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ORIGINAL RESEARCH

A two-level discount model for coordinating a decentralized supply chain considering stochastic price-sensitive demand

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Abstract In this paper, a discount model is proposed to coordinate pricing and ordering decisions in a two-echelon supply chain (SC). Demand is stochastic and price sensitive while lead times are fixed. Decentralized decision making where downstream decides on selling price and order size is investigated. Then, joint pricing and ordering decisions are extracted where both members act as a single entity aim to maximize whole SC profit. Finally, a coordination mechanism based on quantity discount is proposed to coordinate both pricing and ordering decisions simultaneously. The proposed two-level discount policy can be characterized from two aspects: (1) marketing viewpoint: a retail price discount to increase the demand, and (2) operations management viewpoint: a wholesale price discount to induce the retailer to adjust its order quantity and selling price jointly. Results of numerical experiments demonstrate that the proposed policy is suitable to coordinate SC and improve the profitability of SC as well as all SC members in comparison with decentralized decision making.

Keywords Two-level discount · Supply chain coordination · Stochastic price-sensitive demand · Multi-echelon inventory systems

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Introduction

Under traditional decision making, each supply chain (SC) member makes decisions based on its own interests. Since SC members are affected by each other, it is necessary to find mechanisms that improve the performance of all SC parties beyond those in the traditional decision making. Among all SC decisions, the importance of replenishment and marketing decisions is undeniable. To ensure the satisfying customer demand without delay and scrimping the uncertain events' impact, it is essential to keep the inventory in a rational level. Replenishment policies consist of two categories (1) decision on order quantity (lot size) or production rate and (2) decision on reorder point. The first one is one of this paper's concerns.

On the other hand, in the supply chain management, optimal pricing strategy improves the profitability of system significantly (Gallego and Van Ryzin 1994). Pricing strategies play an important role when customer demand is price sensitive and also when production/distribution decisions can be complemented with pricing strategies in manufacturing environments (Simchi-Levi et al. 2014). Pricing strategies has a great impact on retail and manufacturing industries that use the internet and Direct-to-Customer models such as Dell Computers and Amazon.com (Chan et al. 2004). Optimal decision making on pricing strategy requires knowing the customer demand at a specific price. Market demand in addition to the price can depend on other variables such as brand name, quality of product or delivery length, but here we focus on a model the demand of which is sensitive only to the product price.

Integration of individual decisions on ordering and pricing policies throughout the SC can improve the performance of SC in a same way that revenue management has enhanced the efficiency of the rental car companies,

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hotels and airline (Chan et al. 2004). In particular, this may be more applicable for e-commerce operating systems, since price can be changed easily and data for demand are available readily (Chan et al. 2004).

This paper aims to coordinate ordering and pricing strategies throughout the SC simultaneously using an intensive discount scheme. The proposed coordination mechanism improves SC profitability from two aspects: (1) increasing revenue from the customers by selling more and (2) reducing cost of material flow between SC members.

The literature includes some works that theoretically studied the importance of integration SC ordering and pricing policies to mitigate the traditional decision-making disadvantages, but the coordination of these decisions simultaneously with a finite production rate and lost sales inventory system is often neglected. We refer the readers to Glock (2012) for a comprehensive literature review on integrated ordering policies and to Chen and Simchi-Levi (2012) for a comprehensive review on integrated inventory planning policies and pricing strategies.

Starting from the evaluation of individual decision making on ordering and pricing policies in a one-retailer and one-supplier system, this paper presents and evaluates a two-level discount scheme to coordinate SC members decisions simultaneously to maximize whole SC profit as well as all members profitability. The results show a dramatic improvement in the SC and its members performance when the proposed incentive plan is placed.

The rest of this paper is organized as follows. "Literature review" presents a literature review on the supply chain coordination which involved ordering, pricing and price discount in inventory systems. In "Mathematical modeling", a description of the proposed model is provided. The optimization models in decentralized and centralized modes are presented, the proposed coordination plan is explained and a method based on Li and Liu's work (Li and Liu 2006) is developed to divide the increased profit raised from joint decision making between both sides of SC. "Numerical experiments" presents numerical experiments to demonstrate the capability of proposed scheme and finally "Conclusions" concludes the paper.

Literature review

This work is involved with two main categories in the literature: (1) the supply chain coordination that examines how to align SC members' decisions to improve their profitability and subsequently the profitability of the whole SC and (2) the ordering policies, pricing strategies and price discount in inventory systems.

If SC members coordinate their decision based on the whole SC optimal objective, the performance of SC will be improved (Cachon 2003). A comprehensive literature review of supply chain coordination through contracts has been provided by Cachon (2003). Among various SC coordination mechanisms, buyback policies (Yao et al. 2011; Ai et al. 2012), revenue-sharing mechanisms (Palsule-Desai 2013), sales rebate contracts (Saha 2013), quantity-flexibility models (Karakaya and Bakal 2013) and quantity discount contracts are the most common ones.

Quantity discount contracts are more common in practice among aforementioned contracts. Under quantity discount contracts, the supplier improves his performance through more sales of products and reduction of his operational costs; on the other hand, the retailer benefits from a discount in wholesale price (Sadrian and Yoon 1994; Munson and Rosenblatt 1998; Lin 2008, 2013; Chung et al. 2014; Taleizadeh and Pentico 2014).

Monahan (1984) demonstrated that the quantity discount offered by a vendor is able to induce the buyer to increase his order quantity. Recent researches, by considering this fact, have shown that this incentive tool could be used to coordinate SC decisions. Weng (1995) investigated a supplier–buyer system faced with the deterministic demand. It has been shown that a quantity discount policy regardless of its form (incremental or all-unit discount policy) is an effective plan to induce the buyer to align its decision variables with the joint decision-making plan. Qi et al. (2004) investigated a supplier–retailer SC faced with the demand disruption during the planning horizon. They showed that a wholesale quantity discount policy is able to revise the production plan and coordinate the SC. Li and Liu (2006) designed a wholesale quantity discount scheme under demand uncertainty to induce the retailer to increase its order quantity in a one-supplier one-retailer supply chain. They showed there is feasible solution that the supplier and retailer accept to participate in the joint decision making. Further, the authors designed a method to divide the increased profit raised from joint decision making between both sides of SC.

Xie et al. (2010) studied an early order commitment (EOC)-based discount to coordinate a SC consisting of one manufacturer and multiple independent retailers. The authors demonstrated that the wholesale price discount policy can encourage the retailers to participate in EOC that result in lower SC costs. Sinha and Sarmah (2010) developed a multiple pricing schedule to influence ordering polices of a group of heterogeneous buyers to participate in coordination plan provided by a supplier. Their study showed that by increasing the number of pricing schedules, the benefits of coordination plan would increase. Wang et al. (2011) developed an all-unit discount coordination scheme, for a one-supplier multiple-buyer SC. To reduce warehousing costs, the supplier should encourage the buyers to synchronize their orders with the supplier's

replenishments policies; in their model, encouragement is planned by providing an all-unit price discount.

Du et al. (2013) studied a hybrid credit-wholesale price discount scheme to induce the SC members—consisting of one supplier and one buyer—to make their decisions in a way to improve the entire SC profitability in addition to their own profit. Peng and Zhou (2013) investigated efficiency of a quantity discount scheme to coordinate a fashion supply chain faced with stochastic demand and uncertain yields. The results of their study showed that the negative effects of uncertain yields and stochastic demand can be reduced by implementing the proposed policy. Zhang et al. (2014b) proposed a quantity discount scheme to coordinate and improve the efficiency of an integrated production-inventory system. They assumed that the product has a fixed lifetime and production rate is finite. A coordination plan was developed in which a vendor by proposing an all-unit quantity discount induces the buyer to purchase larger lot size in an integrated inventory system involving defective items (Lin and Lin 2014).

Zhang et al. (2014a) investigated a manufacturer–retailer supply chain where the manufacturer is risk averse and delivers lower amount of the retailer's order quantity when the retailer has a delay in payments. In this situation, they proposed a modified quantity discount in which the manufacturer induces the retailer to increase its order quantity while having an advanced payment. Heydari (2014) proposed a time-based temporary price discount coordination plan for a two-echelon SC, in which the seller tries to induce the buyer to globally optimize safety stock. Yin et al. (2014) developed a game theoretic model to help a manufacturer for supplier selection and having a long-term relationship using a quantity discount coordination scheme. Yang et al. (2014) discussed about three plans: quantity discount, credit period and centralized SC in a two-echelon system consisting of one manufacturer and one retailer where demand depends on stock level of the retailer. The authors showed that the credit period contract is preferred to induce the retailer to increase its order quantity when the manufacturer interest rate is less than the retailer interest rate. In that model, irrespective of interest rates, centralized decision making result in equal or higher SC profit than two other ones.

A price discount policy was developed for a firm with an opaque selling strategy to encourage flexible customers to postpone their demand (Wu and Wu 2015). Results showed that using this intensive scheme, the firm reduces shortages and capacity wastes. Saha and Goyal (2015) analytically investigated three SC coordination plans namely wholesale price contract, cost sharing contract and joint rebate contract to coordinate one-manufacturer one-retailer SC faced with a price–stock-sensitive demand. Results showed that the preferences of parties between three aforementioned

plans are not always aligned. Furthermore, they found out that the retailer with a higher bargaining power prefers the wholesale price contract.

In addition to discount contracts as an intensive scheme to integrate system's decisions, we review some recent papers in the field of pricing, replenishment policies and coordinating SC with price-sensitive demand.

Chen and Bell (2011) investigated a single manufacturer–single retailer system where customer demand is price sensitive and the system experiences customer returns. They proposed an incentive contract that includes two buyback prices: (1) for unsold products and (2) for returns of customer. Taleizadeh and Noori-daryan (2014) investigated a decentralized supply chain consisting of one supplier, one producer, and some retailers by considering price-sensitive demand. They aim to reduce system-wide cost by coordinating SC decisions that includes: supplier and producer price and shipment numbers received by the supplier and the producer. Lin and Wu (2014) designed an integrated system operations policy under uncertain and price-sensitive demand that simultaneously determines the price of product and operational levels of procurement, production, and distribution. Taleizadeh et al. (2015a) developed an economic production and inventory model for a system consisting of one distributor, one manufacturer, and one retailer. They determined the distributor order quantity and its selling price and also the manufacturer and the retailer selling price with the aim of maximizing all system members' profit.

In another work, Taleizadeh et al. (2015c) investigated a problem of joint determination of replenishment lot size, selling price, and shipment numbers with assuming rework on defective items for an economic production quantity (EPQ) model. Taleizadeh et al. (2015b) developed a vendor managed inventory (VMI) model for a two-echelon system including one supplier and several noncompeting retailers facing a price-sensitive demand. They proposed an inventory model to jointly optimize the replenishment frequency of raw material, the production rate, the replenishment cycle of the product, and the retail price. They assumed different deterioration rates for the raw material and the finished product.

Extending the previous studies, this paper considers a two-echelon supply chain consisting of one-supplier and one-retailer faced with a stochastic price-sensitive demand. In this model, coordination of pricing and ordering decisions simultaneously with considering a finite supplier production rate and lost sales inventory system, is investigated. To achieve it, a two-level price discount scheme is proposed: (1) discount in selling price to induce customers to buy more, and (2) a wholesale price discount to induce the retailer to adjust its order quantity and selling price based on joint decision making. In the proposed model, the

SC operates in a competitive market where demand will be lost if it is not met immediately.

In particular, in the proposed model, the retailer has the authority of deciding on order quantity and selling price but both of these decision variables have a substantial impact on the supplier profitability. The retailer makes decisions on both the order quantity and selling price in order to maximize its own profit. On the other hand, in a price-sensitive market, the retailer should provide a selling price to maximize its revenue from the customers. Optimizing order quantity and selling price by the retailer maximizes its profit regardless of other SC members' profit. On the other hand, optimizing order quantity and retail price from the viewpoint of whole SC may reduce the retailer profit. In this study by modeling of SC members cost structure, a two-level discount scheme is proposed to coordinate SC members' decisions to maximize whole SC profit as well as all members' profitability. The proposed two-level discount consists of two parts: (1) a retail price discount initiated by the retailer to induce customers to buy more and (2) a wholesale quantity discount initiated by the supplier to convince the retailer to globally optimize retail price and also the size of orders to reduce SC operational costs. Decision models are developed in three cases decentralized, centralized, and coordinated structures. Under the decentralized decision structure, the retailer decides on both order quantity and retail price based on its own cost structure. Under centralized structure, it is assumed that there is a single decision maker who aims to maximize whole SC profit. Finally, a coordination mechanism is proposed to induce both members to decide based on whole SC viewpoint while both members' profitability is guaranteed. A two-level discount model is proposed as incentive scheme to share benefit of coordinated decision making between SC members fairly.

Mathematical modeling

The considered SC consists of one supplier and one retailer. The supplier manufactures the product. The retailer faces with a stochastic price-sensitive demand with normal distribution. Lead time is deterministic and constant. Demand will be lost if the customer's needs are not met immediately. The retailer uses a (r, Q) continuous review system. Based on demand information and its cost structure, the retailer decides on the selling price to the customers and order quantity from the supplier. After retailer's ordering, the supplier determines the quantity of production. It is assumed that the supplier has enough and finite capacity to meet the retailer's orders. In addition, expected customer's demand is considered as a linear function of
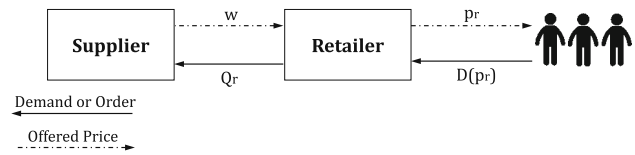

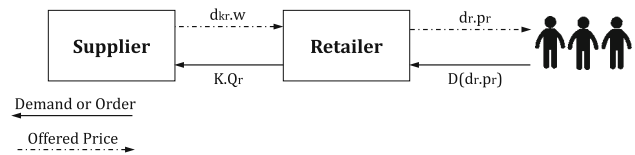**Fig. 1** Supplier–retailer–customers interactions under decentralized model

Fig. 2 Supplier–retailer–customers interactions under coordinated model

retail price, while the standard deviation of demand is fixed and it is not a function of retail price.

Figures 1 and 2 illustrate the investigated problem under decentralized and coordinated structures, respectively.

The following notations are applied in the mathematical models:

p	Retailer's unit selling price (decision variable)
$E[D(p)]$	Expected customer's demand rate per year at selling price p
σ_D	Standard deviation of customer's demand
Q	Retailer's order quantity (decision variable)
S_r	Retailer's ordering costs per order
h_r	Retailer's unit holding cost per year
π	Retailer's unit shortage cost
L	Retailer's replenishment lead time
k	Safety stock factor
h_s	Supplier's unit holding cost per year
w	Supplier's wholesale price before applying discount
S_s	Supplier's fixed cost that is incurred with each handling the retailer's order
c	Supplier's unit production cost
n	Supplier's lot size multiplier (decision variable)
R	Supplier's annual production capacity

Decentralized decision making

Under decentralized decision making, each member tries to maximize its own profit function individually regardless of other member. The expected annual customer demand is a linear function of retail price given by $E[D(p)] = a - bp$, where a is market size and b is the price-elasticity coefficient of demand. Let $Pr_r(Q, p)$ denote the retailer's expected annual profit function, then it can be formulated as:

$$Pr_r(Q,p) = (p-w)(a-bp) - \frac{(a-bp)}{Q}S_r$$
$$- h_r\left(\frac{Q}{2} + \sigma_{D_L}(k + Gu(k))\right)$$
$$- \frac{(a-bp)}{Q}(\pi + p - w)\sigma_{D_L}Gu(k) \qquad (1)$$

where the first term denotes retailer's expected annual gross profit. The second and third terms denote expected annual ordering cost and annual holding cost, respectively. The last term denotes expected annual lost sales penalty and opportunity costs. σ_{D_L} is standard deviation of customer's demand during lead time:

$$\sigma_{D_L} = \sigma_D\sqrt{L} \qquad (2)$$

Since demand comes from normal probability distribution, then expected shortages will be $\sigma_{D_L}Gu(k)$ (Silver et al. 1998, pp. 722–723):

$$Gu(k) = \int_k^\infty (z-k)\frac{1}{\sqrt{2\pi}}\exp(-z^2/2)\,dz \qquad (3)$$

According to Eq. (1), the retailer decides on Q and p to maximize its own profit function.

Proposition 1 *The retailer profit function is concave with respect to Q and p simultaneously when*:

$$2b\left(\frac{2(a-bp)((\pi+p-w)\sigma_{D_L}Gu(k)+S_r)}{Q^3}\right)\left(1 - \frac{Gu(k)\sigma_{D_L}}{Q}\right)$$
$$> \left(\frac{(b(-\pi-2p+w)+a)Gu(k)\sigma_{D_L} - bS_r}{Q^2}\right)^2$$

Proof See "Appendix".

Let Q_r^* and p_r^* denote the optimal values of retailer decision variables that maximize its profit function under the decentralized model. Using first-order condition, we have:

$$Q_r^* = \frac{\sqrt{2(a-bp_r^*)h_r((\pi+p_r^*-w)\sigma_{D_L}Gu(k)+S_r)}}{h_r} \qquad (4)$$

$$p_r^* = \frac{Gu(k)\sigma_{D_L}(\pi-w)+Q_r^*w+S_r}{2(Q_r^* - Gu(k)\sigma_{D_L})} + \frac{a}{2b} \qquad (5)$$

Since the values of Q_r^* and p_r^* are circularly depending on each other, then an iterative procedure can be used to calculate optimal values as follows:

Step 1 Set $p_r = 0$ (minimum feasible value for p_r)
Step 2 Calculate Q_r using (4)
Step 3 Calculate p_r using (5) based on obtained Q_r
Step 4 Repeat second and third steps to converge
Step 5 The obtained values of Q_r and p_r are optimum. Check optimality condition of Proposition 1 to ensure global optimization.

Now let $Pr_s(n)$ be the supplier's expected annual profit function, then it can be formulated as:

$$Pr_s(n) = (w-c)(a-bp_r^*)\left(1 - \frac{\sigma_{D_L}Gu(k)}{Q_r^*}\right)$$
$$- \frac{(a-bp_r^*)}{nQ_r^*}\left(1 - \frac{\sigma_{D_L}Gu(k)}{Q_r^*}\right)S_s$$
$$- \frac{1}{2}h_sQ_r^*\left(n-1-(n-2)(a-bp_r^*)\left(1 - \frac{\sigma_{D_L}Gu(k)}{Q_r^*}\right)\right)\Big/R \qquad (6)$$

Proposition 2 *The supplier profit function is concave with respect to n.*

Proof See "Appendix".

n_s^* is the supplier lot size multiplier so that it maximizes its profit function under the decentralized model. Let n_s be a value of n that maximizes the supplier profit releasing the constraint that n is an integer, it can be calculated as:

$$n_s = \frac{1}{Q_r^*}\sqrt{\frac{2RS_s\left(Q_r^* - Gu(k)\sigma_{D_L}\right)(a-bp_r^*)}{h_s\left((bp_r^*+R-a)Q_r^* + Gu(k)\sigma_{D_L}(a-bp_r^*)\right)}} \qquad (7)$$

Since n must be an integer variable, either the smallest following integer or largest previous integer of n_s whichever results in larger value of $Pr_s(n)$ will be optimum value of n from the supplier perspective (i.e., n_s^*).

Centralized decision making

Under centralized decision making, a central decision maker aims to maximize the whole SC profit. In this situation, sale and replenishment policies are determined from viewpoint of the entire SC. Let $Pr_{sc}(Q, p, n)$ be the expected annual profit function of SC that is the sum of the supplier and retailer annual expected profit:

$$Pr_{sc}(Q,p,n) = Pr_r(Q,p) + Pr_s(n)$$
$$= (p-c)(a-bp)\left(1 - \frac{\sigma_{D_L}Gu(k)}{Q}\right) - \frac{(a-bp)}{Q}$$
$$\times \left(S_r + \sigma_{D_L}Gu(k)\pi + \left(1 - \frac{\sigma_{D_L}Gu(k)}{Q}\right)\left(\frac{S_s}{n}\right)\right)$$
$$- h_r\left(\frac{Q}{2} + \sigma_{D_L}(k + Gu(k))\right)$$
$$- \frac{1}{2}h_sQ\left(n-1-(n-2)(a-bp)\left(1 - \frac{\sigma_{D_L}Gu(k)}{Q}\right)\right)\Big/R \qquad (8)$$

Observation 1 The SC expected annual profit function is concave with respect to Q, p and n under some circumstances (for details see "Appendix").

Let Q_{sc}^*, p_{sc}^* and n_{sc} denote the values of SC decision variables that maximize SC profit function releasing the constraint that n is an integer. Then, we have:

$$Q_{sc}^* = -\frac{1}{3}\frac{\theta^{\frac{1}{3}}}{\left(h_s\left(n_{sc}-1-\frac{(a-bp_{sc}^*)(n_{sc}-2)}{R}\right)+h_r\right)} - \frac{2(a-bp_{sc}^*)}{\theta^{\frac{1}{3}}}\left(\frac{S_s}{n_{sc}}+S_r+(p_{sc}^*-c+\pi)\sigma_{D_L}Gu(k)\right)$$

where,

$$\theta = \left(12\sqrt{3}\sqrt{108\left(\frac{a-bp_{sc}^*\sigma_{D_L}Gu(k)S_s}{n_{sc}}\right)^2 - \frac{8(a-bp_{sc}^*)^3\left(\frac{S_s}{n_{sc}}+S_r+(p_{sc}^*-c+\pi)Gu(k)\right)^3}{\left(h_s\left(n_s-1-\frac{(a-bp_{sc}^*)(n_{sc}-2)}{R}\right)+h\right)}} + \frac{216(a-bp_{sc}^*)\sigma_{D_L}Gu(k)S_s}{n_{sc}}\right) \times \frac{1}{4}\left(h_s\left(n_{sc}-1-\frac{(a-bp_{sc}^*)(n_{sc}-2)}{R}\right)+h_r\right)^2 \tag{9}$$

$$p_{sc}^* = \frac{1}{2}\left(\left(\frac{a}{b}+c\right)+\frac{S_s}{n_{sc}Q_{sc}^*}-\frac{1}{2}\frac{h_sQ_{sc}^*(n_{sc}-2)}{R}+\frac{(\pi\sigma_{D_L}Gu(k)+S_r)}{(Q_{sc}^*-\sigma_{D_L}Gu(k))}\right) \tag{10}$$

$$n_{sc} = \frac{1}{Q_{sc}^*}\sqrt{\frac{2RS_s(Q_{sc}^*+Gu(k)\sigma_{D_L})(a-bp_{sc}^*)}{h_s((bp_{sc}^*+R-a)Q_{sc}^*+Gu(k)\sigma_{D_L}(a-bp_{sc}^*))}} \tag{11}$$

Since the values of Q_{sc}^*, p_{sc}^* and n_{sc} are circularly depending on each other, then an iterative procedure can be applied to calculate optimum values of decision variables as follows:

Step 1 Set $p_{sc} = 0$ (minimum feasible value for p_{sc})
Step 2 Set $n_{sc} = 1$ (minimum feasible value for n_{sc})
Step 3 Calculate Q_{sc} using Eq. (9)
Step 4 Calculate p_{sc} using Eq. (10)
Step 5 Calculate n_{sc} using Eq. (11)
Step 6 Repeat third, fourth and fifth steps to converge values of Q_{sc}^*, p_{sc}^* and n_{sc}
Step 7 Calculate SC profit function at the smallest following integer and largest previous integer of n_{sc}; whichever results in larger value of $Pr_{sc}(Q, p, n)$ is selected as n_{sc}^*
Step 8 Obtained values of Q_{sc}^*, p_{sc}^* and n_{sc}^* are optimum. Check optimality condition of Observation 1 to ensure global optimum solution.

Although the above procedure finds optimal decisions for the whole SC, the found solution does not create more profitability than decentralized model for each SC member. To guarantee participation of both SC members in the joint decision-making process, it is necessary to designate an incentive scheme to encourage SC members through increasing their profit.

Coordination mechanism and incentive scheme

It is clear that substituting the optimal values of whole SC decision variables for values of decision variables that are made individually by members, SC profit will be improved. Nevertheless, it cannot guarantee improvement of all SC members' profit. To encourage the retailer and supplier to enter joint decision making, their expected annual profit must be improved in comparison with decentralized mode. Both decision variables Q and p are under the authority of the retailer, while decision variable n has no effect on the retailer profit function. In the decentralized model, the retailer decides on both decision variables based on its own profitability; therefore, changing decisions from decentralized mode cause profit loss for the retailer. In this section, we propose a mechanism that requires the supplier to reduce wholesale price to encourage the retailer to change its decisions on Q and p with respect to the whole SC optimal decisions.

Consider that the retailer is required to change its order size from Q_r^* to Q_{sc}^* and also its selling price from p_r^* to p_{sc}^*. Let K be a coefficient that the retailer is required to apply on its order size and d_r be a coefficient to apply on its selling price, then we have:

$$K = \frac{Q_{sc}^*}{Q_r^*} \tag{12}$$

$$d_r = \frac{p_{sc}^*}{p_r^*} \qquad (13)$$

On the other hand, the supplier is required to apply coefficient d_{kr} on its wholesale price w to create a discounted wholesale price. Substituting KQ_r^* for Q_r^* and $d_r p_r^*$ for p_r^* and $d_{kr}w$ for w in the retailer profit function (Eq. (1)), we get the improved profit of the retailer in migrating from decentralized model toward joint decision making as:

$$\begin{aligned}
&Pr_r\big(d_{kr}, KQ_r^*, d_r p_r^*\big) - Pr_r\big(Q_r^*, p_r^*\big) \\
&= \big(d_r p_r^* - d_{kr}w\big)\big(a - b d_r p_r^*\big) \\
&\quad - \frac{\big(a - b d_r p_r^*\big)}{KQ_r^*} S_r - h_r\left(\frac{KQ_r^*}{2} + \sigma_{D_L}(k + Gu(k))\right) \\
&\quad - \frac{\big(a - b d_r p_r^*\big)}{KQ_r^*}\big(\pi + d_r p_r^* - d_{kr}w\big)\sigma_{D_L} Gu(k) - Pr_r\big(Q_r^*, p_r^*\big)
\end{aligned} \qquad (14)$$

The retailer accepts an agreement that makes Eq. (14) greater than zero, thus we should have:

$$\begin{aligned}
&d_{kr}w\big(a - b d_r p_r^*\big)\left(1 - \frac{\sigma_{D_L} Gu(k)}{KQ_r^*}\right) \\
&\leq -\frac{\big(a - b d_r p_r^*\big)}{KQ_r^*}\big((\pi + d_r p_r)\sigma_{D_L} Gu(k) d_r p_r^*\big(a - b d_r p_r^*\big) + S_r\big) \\
&\quad - h_r\left(\frac{KQ_r^*}{2} + \sigma_{D_L}(k + Gu(k))\right) - Pr_r\big(Q_r^*, p_r^*\big)
\end{aligned} \qquad (15)$$

In fact, the maximum value of coefficient d_{kr} that can encourage the retailer to participate is:

$$\begin{aligned}
d_{kr}^{max} = &\frac{1}{w\big(a - b d_r p_r^*\big)\left(1 - \frac{\sigma_{D_L} Gu(k)}{KQ_r^*}\right)} \\
&\times \left(\begin{array}{l} -\frac{\big(a - b d_r p_r^*\big)}{KQ_r^*}\big((\pi + d_r p_r)\sigma_{D_L} Gu(k) d_r p_r^*\big(a - b d_r p_r^*\big) + S_r\big) \\ -h_r\left(\frac{KQ_r^*}{2} + \sigma_{D_L}(k + Gu(k))\right) - Pr_r\big(Q_r^*, p_r^*\big) \end{array}\right)
\end{aligned} \qquad (16)$$

By the similar procedure, the minimum acceptable value of coefficient d_{kr} that guarantees more profitability for the supplier can be calculated as:

Let IPr_{sc} be the increased profit of whole SC after joint decision making:

$$IPr_{sc} = Pr_{sc}\big(Q_{sc}^*, p_{sc}^*, n_{sc}^*\big) - \big(Pr_r\big(Q_r^*, p_r^*\big) + Pr_s\big(n_s^*\big)\big) \qquad (18)$$

If the supplier implements d_{kr}^{min}, the whole IPr_{sc} will be assigned to the retailer, while implementing d_{kr}^{max} assigns all profits to the supplier.

Assume that α $(0 < \alpha < 1)$ fraction of IPr_{sc} is assigned to the retailer and subsequently $1 - \alpha$ fraction is assigned to the supplier where α is bargaining power of the retailer with respect to the supplier. Let IPr_r be the increased profit of the retailer after implementation of quantity discount policy. Based on the above-mentioned analysis, we get:

$$\begin{aligned}
IPr_r(d_{kr}, d_r, K) &= \alpha\, IPr_{sc} = \alpha\, IPr_r\big(d_{kr}^{min}, d_r, K\big) \\
&\quad + (1 - \alpha)IPr_r\big(d_{kr}^{max}, d_r, K\big)
\end{aligned} \qquad (19)$$

Simplifying Eq. (19), we can calculate d_{kr} based on bargaining power α as:

$$d_{kr} = \alpha\, d_{kr}^{min} + (1 - \alpha)d_{kr}^{max} \qquad (20)$$

Normally, the retailer attends d_{kr}^{min} while the supplier wants to implement d_{kr}^{max}. Using Eq. (20) according to bargaining power α, the value of d_{kr} will be specified.

Numerical experiments

Using a set of test problems, the performance of the proposed model is demonstrated. Table 1 shows the three investigated test problems.

The results of running the proposed model on the test problems show that the proposed intensive scheme is able to coordinate SC. Coordination scheme improves profitability of both SC members as well as whole SC profit.

Table 2 compares the values of decision variables and profit functions obtained from the decentralized, centralized, and coordinated decision-making models. Note that since variable n should take an integer value, then obtained

$$\begin{aligned}
d_{kr}^{min} = &\frac{1}{w\big(a - b d_r p_r^*\big)\left(1 - \frac{\sigma_{D_L} Gu(k)}{KQ_r^*}\right)} \\
&\times \left(\begin{array}{l} \frac{1}{2}K h_s Q_r^*\left(n_{sc}^* - 1 - \big(a - b d_r p_r^*\big)\big(n_{sc}^* - 2\big)\left(1 - \frac{\sigma_{D_L} Gu(k)}{KQ_r^*}\right)\Big/R\right) \\ \frac{\big(a - b d_r p_r^*\big)}{KQ_r^*}\left(1 - \frac{\sigma_{D_L} Gu(k)}{KQ_r^*}\right)\big(S_s/n_{sc}^*\big) + Pr_s\big(n_s^*\big) \end{array}\right) + \frac{c}{w}
\end{aligned} \qquad (17)$$

Table 1 Data for the investigated test problems

	Test problem 1	Test problem 2	Test problem 3
w	200	205	210
a	3000	4000	9000
b	10	11	35
h_r	40	32	35
h_s	35	32	37
S_r	8000	6000	4000
S_s	9000	6500	6550
π	4	2	3
c	150	160	185
L	4	9	1
σ_D	40	30	50
R	4500	5500	10,000
k	0.95	0.95	0.95

Table 2 The results of decentralized, centralized, and coordinated decision-making models

	Test problem 1	Test problem 2	Test problem 3
Decentralized mode			
Q_r^*	411.94	584.80	390.24
p_r^*	259.92	289.54	238.78
$n_s \rightarrow n_s^*$	1.143 → 1	1.06 → 1	1.26 → 1
Pr_r	4204.99	47192.39	3017.65
Pr_s	10451.50	25863.66	4760.76
Pr_{sc}	14656.49	73056.47	7778.40
$E[D(p)]$	400.80	815.06	642.70
Centralized mode			
Q_{sc}^*	849.46	970.76	1256
p_{sc}^*	239.45	271.33	229.79
$n_{sc} \rightarrow n_{sc}^*$	0.70 → 1	0.73 → 1	0.49 → 1
Pr_r	−2364.49	41953.02	−7955.16
Pr_s	21671.02	33982.37	16651.27
Pr_{sc}	19306.53	75935.39	8696.11
$E[D(p)]$	605.62	1015.39	956.93
Coordinated mode			
K	2.0621	1.6600	3.2134
d_r	0.9212	0.9371	0.9624
d_{kr}^{max}	0.9453	0.9746	0.9452
d_{kr}^{min}	0.9065	0.9522	0.9406
d_{kr}	0.9259	0.9634	0.9429
Pr_r	6531.64	49500.74	3476.50
Pr_s	12774.89	26434.65	5219.61
Pr_{sc}	19306.53	75935.39	8696.11

decimal values for this variable are infeasible. As discussed before, the smallest following and largest previous integer of n may be optimal. As shown in Table 2, in the centralized model, the retailer should increase its order
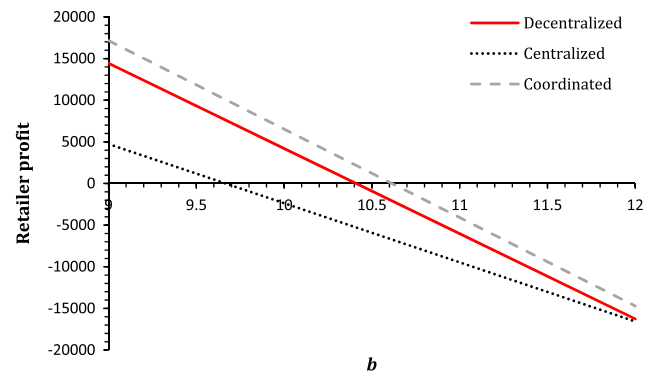

Fig. 3 The effect of b on the retailer profit

quantity and reduce the selling price compared to decentralized model.

The results show the expected profit of SC in centralized model is increased in comparison with decentralized model. In centralized model, the expected profit of supplier is increased but the retailer loses money. Thus, normally the retailer refuses to change its mind about its decision variables Q and p. A mechanism that increases both SC members profit should be designated.

The maximum and minimum values of wholesale discount coefficient d_{kr} are obtained so that both the retailer and supplier benefit from increased profit of SC. Wholesale discount coefficient d_{kr} can be determined based on bargaining power α using Eq. (20). In Table 2, wholesale discount coefficient d_{kr} is determined supposing equal bargaining power for SC members (i.e., $\alpha = 0.5$). With agreement of SC members to participate in coordination plan, the retailer should apply rates K and d_r on its order quantity and selling price, respectively. On the other hand, the supplier should apply rate d_{kr} on its wholesale price.

Table 2 shows that the retailer and supplier have a more profit beyond those in the decentralized decision making. Thus, the proposed incentive scheme is applicable. In addition, the proposed model increases SC profit same as centralized decision making in all investigated test problems, therefore the proposed model is able to achieve channel coordination.

To investigate the impact of two significant parameters b and L on the profitability of SC and its members, a set of sensitivity analyses is conducted. The required data for sensitivity analyses are taken from test problem 1. Figures 3 and 4 illustrate changing of the retailer and the supplier profit functions in the decentralized, centralized, and coordinated models over increasing b. As expected, both members of SC have a lower profit when b has a larger value that subsequently results in lower SC profit. In addition, both members' profitability under coordinated model is greater than decentralized model which implies that the coordination model is applicable from both
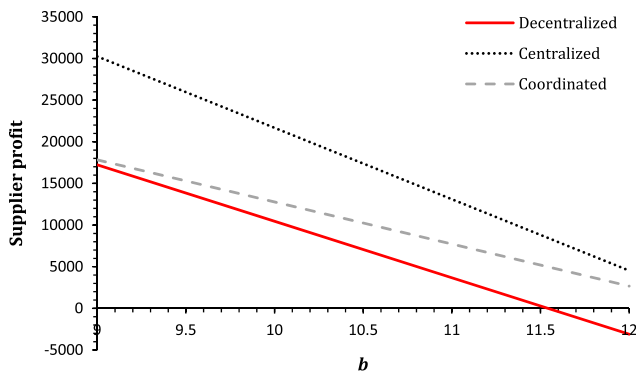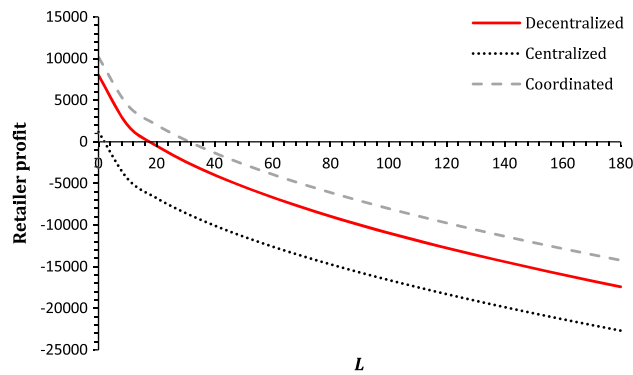
Fig. 4 The effect of *b* on the supplier profit

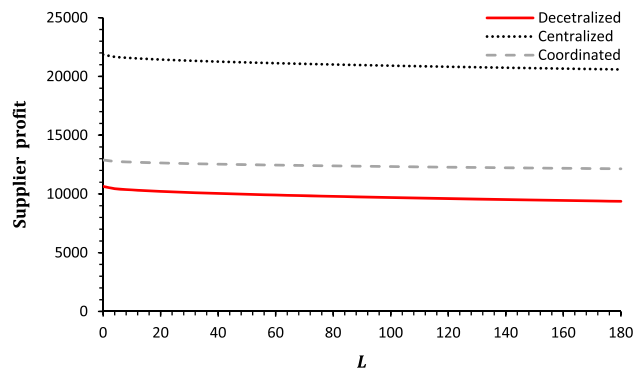**Fig. 5** The effect of *L* on the retailer profit

Fig. 6 The effect of *L* on the supplier profit
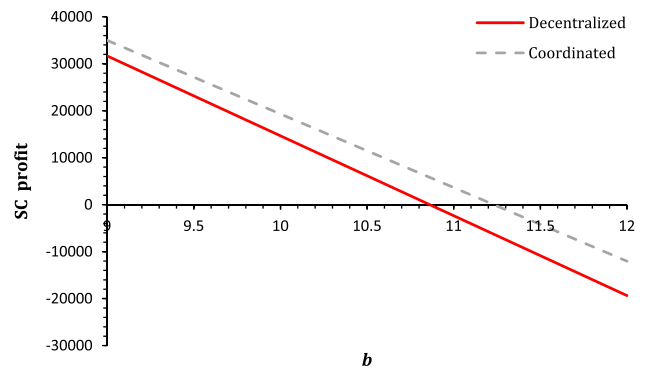

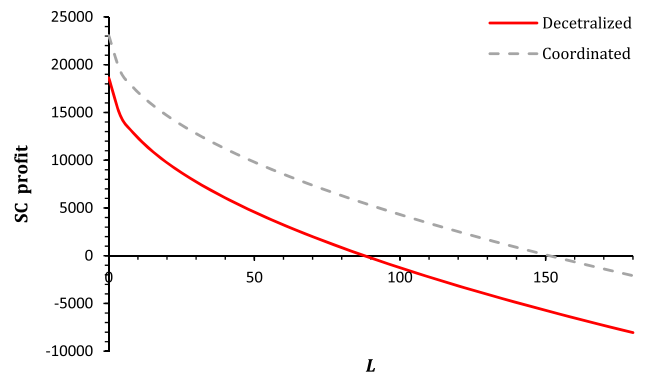**Fig. 7** The effect of *b* on the SC profit

Fig. 8 The effect of *L* on the SC profit

Figures 7 and 8 illustrate changes on whole SC profit by increasing *b* and *L*, respectively. As expected, by increasing *b* and *L*, SC profitability decreases. Nevertheless, the proposed coordination model mitigates negative effects of increasing *b* and *L* on SC profitability with respect to decentralized decision-making model.

Conclusions

This paper has investigated a two-level price discount as a coordination plan which is characterized by two different aspects: (1) Marketing management: discount in retail price to increase the demand, and (2) Operations management: discount in wholesale price to induce the retailer to adjust its order quantity and retail price based on joint decision making. In the proposed model, SC faces with a stochastic price-sensitive demand. In the decentralized decision-making model, the retailer decides on the order quantity and selling price based on its own expected profit function. However, centralized modeling of SC decisions revealed that there is a better solution from the whole SC viewpoint. Our numerical experiment results show that the profitability of the entire SC and supplier improves in the centralized mode, but the retailer loses money. In this situation, by applying a discount on wholesale price, the increased profit

members' perspective. Based on Figs. 3 and 4, there is a threshold of *b* beyond that the business is disadvantageous; the proposed coordination model is able to enhance this threshold for both members with respect to decentralized decision-making model.

Figures 5 and 6 illustrate the effect of lead time *L* on the retailer and supplier profitability under the decentralized, centralized, and coordinated models. As the Figures show, the profitability of both SC members decreases with increasing *L* that is the direct result of more lost sales by prolonging the lead times. However, profitability of both members under coordinated decision making is always greater than decentralized model.

would be divided between SC members so that the supplier and retailer improve their profit. Upper bound and lower bound for the wholesale price discount from both the retailer and supplier perspectives are extracted. Numerical experiments and sensitivity analyses show that the proposed model has the ability of achieving channel coordination. The proposed model can be extended to consider other demand functions. Also, in addition to sensitivity of the demand to price, other parameters such as lead time length and product quality that can affect the demand can be considered. Another alternative for future study is about considering the impact of selling price on demand standard deviation.

Appendix

Proof of Proposition 1 To prove concavity of the retailer profit function with respect to Q and p, the Hessian matrix of the retailer's expected annual profit function should be calculated. If the Hessian matrix is negative definite, the proposition will be proved. We have:

$$H(Pr_r(Q,p)) = \begin{bmatrix} \partial^2 Pr_r(Q,p)/\partial Q^2 & \partial^2 Pr_r(Q,p)/\partial Q \partial p \\ \partial^2 Pr_r(Q,p)/\partial p \partial Q & \partial^2 Pr_r(Q,p)/\partial p^2 \end{bmatrix}$$

where

$$\frac{\partial^2 Pr_r(Q,p)}{\partial Q^2} = -\frac{2(a-bp)((\pi+p-w)\sigma_{D_L}Gu(k)+S_r)}{Q^3}$$

$$\frac{\partial^2 Pr_r(Q,p)}{\partial p^2} = -2b\left(1-\frac{Gu(k)\sigma_{D_L}}{Q}\right)$$

$$\frac{\partial^2 Pr_r(Q,p)}{\partial Q \partial p} = \frac{\partial^2 Pr_r(Q,p)}{\partial p \partial Q} = \frac{(b(-\pi-2p+w)+a)Gu(k)\sigma_{D_L}-bS_r}{Q^2}$$

Since $p > w$, then the first element of the main diagonal is negative. On the other hand, we would have a profitable business if $Gu(k)\sigma_{DL} < Q$ then the second element of the

main diagonal is also negative. Therefore, the requisite for Hessian matrix to be negative definite is satisfied.

The first principal minor of the above Hessian matrix is the same as the first element of the main diagonal that has a negative value. The second principle minor is positive when:

$$2b\left(\frac{2(a-bp)((\pi+p-w)\sigma_{D_L}Gu(k)+S_r)}{Q^3}\right)\left(1-\frac{Gu(k)\sigma_{D_L}}{Q}\right)$$
$$> \left(\frac{(b(-\pi-2p+w)+a)Gu(k)\sigma_{D_L}-bS_r}{Q^2}\right)$$

Satisfying the above condition results in Hessian matrix of the retailer's expected profit function to be negative definite. With respect to rational values of parameters, this condition would be satisfied.

Proof of Proposition 2 It is enough to show that the second-order derivative of the supplier's expected annual profit function with respect to n is negative:

$$\frac{\partial^2 Pr_s(n)}{\partial n^2} = -\frac{2(a-bp)\left(1-\frac{\sigma_{D_L}Gu(k)}{Q}\right)S_s}{n^3Q}$$

As mentioned, we have a profitable business when $Gu(k)\sigma_{DL} < Q$ then above term is negative.

Details of observation 1 To show concavity of SC profit function with respect to Q, p, and n the Hessian matrix of the SC expected annual profit function should be calculated. If the Hessian matrix is negative definite, the observation will be proved. We have:

$$H(Pr_{sc}(Q,p,n)) = \begin{bmatrix} \partial^2 Pr_{sc}(Q,p,n)/\partial Q^2 & \partial^2 Pr_{sc}(Q,p,n)/\partial Q \partial p & \partial^2 Pr_{sc}(Q,p,n)/\partial Q \partial n \\ \partial^2 Pr_{sc}(Q,p,n)/\partial p \partial Q & \partial^2 Pr_{sc}(Q,p,n)/\partial p^2 & \partial^2 Pr_{sc}(Q,p,n)/\partial p \partial n \\ \partial^2 Pr_{sc}(Q,p,n)/\partial n \partial Q & \partial^2 Pr_{sc}(Q,p,n)/\partial n \partial p & \partial^2 Pr_{sc}(Q,p,n)/\partial n^2 \end{bmatrix}$$

where:

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial Q^2} = -\frac{2(a-bp)}{Q^3}((p-c+\pi)\sigma_{D_L}Gu(k)+S_r)$$
$$+\frac{2(a-bp)(3\sigma_{D_L}Gu(k)-Q)}{nQ^4}S_s$$

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial p^2} = -2b\left(1-\frac{\sigma_{D_L}Gu(k)}{Q}\right)$$

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial n^2} = -\frac{2(a-bp)(Q-\sigma_{D_L}Gu(k))S_s}{n^3Q^2}$$

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial Q \partial p} = \frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial p \partial Q}$$

$$= \frac{(a - bp - b(p-c))\sigma_{D_L} Gu(k) - b(S_r + \pi\sigma_{D_L} Gu(k))}{Q^2}$$

$$+ \frac{b(2\sigma_{D_L} Gu(k) - Q)S_s}{nQ^3} - \frac{1}{2} h_s b(n-2)$$

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial Q \partial n} = \frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial n \partial Q}$$

$$= \frac{(a - bp)(2\sigma_{D_L} Gu(k) - Q)S_s}{n^2 Q^3}$$

$$- \frac{1}{2} h_s p \left(1 - \frac{(a-bp)}{R}\right)$$

$$\frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial p \partial n} = \frac{\partial^2 Pr_{sc}(Q,p,n)}{\partial n \partial p} = -\frac{b(Q - \sigma_{D_L} Gu(k))S_s}{n^2 Q^2}$$

$$- \frac{1}{2} \frac{bh_2(Q - \sigma_{D_L} Gu(k))}{R}$$

The first element of the main diagonal is negative under below condition. However, with respect to rational values of the model parameters, this condition would be satisfied.

$$\frac{3\sigma_{D_L} Gu(k) - Q}{nQ} S_s < (p - c + \pi)\sigma_{D_L} Gu(k) + S_r \qquad (21)$$

According to the preceding explanation, the second and third elements of main diagonal are negative. Therefore, the requisite for Hessian matrix to be negative definite is satisfied.

The first principal minor of the above Hessian matrix is the same as the first element of the main diagonal that has a negative value under condition 21. The second principle minor is positive when:

$$2b \left(\frac{2(a-bp)}{Q^3}((p-c+\pi)\sigma_{D_L} Gu(k) + S_r)\right.$$

$$\left. - \frac{2(a-bp)(3\sigma_{D_L} Gu(k) - Q)}{nQ^4} S_s\right)\left(1 - \frac{\sigma_{D_L} Gu(k)}{Q}\right)$$

$$> \left(\frac{(a - bp - b(p-c))\sigma_{D_L} Gu(k) - b(S_r + \pi\sigma_{D_L} Gu(k))}{Q^2}\right.$$

$$\left. + \frac{b(2\sigma_{D_L} Gu(k) - Q)S_s}{nQ^3} - \frac{1}{2} h_s b(n-2)\right)^2 \qquad (22)$$

And the third principle minor is negative when:

$$\begin{pmatrix} 4b\left(-\dfrac{2(a-bp)}{Q^3}((p-c+\pi)\sigma_{D_L} Gu(k) + S_r) + \dfrac{2(a-bp)(3\sigma_{D_L} Gu(k) - Q)}{nQ^4} S_s\right) \\[2ex] \times (Q - \sigma_{D_L} Gu(k)) \dfrac{(a-bp)(Q - \sigma_{D_L} Gu(k))S_s}{n^3 Q^3} \end{pmatrix}$$

$$+ 2b \left(1 - \frac{\sigma_{D_L} Gu(k)}{Q}\right)\left(\frac{(a-bp)(2\sigma_{D_L} Gu(k) - Q)S_s}{n^2 Q^3} - \frac{1}{2} h_s p \left(1 - \frac{(a-bp)}{R}\right)\right)^2$$

$$+ \begin{pmatrix} \left(-\dfrac{2(a-bp)}{Q^3}((p-c+\pi)\sigma_{D_L} Gu(k) + S_r) + \dfrac{2(a-bp)(3\sigma_{D_L} Gu(k) - Q)}{nQ^4} S_s\right) \\[2ex] \left(\dfrac{b(Q - \sigma_{D_L} Gu(k))S_s}{n^2 Q^2} + \dfrac{1}{2} \dfrac{bh_s(Q - \sigma_{D_L} Gu(k))}{R}\right)^2 \end{pmatrix}$$

$$+ \begin{pmatrix} \left(\dfrac{(a - bp - b(p-c))\sigma_{D_L} Gu(k) - b(S_r + \pi\sigma_{D_L} Gu(k))}{Q^2} + \dfrac{b(2\sigma_{D_L} Gu(k) - Q)S_s}{nQ^3} - \dfrac{1}{2} h_s b(n-2)\right)^2 \\[2ex] \times \dfrac{2(a-bp)(Q - \sigma_{D_L} Gu(k))S_s}{n^3 Q^2} \end{pmatrix}$$

$$< \begin{pmatrix} 2\left(\dfrac{(a - bp - b(p-c))\sigma_{D_L} Gu(k) - b(S_r + \pi\sigma_{D_L} Gu(k))}{Q^2} + \dfrac{b(2\sigma_{D_L} Gu(k) - Q)S_s}{nQ^3} - \dfrac{1}{2} h_s b(n-2)\right) \\[2ex] \left(\left(\dfrac{(a-bp)(2\sigma_{D_L} Gu(k) - Q)S_s}{n^2 Q^3} - \dfrac{1}{2} h_s p \left(1 - \dfrac{(a-bp)}{R}\right)\right)\right)\left(\dfrac{b(Q - \sigma_{D_L} Gu(k))S_s}{n^2 Q^2} + \dfrac{1}{2} \dfrac{bh_s(Q - \sigma_{D_L} Gu(k))}{R}\right) \end{pmatrix} \qquad (23)$$

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These conditions are tested numerically and observed that it would be satisfied for reasonable parameter values. Then, by satisfying conditions 21, 22 and 23 Hessian matrix of the SC expected annual profit function is negative definite.

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