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Thomas O'Brien

Institutions: Bowling Green State University

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A TWO PARAMETER FAMILY OF PENSION CONTRIBUTION FUNCTIONS AND STOCHASTIC OPTIMIZATION

BY

THOMAS O'BRIEN

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INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
UNIVERSITY OF MINNESOTA
514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455

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A TWO PARAMETER FAMILY OF PENSION CONTRIBUTION FUNCTIONS AND STOCHASTIC OPTIMIZATION

Thomas O'Brien

In a previous article the author has suggested a linear function of $A(t)$ (present value of future benefits) and $F(t)$ (fund) as pension contribution function in place of the form given in Trowbridge (1963) which is a one-parameter family of funding methods. Here we provide some theoretical justification for such a method by showing that, in the simplified model of this paper, the optimal solution of a stochastic control problem yields, as contribution function, an affine function of $A(t)$ and $F(t)$.

Keywords: Pension funding dynamics, Stochastic Control.

1. INTRODUCTION

In Trowbridge (1963), the unfunded present value family of pension funding methods is defined by the contribution formula

$$C_t = (k + d)(V_t - F_{t-1})$$

where V_t is the present value of benefits at time t, F_{t-1} is the fund amount at time $t - 1$, d is the discount rate, and k is a positive number less than 1. By suitable choice of k this method can be used to achieve any level of funding from class I through class V as described in Trowbridge (1952). However, the convergence to the ultimate funding level may be quite slow. This is especially true for small values of k. In Gasiewski (1985), computer simulations are carried out illustrating the use of a 2-parameter funding family. The contribution family is of the form $C(t) = c_1A(t) - c_2F(t)$ with c_1 and c_2 positive. By varying the parameters c_1 and c_2 both the ultimate level of funding and the rate of convergence to this level are controlled.

In the second section of this paper we outline the dynamic programming approach in controlled diffusion problems. In the final section we model the pension problem as a controlled diffusion process with the contribution function as control and then solve the problem obtaining as contribution function an affine function of $A(t)$ and $F(t)$.

2. CONTROLLED DIFFUSION PROCESSES

We give a brief sketch of the technique to be used in section 3. For more details see Arnold (1974) and Fleming and Rishel (1975).

We consider a system described by a stochastic differential equation

$$dX_t = f(t,X_t,u(t,X_t))dt + G(t,X_t,u(t,X_t))dW_t \qquad (2.1)$$
$$X_{t_0} = c \qquad t \geq t_0$$

where X_t and $f(t,X_u)$ assume values in R^d, $G(t,x,u)$ is $d \times m$ matrix valued and W_t is an m-dimensional Wiener process. If f and G are continuous and satisfy growth and Lipschitz conditions on $[t_0,T]$ then 2.1 has a unique solution and the resulting stochastic process, X_t, is a d-dimensional diffusion process. The control function, u, takes values in some R^p. The infinitesimal generator of this process is the differential operator

$$L^u = \frac{\partial}{\partial s} + \sum_{i=1}^{d} f_i(s,x,u) \frac{\partial}{\partial x_i} + \frac{1}{2} \sum_{i=1}^{d} \sum_{j=1}^{d} (G(s,x,u)G(s,x,u)')_{ij} \frac{\partial^2}{\partial x_i \partial x_j} . \qquad (2.2)$$

The **costs** arising from the choice of control function u starting at time s in state X up to the instant T are given by

$$J(s,x,u) = E_{s,x}(\int_s^T k(r,X_t,u(r,X_r))dr + M(T,X_T)) \qquad (2.3)$$

Here $E_{s,x}$ means expectation conditioned on being in state X at the time s. The function k represents running costs and M represents costs associated with being at X_T at the final time T. In our appliction, M will be zero. We seek an <u>optimal control function</u>, that is a control u^* from among a set U of admissible control functions which minimizes the function 2.3. We will write

$$W(s,x) = \min_{u \in U} J(s,x,u).$$

It follows from Bellman's optimality principal (a control function is optimal on $[t_0,T]$ iff it is optimal on every subinterval of the form $[s,T]$ where $t_0 \leq s \leq T$) that $W(s,x)$ satisfies Bellman's equation

$$0 = \min_u (L^u W(s,X) + k(s,x,u)) \qquad t_0 \leq s \leq T \qquad (2.4)$$

with the end condition $W(T,X) = M(T,X)$. Thus the technique is to find the function $u^*(s,x,W)$ which minimizes $L^u W(s,x) + k(s,x,u)$ for each (s,x) in

$[t_0,T] \times R^d$. Substitution of this function u^* into the Bellman equation yields a partial differential equation

$$L^{u^*}W(s,x) + k(s,x,u^*) = 0 \qquad t_0 \leq s \leq T \tag{2.5}$$

with end condition $W(T,X) = M(T,X)$. The solution W is then inserted into the function u^* yielding the optimal control function which, slightly abusing notation, we write as

$$u^*(s,x) = u^*(s,x,W(s,x)).$$

3. OPTIMAL PENSION FUNDING

The genesis of this work may be found in the deterministic pension fund models of Bowers, Hickman, and Nesbitt (1976, 1979). In a previous paper the current author studied a stochastic pension fund model derived from the Bowers, Hickman, Nesbitt work. This effort also makes use of the basic framework of these earlier papers. The fund is for active plan members and the basic functions required for our model are:

$T(t)$	Annual rate of terminal funding normal cost for the plan at time t.
$A(t)$	Present value of future benefits at time t for the active members.
$F(t)$	The fund at time t.

In the above mentioned papers, population and salary were assumed to follow exponential growth paths. As a result, the terminal funding normal cost function had the form $T(t) = e^{\tau t}T_0$ where T_0 was the initial value. Here, in order to avoid later linearizations we will assume a linearly growing terminal funding normal cost. Thus $T(t) = (\tau t + 1)T_0 \quad t \geq 0$. It is assumed that all members enter the plan at age a and that all retirements occur at age r. Thus

$$A(t) = \int_a^r e^{-\delta_1(r-x)} T(t + r - x)dX = T_0\int_a^r e^{-\delta_1(r-x)} (\tau(t + r - x) + 1)dX$$

Here δ_1 is the fixed rate at which we discount for valuation purposes. Carrying out the integration and then differentiating with respect to t we obtain

$$\frac{dA(t)}{dt} = \frac{\tau}{\delta_1} T_0(1 - e^{-\delta_1(r-a)}) \tag{3.1}$$

Now let $X_t = (A(t),F(t))'$, the state vector of our system and let the contribution function (thought of as a control) be denoted $u(t,X_t)$. We then have

$$\frac{dF(t)}{dt} = u(t,X_t) + \delta F(t) - (\tau t + 1)T_0 \tag{3.2}$$

We next consider the growth rate τ and the fund earning rate δ as stochastic variables with $\tau = \tau_0 + B_1\xi_1$ and $\delta = \delta_0 + B_2\xi_2$ where ξ_1 and ξ_2 are independent white noise processes. Substituting these into 3.1 and 3.2 and combining the two into a single vector equation we obtain

$$dX_t = \begin{bmatrix} \alpha_1 \\ u(t,X_t) + \delta_0 F(t) - (\tau_0 t + 1)T_0 \end{bmatrix} dt + \begin{bmatrix} \beta_1 & 0 \\ -B_1T_0t & B_2F(T) \end{bmatrix} dW_t$$

where

$$\alpha_1 = \frac{\tau_0}{\delta_1} T_0(1 - e^{-\delta_1(r-a)})$$

$$\beta_1 = \frac{B_1}{\delta_1} T_0(1 - e^{-\delta_1(r-a)})$$

and W_t is a 2-dimensional Wiener process.

We now consider the form of the cost function to be minimized. The control function u represents a real cost. Further, we are aiming for a certain level of funding as measured by the fund ratio $\frac{F(t)}{A(t)}$. Thus suppose we want a fund ratio $\frac{F(t)}{A(t)} = \eta$. Our cost function will penalize deviation from this goal. Consider a finite horizon from initial time 0 to time T and take

$$J(s,x,u) = E_{s,y} \int_s^T e^{-\rho t}(u^2 + \beta(\eta A - F)^2)dt$$

Here, as in much of the computation below we delete the arguments of some of the functions involved in order to avoid cumbersome expressions. With $W(s,x) = \min_{u \varepsilon U} J(s,x,u)$ the Bellman equation is

$$0 = W_s + \min_{u \varepsilon U} [\alpha_1 W_A + (u + \delta_0 F - (\tau_0 s + 1)T_0)W_F + \frac{1}{2}\beta_1^2 W_{AA}$$
$$- B_1\beta_1 T_0 s W_{AF} + \frac{1}{2}(B_2^2F^2 + B_1^2T_0^2s^2)W_{FF} + e^{-\rho s}[u^2 + \beta(\eta A-F)^2]] \tag{3.3}$$

Elementary calculus yields the minimizing u,

$$u^* = \frac{-W_F e^{\rho s}}{2} \tag{3.4}$$

Substituting u^* into 3.3 we get the partial differential equation

$$0 = W_s + \alpha_1 W_A + (\frac{-W_F e^{\rho s}}{2} + \delta_0 F - (\tau_0 s + 1)T_0)W_F + \frac{1}{2}\beta_1^2 W_{AA}$$
$$- \beta_1 B_1 T_0 s W_{AF} + \frac{1}{2}(B_2^2F^2 + B_1^2T_0^2s^2)W_{FF} + e^{-\rho s}[\frac{W_F^2 e^{2\rho s}}{4} + \beta(\eta A - F)^2] \tag{3.5}$$

with boundary condition $W(T,X) \equiv 0$.

We try a solution of the form $X'QX + P'X + r(s)$ where

$$Q = \begin{bmatrix} q_1(s) & q_2(s) \\ q_2(s) & q_3(s) \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} p_1(s) \\ p_2(s) \end{bmatrix}$$

The function W and its needed partials are then:

$$W = q_1(s)A^2 + 2q_2(s)AF + q_3(s)F^2 + p_1(s)A + p_2(s)F + r(s)$$

$$W_s = \dot{q}_1(s)A^2 + 2\dot{q}(s)AF + \dot{q}_3(s)F^2 + \dot{p}_1(s)A + \dot{p}_2(s)F + \dot{r}(s)$$

$$W_A = 2(q_1(s)A + q_2(s)F) + p_1(s); \quad W_F = 2(q_2(s)A + q_3(s)F) + p_2(s)$$

$$W_{AA} = 2q_1(s) \; ; \quad W_{FF} = 2q_3(s) \; ; \quad W_{AF} = 2q_2(s)$$

Substituting these into 3.5 yields

$$\begin{aligned} 0 = {} & \dot{q}_1(s)A^2 + 2\dot{q}(s)AF + \dot{q}_3(s)F^2 + \dot{p}_1(s)A + \dot{p}_2(s)F + \dot{r}(s) \\ & + \alpha_1(2(q_1(s)A + q_2(s)F) + p_1(s)) - \frac{e^{\rho s}}{4}[2(q_2(s)A + q_3(s)F) + p_2(s)]^2 \\ & + (\delta_0 F - (\tau_0 s + 1)T_0)(2(q_2(s)A + q_3(s)F) + p_2(s)) + \beta_1^2 q_1(s) \\ & - 2B_1\beta_1 T_0 s q_2(s) + (B_2^2F^2 + B_1^2T_0^2s^2)q_3(s) + e^{-\rho s}\beta(\eta A - F)^2 \end{aligned}$$

Equating the coefficients of A^2, AF, F^2, A, F, and 1 to 0 yields the system of ordinary differential equations:

1) $\dot{q}_1(s) - e^{\rho s}q_2^2(s) + e^{-\rho s}\beta\eta^2 = 0$

2) $\dot{q}_2(s) - e^{\rho s}q_2(s)q_3(s) - e^{-\rho s}\beta\eta + \delta_0 q_2(s) = 0$

3) $\dot{q}_3(s) - e^{\rho s}q_3^2(s) + 2\delta_0 q_3(s) + B_2^2 q_3(s) + e^{-\rho s}\beta = 0$

4) $\dot{p}_1(s) + 2\alpha_1 q_1(s) - e^{\rho s}q_2(s)p_2(s) - 2(\tau_0 s + 1)T_0 q_2(s) = 0$

5) $\dot{p}_2(s) + 2\alpha_1 q_2(s) - e^{\rho s}q_3(s)p_2(s) + \delta_0 p_2(s) - 2(\tau_0 s + 1)T_0 q_3(s) = 0$

6) $\dot{r}(s) + \alpha_1 p_1(s) - \frac{e^{\rho s}}{4}p_2^2(s) - (\tau_0 s + 1)T_0 p_2(s) + \beta_1^2 q_1(s) -$

$$- 2B_1 T_0 \beta s q_2(s) + B_1^2 T_0^2 s^2 q_3(s) = 0$$

The boundary conditions are: $q_1(T) = q_2(T) = q_3(T) = p_1(T) = p_2(T) = r(T) = 0$.

Note that equation 3) involves only the function q_3. If we can solve it, then plugging the resulting function into equation 2) yields a first order linear equation which can be solved. We then can substitute the functions q_2 and q_3 into equation 5) obtaining a first order linear equation for p_2. Since $u^* = \frac{-W_F e^{\rho s}}{2}$ involves only the functions q_2, q_3 and p_2 we need not complete the solution of the system (though we observe that by the same method, q_1, p_1, and r may be obtained.

To solve 3), make the substitution $h(s) = [e^{\rho s}q_3(s)]^{-1}$. Equation 3) then becomes

$$-\dot{h}(s)\,\frac{e^{-\rho s}}{h^2(s)} - \frac{\rho e^{-\rho s}}{h(s)} - \frac{e^{-\rho s}}{h^2(s)} - \frac{2\delta_0 e^{-\rho s}}{h(s)} + \frac{B_2^2 e^{-\rho s}}{h(s)} + e^{-\rho s}\beta = 0.$$

Multiplying through by $-e^{\rho s}h^2(s)$ we obtain

$$\dot{h}(s) - \beta h^2(s) + (\rho - 2\delta_0 - B_2^2)h(s) + 1 = 0.$$

This is a separable equation and the quadratic polynomial $\beta h^2 - (\rho - 2\delta_0 - B_2^2)h - 1$ will have roots r_1 and r_2 with $r_1 < 0 < r_2$. This is so because β is positive and since ρ and δ_0 are both interest rates we will assume that $\rho < 2\delta_0$ and we then have the coefficient of h is positive. We carry out the integration and then substitute back to obtain q_3. The choice $-\beta T$ of constant of integration will satisfy the initial condition $q_3(T) = 0$. We finally have

$$q_3(s) = \frac{e^{-\rho s}(1 - e^{\beta(s-T)(r_2-r_1)})}{r_2 - r_1 e^{\beta(s-T)(r_2-r_1)}}$$

<u>Remark:</u> Since $r_2 > r_1$ and $0 < s < T$, $q_3(s)$ is positive.

We next solve for q_2 as described earlier subject to $q_2(T) = 0$. The solution may be written:

$$q_2(s) = e^{-\int_T^s (\delta_0 - e^{\rho t} q_3(t))dt} [\int_T^s e^{-\rho t} \beta n e^{\int_T^t (\delta_0 - e^{\rho u} q_3(u))du} dt]$$

Remark: Since the principal integrand in this expression is positive and we are integrating from right to left, q_2 is negative.

Continuing, we solve for p_2 in equation 5)

$$p_2(s)=e^{-\int_T^s (\delta_0 - e^{\rho t} q_3(t))dt} [\int_T^s [2(\tau_0 t+1)T_0 q_3(t) - 2\alpha_1 q_2(t)] e^{\int_T^t (\delta_0 - e^{\rho u} q_3(u))du} dt].$$

Remark: For the same reason as for q_2, p_2 is negative.

We have now found the optimal control function.

$$u^* = \frac{-W_F e^{\rho s}}{2} = e^{\rho s}\{-q_2(s)A_s - q_3(s)F_s - \frac{1}{2} p_2(s)\}$$

It is given as an affine function of the state variables of the system, that is of the present value of future benefits and the fund at time s. The coefficients of A_s and F_s are positive and negative respectively in agreement with the form of the contribution function used by Gasiewski.

Conclusion

It is clear that a two parameter family of funding methods offers greater flexibility than the unfunded present value family. It is interesting that such a function should occur as the solution to an optimal control problem where the contribution function is the control. Though by no means is this result definitive it would seem that at the very least stochastic control theory offers a valuable way of thinking about pension funding and may in the future, with further development, become a practical method for its accomplishment.

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