas at a rate approaching the local sound speed, the resulting shear stress acting at the walls of the channel ($\rho^* c_s c \sim p^* c/c_s$) would completely disrupt the flow. Other (or milder) fluid-dynamical instabilities would not necessarily invalidate our general model (especially as the radio sources to which we apply it generally do have an irregular appearance). The growth times of some (e.g. Rayleigh–Taylor) are long compared to the time scale of the relativistic flow; and small-scale Kelvin–Helmholtz instabilities could be so rapid that one could still regard the overall mean flow as steady (rather as some hydrodynamic jets look 'ragged' on short exposure photographs, but smooth when long exposures are used).

3.3 *Properties of 'hot' fluid near galactic nucleus*

The incorporation of dynamically significant magnetic fields (and consequent pressure anisotropies) may also help to stabilize the real situation. Magnetic fields that are disordered on scales small compared with the dimensions of the system exert a mean pressure equal to one-third of their energy density. However, if magnetic fields are frozen into the flow within the nucleus, which is not unreasonable if nuclear conditions are like a scaled-up version of the Crab Nebula, then the field lines, assumed randomly oriented within the nucleus, will be extremely stretched along the flow direction by the time the flow reaches the nozzle. If the magnetic field is not initially strong enough to influence the dynamics seriously, then the longitudinal component of the field strength will *increase* proportional to r^{-2}, whilst the transverse component will *decrease* proportional to r. Thus, the magnetic stresses will become anisotropic as the nozzle is approached, and the transverse pressure will increase $\propto r^{-4}$, whilst the gas pressure will fall by a factor $\lesssim \frac{1}{2}$. (There will be a corresponding increase in the magnetic tension along the channel.) Magnetic stresses may therefore start to dominate the particle pressure before the nozzle is reached. If the magnetic pressure is comparable with the gas pressure in the vicinity of the nozzle, the growth of shearing instabilities *may* be inhibited.

If the magnetic field does approach an equipartition value in the vicinity of the nozzle, $B_{eq} \sim 2 \times 10^{-2} p_{-4}{}^{1/2}$ gauss, and the corresponding cooling time for an electron of energy $\gamma_e m_e c^2$ is $\sim 3 \times 10^4 p_{04}{}^{-1} \gamma_e{}^{-1}$ yr. This must be compared with the time for the fluid to pass through the nozzle $\sim R^*/2c \sim 150R^*_2$ yr. Thus, if $R^* \sim 220$ pc (appropriate for Cygnus A), $\gamma_e \lesssim 100$ after passing through the nozzle (these energies are measured in the co-moving frame). The observed radio luminosity of 10^{42} erg s^{-1} from the galactic nucleus (Hargrave & Ryle 1974) can in fact be explained in these terms, as well as perhaps the much higher ultraviolet flux needed to excite the emission lines (Mitton & Mitton 1972).† The proton cooling-time is far too long to be significant and so, provided the protons are initially ultra-relativistic and are not strongly coupled to the electrons, the random energy per proton at the nozzle will have decreased in the same ratio as the total pressure ($\sim \frac{1}{2}$) and so most of the internal energy of the particles will be due to protons hereafter. The presence of appreciable field strengths will probably also preclude the co-existence of low frequency electromagnetic waves through synchrotron self-absorption (Rees & Gunn 1974; Blandford 1974, in preparation). In the absence of magnetic fields, decay instabilities (Arons & Max 1974) and statistical acceleration (Blandford 1973a) can also ensure mean free paths for low frequency photons and plasmons that are short compared with the length of the channel. This suggests that irrespective of the precise constitution of the relativistic outflows, we are probably justified in applying fluid-dynamical concepts.

3.4 *How is the flow pattern set up?*

Finally we consider the initial formation of the channels. As already indicated, it seems plausible to associate the direction of the beam with the rotation axis of the gas cloud—and infalling gas need not have very much angular momentum in order to be significantly rotationally flattened when it has formed a cloud only ~ 100 pc across. The isobaric surfaces will not be spherical, but approximately oblate ellipsoids. If 'violent activity' is initiated within such a cloud, the resulting bubble of relativistic fluid will expand most rapidly along the rotation axis. The bubble will probably expand until it has dimensions comparable with the scale height in the gas cloud. At this point a Rayleigh–Taylor instability should cause a neck to develop perpendicular to the rotation axis. This is essentially the same mechanism (though on a much smaller scale) as that proposed by Gull & Northover (1973). However, if the central energy supply is maintained (i.e. $\tau_{\rm prod} \gg \tau_{\rm det}$), it seems more likely that a continuous flow rather than a succession of bubbles should be produced. Indeed, one can show that a string of bubbles, with individual velocities $< c_s$, cannot possibly transport an energy flux $\sim 10^{46}$ erg s^{-1} through a gas cloud with the parameters that we envisage. If the nuclear luminosity is so high that the surface of the relativistic fluid moves supersonically with respect to the external gas, the expansion will still be most rapid along the rotation axis, because this is where the density gradient is steepest. As long as the expansion is subrelativistic, we can regard the pressure $p(t)$ within the relativistic fluid as uniform, and equal at all points of the surface to the local ram pressure; in this case too, the bubble of relativistic gas clearly elongates along the inner axis of the surrounding gas cloud and eventually squirts out in the form of two jets. The equatorial expansion then becomes subsonic and the situation eventually adjusts

† If the gas cloud is indeed in a steady state, a heat input $\gtrsim 10^{45}$ erg s^{-1} is needed to balance its X-ray emission, and this must either be a by-product of the central violent activity or result from continuing infall of gas from the rest of the galaxy. The heat input due to the passage of stars through the gas cloud is negligible.

to a quasi-stationary pressure equilibrium in which the dimensions of the central cavity can adjust slowly to accommodate variations in the power output L.

4. SUPERSONIC FLOW AND THE EXTENDED RADIO COMPONENTS

Beyond R^* the outflow is supersonic, and when $p \ll p_0$, equation (2.3) yields, for a relativistic fluid, the approximate relations

$$\begin{aligned} r &\sim 0{\cdot}6\gamma r^* \\ &\sim 0{\cdot}6(p_0/p)^{1/4}\, r^* \\ &\sim 150 L_{46}{}^{1/2} p_{0-4}{}^{-1/4} p_{-10}{}^{-1/4}\ \mathrm{pc}, \end{aligned} \tag{4.1}$$

γ being the Lorentz factor corresponding to the bulk velocity v. The fact that r depends on p to the power $-\frac{1}{4}$ means that the cross-section widens rather gradually, even though the external pressure may fall by several orders of magnitude as the beam moves from the inner region of the galaxy out into intergalactic space.

The average internal energy per particle, $(3p/n)$, is clearly reduced by a factor γ^{-1} from its value in the nucleus. However, the mean total energy per particle remains constant and so the nozzle is bringing about an adiabatic conversion of random into bulk kinetic energy, which has correspondingly increased by a factor $\sim\gamma$. That such a conversion is possible without doing catastrophic amounts of work on the surrounding medium has been emphasized by Longair *et al.* (1973).

4.1 *Properties of fluid in collimated beams*

In order that the flow be adequately described by a velocity that is solely a function of the distance along the channel, it is necessary that the channel does not widen too abruptly. We require that $dr/dR \ll \gamma^{-1}(c_r/c)$ where c_r is the internal sound speed measured in the co-moving frame. This can be translated into a condition that the external pressure gradient must not get too large. Assuming that the fluid remains relativistic, we obtain

$$\left|\frac{dR}{d\ln p}\right| \gg 0{\cdot}15\gamma^2 r^* \sim 0{\cdot}4 r^2/r^*,$$

as a condition on the local scale height. A similar condition must be satisfied for us to ignore the extra 'centripetal' pressure that would have to be exerted by the walls of the channel. This is of magnitude $\sim d^2r/dR^2.(L/rc)$ and is negligible by comparison with the thermal pressure $\sim Lr^{*2}r^{-4}c^{-1}$ as long as the local scale height $\gg r^2/r^*$. That these two conditions are essentially identical is not unexpected because a significant centrifugal pressure would result in a pressure gradient transverse to the axis of the channel, which would give non-uniform density and velocity profiles. If the scale height in the external gas is too small to satisfy this inequality, as might well be the case near the nozzle, the problem becomes essentially two-dimensional. There is no reason why this should significantly affect the isentropic character of the flow but a more detailed analysis is required (see, for instance, Courant & Friedrichs 1948). Note also that, even if the external pressure dropped to zero, the beam would remain collimated within an angle $\sim\gamma^{-1}$.

Close to the nozzle, within the galaxy, we anticipate that the channel should be maintained by the direct thermal pressure of the gas cloud. However, outside the galaxy it is quite possible that (Fig. 2) the channel is surrounded by a 'cocoon' of

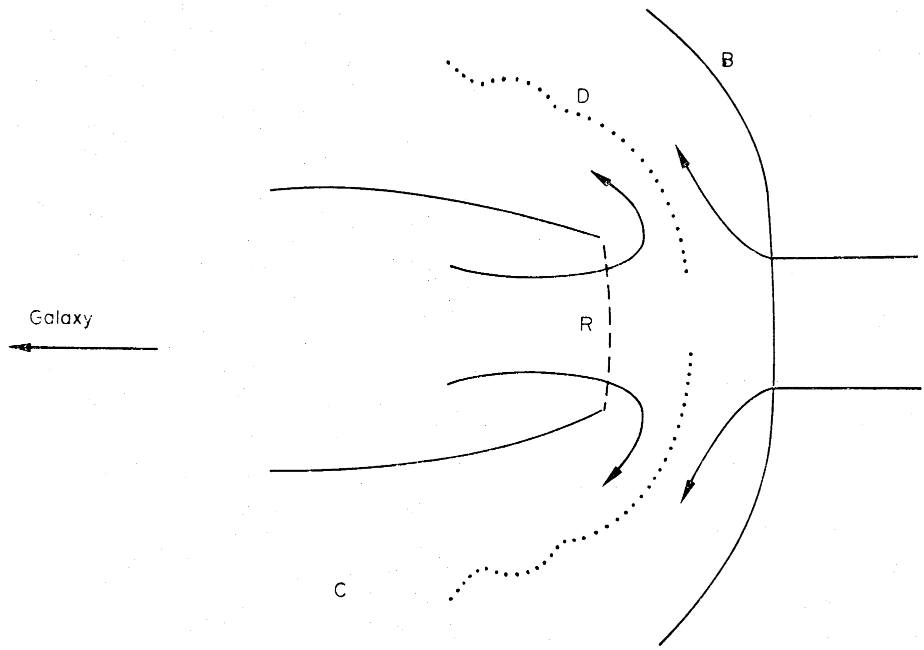

Fig. 2. *Schematic representation of the ' working surface ' of a strong radio source. Flow lines are indicated in the frame moving away from the galaxy with speed V (equation (4.2)). The bulk kinetic energy of the beam is randomized at the shock (R) and it meets the intergalactic medium (which has passed through shock (B)) at a contact discontinuity (D) where mixing and further particle acceleration may take place. The energy contained in the head of the source is confined by the ram pressure of the ' intergalactic medium ', but the cocoon (C) is eventually confined by thermal pressure. For an acceptable model of Cygnus A, the beam diameter would be 1–3 kpc. The transverse pressure due to the cocoon will probably increase as the ' working surface ' is approached, causing some reconvergence of the beam.*

' waste ' energy which is in turn confined by the thermal pressure of the extragalactic medium (Scheuer 1974; Blandford 1973b). This possibility—together with most of the discussion in this section—actually applies to any ' supersonic beam ' model irrespective of whether the initial collimation is achieved via the nozzle mechanism proposed in Sections 2 and 3.

So far we have assumed that the internal kinetic motions of the fluid have remained ultra-relativistic. Equation (4.1) shows that, even if this is so initially, it will no longer be the case if r/r^* comes to exceed the mean Lorentz factor of the protons within the nuclear region.† The protons will then become subrelativistic (in the co-moving frame) before they reach the head of the source. This alters the effective specific heat ratio and changes the properties of the flow. As long as neither magnetic fields nor electrons are dynamically significant, the enthalpy per proton (chemical potential), w/n, when the protons have become non-relativistic, will be given by $\sim(m_p c^2 + 5p/2n)$, which is effectively constant. This implies that the bulk Lorentz factor will no longer significantly increase and the pressure will simply fall as a result of the sideways expansion of the channel. Thus the radius of the channel will increase as $r \sim r^+(p^+/p)^{3/10}$ (where the superscript + refers to the transition to a subrelativistic equation of state), somewhat less slowly than if the fluid had remained ultra-relativistic. If there is a negligible magnetic field and the

† If the protons initially have a power law spectrum, $N(\gamma)\, d\gamma \propto \gamma^{-p}\, d\gamma$, this statement applies to the *mean* particle energy (which is close to the low energy cut-off if $p > 2$).

electrons are accelerated in the nucleus with the same energy as the protons, they will cool at the same rate until the protons become subrelativistic. Thereafter they will cool at a slower rate and be responsible for most of the pressure. In this case $r \sim r^+(p^+/p)^{3/8}$.

In Section 3 we discussed the possibility that the wind be magnetized. The strength of the magnetic field after passage through the nozzle is an unknown quantity; it may even be strong enough to govern the dynamics of the flow. Near the nozzle the field will be predominantly longitudinal and, as the channel expands, we expect $B_\parallel \sim B_\parallel{}^*(r^*/r)^2$, in obvious notation. The longitudinal field is of course measured to be the same in both the co-moving and the observed frames. For the perpendicular component of the field, measured in the co-moving frame, we have $B_\perp \sim B_\perp{}^*(nr/n^*r^*) \sim B_\perp{}^*(r^*/r)^2$, as long as the field is not strong enough to influence the flow appreciably and the ultra-relativistic equation of state remains applicable. The associated electromagnetic transverse pressure will thus decay as r^{-4}, in the same way as the particle pressure. However, in the observer frame this will be manifest as a balance between a magnetic pressure and an electric tension. One consequence of this is that if the longitudinal magnetic field is dominant over the transverse field when the flow becomes relativistic near the nozzle, it will remain so as long as the protons have relativistic random motions. When the protons become subrelativistic and γ ' saturates ', $B_\perp \sim B_\perp{}^+(r^+/r)$, and the transverse magnetic pressure diminishes $\propto r^{-2}$—slower than either the longitudinal magnetic field pressure or the particle pressure. So, if the protons become sub-relativistic, there is a second opportunity for the magnetic stresses to dominate the dynamics of the flow. Should this happen, we would expect the field to ' untangle ' and become much more ordered.

4.2 *The radio components*

The head of the radio source, of transverse diminisions r_{h}, advances into the intergalactic medium with a velocity V (assumed non-relativistic) satisfying

$$L/\pi r^2{}_{\mathrm{h}}c \sim \rho_{\mathrm{ext}}V^2. \tag{4.2}$$

There will be a shock at which the bulk kinetic energy in the flow is reconverted into random energy, this time with a large increase in entropy. The dominant physical processes operating, and the structure of the shock, are to some extent a disjoint subject from the fluid dynamics and we do not consider them in much detail here. However, from general arguments we expect that collisionless plasma interactions should be able to restrict the width of the shock to the (relativistic) proton Larmor radius. On passage through the shock, the protons will acquire random energies similar to those that they possessed in the nucleus.

There will also (see Fig. 2) be a second shock in the intergalactic medium where, in the shock frame, the incoming bulk kinetic energy of the intergalactic medium is partially randomized. For typical strong source parameters we expect that V greatly exceeds the sound speed in the intergalactic medium, and that the cooling time for this medium is so long that the shock is adiabatic. The density of the compressed extragalactic medium will then be $\sim 4\rho_{\mathrm{ext}}$. The strong shear at the ' contact discontinuity ' separating the two fluids makes it likely that there should be some mixing between the randomized relativistic fluid and the shocked extragalactic medium in the intershock region. (We can tentatively identify this mixing region as being responsible for injecting the non-relativistic matter that is probably required in at least some radio sources, including Cygnus A, to being about Faraday depolarization.)

4.3 *Particle acceleration, etc, at the 'working surface'*

Adiabatic and radiative losses would have reduced the *random* energies of particles in the beam to such an extent that no traces would survive of any power law spectrum which might have been established in the nucleus. When the beam encounters the shock (denoted by R in Fig. 2), its bulk energy is re-randomized, and we suspect that it is soon after this point that the synchrotron-emitting relativistic electrons are accelerated.

If the bulk velocity of the unshocked flow is relativistic, and the shock is strong in the sense that the internal energy increases by a large factor, the shocked fluid (before mixing into the intergalactic medium) will be ultra-relativistic and have a bulk velocity $\sim c/3$ (see the Appendix). The pressure ratio across the shock front is given by

$$p_2/p_1 = \tfrac{8}{3}\gamma_1{}^2 - 9. \tag{4.3}$$

A longitudinal magnetic field is unchanged on passage through the shock. However, a transverse magnetic field, whilst being only slightly changed in the observer frame, increases by a factor $\sim\gamma_1$ in strength in the co-moving frame, so if the internal (thermal) motions in the incident fluid are ultra-relativistic, the ratio of co-moving magnetic energy density to that in relativistic particles would be approximately unchanged on passage through even a strong shock. If the unshocked fluid has a non-relativistic equation of state, the particle pressure will increase by a factor $\gg\gamma_1{}^2$. In the co-moving frame, the abrupt increase in the transverse magnetic field will probably be associated with an equally large generation of transverse electric fields, strengthening the argument that the intershock region is the site of most of the particle acceleration. If the beam were relativistic, then the protons would all have random energies $\gtrsim 1$ GeV after passing through the shock. Merely bringing a small fraction of the electrons up to the same *temperature* as the protons, by, for instance, electrostatic coupling, would produce (in the presence of equipartition magnetic fields) sufficiently energetic electrons to account for the observed radio emission except perhaps at the highest frequencies.

To transform the spectrum of these relativistic electrons into a power law would require some further acceleration process of a presumably stochastic character. The simplest such process is the well-known Fermi mechanism; and it is easily seen (by an extension of arguments due to Syrovatski (1961) and Burn (1974,) that a power-law electron spectrum with a slope in the observed range can indeed be expected under the following hypotheses: (i) On passage through the shock, a fraction f_1 of the power L in the beam is transformed into turbulence; (ii) A power f_2L is injected in the form of relativistic electrons with energies $\sim\gamma_{\mathrm{inj}}m_ec^2$ (these electrons having initially a Maxwellian or δ-function spectrum); and (iii) The turbulent energy is all transferred to the relativistic particles via a Fermi-type mechanism (i.e. one for which $\dot{\gamma} \propto \gamma$—this would generally require that the scale of the turbulence should exceed the Larmor radii of even the most energetic particles). Assumption (iii) ensures that the electrons will acquire a power-law spectrum provided that their probability of escape from the region is independent of γ, and that radiative energy losses are unimportant during the acceleration processes. Assumptions (i) and (ii) then require that the mean final particle energy (of a sample of particles either in the accelerating region or escaping) is $[(f_1+f_2)/f_2]\,\gamma_{\rm inj} mc^2$, which implies a spectrum

$$N(\gamma)\,d\gamma \propto \gamma^{-p}\,d\gamma(\gamma \gtrsim \gamma_{\rm inj})$$

with $p = (2f_1+f_2)/f_1$. If $f_1 \gtrsim f_2$ the resulting slope is $2 < p \lesssim 3$. (The inclusion of possible radiation losses and the diffusive contribution to the Fermi process both tend to flatten the slope.) One of the difficulties with most astrophysical applications of the Fermi mechanism is that one needs to invoke some preliminary injection process, because typical particles have such low energy that the 'ionization' loss rate exceeds the acceleration rate: in the present model, however, *all* particles in the region have in effect been 'injected', via the shock, with relativistic energies. A second traditional weakness of the Fermi mechanism is that the resultant power law depends on the ratio of the acceleration time scale ($t_{\rm acc} = \gamma/\dot{\gamma}$) and the escape time scale ($t_{\rm esc}$)—in fact $p = 1 + t_{\rm acc}/t_{\rm esc}$—which may vary wildly between one object and another. Here, however, this ratio is in effect determined by the value of f_1/f_2. The rate of consumption of turbulent energy must, in a steady state, equal $f_1 L$. This determines the slope; so, if we regard $t_{\rm esc}$ as given, the turbulent energy density must adjust itself so that $t_{\rm acc}$ has the self-consistent value.

It seems inevitable that much of the kinetic energy in the beam will, on passing through the shock, go into either turbulence or relativistic particles (i.e. $f_1+f_2 \simeq 1$). Provided the other assumptions are approximately valid—and this question of course cannot be settled without further study—the production of a power-law spectrum in radio components could be understood. We note also that, as the above process depends only on the particle's rigidity and not its rest mass, protons should be similarly accelerated in *energy*, and so there would be approximate equipartition between the electron and proton components. There are, however, rather more complicated possibilities for particle acceleration. Between the relativistic shock and the contact discontinuity we expect rapid generation of electrostatic and electromagnetic waves (with wavelengths much shorter than the length-scales of turbulent motions which are relevant to ordinary Fermi acceleration). These modes could accelerate electrons efficiently by resonant processes (e.g. Landau damping or synchrotron self-absorption) and wave–wave interactions (e.g. induced Compton effects). If the beam contains low frequency waves, these will be efficiently absorbed at the shock, energy thus being transformed into relativistic electrons in a manner analogous to that which might occur at the 'wisps' in the Crab Nebula (Rees & Gunn 1974). (Low frequency waves might in fact be generated in the flow from magnetic field irregularities on length scale l, which cannot be supported without a displacement current after the plasma frequency has fallen below $\sim c/l$.)

We now discuss entrainment of the surrounding gas through the channel walls, whilst the flow is supersonic. There are two principal consequences of entrainment—the reduction in the bulk velocity of the flow due to the addition of rest mass; and the increase in pressure, resulting in transverse expansion of the channel. The total momentum of the flow will not be altered by the former effect, and it is easy to show that the second effect is the more serious. The maximum entrained mass flux through the channel walls is $\sim \rho_{\rm ext} c_{\rm s} \sim p c_{\rm s}^{-1}$, where $c_{\rm s}$ is the external sound speed. Thus, for a given pressure, surrounding the channels with a 'cocoon' of waste energy diminishes the importance of entrainment. If the average mass flux through the walls is a fraction α of the maximum value, and the bulk velocity has become sub-relativistic, the rate of increase of internal energy density is given by

$\alpha p v^2 r^{-1} c_s^{-1}$ and so the pressure (and hence r) changes significantly in a distance $r c_s/\alpha v \sim r/\alpha$ if the flow is supersonic. Thus, if the beam is to propagate a significant distance into the extragalactic medium, $\alpha \ll 1$. This conclusion is unaffected if the bulk speed is relativistic.

This means that entrainment into the supersonic part of the beam ($R \gg R^*$) certainly cannot proceed at the maximum possible rate. (We showed in Section 3 that a similar result applied to the flow at $R \sim R^*$.) We suspect that longitudinal magnetic fields may be the primary agents for suppressing entrainment. There are, however, two arguments which suggest that in Cygnus A (and probably in other sources also) some entrainment *does* occur and the beam is not entirely isentropic.

4.4 *Specific application to Cygnus A*

The first argument comes from the sizes of the ' hot spots ' which are ~ 2 kpc in linear size. There is no evidence for any radio structure of smaller scale. We assume that the pressure in the radio-emitting regions is in fact given by the minimum pressure inferred from the observations of Hargrave & Ryle (1974) which is $\sim 3 \times 10^{-10}$ dyne cm^{-2} for the low brightness regions, and should be comparable with the pressure in the unshocked fluid. Using the formula (4.1) and adopting a central pressure of $p_0 = 3 \times 10^{-5}$ dyne cm^{-2}, we find that the channel radius on the basis of isentropic flow is ~ 100 pc, ten times smaller than the hot spots. The most probable explanation of this discrepancy is that entrainment has widened the channel. In fact, if the flow were completely isentropic and the channel radius only ~ 100 pc, the momentum flux density would almost certainly exceed $\rho_{ext} c^2$ and the head of the source would advance relativistically.

The second argument relies on the assumptions that the fluid remains relativistic and that the pressure ratio between the ' hot spots ' and the low brightness region equals the pressure ratio across the shock. Observationally this is ~ 10 if we assume that the pressure is a constant multiple of the minimum pressure. From (4.3) we then derive an estimate for the Lorentz factor in the beam $\gamma_1 \sim 3$, which is far below the value $\sim r/r^*$ calculated on the assumption of relativistic isentropic flow.

As the total radio luminosity from the ' hot spots ' of Cygnus $A \ll 10^{46}$ erg s^{-1}, the energy discharge into the ' cocoon ' from the head of the source (predominantly in the form of relativistic particles and magnetic fields) must be comparable with L. We can then interpret a particular morphological feature of the map of Cygnus A. In the immediate post-shock region, the fluid has a subsonic flow speed $\sim c/3$ which must be reduced to the velocity, V, at which the shocked intergalactic medium sweeps past the source. The observed symmetry of the sources and the inferred absence of kinematic and evolutionary effects (*cf.* Mackay 1973) suggest that $V < c/3$ and therefore the flow lines must diverge away from the head of the source. The radio contours should then display a rather blunt appearance, which is qualitatively what is observed.

(In the above, we have assumed that a steady flow is set up. This may well not be the case either because of non-uniformity in the nuclear energy generation or because of instabilities at the head of the source. As the energy is supplied supersonically, it is possible that the minimum pressure in the hot spots *exceeds* the current ram pressure and the hot spots are somewhat transient features which are at present expanding.)

The shocked intergalactic medium must be heated sufficiently to withstand the ram pressure on one side and the pressure of the hot spots on the other. If we assume a pressure of $10^{-8}p_{-8}$ dyne cm^{-2} and an ambient external electron density of $10^{-3}n_{e-3}$ cm^{-3}, we obtain

$$T_s \sim \frac{p}{8n_e k} \sim 10^{10} p_{-8} n_{e-3}^{-1} \text{ K},$$

and $V \sim 2 \cdot 5 \times 10^9 p_{-8}^{1/2} n_{e-3}^{-1/2}$ cm s^{-1}: thus the shocked intergalactic medium might be a source of hard (up to 1 MeV) X-rays. The emitting volume is uncertain, but we estimate 10^{67} cm^3, which gives a hard X-ray luminosity $\sim 10^{39}$ erg s^{-1} from the components of Cygnus A. This is rather too faint to be detected in the foreseeable future. Heating by shocks associated with radio sources may, however, have an important cumulative effect on the intergalactic medium (Bollea & Cavaliere 1974), and can give rise to thermal emission with temperatures higher than the 'virial' temperature in clusters of galaxies.

As shown in Fig. 2, the shocked relativistic fluid and intergalactic medium should be partially mixed and flow around the beams to form an enveloping cocoon bounded by a shock that gradually weakens away from the head of the source. Scheuer (1974) assumed that within the cocoon the sound speed was $\sim c/\sqrt{3}$ and the pressure uniform. Alternatively, sufficient mixing of the shocked external medium may occur for the sound speed in the cocoon to be less than the speed, V, of advance of the head of the source (Blandford 1973b). The observation of depolarization in the extended low brightness regions, requiring electron densities $\sim 10^{-3}$ cm^{-3}, supports this idea. (In fact the ratio of the internal to external density need only exceed $\sim M^{-2}$, where M is the Mach number for the speed of advance of the shock at the head of the source.) Using the simplest approximation that the magnetized plasma expands with a specific heat ratio of 4/3 from the neighbourhood of the hot spots (radius r_h) to fill the cocoon of radius r_c, we can obtain a second estimate of the ratio of the pressures in the hot spots and the cocoon from

$$\frac{p_h}{p_c} \sim \left(\frac{r_c^2 \cdot 3V}{r_h^2 c}\right)^{4/3}.$$

Observationally, $r_c/r_h \sim 6$, and if $V \sim 0 \cdot 1$ c, we obtain $p_h/p_c \sim 20$ which is consistent with the estimate obtained from the relative brightness temperatures.

Assuming energy equipartition, the lifetime of a 5 GHz electron in the cocoon is estimated by Hargrave & Ryle (1974) to be $\sim 2 \times 10^5$ yr which is $0 \cdot 1 \times$ the estimated age of the source for $V \sim 0 \cdot 1$ c. Thus, the absence of low brightness emission extending down to and enveloping the galaxy as discussed in Scheuer (1974) can, in this source at least, be satisfactorily explained in terms of radiation losses without having to invoke a non-uniform external pressure.

In summary, the overall dynamics of the model we have proposed seems consistent with present observations of the powerful sources, in particular Cygnus A, and can account quite naturally for several qualitative features. The least satisfactory aspect of the model is our ignorance of the effect of instabilities (*cf.* Scheuer 1974). It seems highly unlikely that all instabilities could be suppressed, but whether they could be allowed to grow sufficiently to alter completely the nature of the flow is very much an open question.†

† It may be possible to perform a relevant experimental investigation by blowing a jet of air through a couple of scale heights of water, either in a lake (depths ~ 30 m would be required) or under reduced pressure in the laboratory. However, this would not be completely satisfactory because, as we have shown, the influence of magnetic fields and a spherical rather than a plane geometry might well be crucial.

5. FURTHER APPLICATIONS OF THE MODEL

So far we have mostly restricted the application of the model to Cygnus A, for which detailed structural information is available. But one must remember that Cygnus A is *extreme* rather than typical in its properties. There is also available a wide variety of morphological, spectral and polarization data for a large sample of sources. Some interesting correlations have emerged (Mackay 1971; Fanaroff & Riley 1974). In particular the luminosity of a source seems to be strongly correlated with the relative positions of the high and low brightness regions in the sense that, in a low luminosity source, the region of highest brightness (on one side of the galaxy) is closer to the galaxy than the centre of the low brightness region, while for the high luminosity sources such as Cygnus A, the opposite is the case. The most appealing interpretation of these results (*cf.* Longair *et al.* 1973) is that extended low brightness regions are always confined by the external medium and are expanding subsonically with the spectrum continually steepening as a result of synchrotron losses. Hot spots at the leading edges of components indicate that the energy supplied by the nucleus possesses sufficient momentum to push out the intergalactic medium supersonically: in the low luminosity sources, the power supply has either ceased, or else is unable to reach the head of the component before being dissipated. In some sources, e.g. 3C 236 (Willis, Strom & Wilson 1974), the beam must have persisted long enough for the components to move several Mpc from the central galaxy. Lower luminosity sources do not necessarily display the double structure characteristic of powerful sources. In the 'radio trails' (Miley *et al.* 1972; Jaffe & Perola 1973), the relativistic outflow is perhaps so weak, or so poorly collimated, that V (equation (4.2)) is lower than the speed of the galaxy through the external gas.

If the dominant radiation mechanism is ordinary synchrotron emission, polarization studies provide direct evidence on the magnetic field structure. The magnetic field in extended sources could have been generated in the galactic nucleus along with the relativistic particles and transported out by the beam (*cf.* the discussion by Rees & Gunn (1974) of the origin of the field in the Crab Nebula)—there is no need to invoke a pre-existing external field. The correlation length of the field in the outflowing material would depend on whether the energy came from one supermassive object; or successive, independent, stellar-mass outbursts. It was shown in Section 4 that, if magnetic stresses in the moving frame became dominant over particle pressures, the field could become *more* ordered as it was transported outwards, and that the transverse component should be amplified after passage through the shock at the 'working surface'. We thus expect that that synchrotron emission from the 'hot spots' at the outer edge of the components should be highly polarized, the magnetic field being predominantly transverse unless $B_{\parallel}/B_{\perp}$ were $\gg 1$ in the unshocked region. However, plasma modes excited by the shock may contribute to the electromagnetic energy density, and so affect the character and polarization of the radio emission, especially in the 'hot spots'. We have at the moment no grounds for expecting any particular polarization direction in the low surface brightness 'cocoon' regions.†

It is natural to try and relate the above ideas to more general theoretical questions concerning active galaxies and quasars, and to different types of radio sources.

† We do not generally expect much radio emission from the beams themselves (except perhaps from the vicinity of the nozzles, which could be detectable as a small 'double source' aligned with the source axis), because of the ordered character of the particle motions.

The hypothesis of a relativistic wind emanating from a central active region suggests a possible interpretation of multiple redshift absorption line systems. Cool filaments, provided only that they were larger than the proton Larmor radius, would ' feel ' the full pressure of the flow and be efficiently accelerated. The physics behind this idea is also to some extent independent of the nature of the central active region.

The jets in M87 can be interpreted as early stages in the formation of a radio source. The principal energetic problem is with the optical emission. If we take an equipartition field $\sim 7 \times 10^{-3}$ gauss for the brightest knot, and a linear size ~ 30 pc (Okoye 1973), we see that the minimum pressure $\sim 10^{-6}$ dyne cm^{-2}. Ram pressure confinement could be achieved with an external particle density $\sim 10^{-2}$ cm^{-3} and a speed of advance $\sim c/3$. The first figure, though approximately $10\times$ the cluster density estimated on the assumption of a thermal bremsstrahlung model for the X-ray emission (Griffiths *et al.* 1974), is acceptable within the galaxy itself; and the second is probably the maximum velocity consistent with kinematic restrictions following from the similarities of the two radio jets. The cooling time for an electron emitting optical synchrotron radiation in a field of 7×10^{-3} gauss at 10^{14} Hz is ~ 3 yr which is much less than the light travel time across the knot (~ 100 yr). The necessary power that must be supplied by the nucleus ($c\times$ the momentum supplied per second to push out the surrounding medium, assuming that it is supplied relativistically) is $\sim 3 \times 10^{44}$ erg s^{-1}. The observations of Griffiths *et al.* (1974) give an upper limit of 3×10^{42} erg s^{-1} for the X-ray flux from the nuclear region of M87. If we try and use the model outlined in Section 3 we find that $p_0 \sim 3 \times 10^{-6}$ dyne cm^{-2}, $T \sim 5 \times 10^8$ K, $r^* \sim 6$ pc, $R_0 \sim 100$ pc, $R^* \sim 100$ pc, $\rho_s \sim 4 \times 10^4 M_\odot$ pc^{-3}, are a self-consistent set of parameters for the central region. The resolution of the one optical jet into knots and the absence of optical emission from the other radio jet can possibly be attributed to the development of some long-wavelength instability in only one of the jets (*cf.* Stewart 1971).†

An essential feature of our proposal is that active galactic nuclei contain dense thermal gas in addition to the material right at the centre whose gravitational(?) energy powers the violent activity. The required mass of gas could easily be supplied by mass loss from stars throughout the galaxy, or else could represent material that never condensed into stars. In the absence of the central energy source, however, the quiescent gas would cool and collapse. Thus (unless its formation predated the initiation of the ' violent activity ' and it had pre-existed in the form of a rotationally-supported cool disc) it may be more plausible to suppose that the accumulation of a massive gas cloud quickly triggered the violent activity in its core. The calculations of Larson (1974) on the evolution of the gas content of elliptical galaxies are relevant

† An essentially similar, though drastically scaled down, version of our model can also be applied to the double radio source (positionally) identified with Sco X-1 (Braes & Miley 1971). Assuming that the double source is not a background source, there seems to be no energetic objection to the energy in the outer components being supplied by Sco X-1. The integrated radio luminosity ($\sim 10^{28}$ erg s^{-1}) is much less than either the optical or X-ray powers. The source is unresolved as yet, but if we assume a size-to-separation ratio similar to those of extragalactic radio sources, we obtain a minimum pressure within the components comparable with those in the most luminous extragalactic sources, and therefore quite easily confined by ram pressure in the interstellar medium even with $V \lesssim 0{\cdot}01$ c. If Sco X-1 is a rapidly rotating neutron star surrounded by a dense cloud of hot gas that is responsible for the X-ray emission, as has been proposed by Davidson, Pacini & Salpeter (1971), magnetized plasma generated by the neutron star could perhaps escape axially in a manner analogous to that proposed for the extragalactic sources.

to this possibility. If the relativistic plasma were supplied by $\sim 10^7$ 'events' in the galactic nucleus, each of which resembled a supernova and ejected 1–10 $M_\odot$ of thermal gas, this material would itself suffice to provide the cloud.

Our model predicts that a galactic nucleus generates a well-collimated double radio source only if the thermal gas density in the central 10–1000 pc is sufficiently high. This gas (with $T \sim 10^8$ K) should manifest itself most directly in its X-ray emission. If the cooler ($T \sim 10^4$–10^5 K) gas, responsible for the optical emission lines, is related to this hot cloud, correlations of radio and optical properties might allow further test of our proposals.

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APPENDIX

ONE-DIMENSIONAL FLOW OF AN ULTRA-RELATIVISTIC FLUID

In this Appendix we demonstrate that several of the results characterizing one-dimensional flow of a non-relativistic isentropic fluid through a channel of variable cross-section (see, for example, Landau & Lifshitz 1959, Chapter X) also hold when the fluid is ultra-relativistic. We use the notation of Landau & Lifshitz (1959), Chapter XV, and define w as the enthalpy per unit proper volume of the fluid. For an isotropic, ultra-relativistic fluid, $w = 4p \propto n^{4/3}$ where p is the pressure measured in the co-moving frame and n is the proper particle density. The energy-momentum tensor of a portion of the fluid is given by

$$T_{ik} = wu_iu_k + pg_{ik},$$

where u^i is the fluid 4-velocity and g_{ik} is the flat space metric tensor with $g_{11} = g_{22} = g_{33} = -g_{00} = 1$, and the other components zero. $T_{\alpha\beta}$ is the momentum flux density tensor and $cT_\alpha{}^0$, the energy flux density, where $\alpha, \beta = 1, 2, 3$.

Thus, if $u^i = (\gamma, u, 0, 0)$ and A is the cross-sectional area of the channel, normal to the flow, the momentum, energy and particle discharges are given respectively by

$$Q = (wu^2+p)\,A$$
$$L = wu\gamma cA$$
$$J = nuA.$$

For the present application, we assume that there is no transfer of heat (i.e. the flow is isentropic), nor of particles, through the walls of the channel; and that the flow is stationary. Thus L and J are constant, which implies that $w\gamma/n$ is constant, the relativistic Bernouilli equation. Substitution of the equation of state, $p \propto n^{4/3}$, gives

$$p = p_0\gamma^{-4},$$

where p_0 is the stagnation pressure, when $\gamma = 1$, and

$$A = \tfrac{1}{4}Lp^{-3/4}p_0^{-1/4}[(p_0/p)^{1/2} - 1]^{-1/2}\,c^{-1}.$$

Now A has a minimum value of $3\sqrt{3}L/(8p_0c)$ when $p = p^* = 4/9p_0$. Thus if the pressure decreases monotonically along the channel, a nozzle forms when the pressure has fallen to 4/9 the stagnation pressure. At the nozzle the bulk Lorentz factor is $\surd(3/2)$ and the corresponding bulk speed is $c/\surd 3$, identical to the sound speed in an ultra-relativistic fluid. Thus the flow is trans-sonic at the nozzle, just as in the non-relativistic case. If the channel is cylindrical with radius r,

$$r = 2^{1/2}3^{-3/4}(p/p_0)^{-1/4}[1-(p/p_0)^{1/2}]^{-1/4}\, r^*,$$

where r^* is the channel radius at the nozzle.

As the channel walls are not parallel to the bulk velocity, the fluid can exchange momentum with the walls and Q will not be constant. In fact

$$Q = \frac{(u^2+\frac{1}{4})}{u\gamma}\frac{L}{c} = \frac{Q^*}{2}\left(\frac{v}{v^*}+\frac{v^*}{v}\right),$$

where

$$Q^* = \frac{\surd 3L}{4c} = \frac{(\Gamma-1)}{\Gamma}\frac{L}{v^*}$$

is the minimum value of Q which occurs at the nozzle, $v = u\gamma^{-1}c$ is the bulk velocity of the flow, $v^* = c/\surd 3$ is the bulk velocity at the nozzle and Γ is the ratio of specific heats. Exactly the same formula holds for non-relativistic flow.

Finally we derive some results quoted in the text, pertaining to a strong, ultra-relativistic shock, when the ultra-relativistic equation of state is valid on both sides of the shock. We use the expressions derived in Landau & Lifshitz (1959), Chapter XV, for quantities measured in the frame in which the discontinuity is stationary. If p_1, p_2 are the pressures of the unshocked and shocked fluid respectively, then

$$\frac{v_1}{c} = \sqrt{\frac{p_1+3p_2}{3(3p_1+p_2)}}, \qquad \frac{v_2}{c} = \sqrt{\frac{3p_1+p_2}{3(p_1+3p_2)}}$$

and so $p_2/p_1 = \frac{8}{3}\gamma_1{}^2-9$.

For a strong shock ($p_2/p_1 \gg 1$, $v_2 \sim c/3$), the energy and momentum flux densities, which are both continuous across the shock, are given respectively by

$$L/A = \tfrac{3}{2}(3p_1+p_2)(p_1+3p_2)\,c$$
$$Q/A = \tfrac{3}{2}(p_1+p_2).$$