A Two Moment RC Delay Metric for Performance Optimization

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Abstract

For performance optimization tasks such as floorplanning, placement, buffer insertion, wire sizing, and global routing, the Elmore RC delay metric [3] remains popular due to its simple closed form expression, fast computation speed, and fidelity with respect to simulation. More accurate delay computation methods are typically either CPU intensive or difficult to implement. To bridge this gap between accuracy and simplicity, we propose the D2M RC delay metric, which is virtually as simple and fast as the Elmore metric but is significantly more accurate. The new metric is theoretically bounded above by the Elmore delay, yet it rarely is more than a few percent below the actual delay. Consequently, the metric behaves like the Elmore metric in that it generally overestimates delay, but with consistently less error. Further, the metric is extremely accurate at the far end of RC lines.

1. Introduction

As process technology scales to the ultra deep sub-micron regime, interconnect delays increasingly dominate gate delays. Thus, physical design tools such as floorplanning, placement, and routing are becoming more "timing-driven". To be effective, such a tool must be able to efficiently compute interconnect delay since millions of delay calculations may be required to optimize a design. Moment matching via asymptotic waveform evaluation (AWE) [14] is very accurate but too computationally expensive to use within a tight optimization loop. Two-pole variants (e.g., [5][16]) of AWE are considerably faster, but still may be too expensive. Closed form delay equations are certainly preferable since they are efficient and easy to implement, as long as accuracy is sufficient. Both Pileggi [13] and Cong *et al.* [2] have recently surveyed timing metrics for RC trees.

The Elmore delay metric [3], or the first moment of the impulse response, is the most widely applied interconnect delay metric. It is also the simplest metric that still captures some amount of metal resistance effects [13]. Rubenstein *et al.* [15] proved that RC trees have monotonic responses and derived best and worst case bounds on the step response waveform. Gupta *et al.* [4] showed that the Elmore metric provides an upper bound on delay for any input waveform.

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The Elmore delay is known to be extremely inaccurate at times because it ignores the resistive shielding of downstream capacitance. For example, in the RC network shown in Figure 1, the Elmore delay to capacitor C_1 is independent of the resistors R_2, R_3, R_4, and R_5. The higher the values of these resistors, the more downstream capacitance is shielded, i.e., the larger the error is for the Elmore approximation. One can actually choose values for the R_i and C_i elements to cause arbitrarily large error. For real deep sub-micron technologies, the Elmore delay can result in errors of several hundred percent [13]. Errors from the Elmore delay are generally much more pronounced for *near-end* nodes (nodes relatively close to the driving source) than for *far-end* nodes (nodes relatively far from the source) since resistive shielding is less prominent for far-end nodes.

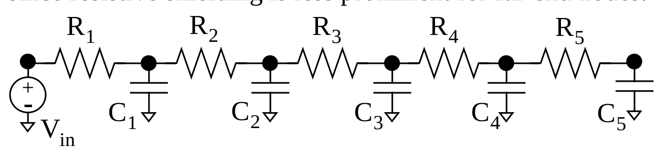

Figure 1 A simple RC tree network.

To achieve greater accuracy than the Elmore delay can provide, additional moments of the impulse response can be employed. Moment matching does not directly produce a delay approximation, but rather a reduced order response which can be solved via nonlinear iterations. These iterations can dominate the runtime of the entire delay computation method [13]. Thus, several works have sought to circumvent iterations by proposing delay approximations metrics that are direct functions of the circuit moments.

Kahng and Muddu [6][8] proposed three delay metrics. The first is obtained by computing two poles and residues from the first two circuit moments, then throwing out the less dominant pole. The second is generated by using the transfer function generated by these two poles and matching it to a 3 dB frequency. The final metric adds the first moment to the standard deviation of the impulse response; however, this metric is better suited for highly inductive transmission lines than for RC trees. Indeed, the metric is always larger than the Elmore delay, which already is an upper bound for RC trees. This metric was also proposed independently in [10].

Tutuianu *et al.* [16] showed how to compute two poles with guaranteed stability from the first three circuit moments. More recently, Kay and Pileggi [9] proposed PRIMO which fits the moments of the impulse response to probability density functions. First, the parameters to the probability density function must be computed from the moments, and then a single table lookup operation is all that is required to produce the delay. Lin *et al*. [11] later proposed the h-gamma metric which subsumes PRIMO by avoiding time-shifting the distribution functions and matching moments to the circuit's homogenous response.

The primary contribution of this work is a new delay metric for RC trees, namely,

$$D2M = \frac{m_1^2}{\sqrt{m_2}}\ln 2 \tag{1}$$

where m_1 and m_2 are the first two moments of the impulse response. D2M stands for "delay with 2 moments". The metric has several advantages:

- It is simpler than previously proposed higher order delay metrics [6][8][9][11][16], making it more amenable for optimization. Its simple form may be suitable for finding optimal algorithms for buffer insertion and wire sizing, which have been discovered for the Elmore delay [2].
- It is significantly more accurate than the closed form metrics proposed in [6][8][16]. In particular, the metric shows remarkable accuracy at the far end.
- It is provably less than the Elmore delay, yet we observe that it typically overestimates delay. Consequently, the metric behaves similarly to the Elmore delay but with significantly reduced error.
- Although not as accurate as the h-gamma metric [11], the new metric avoids the need to carefully construct a 2-dimensional table model. Also, at the near end, h-gamma can severely underestimate delay while our metric tends to overestimate delay.

The remainder of the paper is as follows. Section 2 presents background material and notation, Section 3 explains the derivation and properties of the new delay metric, Section 4 presents experimental results, and we conclude in Section 5.

2. Background

Assume that we are given an RC tree with nodes $\{v_0, \ldots, v_N\}$ where v_0 is the source. Let C_i be the capacitance at node v_i for $0 < i \le N$, and let R_{ki} be the total resistance of the portion of the unique path from v_0 to v_i that overlaps with the unique path from v_0 to v_k. The Elmore delay to node v_i is given by

$$ED_i = \sum_{k=1}^{N} R_{ki}C_k . \tag{2}$$

From the formula, one can see that two tree traversals are sufficient to compute the Elmore delay.

Let $m_j^{(i)}$ be the j^{th} moment of the impulse response for node v_i. We use m_j to denote this value for a generic node. By definition, $m_0 = 1$ for every node [4]. The j^{th} moment for node v_i can be recursively expressed as (for $j \ge 1$)

$$m_j^{(i)} = -\sum_{k=1}^{N} R_{ki}C_k m_{j-1}^{(k)} . \tag{3}$$

Observe that Equations (2) and (3) are equivalent for $j = 1$, except for the sign change. Thus, the Elmore delay is the absolute value of the first moment. Observe from Equation (3) that additional moments can easily be computed recursively via additional tree traversals. Further, one can express any moment directly as a closed form formula in terms of R_{ki} and C_k values.

Moment matching [14] approximates the transfer function by a reduced set of q approximate poles and residues:

$$\hat{H}(s) = \frac{k_1'}{s-p_1} + \frac{k_2'}{s-p_2} + \ldots + \frac{k_q'}{s-p_q} \tag{4}$$

where $k_1', \ldots, k_q'$ are the transfer function *residues* that correspond to the *poles* $0 > p_1 \ge p_2 \ge \ldots \ge p_q$. Let $V(s)$ be the Laplace transform of the voltage response under a step input, i.e., $V(s) = \hat{H}(s)U(s) = \hat{H}(s)/s$. The time domain response $v(t)$ for $V(s)$ can be written as:

$$v(t) = 1 + \frac{k_1'}{p_1}e^{p_1 t} + \frac{k_2'}{p_2}e^{p_2 t} + \ldots + \frac{k_q'}{p_q}e^{p_q t} \tag{5}$$

Let $k_i = -k_i'/p_i$ for $1 \le i \le q$. For the remainder of the discussion, we refer to these k_i coefficients as the residues for mathematical simplicity. Equation (5) can now be rewritten as:

$$v(t) = 1 - k_1 e^{p_1 t} - k_2 e^{p_2 t} - \ldots - k_q e^{p_q t} \tag{6}$$

For a q-pole approximation, there are q unknown poles and q unknown residues. The following system of $2q$ equations determine these values.

$$\left(\sum_{i=1}^{q} k_i = m_0\right), \left(\sum_{i=1}^{q} \frac{k_i}{p_i} = m_1\right), \left(\sum_{i=1}^{q} \frac{k_i}{p_i^2} = m_2\right), \ldots, \left(\sum_{i=1}^{q} \frac{k_i}{p_i^{2q-1}} = m_{2q-1}\right) \tag{7}$$

If $p_1 \gg p_2$, then p_1 is called the *dominant pole*. When p_1 is dominant, then Equation (6) can be approximated by

$$v(t) = 1 - k_1 e^{p_1 t} \tag{8}$$

At the 50% delay point, where $v(t) = 0.5$, one can solve Equation (8) explicitly for the delay t_D to yield the *single-pole delay approximation:*

$$t_D = -\frac{1}{p_1}\ln(2k_1). \tag{9}$$

Solving the system of equations in (7) for $q = 1$ yields $k_1 = 1$ and $p_1 = 1/m_1$. Thus, Equation (9) can be rewritten as $t_D = -m_1\ln(2)$, which is effectively the Elmore delay scaled by $\ln(2)$, or about 0.69. We call this metric the *scaled Elmore delay*. Pileggi [13] notes that shifting the Elmore approximation this way does not change the relative delay error problem, but merely shifts the error.

Two-pole approximations (e.g., [5][16]) are recognized to be considerably more accurate than the Elmore delay and are generally derived from matching the first four equations from the system in (7). Thus, using three moments is much more powerful than using a single moment. The solutions to the 2-pole system may be unstable, i.e., poles may be non-negative; hence, special care has to be taken to ensure stability. Further, Equation (6) cannot be solved explicitly for delay; instead, iterative techniques are required.

3. The D2M RC Delay Metric

As evidenced by the accuracy of two-pole approximations and the h-gamma metric [11], three moments provide sufficient information from which to derive fairly accurate delays. However, these methods have to do quite a bit of computation in order to derive the delay, and in the process lose the intuition contained within the Elmore delay.

Alternatively, we empirically sought a simple function of the first three moments that correlates well with actual delays. Our new D2M delay metric is given by

$$D2M = -m_1 \ln(2) \sqrt{\frac{m_1^2}{m_2}} = \frac{m_1^2}{\sqrt{m_2}} \ln 2 . \qquad (10)$$

The relative values of m_1^2 and m_2 are of particular interest. For a single RC circuit, $m_1^2 = m_2$ so the ratio of m_1^2 to m_2 is one. In general, for RC lines, we observed that this ratio was significantly less than one at the near end, and slightly greater than one at the far end. Similarly, we observed that the ratio of actual delay to the scaled Elmore delay is much less than one at the near end, and slightly greater than one at the far end. Consequently, adjusting the scaled Elmore delay by some function of m_1^2/m_2 should result in a metric that is more accurate than the Elmore delay. We attempted to integrate m_3 into the metric, but all attempts resulted in a reduction in accuracy at the far-end.

Theorem 1: The D2M metric is strictly less than the Elmore delay.

Proof: Gupta *et al.* [4] showed that the second central moment $\mu_2 = 2m_2 - m_1^2$ of the transfer function is always non-negative. This implies that $m_1^2/m_2 \le 2$. We have

$$D2M = -m_1 \ln(2) \sqrt{\frac{m_1^2}{m_2}} \le -m_1 \ln(2)\sqrt{2} \cong -0.9802 m_1 \qquad (11)$$

and since $-m_1 = ED$, this quantity is strictly less than the Elmore delay. □

For RC trees, we empirically observe that the new delay metric is never as close as 2% to the Elmore delay, which suggests a tighter upper bound exists. In [7], Muddu and Kahng observe that for an open-ended distributed RLC line that $4m_2 - 3m_1^2 \ge 0$ when the line has no inductance. If it can be shown that this bound holds for RC trees (which it appears to empirically), then the tighter upper bound would become $D2M \le -m_1 \ln(2)\sqrt{4/3} \cong 0.8003 ED$.

An alternative way to view this metric is as a single-pole approximation where the dominant pole p_1 is $-\sqrt{m_2}/m_1^2$ and the residue k_1 is one. Solving Equation (9) at the 50% delay point with these values yields our delay metric. Indeed, single-pole approximations are typically how closed formed metrics are derived (see Section 4.1) since there are many ways to estimate the dominant pole. By viewing D2M as a single-pole approximation, one can extend it to delay points other than the 50% delay. For example, the 10-90 output slew would be given by

$$\frac{m_1^2}{\sqrt{m_2}} \left(\ln 10 - \ln \frac{10}{9} \right) . \qquad (12)$$

We have not empirically tested the accuracy of our metric for other delay points.

To apply the delay metric to ramp inputs, the "step" moments can be suitably modified to yield "ramp" moments. The Laplace transform of the voltage response $V_R(s)$ under a ramp input can be written as

$$V_R(s) = H(s)R(s) = \sum_{i=0}^{\infty} m_i s^i \left(\frac{1}{s^2 \tau} (1 - e^{-s\tau}) \right) \qquad (13)$$

$$= \sum_{i=0}^{\infty} m_i s^i \frac{1}{s^2 \tau} \left(1 - \left(1 - s\tau + \frac{s^2 \tau^2}{2} - \frac{s^3 \tau^3}{6} + \dots \right) \right)$$

$$= \frac{1}{s} \left(1 + \left(m_1 - \frac{\tau}{2} \right) s + \left(\frac{\tau^2}{6} - \frac{m_1 \tau}{2} + m_2 \right) s^2 + \dots \right) = \frac{1}{s} \tilde{H}(s)$$

where τ is the input ramp time. One can now use the modified "ramp" moments $\tilde{m}_1 = m_1 - \tau/2$ and $\tilde{m}_2 = \tau^2/6 - m_1 \tau/2 + m_2$ instead of the "step" moments in the D2M metric (Equation (10)).

4. Experimental Results

We now compare the D2M metric to the Elmore delay and to four other higher-order closed form metrics, which we will call DM1, DM2, DM3, and DM4. All of these metrics were derived from a single-pole approximation (Equation (9)) for a different dominant pole and residue. We now explicitly define these metrics.

4.1 Previous Closed Form Delay Metrics

In [7], Kahng and Muddu illustrated that a two-pole approximation could be derived by using only two moments, instead of three, of the impulse response by adding the constraint that $v'(t = 0) = 0$. The resulting two poles and residues are

$$p_{1,2} = \frac{2}{m_1 \mp \sqrt{4m_2 - 3m_1^2}} \text{ and}$$

$$k_1 = -k_2 = -\frac{1}{\sqrt{4m_2 - 3m_1^2}} . \qquad (14)$$

Solving Equation (9) using the first pole and residue yields (what we call) the DM1 delay metric proposed in [8], namely,

$$\frac{1}{2}\left(-m_1 + \sqrt{4m_2 - 3m_1^2}\right) \ln \left(1 - \frac{m_1}{\sqrt{4m_2 - 3m_1^2}} \right) . \qquad (15)$$

In [6], Kahng and Muddu attempt to approximate these 2-poles with a single-pole by matching the transfer functions up to the 3 dB frequency. They propose using a new pole p_d which is given by

$$\frac{1}{p_d} = -\sqrt{\frac{1}{p_1^2} + \frac{1}{p_2^2}} = -\sqrt{2m_2 - m_1^2} \quad (16)$$

Using a residue of one in Equation (8), yields the second of Kahng and Muddu's delay metrics:

$$DM2 = \sqrt{2m_2 - m_1^2}\ln(2). \quad (17)$$

In [16], Tutuianu *et al.* proposed a "first order delay estimate" to be used as an initial guess to a Newton-Raphson iteration. The authors used the fact that the limit of m_j / m_{j+1} as j goes to infinity converges to the dominant pole to approximate p_1 by m_2 / m_3. Their approximation for a second pole is given by

$$p_2 = \frac{m_2}{m_3}\left|\frac{m_3(m_2 - m_1^2)}{m_1(m_1 m_3 - m_2^2)}\right|. \quad (18)$$

The values for the two corresponding residues are obtained by solving the first two equations of the system in (7). In particular, the residue expressions are

$$k_1 = \frac{(1 - m_1 p_2)p_1}{(p_1 - p_2)} \text{ and } k_2 = \frac{(1 - m_1 p_1)p_2}{(p_1 - p_2)}. \quad (19)$$

The delay estimate DM3 is thus given by substituting the dominant pole m_2 / m_3 and the above corresponding residue k_1 into Equation (9).

The actual delay metric proposed in [16] is obtained by running a single Newton-Raphson iteration for DM3. So the new "improved explicit approximation" is given by

$$DM4 = DM3 + \frac{(0.5 - k_1 e^{p_1 DM3} - k_2 e^{p_2 DM3})}{k_1 p_1 e^{p_1 DM3} + k_2 p_2 e^{p_2 DM3}}. \quad (20)$$

4.2 Single-Sink Comparisons

The first set of experiments seeks to analyze the difference between D2M and other proposed closed form metrics, DM1-4. We begin with a random 10 RC circuit connected in series, e.g., Figure 1 denotes a 5 RC circuit. Each resistor and capacitor was randomly chosen between 1-20 kΩ and 1-20 ff, respectively. These test cases are actually quite challenging for a delay metric since the nodes at the near end will have significant resistive shielding. An RC line also permits one to track accuracy trends from near-end to far-end nodes. Let node 1 be the node closest to the source, node 2 the second closest, etc.

To determine actual delays, we applied the AS/X electrical simulator from IBM [1]. We generated 100 random circuits and computed delay according to each metric and AS/X. Table 1 presents the average delay ratio of each metric to the AS/X over all 100 circuits. A value close to one shows excellent correspondence to AS/X; a value above one shows over-estimated delay, while a value below one shows under-estimated delay. We make several observations.

- DM3 is a poor approximation at the near-end since it returns large negative values. For these cases, we use zero as our initial guess for the DM4 iteration (Equation (20)). Even so, DM4 also gives negative delay results at the near-end. Both DM3 and DM4 show outstanding correlation with AS/X at the far-end.
- DM2 does not appear to be any better than the Elmore delay. It is significantly worse than Elmore for the first two nodes and it under-estimates delay at the far-end.
- Both DM1 and D2M are more accurate than the Elmore delay for every node, while neither significantly under-estimates the delay. D2M is more accurate than DM1 at both the near and far-ends. At the near end (node 1), D2M overestimates delay by a factor of 3.7, but DM1 overestimates delay by twice this value, and Elmore overestimates it by almost four times this value. At the far-end, the D2M delays correspond almost exactly with AS/X, while DM1 is off by roughly 2-4%.

RC Elt	Average Ratio of Delay Metric to AS/X Delays					
	D2M	Elmore	DM1	DM2	DM3	DM4
1	3.734	13.128	7.250	34.853	-298.232	-3.417
2	2.152	4.771	2.809	6.538	-8.189	-1.167
3	1.543	2.839	1.749	2.981	-0.283	-0.241
4	1.246	2.049	1.315	1.804	0.753	3.140
5	1.103	1.684	1.124	1.303	0.928	1.214
6	1.033	1.495	1.037	1.043	0.962	1.058
7	1.006	1.398	1.011	0.896	1.002	1.010
8	0.999	1.354	1.014	0.820	1.000	0.999
9	0.999	1.332	1.029	0.778	1.000	0.999
10	1.000	1.323	1.041	0.757	1.000	0.999

Table 1 Ratio of each delay metric to AS/X averaged over 100 random 10 RC circuits.

RC Elt	Minimum Ratio			Maximum Ratio			Standard Dev.		
	D2M	Elm	DM1	D2M	Elm	DM1	D2M	Elm	DM1
1	0.996	3.287	1.990	26.29	121.8	65.56	3.45	16.4	8.79
2	1.217	2.008	1.283	6.664	19.28	10.86	0.78	2.28	1.27
3	1.123	1.799	1.165	2.579	5.394	3.103	0.29	0.77	0.43
4	1.021	1.483	1.022	1.911	3.918	2.341	0.17	0.42	0.23
5	1.001	1.386	1.004	1.590	2.836	1.760	0.10	0.23	0.12
6	0.992	1.359	0.998	1.207	1.963	1.262	0.04	0.10	0.04
7	0.991	1.330	1.002	1.070	1.587	1.074	0.01	0.05	0.01
8	0.990	1.293	1.003	1.008	1.416	1.051	0.00	0.02	0.01
9	0.984	1.287	1.007	1.003	1.377	1.082	0.00	0.02	0.02
10	0.985	1.252	1.014	1.003	1.362	1.159	0.00	0.02	0.02

Table 2 Minimum, maximum, and standard deviation of the ratio of each metric to AS/X for 100 random 10 RC circuits.

The remaining experiments do not consider DM2, DM3, and DM4 since they have regions where they are worse than the Elmore delay. Table 2 takes a closer look at the results for D2M, Elmore (using Elm to abbreviate Elmore), and DM1. For the 100 circuits, the minimum and maximum ratios of each metric to AS/X are shown, along with the standard deviation of these ratios. We observe that:

- At node 1, for at least one RC circuit, the Elmore delay is a factor of over 100 worse than AS/X. DM1 is about half as bad, while D2M is about 26 times worse. Indeed, this RC circuit is a particularly difficult instance.
- Elmore delay always significantly overestimates delay at node 1, with the best case being a factor of 3.3 more than AS/X. DM1 is always at least twice AS/X, but this behavior is not seen for D2M.
- D2M performs exceptionally at the far end. For nodes 8-10, D2M is never more than 1% above or 1.6% below AS/X over all 100 runs. Further, D2M is more stable than the other metrics as can be seen by its uniformly lower standard deviations.

# RC	S	Average Ratio			Minimum Ratio			Maximum Ratio		
		D2M	Elm	DM1	D2M	Elm	DM1	D2M	Elm	DM1
10	2	1.058	1.522	1.096	0.983	1.268	0.999	2.134	4.227	2.546
	3	1.054	1.536	1.087	0.977	1.266	0.991	1.986	5.282	2.996
	4	1.060	1.554	1.090	0.978	1.280	0.988	4.268	9.723	5.684
	5	1.046	1.519	1.066	0.977	1.274	0.986	2.187	4.174	2.538
20	2	1.070	1.564	1.127	0.983	1.264	0.987	2.471	5.500	3.229
	4	1.106	1.686	1.167	0.977	1.270	0.988	3.435	8.594	4.944
	7	1.079	1.618	1.119	0.979	1.275	0.985	2.818	6.745	3.623
	10	1.079	1.618	1.114	0.979	1.278	0.985	4.193	10.17	5.878
50	5	1.124	1.788	1.217	0.881	1.273	0.973	4.123	10.86	6.030
	10	1.120	1.749	1.188	0.944	1.253	0.979	4.695	11.34	6.562
	15	1.102	1.694	1.153	0.973	1.259	0.979	9.261	24.31	13.89
	20	1.090	1.667	1.137	0.942	1.254	0.980	9.730	22.41	13.08
100	10	1.113	1.777	1.200	0.840	1.252	0.938	5.425	16.15	9.069
	20	1.111	1.760	1.185	0.864	1.260	0.967	6.795	19.55	10.90
	30	1.090	1.676	1.139	0.967	1.262	0.968	12.88	28.64	16.82
	40	1.076	1.643	1.119	0.963	1.245	0.967	8.844	23.02	13.16

Table 3 Ratio of delays for DM1, Elmore, and D2M to AS/X for RC trees. Each row in the table summarizes results for 100 randomly generated tree topologies.

4.3 Multi-Sink Comparisons

We now compare DM1 and the Elmore delay to D2M for RC trees. We wrote a random RC tree generator that takes the number of RC elements (#RC) and number of sinks (S) as input and generates a random tree topology with random values for the resistors and capacitors. For each tree we measured the ratio of each delay metric to AS/X at each sink in the tree. The average, minimum and maximum ratios to AS/X are presented in Table 3. Each row represents the results for 100 random trees. The D2M uniformly dominates Elmore and DM1 in terms of accuracy in the average case. The maximum error is also uniformly better than the other metrics. Not surprisingly, the overall accuracy to tree sinks is less than for far end sinks in the 2-pin case because sinks in trees can behave like near-end nodes. Although not shown, D2M is also uniformly dominant in standard deviation, or stability of results.

However, for 100 RC's and 10 sinks, we observe that D2M can be as much as 16% below AS/X delays, and is 6-14% below AS/X in other cases with more than 50 RC's. By taking a closer looking at these handful of instances (18 out of 18,700 sinks on 9 out of 1600 nets have DM2 delays that are more than 6% below AS/X), we do not observe a discernible pattern to the characteristics of the sinks that cause this underestimation.

4.4 Comparisons to h-gamma

Our final experiments compare D2M to the h-gamma metric [11]. Rather than implement our own version of h-gamma (since the quality of the results returned by h-gamma depends on how well crafted the 2-dimensional lookup table is), Tao Lin [12] ran h-gamma on four RC circuits that we generated and reported back the results. The first was a 7 node, 2-sink RC tree, shown in Figure 2. The other three circuits were chosen from the set of 100 circuits with 10 RC's reported in Section 4.2. We call them A, B, and C where A is a fairly typical random circuit, B has more resistive shielding at the near end, and C has very high resistive shielding at the near end. We also compare with the Elmore delay and DM1 to provide frames of reference.

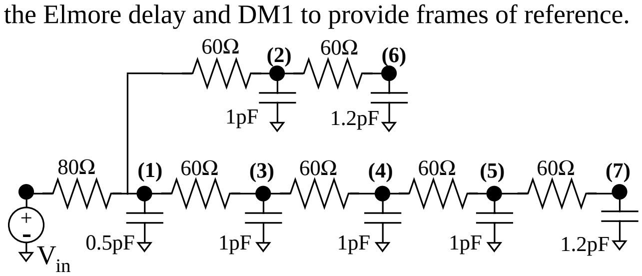

Figure 2 An RC Tree example.

Results for the RC tree are shown in Table 4 (using h-Γ to abbreviate h-gamma). The first five data columns give the computed delays in picoseconds, and the last four columns give the ratio of the metric compared to AS/X. For this example, h-gamma is clearly better at computing near-end delays than the closed form solutions, and its results at the far end are just as good as the other metrics. Notice though that h-gamma can underestimate delay somewhat, by about 5% at nodes 2 and 6. This trend is more noticeable for the circuits A, B, and C.

Table 5 presents delays (ps) at each node for the circuits A, B, and C for D2M, h-gamma, and AS/X. Observe that for circuit A, h-gamma delays are very close to AS/X delays; however, for circuits B and C, one can see that h-gamma can significantly underestimate delay at the near end. In particular, for node 1 in circuit B, h-gamma underestimates delay by 75%, and at node 2 in circuit C, it underestimates delay by 26%. Overall, h-gamma is clearly the more accurate metric; however, D2M is actually more accurate at the far-end (nodes 8-10) for all three circuits.

RC Elt	Reported Delay					Ratio to AS/X delay			
	D2M	Elm	DM1	h-Γ	AS/X	D2M	Elm	DM1	h-Γ
1	299	552	339	194	197	1.517	2.802	1.720	0.984
2	420	684	440	355	374	1.122	1.828	1.176	0.949
3	514	804	527	486	477	1.077	1.685	1.104	1.018
4	696	996	696	701	701	0.992	1.420	0.992	1.000
5	830	1128	839	840	845	0.982	1.334	0.992	0.994
6	492	756	501	431	452	1.088	1.672	1.108	0.953
7	905	1200	936	912	919	0.984	1.305	1.018	0.992

Table 4 Delay metric comparison for the RC tree in Figure 2.

RC Elt	Circuit A			Circuit B			Circuit C		
	D2M	h-Γ	ASX	D2M	h-Γ	ASX	D2M	h-Γ	ASX
1	399	74	64	22	3	12	12	0	2
2	1440	714	750	578	226	230	622	144	200
3	2307	1916	1789	1097	929	869	1250	918	907
4	2605	2304	2162	1322	1216	1158	1473	1152	1206
5	2979	2772	2648	1822	1802	1792	2146	2057	2005
6	3960	3927	3910	2028	2017	2019	2637	2619	1611
7	4617	4614	4636	2080	2072	2077	2892	2885	2893
8	4787	4807	4778	2327	2321	2337	3104	3102	3121
9	5070	5049	5083	2438	2426	2446	3322	3315	3342
10	5106	5082	5117	2500	2484	2505	3468	3453	3483

Table 5 Delays (ps) for D2M, h-gamma and AS/X on the circuits A, B, and C.

5. Conclusions and Future Work

We have proposed D2M, a delay metric for RC trees that is a simple function of two moments of the impulse response. Our metric typically behaves as an upper bound to the real delay, but the magnitudes of the error are significantly less than for the Elmore delay. D2M is also more accurate than other delay metrics that can be written as functions of the first few moments, though is not as accurate as h-gamma. The potential applications for the new metric are widespread. Applications such as timing-driven placement, interconnect synthesis, and global routing could benefit significantly by using D2M instead of Elmore delay within an optimization loop.

There are still some questions about D2M that we wish to answer in future work. We would like to derive the metric theoretically and prove the tighter upper bound $D2M \le 0.8003ED$. We would also like to generalize the metric to more than two moments. This could allow the user to trade off (relatively inexpensive) moment computations for accuracy.

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