A Two Phase Approach for Minimal Diagnostic Test Set Generation[*]

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Abstract

We optimize the full-response diagnostic fault dictionary from a given test set. The smallest set of vectors is selected without loss of diagnostic resolution of the given test set. We give an integer linear program (ILP) formulation using a fault diagnostic table. The complexity of the ILP is made manageable by two innovations. First, we define generalized fault independence. This property identifies many fault pairs that are guaranteed to be distinguished, significantly reducing the number of ILP constraints. Second, we propose a two-phase ILP approach. An initial phase, which uses existing procedures, selects a minimal detection test set. In a final phase, additional tests are then selected for the undiagnosed faults using a new diagnostic ILP. The overall minimized test set may be only slightly longer than that obtained from a one-step ILP optimization, but has advantages of significantly reduced computation complexity and reduced test time. Benchmark results show potential for very small diagnostic test sets.

Keywords - Fault diagnosis, integer linear programming, generalized fault independence, fault dictionary, test minimization.

1. Introduction

The process of determining the cause of a failure in a chip is known as *failure analysis*. Failure analysis often leads to improvement in the design of the chip and/or the manufacturing process. *Fault diagnosis* is the first step in failure analysis, which by logical analysis gives a list of likely defect sites or regions. Basically, fault diagnosis narrows down the area of the chip on which physical examination needs to be done to locate defects.

Diagnosis algorithms are broadly classified into two types: *effect-cause fault diagnosis* and *cause-effect fault diagnosis*. As the name suggests the effect-cause algorithm directly examines the response of the failing chip and then derives the fault candidates [1] using path-tracing algorithms. The fault candidate here usually is a logical location or area of the chip.

On the other hand, the cause-effect algorithm starts with a particular fault model and compares the signature of the observed faulty behavior with the simulated signatures for each fault in the circuit. A *fault signature* or *syndrome* is a list of failing vectors and outputs at which errors are detected [4]. Cause-effect algorithms can be classified as either static in which fault simulation is done in advance and all fault signatures are stored in a fault dictionary, or dynamic where simulation is performed only as needed during the diagnosis process. A cause-effect algorithm is based on a fault model and real defects on the chip may not behave similar to the fault model used; the observed signature may not match with any of the simulated signatures. In such cases sophisticated techniques are used to select a set of signatures that best match the observed signature [4], [11].

Despite its overwhelming data requirements, the fault dictionary based diagnosis has been popular as it facilitates faster diagnosis by comparing the observed behavior with pre-computed signatures in the dictionary [4]. The most detailed form of fault dictionary that considers fault detection at multiple outputs of a circuit is a *full-response dictionary*. It consists of all output responses for each fault by each test. On the other hand, the most compact form of fault dictionary is a *pass-fail dictionary*, which stores a single pass or fail bit for a fault-vector pair, ignoring detections at separate outputs of the circuit. The disadvantage with a pass-fail dictionary is that since the failing output information is ignored, faults that fail same set of tests but at different outputs cannot be distinguished [12]. Thus pass-fail dictionaries are less effective in fault diagnosis.

There has been a lot of work done to reduce the size of the full-response dictionary [5], [12], [15]. Most of these techniques concentrate on reducing the size by managing the organization and encoding of the dictionary. Dictionary organization is the order and content of the information, and dictionary encoding is the data representation format in the dictionary. Very little work has been done on reducing the size of the dictionary by compaction of the diagnostic test set [8]. In this work we explore the idea of using a minimal test set for fault diagnosis.

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We give an integer linear program (ILP) formulation to minimize test sets for a full-response dictionary based diagnosis. The ILP solution is a test set with diagnostic characteristics identical to that of the original unoptimized test set. Having a smaller test set not only reduces the dictionary size, but also reduces the time for debugging the faulty chip. An ideal test set for diagnosis is one which distinguishes all faults. Thus during diagnostic test set minimization it should be ensured that the resulting test set consists of at least one vector to distinguish every pair of faults. Notice that the number of fault pairs is proportional to the square of the number of faults. This results in a very large number of constraints in the ILP. We define a new generalized fault independence relation to reduce the number of fault pairs to be considered. We then propose a two-phase method for generating a minimal diagnostic test set from any given test set. In the first phase we use existing ILP minimization techniques [17] to obtain a minimal detection test set and find the faults not diagnosed by that test set. In the second phase we use the diagnostic ILP to select a minimal set of vectors capable of diagnosing the undiagnosed faults from Phase-1. The resulting minimized test set combined with the minimal detection test set of Phase-1 serves as our complete diagnostic test set.

The rest of the paper is organized as follows. Section 2 gives the diagnostic ILP formulation and illustrates its complexity. Section 3 introduces a new generalized fault independence relation to reduce the number of constraints in the diagnostic ILP. Section 4 describes the two-phase method for generating a minimal diagnostic test set. Section 5 gives the results and Section 6 gives the conclusion.

2. ILP for Diagnostic Test Minimization

Integer linear programming (ILP) is an effective mathematical method for test optimization. It gives global optimization and has been used for both combinational and sequential circuits [6], [7] as well as for minimizing N-detect tests [9]. These ILP formulations use a fault detection table which contains information about faults detected by each vector. The fault detection table is obtained by fault simulation without fault dropping. Note that the information in a fault detection table is similar to that in a pass-fail dictionary.

2.1. Fault Diagnostic Table for Diagnostic ILP

The ILP formulation for minimizing test sets used for full-response dictionary based diagnosis requires a matrix representation that not only tells which tests detect which faults, but also at which outputs the discrepancies were observed for each fault-test pair. For this reason we define a new *fault diagnostic table*. We illustrate the construction of this table with the following example.

	t1		t2		t3		t4		t5	
	o1	o2	o1	o2	o1	o2	o1	o2	o1	o2
f1	1	0	1	0	1	0	1	0	0	0
f2	1	1	1	1	1	0	1	1	0	0
f3	1	1	1	1	1	0	0	0	0	0
f4	0	1	0	1	0	0	0	1	0	0
f5	0	0	0	0	0	1	0	0	1	1
f6	0	0	0	0	0	1	0	0	0	0
f7	0	0	0	0	0	1	0	0	0	1
f8	0	0	1	0	1	0	1	0	0	0

Figure 1. Full-response fault dictionary.

	t1	t2	t3	t4	t5
f1	1	1	1	1	0
f2	2	2	1	2	0
f3	2	2	1	0	0
f4	3	3	0	3	0
f5	0	0	2	0	1
f6	0	0	2	0	0
f7	0	0	2	0	2
f8	0	1	1	1	0

Figure 2. Fault diagnostic table.

Let us consider a circuit with 2 outputs, having 8 faults detected by 5 test vectors. A sample full response dictionary for this circuit is shown in the Figure 1. Here ‘0’ stands for pass and ‘1’ stands for fail.

We use integers to represent the output response for each test vector. As faults detected by different test vectors are already distinguished, there is no need to compare the corresponding output responses. Hence we assign indices for the failing output responses for each test vector. In the example, for test t1 the 3 different failing output responses (“10”, “11”, and “01”) are indexed by integers 1, 2 and 3, respectively, in the fault diagnostic table as shown in Figure 2. The largest integer needed to index an output response is *minimum* $\{ 2^{No.\ of\ primary\ outputs} - 1,$ *highest number of faults detected by any test vector*$\}$. However, it should be noted that output responses to a particular vector are likely to repeat across a fault set as faults in the same output cone can have identical output responses for a particular test. For this reason the largest integer needed to index an output response observed in our experiments was much smaller than the highest number of faults detected by any test vector.

2.2. Diagnostic ILP Formulation

Suppose a combinational circuit has K faults. We are given a vector set V of J vectors and we assign a [0, 1] integer variable v_j , j = 1, 2, . . . , J to each vector. The variables v_j have the following meaning: If v_j = 1, then vector j is included in the selected vector set. If v_j = 0, then vector j is discarded.

Without loss of generality, we assume that all K faults are detected by vector set V and are also distinguishable from each other. Our problem then is to find the smallest subset of these vectors that distinguish all fault pairs. We simulate the fault set and the vector set without dropping faults and the fault diagnostic table is constructed as explained in the previous section. In this table, an element $a_{kj} \geq 1$ only if fault k is detected by vector j. The diagnostic ILP problem is stated as follows:

$$\text{Minimize} \sum_{j=1}^{J} v_j \tag{1}$$

subject to,

$$\sum_{j=1}^{J} v_j \; a_{ij} \geq 1; \quad for \; i = 1, 2, ..., K \tag{2}$$

$$\sum_{j=1}^{J} v_j \; |a_{kj} - a_{pj}| \geq 1 \tag{3}$$

for, $k = 1, 2, .., K - 1$ and $p = k + 1, .., K$

$$v_j \in integer[0, 1], \quad j = 1, ..., J \tag{4}$$

The inequality set (2) consists of K constraints, called *detection constraints*, which ensure that every fault is detected by at least one vector. The inequality set (3) consists of $K(K - 1)/2$ constraints - one constraint for every fault pair. These are called the *diagnostic constraints*. A diagnostic constraint consists of vector variables corresponding to non-zero $|a_{kj} - a_{pj}|$, i.e., the vectors that produce different output responses for the k^{th} and p^{th} faults. It forces at least one of those vectors to be selected since the inequality is greater than or equal to 1. Thus the diagnostic constraint set insures that k^{th} fault is distinguished from the p^{th} fault by at least one vector in the selected vector set. Additionally, the provable ability of this ILP to find the optimum provided its execution is allowed to complete guarantees the smallest size test set. Note that the total number of constraints here is $K(K+1)/2$, which is proportional to the square of the number of faults.

3. Generalized Fault Independence

One clear disadvantage of the diagnostic ILP is that the number of constraints is a quadratic function of the number of faults. Thus, for large circuits the number of constraints would be unmanageable. To overcome this, we define a relation between a pair of faults which allows us to drop the diagnostic constraints in the ILP corresponding to many fault pairs. We have generalized the conventional *fault independence relation* given in the literature by considering the detection of faults at different primary outputs and relative to a vector set. Conventionally [3], a pair of faults is called *independent* if the faults are not detected by any common vector. This definition does not account for the detection of the faults at specific outputs. Also, it implies "absolute" independence, which is with respect to the exhaustive vector set. We generalize the definition of fault independence by saying that two faults detected by the same vector can still be called independent, provided the output responses of the two faults to that vector are different.

	t1	t2	t3	t4
f1	1	0	1	0
f2	1	1	0	0
f3	0	0	1	1

Figure 3. Fault detection table.

Table 1. Independence relation.

Fault pair	Independence relation	Reason
f1, f2	NO	Both faults detected by t1
f1, f3	NO	Both faults detected by t3
f2, f3	YES	No vector detects both faults

Definition: Generalized Fault Independence - Two faults detectable by a vector set V are said to be independent with respect to vector set V, if there is no single vector that detects both the faults and produces identical output responses.

The conventional independence can be viewed as a special case of the generalized independence, for a single output circuit and conditional to an exhaustive vector set.

Example: Consider a fault detection table with 3 faults and 4 test vectors as shown in Figure 3. The independence relation between every fault pair is given in Table 1. Now consider a fault diagnosis table for the same set of faults and vectors as shown in Figure 4. Recall that the fault diagnosis table takes into account the output responses for each fault-vector pair. It is constructed as explained in Section 2.1. The generalized independence relations for all pairs of faults are given in Table 2.

In the context of the diagnostic ILP, the generalized independence relation plays an important role in reducing the number of constraints to be used in the formulation. When two faults are independent, any vector that detects either of the faults will be a distinguishing vector for the two faults. Thus, in the constraint set (3), a constraint for an independent fault pair will have vector variables corresponding to all the vectors that detect any one or both faults. In the presence of detection constraints (2), which guarantee a test for every fault, a diagnostic constraint for an independent fault pair is redundant. Also, such a constraint will be covered by other diagnostic constraints corresponding to non-independent fault pairs containing a fault from the independent fault pair.

	t1	t2	t3	t4
f1	1	0	1	0
f2	2	1	0	0
f3	0	0	1	1

Figure 4. Fault diagnostic table.

Table 2. Generalized independence relation.

Fault pair	Generalized indep. relation	Reason
f1, f2	YES	Different output responses for t1 detecting both faults
f1, f3	NO	Identical output responses for t3 detecting both faults
f2, f3	YES	No vector detects both faults

Table 3 shows the reduction in the constraint set sizes by considering diagnostic independent faults for a 4 bit ALU and several ISCAS85 benchmark circuits. We observe that there is an order of magnitude reduction in the constraint set sizes due to the elimination of constraints corresponding to generalized independent faults. However, the constraint set sizes still are large and need to be reduced to manageable proportions.

4. Two-Phase Minimization

Given an unoptimized test set, we proceed as [16]:

Phase 1: *Use existing ILP minimization techniques [17] to obtain a minimal detection test set from the given unoptimized test set. Find the faults not diagnosed by the minimized test set.*

Phase 2: *Run the diagnostic ILP on the remaining unoptimized test set to obtain a minimal set of vectors to diagnose the undistinguished faults from Phase-1. The resulting minimized test set combined with the minimal detection test set of Phase-1 serves as a complete diagnostic test set.*

In the context of diagnostic ILP of Phase-2, the Phase-1 along with the generalized independence relation helps in reducing the number of constraints to manageable levels. This is because diagnostic constraints are now needed only for the undiagnosed fault pairs of Phase-1. Also, there will be a further reduction in the number of diagnostic constraints due to independent fault pairs that could be present. We can also drop the detection constraints as we have started with a minimal detection test set that detects all targeted faults.

There is an additional benefit of the two-phase approach [16]. For all good chips, testing can be stopped at the end of the Phase-1 detection test set, which is minimal. Only for bad chips whose number will depend on the yield, we need to apply the remaining tests for diagnosis.

Table 3. Constraint set sizes.

Circuit	No. of faults	Initial constraint set size	No. of generalized independent fault pairs	Final constraint set size
4 alu	227	25,651	22,577	3,074
c17	22	231	170	61
c432	520	125,751	111,589	14,162
c499	750	271,953	138,255	133,698
c880	942	392,941	344,180	48,761
c1908	1870	1,308,153	1,201,705	106,448

5. Results

In our experiments we have used the ATPG ATALANTA [13] and fault simulator HOPE [14]. We have used AMPL package for ILP formulation.

Table 4 gives the results of Phase-1 and Phase-2 of the two-phase minimization approach. First column lists the names of the ISCAS85 circuits. The second column gives the number of faults in the target fault list. These faults are equivalence collapsed single stuck-at faults, excluding those identified as redundant or aborted by the ATPG program. We have used the minimal detection test sets obtained using the primal-dual ILP algorithm [16], [17]. The primal-dual ILP algorithm creates unoptimized test sets, which essentially consist of N-detect tests, and then minimizes them to give the minimal detection test sets. The sizes of the unoptimized and minimized vector sets are given in columns 3 and 4 of the table. The column *Undiag. Faults* gives the number of faults not diagnosed by the minimal detection vectors. A fault whose syndrome is shared by other faults is said to be *undiagnosed*. The undiagnosed faults obtained in this phase are the target faults for Phase-2 of our algorithm. The next 3 columns give the results from Phase-2 in which diagnostic ILP is used to minimize the tests for the undistinguished fault pairs of Phase-1. The diagnostic ILP is run on the unoptimized test sets (excluding the minimal detection tests) of Phase-1. The next column gives the number of constraints generated during the ILP formulation. It can be seen that the constraint set size is very small even for the larger benchmark circuits like c7552 and c6288. The column *Minimized additional vectors* gives the result of the diagnostic ILP. These vectors combined with the minimal detection vectors of Phase-1 constitute the complete diagnostic test set. The last column gives the sizes of the complete diagnostic test sets obtained by the two-phase approach. Notice that these test sets are just a little bigger than the minimal detection test sets of Phase-1. Thus failed chips can be diagnosed very quickly as the detection tests would have already been applied during testing.

Table 5 gives the results and statistics of the fault

Table 4. Results of two-phase minimization.

Circuit	Total faults	Phase-1			Phase-2			Complete diagnostic test set
		Original unoptim. vectors	Minimal detection tests	Undiag. faults	No. of unoptim. vectors	No. of constraints	Minimized additional vectors	
4b ALU	227	270	12	43	258	30	6	18
c17	22	32	4	6	28	3	2	6
c432	520	2036	30	153	2006	101	21	51
c499	750	705	52	28	652	10	2	54
c880	942	1384	24	172	1358	41	7	33
c1355	1566	903	84	1172	1131	12	2	86
c1908	1870	1479	107	543	819	186	21	127
c2670	2630	4200	70	833	4058	383	51	121
c3540	3291	3969	95	761	3874	146	27	122
c5315	5291	1295	63	1185	1232	405	42	105
c6288	7710	361	16	2416	345	534	12	28
c7552	7419	4924	122	1966	4802	196	31	153

Table 5. Diagnosis with complete diagnostic test set.

1	2	3	4	5	6	7	8	9
Circuit	Total diagnostic vectors	No. of faults	Uniquely diagnosed faults	No. of CEFS	Undiagnosed faults (3 - 4)	No. of syndromes (4 + 5)	Maximum faults per syndrome	Diagnostic resolution (3 / 7)
4b ALU	18	227	227	0	0	227	1	1
c17	6	22	22	0	0	22	1	1
c432	51	520	488	16	32	504	2	1.032
c499	54	750	726	12	24	738	2	1.016
c880	33	942	832	55	110	887	2	1.062
c1355	86	1566	397	532	1169	929	3	1.686
c1908	127	1870	1380	238	490	1618	8	1.156
c2670	121	2630	2027	263	603	2290	11	1.149
c3540	122	3291	2720	313	571	3033	8	1.085
c5315	105	5291	4496	381	795	4877	4	1.085
c6288	28	7710	5690	1009	2020	6699	3	1.151
c7552	153	7419	5598	848	1821	6446	7	1.151

dictionary obtained by using the complete diagnostic test set. The *total diagnostic vectors* are the combined vector sets from Phases 1 and 2. Column 3 gives the number of faults in the target fault list. Column 4 gives the number of uniquely diagnosed faults. A fault is *uniquely diagnosed* if it has a unique syndrome [4]. Faults with identical syndromes are grouped into a single set called an equivalent fault set. Note that such an equivalent fault set is dependent on the vector set used for diagnosis, thus it is called a *Conditional Equivalent Fault Set (CEFS)*. The column, $No.\ of\ CEFS$ gives the number of such sets. There is one CEFS for every non-unique syndrome, consisting of the undiagnosed faults associated with that syndrome. Thus, the total number of syndromes listed in column 7 is the sum of the number of uniquely diagnosed faults and the number of CEFS. *Maximum faults per syndrome* is the largest number of faults associated with any syndrome. *Diagnostic resolution* (DR) has been defined [2] as the average number of faults per syndrome. It is obtained by dividing the total number of faults by the total number of syndromes. These two parameters quantify the effectiveness of diagnosis since DR indicates how well faults are distributed among all syndromes and the maximum faults per syndrome indicate the worst distribution among all syndromes.

The unoptimized test sets used in our experiments are essentially N-detect tests. It should be noted that using an unoptimized test set consisting of diagnostic ATPG vectors [18] will be more effective in achieving a good DR, as these vectors are generated for the sole purpose of distinguishing pairs of faults. Also, it is recognized that the complexity of ILP would be too high even for medium size circuits. This problem can be overcome by using reduced-complexity approximate solutions of ILP [10].

Table 6 gives a comparison between the two-phase minimization and another test compaction algorithm for pass-fail dictionary [8]. For both algorithms an initial unoptimized set of 1024 random vectors is used. The authors of [8] measure the diagnostic effectiveness of the compacted test set in terms of number of undiagnosed

Table 6. Two-phase minimization versus previous work [8].

Circuit	Pass-fail dictionary compaction [8]				Two-phase approach (this work)			
	Fault coverage %	Minim. vectors	Undist. fault pairs	CPU* s	Fault coverage %	Minim. Vectors	Undist. Fault pairs	CPU** s
c432	97.52	68	93	0.1	98.66	54	15	0.94
c499	-	-	-	-	98.95	54	12	0.39
c880	97.52	63	104	0.2	97.56	42	64	2.56
c1355	98.57	88	878	0.8	98.6	80	766	0.34
c1908	94.12	139	1208	2.1	95.69	101	399	0.49
c2670	84.4	79	1838	2.8	84.24	69	449	8.45
c3540	94.49	205	1585	10.6	94.52	135	590	17.26
c5315	98.83	188	1579	15.4	98.62	123	472	25.03
c6288	99.56	37	4491	1659	99.56	17	1013	337.89
c7552	91.97	198	4438	33.8	92.32	128	1289	18.57

*Pentium IV 2.6 GHz machine **SUN Fire 280R, 900 MHz Dual Core machine

fault pairs. The pass-fail dictionaries have inherently lower resolution than the full-response dictionaries. Thus, there may not be a one-to-one comparison between the two results. However, we still notice the compactness of the diagnostic test sets and the computing efficiency of the two-phase method.

6. Conclusion

We have presented an integer linear program (ILP) formulation for compaction of the test set used in full-response dictionary based fault diagnosis. The compaction is carried out without any compromise on the diagnostic resolution of the initial test set. The newly defined generalized independence, which identifies fault pairs that need not be distinguished, is very effective in reducing the number of constraints in the diagnostic ILP. The two-phase approach further improves the efficiency of the procedure. These diagnostic test sets are very small and lead to significant reductions in the fault dictionary size and diagnosis time. Also, the minimized fault dictionary can be further compacted by other compaction techniques that employ encoding of the data in the dictionary.

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