



A two-phase stochastic programming approach to biomass supply planning for combined heat and power plants

Guericke, Daniela; Blanco, Ignacio; Morales, Juan M.; Madsen, Henrik

Published in:
OR Spectrum

Link to article, DOI:
[10.1007/s00291-020-00593-x](https://doi.org/10.1007/s00291-020-00593-x)

Publication date:
2020

Document Version
Peer reviewed version

[Link back to DTU Orbit](#)

Citation (APA):
Guericke, D., Blanco, I., Morales, J. M., & Madsen, H. (2020). A two-phase stochastic programming approach to biomass supply planning for combined heat and power plants. *OR Spectrum*, 42, 863–900.
<https://doi.org/10.1007/s00291-020-00593-x>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

A two-phase stochastic programming approach to biomass supply planning for combined heat and power plants

Daniela Guericke · Ignacio Blanco ·
Juan M. Morales · Henrik Madsen

the date of receipt and acceptance should be inserted later

Abstract Due to the new carbon neutral policies, many district heating operators start operating their combined heat and power (CHP) plants using different types of biomass instead of fossil fuel. The contracts with the biomass suppliers are negotiated months in advance and involve many uncertainties from the energy producer's side. The demand for biomass is uncertain at that time, and heat demand and electricity prices vary drastically during the planning period. Furthermore, the optimal operation of combined heat and power plants has to consider the existing synergies between the power and heating systems. We propose a solution method using stochastic optimization to support the biomass supply planning for combined heat and power plants. Our two-phase approach determines mid-term decisions about biomass supply contracts as well as short-term decisions regarding the optimal production of the producer to ensure profitability and feasibility. We present results based on ten realistic test cases placed in two municipalities.

Keywords Mixed-integer programming · Stochastic programming · Combined heat and power plants · Biomass supply planning · Operational planning

D. Guericke, I. Blanco, H. Madsen
Technical University of Denmark, Department of Applied Mathematics and Computer Science, Richard Petersens Plads, 2800 Kgs. Lyngby, Denmark

Juan M. Morales
Málaga University, Department of Applied Mathematics, Escuela de Ingenierías Industriales, 29071 Málaga, Spain

D. Guericke
Phone: +45 4525 3428
E-mail: dngk@dtu.dk

1 Introduction

The integration of different energy systems is one step towards a fossil-free energy system, which many developed countries target today. By integrating different energy systems, such as heat and power, a higher share of volatile renewable energies, e.g., wind energy, can be used efficiently [21]. In areas with large district heating networks, one way to achieve this integration is using combined heat and power (CHP) plants that produce heat and power simultaneously. By co-optimizing the production of both, the efficiency of the system is increased while providing flexibility to the power grid and satisfying the heat demand in the district heating network. Due to the neutral carbon policies imposed by the authorities, a shift from traditional fuels to renewable resources is taking place. Denmark has a widespread use of district heating and CHP plants and the government supports the use of biomass to produce heat and power. With subsidies and tax benefits, it has become profitable for large CHP plants to change from, e.g., coal or natural gas to biomass [10].

The use of biomass as fuel for CHP plants raises some challenges in the planning of the supply and in the operation of the plant. Many different types of biomass are used to produce heat and power [35] but the most common type of biomass used for large-scale CHP producers is wood pellets. Due to their high energy content, wood pellets facilitate a more efficient transport because smaller volumes are required. In addition, the low moisture content of wood pellets allows a better conservation of the product resulting in a larger storage capacity [31]. In combination with neutral carbon policy incentives for biomass, the wood pellet is becoming a candidate to substitute coal in CHP plants. The supply of wood pellets, or biomass in general, has some disadvantages compared to the supply of natural gas. First, natural gas prices have been dropping since 2008. Second, natural gas has a well-developed infrastructure that allows the producer to be directly connected to the gas network. On the contrary, biomass has to be transported for long distances and contracts with the supplier must be agreed beforehand for a long horizon (one to three years) involving a high degree of uncertainty at the time of negotiation because the final amount is unknown. With the maturing of the biomass markets also new types of contracts are emerging, e.g., to include some flexibility to the contracted amount in terms of options. Therefore, it is crucial for CHP operators to optimize their biomass contracts to be competitive with gas-fired plants.

In this work, we propose a solution approach based on stochastic programming [4] to optimize the yearly biomass contracting decisions for a CHP operator taking into account the uncertainty at the point of negotiation. Furthermore, the approach also determines the optimal operation of the plant to maximize profits and satisfy the heat demand throughout the year while taking the long-term biomass contract decisions into account.

2 Literature review

Several models for the optimal operation of CHP systems, where different aspects of the problem are highlighted, have been proposed. We refer for example to [3, 28, 14, 8, 29, 25]. These solution approaches determine the optimal production of both commodities (heat and power) at different levels of detail, but do not consider uncertainties and supply contracts for fuel explicitly.

Since then several approaches that apply stochastic programming to the operational planning were developed. [2] solve the operational scheduling for an industrial customer that owns an integrated system formed by CHP units, conventional power production and heat only units. The method uses electricity market sales and demand response programs to integrate the uncertainty caused by electricity prices and load. An optimal operation of a portfolio of different CHP systems in a district heating network is studied in [24]. The authors consider uncertain heat demand and electricity prices and show that the system profits from leveraging a thermal storage to handle this uncertainty. [19] present a multi-stage stochastic program for optimizing the operation of a gas-fired CHP plant and deriving bids for the German spot and balancing markets. The considered uncertainty are electricity prices. [12] propose a stochastic program including technical aspects of an extraction-condensing CHP plant to optimize the hourly operation under price and demand uncertainty. The authors use this model to determine bidding curves for the day-ahead market. In [13] this model is revisited with more focus on the joint production scheduling of two CHP plants. The operational planning problem in our work is similar to these two formulations, but extended with further characteristics regarding the biomass contracts deliveries and technical constraints.

The above mentioned publications assume instantaneous fuel supply and, therefore, do not consider fuel supply decisions. Another stream of publications explicitly focuses on the biomass supply chain planning for power generation considering processing of biomass, transport and logistics aspects. The OPTIMASS model for strategic and tactical biomass supply chain planning is presented in [11]. The formulation is based on a facility location planning problem that includes the processing of the biomass to determine the locations and capacities of facilities in the supply chain and the allocation of biomass sites to conversion facilities. The final usage of biomass in electricity production is not part of this study. [16] present a decision support system for a forest biomass supply chain deciding on the locations and capacities as well as assignment of biomass sources to power plants. [27] present a two-stage stochastic program with chance constraints for biomass supply chain planning under biomass availability uncertainty. The demand is based on markets and not single plants. [6] model the biomass-based energy production process, which includes deciding the location of plants as well as flow and conversion of commodities where one commodity is electricity. The model focuses on long-term decisions. [18] consider the supply chain connected to a biogas CHP plant and use a network flow model formulation. The model includes conversion to biogas and production with a CHP or heat boiler as well as transportation

costs. All publications mentioned above have a setting where the biomass is not traded based on contracts, because the biomass delivery is part of their own supply chain. However, this is often not the case when we consider e.g. wood chips or wood pellets.

Therefore, in this work the perspective of a CHP plant that receives biomass from third party suppliers is considered. Furthermore, we investigate the integration of long-term biomass supply decisions with the operational planning of the production. Similar settings have been studied in the following publications. [23] consider the fuel supply of gas for a consumer having a micro CHP and a heat boiler. Their multi-stage stochastic program decides on how much gas to buy on the spot or the monthly and weekly futures market, while electricity can be sold with similar market instruments. The model has a monthly planning horizon and abstracts from more detailed considerations regarding the operation of the system. In [32], a general overview of the benefits of using stochastic programming to incorporate the uncertainty involved in the biomass supply chain for a power producer on a tactical planning level is given. The authors formulate a one year planning problem considering the amount of biomass supply from different suppliers, storage and the expected power production on a monthly basis, i.e., the model abstracts from considering operational implications of the biomass supply. [30] address biomass supplier selection combining an analytic hierarchy process (AHP) with a chance constraint program to address stakeholders and uncertainties in this setting. Their focus is ensuring the quality of the biomass by blending biomass of different kinds and suppliers to fulfill the overall demand. The solution approach disregards the production level and delivery times. Finally, [7] use stochastic programming for optimal biomass contracting decisions in a long-term planning horizon. The model decides which biomass contracts should be settled with the suppliers. They model the contracts as well as the deliveries and production to provide a basis for this decision. Due to the planning horizon and short time periods, the model results in a computationally hard two-stage stochastic program. To the best of our knowledge, [7] are the first and only approach to handle biomass contracts and the production of the CHP in one solution approach.

Our work differs from [7] regarding the modeling of contracting decisions and the overall solution approach. Delivery times and amounts for contracts in [7] are fixed and the decision-maker can just decide which contracts are selected. On the contrary, our approach allows more flexibility to decide on the amount to be supplied and the delivery time. As a consequence, the exact delivery time and precise quantity are determined closer to the actual time of energy delivery. Furthermore, we reduce the computational complexity of the planning problem by presenting a two-phase approach. Finally, [7] only solve the problem for a planning horizon of year, i.e., they do not investigate and model the implications of the biomass contract selection on the operational planning of the CHP plant.

The main contributions of our work are the following:

1. We propose a two-phase solution approach that combines biomass contracting decisions with the optimal operation of the CHP plant. Therefore, it provides two models that can be used by an operator for long-term and operational planning, respectively. The first phase concentrates on the biomass contract selection at the beginning of the year considering production on a weekly less detailed basis and, therefore, reducing the complexity of the problem. The second phase optimizes the weekly operation of the system on a detailed hourly basis and takes the biomass contract decisions into account. The overall solution approach considers relevant technical requirements and resembles the planning process in practice.
2. Our modeling of biomass contracts offers a high degree of flexibility. Completely fixed contracts can be investigated as well as more flexible contracts regarding amounts of deliveries. We include the possibility to buy options on the biomass amount to be able to adjust the delivery quantity during the course of the year. This is a new model feature that is worth of investigation, at least from the standpoint of a CHP producer.
3. Our two-phase solution approach enables us to investigate the implications of biomass contract decisions of the long-term planning problem on the operational planning problem. This has not been analyzed before.
4. Furthermore, we use a rolling horizon approach to improve the results of our weekly operational planning, because it is important to take initial information from previous weeks into account and have a feasible transition. This also allows us to update the scenarios with new information.

The remainder of this publication is organized as follows. A detailed description of the planning problem is given in Section 3. Our solution approach and the respective model formulations are presented in Section 4. In Section 5, we analyze two realistic case studies each having five test cases. The section includes a description of the data, experimental setup and scenario generation. The numerical results are stated in Section 6. Finally, Section 7 summarizes our work and gives an outlook.

3 Problem description

In this section, we describe the biomass supply planning problem including used sets and parameters. For quick reference, we also provide an overview of parameters and sets in Table 1¹.

An overview of the components in the planning problem is given in Fig. 1. We consider a power and heat producer directly connected to a district heating network. The producer operates a CHP plant fueled by biomass and an auxiliary heat producing unit (e.g. gas boiler, electric boiler or heat pump). Both units can supply the district heating network directly but are also connected to a thermal storage, which can store hot water for later heat supply.

¹ In general, the notation follows the following pattern: sets are denoted by calligraphic capital letters, parameters are denoted by capital letters, uncertain parameters are marked by \sim and decision variables are denoted by small letters.

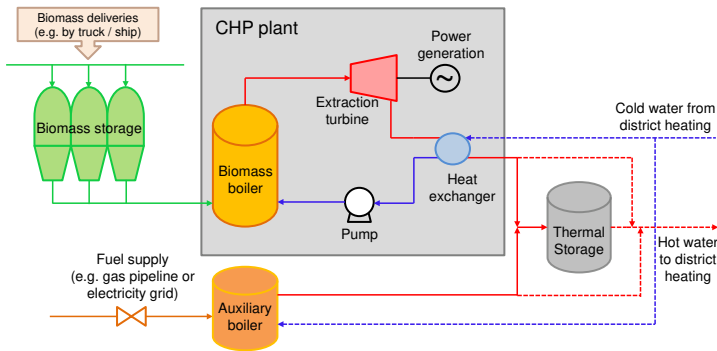


Fig. 1: Overview of components in the planning problem

The biomass delivered by suppliers according to the contracts is unloaded into the biomass storage and withdrawn from the storage for later use (i.e. no direct supply to the boiler). We assume that fuel for the additional heat-only unit is provided directly and instantaneously without storage and deliveries. This assumption stems from the setting of a gas boiler connected to the gas network or an electric boiler connected to the electricity grid.

In practice, biomass contracts are often agreed for a period of one year or more, defining the amount of biomass and a preliminary delivery schedule. The actual delivery time is revised during the course of the year. We model two different types of contracts, namely fixed and flexible. *Fixed* contracts are cheaper but offer no possibility to alter the delivery amount afterward. *Flexible* contracts are more expensive than fixed contracts, but the operator has the opportunity to buy an option of changing the amount. In the beginning of the year, in addition to the delivery amount, the options for up- and/or down-scaling the amount are settled, but the producer has to pay extra for those options. The possibility of buying options to change the biomass delivery amount is a new concept from industry that is studied in this paper. It provides the power producer with additional flexibility that can be beneficial especially in the long term when the actual demand is still uncertain. Also from the supplier's side this is interesting instrument, because it offers additional incomes from selling options while the amounts can be shifted between different customers. However, the supplier side is not the focus of this paper.

The input to our solution approach is a set of possible contracts $j \in \mathcal{J}$, a set of scenarios $\omega \in \Omega$ and a set of periods $t \in \mathcal{T} = \{1, \dots, |\mathcal{T}|\}$. The first planning period is always denoted with $t=1$, so that initial values are given values for period $t=0$. For example, the storage level of the biomass storage at the beginning of the planning horizon in scenario $\omega \in \Omega$ is denoted by $\delta_{0,\omega}$ and the initial storage level of the thermal storage by $s_{0,\omega}$. Each contract $j \in \mathcal{J}$ has a minimum and maximum amount per delivery $(\underline{B}_j, \overline{B}_j)$, a minimum and maximum number of deliveries per planning horizon $(\underline{N}_j, \overline{N}_j)$ and a minimum time between deliveries (F_j) . If contract $j \in \mathcal{J}$ offers up-scaling and down-

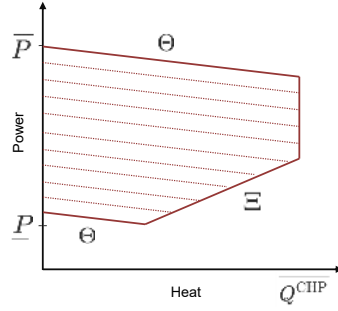


Fig. 2: Feasible production region of extraction-condensing unit in CHP plant

scaling options, the maximum limitations are given by O_j^+ and O_j^- (in percent deviation from the nominal amount), respectively. For fixed contracts these parameters are set to zero ($O_j^+ = O_j^- = 0$). The cost for the fixed, up-scaling and down-scaling amount are given by C_j^B , C_j^{B+} and C_j^{B-} , respectively. The cost are given per MWt (MW thermal), because the payment in practice is determined based on the energy content of the biomass in Gigajoule, which can be directly transformed to MWt. This means that the payment does not depend only on the amount in tonnes but also on the quality of the biomass, the so-called calorific value. Transportation costs are considered only indirectly, because the supplier has to cover these and can include them in the biomass cost per MWt. Furthermore, we assume that the supplier has the responsibility to deliver the contracted amount. As mentioned above, the biomass is delivered to the biomass storage, which is limited by a minimum safety and maximum storage level ($\underline{\Delta}_t, \bar{\Delta}$). The initial storage level $\delta_{0,\omega}$ is given for period 0 and the outflow per period is restricted to a maximum of Δ^F . To avoid congestion at the storage due to several deliveries at the same time, the time distance between deliveries must be at least Δ^W periods.

Biomass from the storage is used by the CHP plant to produce power and heat. The production of both is limited to the feasible production region of an extraction condensing unit depicted with the relevant parameters Θ and Ξ [36] in Figure 2. The efficiency of a conversion from biomass to power and heat is denoted by E_P^{CHP} and E_Q^{CHP} , respectively. From one hour to the next, the power production of the CHP can be ramped up or down but only in the limits of the parameters R^U and R^D . If the unit is started up or shut down it has to be in that state for at least M^U or M^D time periods. Starting up and shutting down is priced with C^{SU} and C^{SD} , respectively. The operation of the CHP itself has a cost of C^{CHP} . The power produced is sold on the electricity market and the profit depends on the market price $\tilde{L}_{t,\omega}^E$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$. In Denmark, the production of electricity by biomass is supported with an incentive of I , while the production of electricity with any fuel is taxed with T^{EP} . Thus, the overall cost $\tilde{L}_{t,\omega}$ €/MWe (MW electrical) is given by $\tilde{L}_{t,\omega} = T^{\text{EP}} - I - \tilde{L}_{t,\omega}^E$, where negative values of $\tilde{L}_{t,\omega}$ are profits.

Table 1: Sets and parameters (MWt = MW thermal, MWe = MW electrical, pu = per unit)

\mathcal{J}	Set of biomass contracts j
\mathcal{W}	Set of weeks w ($\mathcal{W} = \{1, \dots, \mathcal{W} \}$)
\mathcal{T}	Set of time periods t
\mathcal{T}_w	Set of time periods t in week w
Ω	Set of scenarios ω
Π_ω	Probability of scenario ω
$\tilde{D}_{t,\omega}$	Heat demand in period t in scenario ω [MWt/period]
$\tilde{L}_{t,\omega}$	Negative costs, i.e. profit, for selling electricity in period t in scenario ω [€/MWe]
$\tilde{C}_{t,\omega}^{\text{AUX}}$	Operational cost of auxiliary boiler in period t in scenario ω [€/MWt]
C^{CHP}	Operational cost of CHP plant [€/MWt]
C^{SU}	Start up cost for CHP [€/MWt]
C^{SD}	Shut down cost for CHP [€/MWt]
C^{I}	Inventory cost for biomass storage [€/MWt]
$C_{\text{AUX}}^{\text{O\&M}}$	Operational cost for auxiliary boiler [€/MWt]
T^{EP}	Tax for electricity production [€/MWe]
T^{AUX}	Tax for production with auxiliary boiler [€/MWt]
T^{CO_2}	CO ₂ emission tax [€/MWt]
C_j^{B}	Cost for biomass in contract j [€/MWt]
$C_j^{\text{B}+}$	Cost for up-scaling biomass amount in contract j [€/MWt]
$C_j^{\text{B}-}$	Cost for down-scaling biomass amount in contract j [€/MWt]
$\underline{B}_j, \overline{B}_j$	Minimum/maximum amount biomass offered per delivery by contract j [MWt]
$\underline{N}_j, \overline{N}_j$	Minimum/maximum number of deliveries offered by contract j
F_j	Frequency of deliveries in contract j [hours]
O_j^+, O_j^-	Maximum up-scaling/down-scaling option offered in contract j [pu]
Δ	Maximum biomass storage level [MWt]
$\underline{\Delta}_t$	Safety storage level of biomass in period t [MWt]
Δ^{F}	Maximum outflow from biomass storage per period [MWt/period]
Δ^{W}	Time distance between deliveries to biomass storage [periods]
$\underline{S}, \overline{S}$	Minimum/maximum thermal storage level [MWt]
S^{F}	Maximum in/outflow to/from thermal storage per period [MWt/period]
$\underline{P}, \overline{P}$	Minimum/maximum production of CHP plant per period [MWe/period]
$\overline{Q}^{\text{CHP}}$	Maximum heat production of CHP plant per period [MWt/period]
E_P^{CHP}	Electric efficiency of the CHP plant [pu]
E_Q^{CHP}	Heat efficiency of the CHP plant [pu]
E^{B}	Calorific value of the biomass [MWt/tonnes]
Θ	Fraction of power reduction
Ξ	Maximum heat to power ratio
M^U, M^D	Minimum up time / down time of CHP plant [periods]
R^U, R^D	Ramp-up and ramp-down limits of CHP plant [MWe/period]
$\overline{Q}^{\text{AUX}}$	Maximum heat production of auxiliary boiler per period [MWt/period]
E^{AUX}	Auxiliary boiler efficiency [pu]
P^{B}	Target percentage of heat produced by biomass [pu]
Φ^{Sto}	Penalty for excess of storage at the end of time horizon [€]
Φ^{Miss}	Penalty for missed heat demand [€]

The auxiliary boiler has a maximum capacity of $\overline{Q^{\text{AUX}}}$ with an efficiency of E^{AUX} . The operational costs $\tilde{C}_{t,\omega}^{\text{AUX}}$ of the boiler consists of several components and is dependent on the scenario ω due to the uncertain fuel (e.g. gas or electricity) spot price $\tilde{C}_{t,\omega}^{\text{F}}$. Further components are the operation and maintenance costs $C_{\text{AUX}}^{\text{O\&M}}$, taxes T^{AUX} and CO₂ taxes T^{CO_2} . Thus, the overall operational costs are given by $\tilde{C}_{t,\omega}^{\text{AUX}} = \tilde{C}_{t,\omega}^{\text{F}} + C_{\text{AUX}}^{\text{O\&M}} + T^{\text{AUX}} + T^{\text{CO}_2}$.

Both units can feed the thermal storage. In the beginning of the planning horizon (period 0), the heat storage has a given level of $s_{0,\omega}$ and the level has to be always between \underline{S} and \overline{S} . The in-/outflow per period is limited to S^{F} .

The producer is obliged to fulfill the heat demand in each period $t \in \mathcal{T}$ in the district heating network $\tilde{D}_{t,\omega}$, which is modeled in scenarios $\omega \in \Omega$. The probability of scenario $\omega \in \Omega$ is given by Π_ω . To sum up the uncertain parameters, a scenario ω resembles the heat demand $\tilde{D}_{t,\omega}$, the electricity price $\tilde{L}_{t,\omega}$ and the fuel spot price for the auxiliary boiler $\tilde{C}_{t,\omega}^{\text{AUX}}$.

The overall objective of the solution approach is to select the portfolio of biomass contracts and their configurations to minimize the cost while fulfilling the heat demand taking the technical characteristics of the plant into account. In this paper, we consider a planning horizon of one year ranging from summer to summer as it is often done in practice. Thus, the heating seasons lies in the middle of the planning horizon. However, in general the method can be used with any length of the planning horizon starting and ending at an arbitrary point in time during the year.

4 Two-phase solution approach

The time scales in the above mentioned planning problem have a broad range. As the contracts are often agreed for up to one year, this results in a medium-term planning problem. However, many technical characteristics of the CHP unit and the electricity market relate to an hourly level. Additionally, the production does not need to be scheduled more than one week in advance, because then information especially regarding the heat demand gets more accurate. Therefore, we divide the overall planning problem into two-phases. Each phase is modelled by a stochastic program to incorporate the relevant uncertainties as scenarios into the planning.

Biomass contract selection: This model decides which suppliers should be contracted for the next year and which amount of biomass they should deliver (including options). These are the first-stage decisions of the two-stage stochastic model. The model includes the production by the CHP plant on a weekly time scale as second-stage decisions excluding ramping and unit commitment decisions. The uncertainty in the model is based on heat demand scenarios. The thermal heat storage is excluded from this model, because it is not reasonable to model the flows on a weekly scale due the small size of those storages. The goal of the planner is to cover most of the heat demand with biomass production. Therefore, we do not consider the auxiliary boiler in this model as it should be used in peak demand situations. The uncertain cost of

the peak boiler should not influence the biomass contract selection for a long planning horizon. The same holds for the uncertain income from the electricity markets. Set \mathcal{T} represents weekly periods in this model. The formulation is presented in Section 4.1.

Operational planning problem: Here the input of biomass is fixed based on the contracts selected in phase 1, but the amounts of contracts with agreed options can still be altered. Therefore, the final delivery amounts are the first-stage decisions. The model is solved week-by-week taking the input from the previous week into account (storage levels, status of the unit). The second-stage decisions model the production of the CHP plant and auxiliary boiler on an hourly basis incorporating technical requirements and scenario-based price and demand information. Set \mathcal{T} represents hourly periods in this model. The model formulation is described in Section 4.2.

Based on the scenario-based representation of the uncertain parameters, both models are two-stage stochastic programs. The division of the planning problem into two phases not only reduces the complexity of the problem, but also resembles the planning process in practice in a more accurate way. Furthermore, solving the operational planning problem week-by-week enables us to make use of more recent information to update the scenarios for the next week. We do not consider an integrated problem for the entire year in an hourly resolution because the addition of such precise information can negatively affect the solution of the problem towards the real realization of the uncertainty due to forecasting inaccuracies. Furthermore, preliminary experiments showed that the large number of integer variables makes the problem computationally hard and not solvable in a reasonable amount of time. Both is also confirmed by experiments with test cases having a planning horizon of 13 weeks (see Section 6.3).

4.1 Biomass contract selection

The following model represents the biomass contract selection in phase 1. Please note that this model has a weekly time-scale, therefore, the set of period $t \in \mathcal{T}$ consists of weeks. The relevant parameters like capacities and flow restrictions of the units and storage are scaled up to weekly values accordingly. The set of scenarios Ω in this phase contains weekly data for the heat demand $\tilde{D}_{t,\omega}$ for the entire planning horizon of $|\mathcal{T}|$ periods.

The first-stage decision variables in this model decide on the contracts to be selected (u_j) as well as the number of deliveries in each week ($d_{j,t}$) and amounts ($b_{j,t}$) including up- ($b_{j,t}^+$) and down-scaling ($b_{j,t}^-$) options for each contract $j \in \mathcal{J}$ and period $t \in \mathcal{T}$. Based on the second-stage variables, these amounts can be altered with the variables $\bar{b}_{j,t,\omega}^+$ and $\bar{b}_{j,t,\omega}^-$ within the limits of the selected options in the first-stage. Further second-stage variables relate to the biomass storage ($\delta_{t,\omega}$) as well as heat ($q_{t,\omega}^{\text{CHP}}$) and power production ($p_{t,\omega}$). An overview of the variables and their domains is given in Table 2.

Table 2: Variables

$u_j \in \{0, 1\}$	Equals 1, if contract j is used, 0 otherwise
$d_{j,t} \in \mathbb{N}_0$	Number of deliveries by contract j in period t
$\tilde{d}_{j,t} \in \{0, 1\}$	Equals 1, if contract j delivers in period t , 0 otherwise
$b_{j,t} \in \mathbb{R}_0^+$	Amount of biomass contracted in contract j for period t [tonnes]
$b_{j,t}^+ \in \mathbb{R}_0^+$	Up-scaling option contracted in contract j for period t [tonnes]
$b_{j,t}^- \in \mathbb{R}_0^+$	Down-scaling option contracted in contract j for period t [tonnes]
$\bar{b}_{j,t,\omega}^+ \in \mathbb{R}_0^+$	Actual amount used of up-scaling option in contract j [tonnes]
$\bar{b}_{j,t,\omega}^- \in \mathbb{R}_0^+$	Actual amount used of down-scaling option in contract j [tonnes]
$\delta_{t,\omega} \in \mathbb{R}_0^+$	Biomass storage level [MWt]
$\delta_{t,\omega}^+ \in \mathbb{R}_0^+$	Inflow to biomass storage [MWt/period]
$\delta_{t,\omega}^- \in \mathbb{R}_0^+$	Outflow from biomass storage [MWt/period]
$s_{t,\omega} \in \mathbb{R}_0^+$	Thermal storage level [MWt]
$s_{t,\omega}^+ \in \mathbb{R}_0^+$	Inflow to thermal storage [MWt/period]
$s_{t,\omega}^- \in \mathbb{R}_0^+$	Outflow from thermal storage [MWt/period]
$x_{t,\omega} \in \{0, 1\}$	Equals 1, if CHP plant is on in period t , 0 otherwise
$y_{t,\omega} \in \{0, 1\}$	Equals 1, if CHP plant is started up in period t , 0 otherwise
$z_{t,\omega} \in \{0, 1\}$	Equals 1, if CHP plant is shut down in period t , 0 otherwise
$p_{t,\omega} \in \mathbb{R}_0^+$	Power production by CHP [MWe/period]
$q_{t,\omega}^{\text{CHP}} \in \mathbb{R}_0^+$	Total heat production by CHP [MWt/period]
$q_{t,\omega}^{\text{CHP},N} \in \mathbb{R}_0^+$	Heat from CHP flowing to DH [MWt/period]
$q_{t,\omega}^{\text{CHP},S} \in \mathbb{R}_0^+$	Heat from CHP to thermal storage [MWt/period]
$q_{t,\omega}^{\text{AUX}} \in \mathbb{R}_0^+$	Total heat production by auxiliary boiler [MWt/period]
$q_{t,\omega}^{\text{AUX},N} \in \mathbb{R}_0^+$	Heat from auxiliary boiler to DH [MWt/period]
$q_{t,\omega}^{\text{AUX},S} \in \mathbb{R}_0^+$	Heat from auxiliary boiler to thermal storage [MWt/period]
$q_{t,\omega}^{\text{Miss}} \in \mathbb{R}_0^+$	Missed heat demand [MWt/period]
$\delta_{t,\omega}^{\text{EX}} \in \mathbb{R}_0^+$	Amount of biomass above storage capacity [MWt]
$\delta_{\omega}^{\text{T}} \in \mathbb{R}_0^+$	Amount of biomass in excess at the end of the horizon [MWt]

The objective function is given in (1) and minimizes the expected cost of the biomass contract selection. The first part (1a) contains the costs related to the biomass supply and the contract selection. In (1b), operational costs of the system and inventory costs for biomass are modeled. The third part (1c) represents penalty costs. First, we penalize leftover biomass at the end of the planning period ($\Phi^{\text{Sto}} \delta_{\omega}^{\text{T}}$), since we try to empty the storage at the end of the year. Second, missed heat-demand ($\Phi^{\text{Miss}} q_{t,\omega}^{\text{Miss}}$) is penalized.

$$\min \sum_{t \in \mathcal{T}} \left[\sum_{j \in \mathcal{J}} \left(C_j^{\text{B}} b_{j,t} + C_j^{\text{B}+} b_{j,t}^+ + C_j^{\text{B}-} b_{j,t}^- + \sum_{\omega \in \Omega} \Pi_{\omega} C_j^{\text{B}} (\bar{b}_{j,t,\omega}^+ - \bar{b}_{j,t,\omega}^-) \right) \right] \quad (1a)$$

$$+ \sum_{\omega \in \Omega} \Pi_{\omega} \left(C^{\text{CHP}} [p_{t,\omega} - \Theta q_{t,\omega}^{\text{CHP}}] + C^{\text{I}} \delta_{t,\omega} \right) \quad (1b)$$

$$+ \sum_{\omega \in \Omega} \Pi_{\omega} \left(\Phi^{\text{Sto}} \delta_{\omega}^{\text{T}} + \sum_{t \in \mathcal{T}} \Phi^{\text{Miss}} q_{t,\omega}^{\text{Miss}} \right) \quad (1c)$$

Constraints (2) to (9) model the selection of biomass contracts. In constraints (2) the number of deliveries is restricted by the contract limits given by \overline{N}_j and \underline{N}_j . Constraint (3) restricts the number of deliveries per week to a maximum according to the frequency of the contract F_j . The left-hand side sums over several weeks, if the minimum time between visits F_j is longer than one week (168 hours). The right-hand side determines the maximum number of deliveries in that period with at least one delivery or more if the time difference is less than 168 hours. The total amount including up- and down-scaling options is limited between $[\underline{B}_j, \overline{B}_j]$ by constraints (4) and (5) and the use of options to the allowed percentages of deviation O_j^+ and O_j^- in constraints (6) and (7). In constraints (8) and (9), it is ensured that the second-stage alterations respect the first-stage decisions.

$$\underline{N}_j u_j \leq \sum_{t \in \mathcal{T}} d_{j,t} \leq \overline{N}_j u_j \quad \forall j \in \mathcal{J} \quad (2)$$

$$\sum_{\tau=t-\max\{\lfloor \frac{F_j}{168} \rfloor, 1\}}^t d_{j,\tau} \leq \max \left\{ \frac{168}{F_j}, 1 \right\} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (3)$$

$$b_{j,t} + b_{j,t}^+ \leq \overline{B}_j d_{j,t} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (4)$$

$$b_{j,t} - b_{j,t}^- \geq \underline{B}_j d_{j,t} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (5)$$

$$b_{j,t}^+ \leq O_j^+ b_{j,t} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (6)$$

$$b_{j,t}^- \leq O_j^- b_{j,t} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (7)$$

$$\overline{b}_{j,t,\omega}^+ \leq b_{j,t}^+ \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (8)$$

$$\overline{b}_{j,t,\omega}^- \leq b_{j,t}^- \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (9)$$

The biomass storage is modeled by constraints (10) to (14). The model ensures that the storage level is kept within the limits $[\underline{\Delta}_t, \overline{\Delta}_t]$ (10) and calculated correctly based on the previous level and in- and outflows (11). The initial storage level is given by $\delta_{0,\omega}$, which is the same for all scenarios. The inflow from supplier deliveries is calculated in constraints (12), where the incoming biomass is converted from tonnes to MWt using the calorific value of the biomass E^B . The outflow is restricted to Δ^F by constraints (13). Finally, the storage level at the end of the planning horizon is determined in variable δ_ω^T in (14) for penalty cost calculations.

$$\underline{\Delta}_t \leq \delta_{t,\omega} \leq \overline{\Delta}_t \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (10)$$

$$\delta_{t,\omega} = \delta_{t-1,\omega} + \delta_{t,\omega}^+ - \delta_{t,\omega}^- \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (11)$$

$$\delta_{t,\omega}^+ = \sum_{j \in \mathcal{J}} \left(b_{j,t} + \overline{b}_{j,t,\omega}^+ - \overline{b}_{j,t,\omega}^- \right) \cdot E^B \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (12)$$

$$\delta_{t,\omega}^+ \leq \Delta^F \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (13)$$

$$\delta_{|\mathcal{T}|,\omega} \leq \delta_{0,\omega} + \delta_\omega^T \quad \forall \omega \in \Omega \quad (14)$$

The production capacities of the CHP plant are enforced by constraints (15) to (18). In (15) the consumption of biomass from the storage for CHP production is determined based on the corresponding efficiencies E_P^{CHP} and E_Q^{CHP} for power and heat, respectively. The feasible region of the CHP, which was previously presented in Figure 2, is modeled by constraints (16) to (18).

$$\delta_{t,\omega}^- = \frac{p_{t,\omega}}{E_P^{\text{CHP}}} - \Theta \cdot \frac{q_{t,\omega}^{\text{CHP}}}{E_Q^{\text{CHP}}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (15)$$

$$\underline{P} \leq p_{t,\omega} - \Theta \cdot q_{t,\omega}^{\text{CHP}} \leq \bar{P} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (16)$$

$$\Xi \cdot q_{t,\omega}^{\text{CHP}} \leq p_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (17)$$

$$q_{t,\omega}^{\text{CHP}} \leq \bar{Q}^{\text{CHP}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (18)$$

Finally, the uncertain heat demand $\tilde{D}_{t,\omega}$ is ensured in each period t and scenario ω in constraint (19).

$$\tilde{D}_{t,\omega} = q_{t,\omega}^{\text{CHP}} + q_{t,\omega}^{\text{Miss}} \quad \forall t \in \mathcal{T}, \omega \in \Omega \quad (19)$$

4.2 Operational planning

The operational planning model relates to the second phase of the solution approach. For the overall solution approach, the model is solved consecutively week-by-week with a rolling horizon to determine the production schedule and to adjust the biomass deliveries, if possible. Therefore, the planning horizon is $|\mathcal{W}|$ weeks with an hourly resolution. The week in focus is \mathcal{W}_1 and the remaining weeks \mathcal{W}_2 to \mathcal{W}_w are used in a rolling horizon manner to already include predictions for future periods. Thus, the decisions for weeks \mathcal{W}_2 to \mathcal{W}_w can be altered again later, when the respective week comes in focus. Please note that the set \mathcal{T} consists of all hours in the rolling horizon. This is a difference to the biomass contract selection problem where the set consisted of weeks. The set \mathcal{T}_w relates to the hours in the specific week $w \in \mathcal{W}$. This model considers the production of the auxiliary boiler as well as sales to the electricity markets. Therefore, the uncertain parameter of the scenarios does not only include heat demand but also fuel price for the auxiliary boiler and the electricity price. Thus, the set of scenarios Ω in this phase contains hourly data for the heat demand $\tilde{D}_{t,\omega}$, the electricity price $\tilde{L}_{t,\omega}$ and the fuel price of the auxiliary boiler $\tilde{C}_{t,\omega}^{\text{AUX}}$ in the next $|\mathcal{T}|$ periods. The scenarios are updated with new information before each optimization run.

The decision variables for this model decide the amount of biomass ($b_{j,t,\omega}$) including up- and down- scaling options ($\bar{b}_{j,t,\omega}^+$ and $\bar{b}_{j,t,\omega}^-$) and the actual delivery times ($\hat{d}_{j,t,\omega}$) for the deliveries of contract j . Further variables are related to the biomass storage level ($\delta_{t,\omega}$), the thermal storage level ($s_{t,\omega}$), the heat and power production ($q_{t,\omega}^{\text{CHP}}$, $q_{t,\omega}^{\text{AUX}}$ and $p_{t,\omega}$) and the commitment status of the CHP plant ($x_{t,\omega}$, $y_{t,\omega}$ and $z_{t,\omega}$). The variables are included in Table 2.

Table 3: Input parameters from biomass contract selection

$\mathbf{U}_{j,w} \in \mathbb{N}_0$	Number of deliveries of contract j in week w
$\mathbf{B}_{j,w} \in \mathbb{R}_0^+$	Contracted delivery amount of contract j in week w
$\mathbf{B}_{j,w}^+ \in \mathbb{R}_0^+$	Contracted up-scaling of delivery amount of contract j in week w
$\mathbf{B}_{j,w}^- \in \mathbb{R}_0^+$	Contracted down-scaling of delivery amount of contract j in week w

Because the first week of the rolling horizon is the week in focus, the first-stage decisions of the stochastic program are the delivery times and amounts $\hat{d}_{j,t,\omega}$, $b_{j,t,\omega}$, $\bar{b}_{j,t,\omega}^+$ and $\bar{b}_{j,t,\omega}^-$ for periods t in the first week \mathcal{T}_1 . For all other weeks, the decisions can be revised later and are second-stage decisions. To ensure non-anticipativity, we include specific constraints.

The selection of biomass contracts and amounts are input parameters to this model (given in Table 3) and determined by the biomass contract selection model in phase 1. Set \mathcal{J} is reduced to only selected contracts for the corresponding week to limit the number of variables.

Furthermore, the storage levels $\delta_{0,\omega}$ and $s_{0,\omega}$ as well as the unit status of the preceding week $x_{0,\omega}$ are set as initial values, based on the outcome of the previous week.

As in the biomass contract selection model, the objective function (20) minimizes the expected costs composed of biomass contract costs (20a), operational costs (20b) and penalty costs (20c). However, the following changes have to be made. First, the profit for electricity sales ($\tilde{L}_{t,\omega}$) and operational costs for the auxiliary boiler ($\tilde{C}_{t,\omega}^{\text{AUX}}$) are added to the objective function (20b). These are uncertain and depend on the scenarios. Second, the operational cost (20b) now include costs for starting up and shutting down the CHP plant. Third, the term (20c) penalizes unfulfilled heat demand and exceeding the biomass storage capacity in each period. Note that to resemble the total weekly cost of the system, we keep the constant term $C_j^{\text{B}} \mathbf{B}_{j,w} + C_j^{\text{B}+} \mathbf{B}_{j,w}^+ + C_j^{\text{B}-} \mathbf{B}_{j,w}^-$ in (20a).

$$\min \sum_{w \in \mathcal{W}} \sum_{j \in \mathcal{J}} \left(C_j^{\text{B}} \mathbf{B}_{j,w} + C_j^{\text{B}+} \mathbf{B}_{j,w}^+ + C_j^{\text{B}-} \mathbf{B}_{j,w}^- \sum_{t \in \mathcal{T}_w} C_j^{\text{B}} (\bar{b}_{j,t}^+ - \bar{b}_{j,t}^-) \right) \quad (20a)$$

$$+ \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} \Pi_{\omega} \left(C^{\text{CHP}} (p_{t,\omega} - \Theta q_{t,\omega}^{\text{CHP}}) - \tilde{L}_{t,\omega} p_{t,\omega} + C^{\text{SU}} y_{t,\omega} + C^{\text{SD}} z_{t,\omega} \right) \quad (20b)$$

$$+ \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} \Pi_{\omega} \left(\tilde{C}_{t,\omega}^{\text{AUX}} \frac{q_{t,\omega}^{\text{AUX}}}{E^{\text{AUX}}} + C^{\text{I}} \delta_{t,\omega} \right) + \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} \Pi_{\omega} (\Phi^{\text{Sto}} \delta_{t,\omega}^{\text{EX}} + \Phi^{\text{Miss}} q_{t,\omega}^{\text{Miss}}) \quad (20c)$$

The biomass deliveries are handled in constraints (21) to (28). Here we use the input from the biomass contract selection model (see parameters in Table 3). If deliveries were scheduled for the weeks in the planning horizon by phase

1, the operational model decides on the actual delivery times during the week (21). The weekly contracted amount is split on the deliveries in constraints (22). The delivery amount can be altered in the given limits of the options (constraints (23) and (24)), but the total amount must be within the limits of the contract (constraints (25) and (26)). Constraints (27) imposes a maximum frequency F_j on the deliveries associated with each contract, while constraints (28) ensures an elapsed time of at least Δ^W periods between two deliveries irrespective of the supplier.

$$\sum_{t \in \mathcal{T}_w} \hat{d}_{j,t,\omega} = \mathbf{U}_{j,w} \quad \forall j \in \mathcal{J}, \forall w \in \mathcal{W}, \forall \omega \in \Omega \quad (21)$$

$$\sum_{t \in \mathcal{T}_w} b_{j,t,\omega} = \mathbf{B}_{j,w} \quad \forall j \in \mathcal{J}, \forall w \in \mathcal{W}, \forall \omega \in \Omega \quad (22)$$

$$\sum_{t \in \mathcal{T}_w} \bar{b}_{j,t,\omega}^+ \leq \mathbf{B}_{j,w}^+ \quad \forall j \in \mathcal{J}, \forall w \in \mathcal{W}, \forall \omega \in \Omega \quad (23)$$

$$\sum_{t \in \mathcal{T}_w} \bar{b}_{j,t,\omega}^- \leq \mathbf{B}_{j,w}^- \quad \forall j \in \mathcal{J}, \forall w \in \mathcal{W}, \forall \omega \in \Omega \quad (24)$$

$$b_{j,t,\omega} + \bar{b}_{j,t,\omega}^+ \leq \bar{B}_j \hat{d}_{j,t,\omega} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (25)$$

$$b_{j,t,\omega} - \bar{b}_{j,t,\omega}^- \geq \underline{B}_j \hat{d}_{j,t,\omega} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (26)$$

$$\sum_{\tau=t-F_j}^t \hat{d}_{j,\tau,\omega} \leq 1 \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (27)$$

$$\sum_{j \in \mathcal{J}} \sum_{\tau=t}^{t+\Delta^W} \hat{d}_{j,\tau,\omega} \leq 1 \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (28)$$

As the decisions for the biomass delivery in the first week are first-stage decisions of the stochastic program, we have to ensure that they have the same values for each scenario ω for all periods t in the first week \mathcal{T}_1 . This is forced by the non-anticipativity constraints (29) to (30).

$$\hat{d}_{j,t,\omega} = \hat{d}_{j,t,\omega'}, \quad b_{j,t,\omega} = b_{j,t,\omega'} \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}_1, \forall \omega, \omega' \in \Omega, \omega \neq \omega' \quad (29)$$

$$\bar{b}_{j,t,\omega}^+ = \bar{b}_{j,t,\omega'}^+, \quad \bar{b}_{j,t,\omega}^- = \bar{b}_{j,t,\omega'}^- \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}_1, \forall \omega, \omega' \in \Omega, \omega \neq \omega' \quad (30)$$

The inflow to the biomass storage in each period (31) is dependent on the scheduled delivery and adjustments based on the options. The storage level is given by equation (32). The outflow and capacity of the storage is limited in constraints (33) and (34) by Δ^F and $\bar{\Delta}$, respectively. The safety storage $\underline{\Delta}_t$ for biomass is incorporated in constraints (35), but only for periods t in future weeks in the rolling horizon, i.e. $t \in \mathcal{T}_w, w \geq 2$. In the current week \mathcal{T}_1 , the storage can be used for production (36).

$$\delta_{t,\omega}^+ = \sum_{j \in \mathcal{J}} \left(b_{j,t,\omega} + \bar{b}_{j,t,\omega}^+ - \bar{b}_{j,t,\omega}^- \right) \cdot E^B \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (31)$$

$$\delta_{t,\omega} = \delta_{t-1,\omega}^+ + \delta_{t,\omega}^+ - \delta_{t,\omega}^- \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (32)$$

$$\delta_{t,\omega}^- \leq \Delta^F \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (33)$$

$$\delta_{t,\omega} \leq \bar{\Delta} + \delta_{t,\omega}^{\text{EX}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (34)$$

$$\underline{\Delta}_t \leq \delta_{t,\omega} \quad \forall w \in \{2, \dots, |\mathcal{W}|\}, \forall t \in \mathcal{T}_w, \forall \omega \in \Omega \quad (35)$$

$$0 \leq \delta_{t,\omega} \quad \forall t \in \mathcal{T}_1, \forall \omega \in \Omega \quad (36)$$

Constraints (37) to (40) regarding biomass consumption and feasible production region of the CHP unit constraints are similar to constraints (15) to (18) for the biomass selection problem. However, here the production depends also on the status of the unit ($x_{t,\omega} = 1$ means the unit is on). The status of the unit is determined by constraints (41) to (42) while constraints (43) and (44) ensure minimum up- and down times M^U and M^D , respectively. The change of production volume is restricted to the ramping requirements R^U and R^D in constraints (45) and (46). The initial status of the CHP plant depends on the previous week and is given by $x_{0,\omega}$ and $p_{0,\omega}$ as input parameters.

$$\delta_{t,\omega}^- = \frac{p_{t,\omega}}{E_P^{\text{CHP}}} - \Theta \cdot \frac{q_{t,\omega}^{\text{CHP}}}{E_Q^{\text{CHP}}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (37)$$

$$\underline{P} \cdot x_{t,\omega} \leq p_{t,\omega} - \Theta \cdot q_{t,\omega}^{\text{CHP}} \leq \bar{P} \cdot x_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (38)$$

$$\Xi \cdot q_{t,\omega}^{\text{CHP}} \leq p_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (39)$$

$$q_{t,\omega}^{\text{CHP}} \leq \overline{Q^{\text{CHP}}} \cdot x_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (40)$$

$$y_{t,\omega} - z_{t,\omega} = x_{t,\omega} - x_{t-1,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (41)$$

$$y_{t,\omega} + z_{t,\omega} \leq 1 \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (42)$$

$$\sum_{\tau=t-M^U+1}^t y_{\tau,\omega} \leq x_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (43)$$

$$\sum_{\tau=t-M^D+1}^t z_{\tau,\omega} \leq 1 - x_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (44)$$

$$p_{t,\omega} - p_{t-1,\omega} \leq R^U \cdot x_{t-1,\omega} + \underline{P} \cdot y_{t-1,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (45)$$

$$p_{t,\omega} - p_{t-1,\omega} \geq -R^D \cdot x_{t,\omega} - \underline{P} \cdot z_{t,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (46)$$

Constraints (47) set the heat production capacity $\overline{Q^{\text{AUX}}}$ of the auxiliary boiler. The heat storage is modeled by constraints (48) to (54). The inflow is determined by the heat from the CHP unit and auxiliary boiler inserted into the storage (48). The current storage level depends on the inflow, outflow and previous level (49) ($s_{0,\omega}$ for the initial value) and has to satisfy the capacity restrictions $[\underline{S}, \overline{S}]$ (50). Outflow (51) and inflow (52) are limited to S^F and the inflow cannot directly flow out again (53). To avoid emptying the storage at the end, the initial level $s_{0,\omega}$ must be reached again at the end of the rolling horizon, i.e. in period $t = |\mathcal{T}|$ (54).

$$q_{t,\omega}^{\text{AUX}} \leq \overline{Q^{\text{AUX}}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (47)$$

$$s_{t,\omega}^+ = q_{t,\omega}^{\text{CHP,S}} + q_{t,\omega}^{\text{AUX,S}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (48)$$

$$s_{t,\omega} = s_{t-1,\omega} + s_{t,\omega}^+ - s_{t,\omega}^- \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (49)$$

$$\underline{S} \leq s_{t,\omega} \leq \overline{S} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (50)$$

$$s_{t,\omega}^- \leq S^F \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (51)$$

$$s_{t,\omega}^+ \leq S^F \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (52)$$

$$s_{t,\omega}^- \leq s_{t-1,\omega} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (53)$$

$$s|_{\mathcal{T},\omega} = s_{0,\omega} \quad \forall \omega \in \Omega \quad (54)$$

The flow of heat is modelled in constraints (55) to (57). The heat production by both units is used for filling the heat storage and covering the demand. Therefore, the production is split up into those two components in constraints (55) and (56). For fulfilling the uncertain heat demand $\tilde{D}_{t,\omega}$, heat directly fed to the district heating network and heat from the thermal storage is used (57). Any shortfall of heat is modelled by variable $q_{t,\omega}^{\text{Miss}}$ penalized in the objective function.

$$q_{t,\omega}^{\text{CHP}} = q_{t,\omega}^{\text{CHP,N}} + q_{t,\omega}^{\text{CHP,S}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (55)$$

$$q_{t,\omega}^{\text{AUX}} = q_{t,\omega}^{\text{AUX,N}} + q_{t,\omega}^{\text{AUX,S}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (56)$$

$$\tilde{D}_{t,\omega} = q_{t,\omega}^{\text{CHP,N}} + q_{t,\omega}^{\text{AUX,N}} + s_{t,\omega}^- + q_{t,\omega}^{\text{Miss}} \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (57)$$

4.3 Overall solution approach

The overall planning tool for a large-scale CHP producer combines the two above mentioned stochastic programming models. The selection of biomass contracts for the next year has to be decided once a year for the entire coming year. For this long-term planning problem, the producer would use the biomass contract selection model that determines the best set of contracts based on heat demand uncertainty and a weekly production planning. This model is only solved once and the contracting decisions regarding the contract type, options and amounts can not be changed later on (line 1 in Algorithm 1).

The second model, i.e., the operational planning problem, is a planning tool the CHP producer uses for the weekly production planning throughout the year. This means every week of the year, the operational models is solved with updated scenarios using more accurate forecasting information regarding the uncertainty of heat demand and electricity prices (line 2 to 6 in Algorithm 1). We refer to Section 5.2 and Appendix A for more details on the scenario generation in each phase. The model contains a more detailed production planning on an hourly scale covering the market structure and technical requirements. The difference to other operational planning problems in literature is the direct coupling of the model to the decisions of the biomass contract selections. The model uses the contracting decisions as input and can not change those. If and only if biomass options were contracted for a particular week in

Algorithm 1 Two-phase solution approach

- 1: Solve the biomass contract selection model (1)-(19)
 - 2: **for** each week in the overall planning horizon **do**
 - 3: Select the corresponding contract decisions from line 1 and set limits
 - 4: Generate scenarios for the current rolling horizon
 - 5: Solve the operational planning model (20)-(57)
 - 6: **end for**
-

the biomass contract selection, these can be now utilized in the operational production planning to change the final delivery amount of biomass.

5 Case studies

In the following we analyze test cases for two different municipalities in Denmark, named A and B, that are connected to the Aarhus district heating network. We consider 52 weeks starting from 1st of June 2016 as the planning horizon in the numerical results in Section 6. In total, we look at ten test cases (five per municipality) that are created as follows. Test case AY and BY are yearly test cases and cover the entire 52 weeks for the respective municipality. Test cases AQ1, AQ2, AQ3 and AQ4 look at quarterly test cases with 13 weeks planning horizon for municipality A starting in week 1, 14, 27 and 40, respectively. This means test cases AY is the concatenation of test cases AQ1, AQ2, AQ3 and AQ4. Test cases BQ1, BQ2, BQ3 and BQ4 are generated analogously for municipality B. We like to point out that the planning horizon in practice would be one year ranging from summer to summer. We use the quarterly test cases due to two reasons. First, because of the lack of further practical data, we can increase the number of test cases by using these subsets. Second, we can solve an biomass contract selection model with hours as time periods only for the quarterly test cases due to the complexity of the problem.

5.1 Technical data

The heat demand data in the district heating networks is obtained from [1], NordPools' hourly electricity prices for DK1 zone from [15] and daily natural gas prices from [26]. Extreme outlier values in electricity prices are limited to a maximum or minimum of four standard deviations from the mean.

The technical parameters for the CHP and auxiliary units as well as the operation costs are based on [24, 33, 20] and [10] and shown in Tables 4, 5 and 6. Both systems comprise a CHP unit and one auxiliary boiler. Municipality A uses a gas boiler in addition to the CHP, while municipality B uses an electric boiler. The biomass storage minimum level $\underline{\Delta}_t$ is divided in two values. In weeks 20 - 45 (i.e. in the heating season), we have a higher minimum level as in the remaining weeks of the year. The penalty costs for both case are the same and set to $\Phi^{\text{Sto}} = 1000$ and $\Phi^{\text{Miss}} = 10000$.

Table 4: Technical parameters of the CHP unit

	\bar{P}	\underline{P}	$\overline{Q}^{\text{CHP}}$	Θ	Ξ	R^U	R^D	E_P^{CHP}	E_Q^{CHP}	M^U	M^D
A	13.24	3.8	20.8	-0.18	0.55	3.7	3.7	0.62	0.31	6	4
B	35.18	5.72	47.28	-0.12	0.64	4.6	4.6	0.64	0.29	8	5

Table 5: Technical parameters of the auxiliary unit and storages

	Aux. boiler		Thermal storage				Biomass storage					E^B
	E^{AUX}	$\overline{Q}^{\text{AUX}}$	S^0	S^F	\bar{S}	\underline{S}	Δ^F	Δ^0	Δ^W	$\bar{\Delta}$	$\underline{\Delta}_t$	
A	0.97	15	5	3	7	0	35	500	24	20000	4000 (20-45) 2000	4.9971
B	0.99	30	6.5	4.5	9.5	0	70	850	24	35000	7000 (20-45) 3500	4.9971

Table 6: Cost parameters

	CHP					Aux. boiler			Storage
	C^{CHP}	C^{SU}	C^{SD}	T^{EP}	I	$C_{Aux}^{\text{O\&M}}$	T^{AUX}	T^{CO_2}	C^I
A	19.85	14250	0	55.62	20.25	0.07	28.22	6.34	0.0002
B	20.32	16870	0	55.62	20.25	0.5	52.07	0	0.0002

The parameters of the biomass contracts data are given in Table 7, where they are organized from *fixed* contracts at the top of the table and gradually going down to more *flexible* contracts. Both cases use the same set of contracts. The biomass contracts are usually confidential. Therefore, the data for this set of contracts has been determined by analyzing publications on market prices for biomass and their plausibility has been confirmed by an energy utility company in Denmark. Note that the minimum and maximum number of deliveries, $\underline{N}_j, \overline{N}_j$, in Table 7 are given for the yearly test cases AY and BY. For the quarterly test cases, those were adjusted by dividing them by four to have reasonable number of deliveries per quarter.

5.2 Scenario generation

Apart from the deterministic parameters mentioned in the previous section, we have to handle uncertainty regarding heat demands, gas prices and electricity prices to be used in the optimization. Since both municipalities are within the same bidding region in Nordpool (DK1) and the same gas trading region, the electricity and natural gas prices are identical. However, differences exist regarding the heat demand. We use historical data from 1st June 2011 to 31st May 2016 for electricity prices, natural gas prices and heat demands. Based on this data, different techniques for scenario generation are implemented. The resulting scenarios and expected values depend on the municipality due to the

Table 7: Biomass contract data

Contract	C_j^B	C_j^{B+}	C_j^{B-}	O_j^+	O_j^-	\overline{B}_j	\underline{B}_j	F_j	\overline{N}_j	\underline{N}_j
1	150.8	0	0	0	0	19000	18000	2016	4	4
2	156.4	0	0	0	0	17000	12000	1344	5	2
3	170.83	0	0	0	0	15000	11000	1008	8	4
4	181.31	30.56	30.56	0.1	0.1	12000	8000	504	17	15
5	181.43	24.45	24.45	0.15	0.15	12000	8000	504	15	15
6	183.59	30.56	30.56	0.25	0.25	5100	2380	336	25	24
7	183.43	36.67	36.67	0.25	0.25	5100	2380	336	25	15
8	201.89	18.34	18.34	0.5	0.5	1200	1200	168	50	50
9	202.17	18.34	18.34	0.5	0.5	1200	1000	168	50	25
10	204.29	28.12	28.12	0.5	0.5	850	850	120	60	50
11	202.24	28.12	28.12	0.65	0.65	850	500	120	60	30
12	202.05	12.22	12.22	0.75	0.75	350	100	48	100	80
13	202.64	12.22	12.22	0.75	0.75	350	100	48	100	50

different auxiliary boilers and heat consumption in previous years. Furthermore, the input time series varies with the phase of the solution approach regarding time scales and need for scenarios. The scenario generation for both phases is described in Appendix A and shortly summarized here. In phase 1, the biomass contract selection, we use five scenarios based on the historical data due to the long planning horizon. In phase 2, the operational planning problem, we tested different methods to create the scenarios based on time series analysis and historical data. Time series analysis was used here to take advantage of the fact that we have a shorter planning horizon to predict and that we can obtain recent data which allows us new forecasts every week.

The scenarios used in the evaluations are based on a combination of past data and time series forecasts (see description in Appendix A). In the operational planning problem, the scenarios for prices and heat demand consist five scenarios using a time series forecast for the rolling horizon and five scenarios using historical values (denoted as method $F2+P$). This means, the total number scenarios in the operational model is ten. See Appendix B for a comparison of different scenario generation methods.

5.3 Evaluation of solution approach

To evaluate our solution approach, we have to obtain the costs under different realizations of the uncertainty. We use 11 samples, i.e., 11 different realizations of uncertainty, for each test case. Because the samples are different for each municipality, this results in a test set of 22 different samples in total. Please note that these 22 samples do not include the scenarios used for optimization. They are 22 new samples that are used in an out-of-sample setting to evaluate the first-stage decisions in realizations of uncertainty that have not been anticipated at the optimization stage. Sample 0 and 11 are the actual realization of the heat demand, electricity prices and gas prices starting from

1st June 2016 in municipality A and B, respectively. The remaining 20 samples (from 1 to 10 and 11 to 21) are a composite of different real data sets obtained from the same sources as the previous data. The electricity and gas prices are obtained from real data of 2015, 2016 and 2017 from other regions in Nordpool and other European hubs, respectively. The heat consumption is obtained from other municipalities in the Aarhus district heating system and scaled to the size of the system capacity accordingly. The 22 samples can be used for the yearly as well as the quarterly test cases. These leads to a total of 110 evaluations per method (11 samples per test case and 10 test cases).

Evaluating one sample with a configuration of our method requires to extend Algorithm 1 by one step. Each week after the operational problem is solved (line 5 in Algorithm 1), we fix the first-stage decisions and solve the model using the realizations of the uncertainty of the first week. Thus, we obtain the real costs for the first week and the initial status for the next week.

6 Experimental results

For the experimental evaluation, we implemented Algorithm 1 using Python 3.5.4 and Gurobi 9.0.0 (default parameters). All experiments are run on Intel Xeon Processor E5-2660 v3 with 24 GB RAM on the DTU Computing Center². The objective values in this section comprise the real costs summed over all weeks in the planning horizon. Furthermore, we set the length of the rolling horizon to four weeks according to the analysis in Appendix C.

The results in this section are based on the test cases from the two municipalities and show that it is beneficial to use stochastic programming (Section 6.1) and a weekly biomass contract selection model (Section 6.3) in those 10 test cases using a thorough out-of-sample evaluation. The analysis of the real data from the two municipalities (Section 6.2) further illustrates how the model can be used in practice for optimization and evaluation of biomass contracts and system operation.

6.1 Stochastic programming vs. expected value solution

To show the benefit of using stochastic programming instead of using an expected value approach, we compare the results in the following section. The experimental setup analyzes four different combinations of those two approaches. Namely, *StoSto*, *ExpExp*, *ExpSto* and *StoExp* where the first three letters indicate the method used for the biomass contract selection and the latter three letters indicated the method used for the operational rolling horizon approach. For example, *StoExp* means that we solved the biomass contract selection as stochastic program while the operational problem is the expected value approach. The histograms in this analysis contain 110 data points, i.e., each of the

² <https://www.hpc.dtu.dk/>

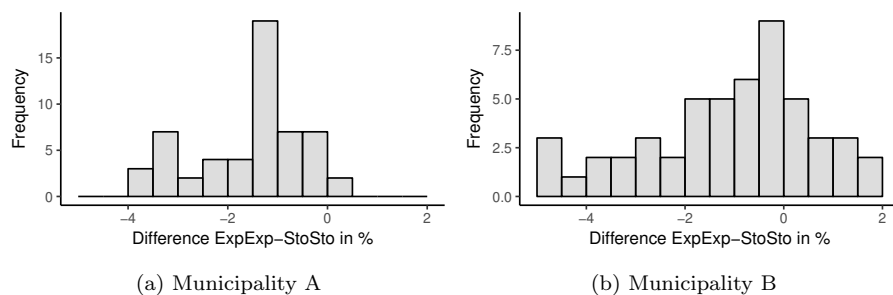


Fig. 3: Difference [%] (ExpExp-StoSto)

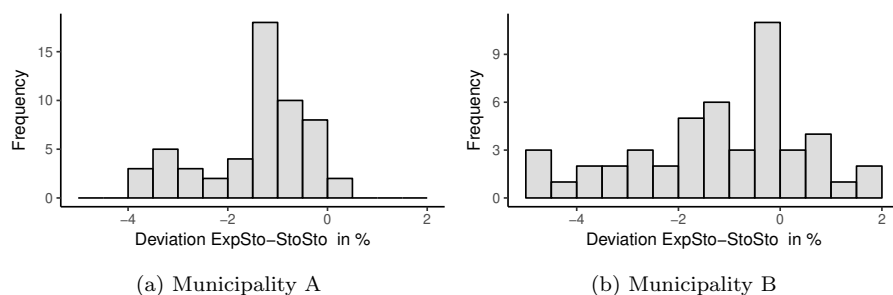


Fig. 4: Difference [%] (ExpSto-StoSto)

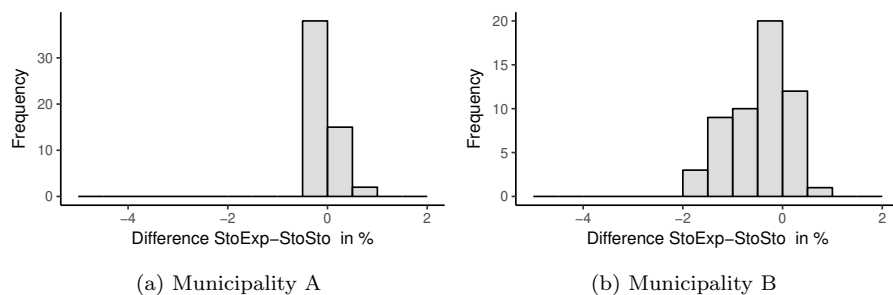


Fig. 5: Difference [%] (StoExp-StoSto)

5 test cases per municipality was evaluated for 11 samples in an out-of-sample setting.

Figure 3 compares the outcomes of a pure expected value method (*ExpExp*) versus a pure stochastic programming method (*StoSto*). The histogram shows the difference of the objective value of *ExpExp* minus the objective value of *StoSto*, i.e., negative differences meant that the stochastic method is better than the expected value method and vice versa. The empirical distribution of the difference shows that the *StoSto* method performs overall better than *ExpExp*. In most of the cases, the difference is below zero. There are some

cases of the 110 evaluations where the *ExpExp* performs slightly better. When looking at the average objective values over all samples in the first row of Table 8, the performance of *StoSto* is confirmed.

We compare the other two possible method setups *StoExp* and *ExpSto* to *StoSto* in Figure 5 and 4, respectively. The figures show that *StoSto* is better when the biomass contract selection model uses an expected value approach (see Figure 4). When only the operational model is solved using an expected value method, the difference becomes smaller (see Figure 5). These observations are also confirmed by the average objective values in Table 8. From this we can conclude that it is important to use a stochastic program especially for the long-term contract decisions. Furthermore, Table 8 shows also the unfulfilled heat demand. Due to the fact that this value does not change with the type of the model used, the improvement of using stochastic programming cannot be explained by avoiding infeasible solutions, i.e., penalty costs, compared to the expected value case. The improvement can rather be explained by the fact that the stochastic solution makes use of options while the expected value solution does not contract any options. For a visualization of the decisions regarding options, we refer to Figure 6 in Section 6.2. The options have a large impact on the operational planning problem, because contracts without any options do not leave any space for reactions in the operational planning problem. The benefit of using options is illustrated in Table 8. If the biomass contract selection model is the expected value model (*Exp**), the actual biomass delivery amount throughout the year equals the contracted biomass amount, i.e., no options are contracted and used. When the biomass contract selection is solved as a stochastic program (*Sto**), the actual delivery is different than the contracted amount which means that the options are actually used. This results in lower cost for the biomass in the end, although we have to pay a slightly higher price for the options in the biomass contract selection. The flexibility of the options in the stochastic setting also leads to a lower biomass storage level at the end of the year, i.e., less unused but paid biomass. Finally, the flexibility of the options allows the model to shift more production to the auxiliary boiler if it is cheaper the cheaper alternative leading to lower production costs. This is in particular the case, if also the operational planning problem is solved as a stochastic program (**Sto*), because the stochastic model includes fuel price scenarios of the auxiliary boiler. The usage of the options is also investigated in the next section for the two samples with real data.

6.2 Interpretation of results for real data from 2016-2017

In this section, we describe the results of the contract selection and operational planning in more detail. As an example, we analyze sample 0 for test case AY and sample 11 for test case BY, which contain the real data from 1st June 2016. We would like to point out that the conclusions drawn in this section coincides with the observations from the other samples.

Table 8: Objective value, missing heat demand, biomass contract, delivery and storage values as well as heat production values for the different combinations of models. The values are averaged over all 11 samples per test case. All values are summed over the entire planning horizon except the storage level.

Method	AQ1	AQ2	AQ3	AQ4	AY	BQ1	BQ2	BQ3	BQ4	BY
Objective value	10.45	20.11	34.51	24.28	87.65	21.33	39.10	74.33	48.61	173.34
[x100000]	10.45	20.06	34.51	24.27	87.40	21.32	38.97	74.34	48.62	173.14
[EUR]	10.35	19.74	34.30	23.52	86.70	20.77	39.52	73.84	47.65	172.81
Missing heat demand	10.37	19.72	34.29	23.51	86.56	20.63	39.33	73.46	47.47	172.59
[MWt]	0.00	0.00	4.91	0.02	4.93	0.00	0.00	5.33	3.23	2.39
Contracted biomass [t]	0.00	0.00	4.91	0.02	4.93	0.00	0.00	5.33	3.23	2.39
	0.00	0.00	4.91	0.02	4.93	0.00	0.00	5.33	3.23	2.39
Contracted biomass [t]	3599.12	6874.81	12311.07	8507.09	30476.45	7387.85	15839.42	28720.36	18747.91	68260.61
Contracted biomass [t]	3562.97	6720.10	12166.78	8735.42	31481.13	8311.86	15497.95	27025.99	19399.62	69121.68
Contracted biomass [t]	3562.97	6720.10	12166.78	8735.42	31481.13	8311.86	15497.95	27025.99	19399.62	69121.68
Contracted options [t]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]	[0.00;0.00]
[Up, Down]	[278.27; 58.89]	[968.81; 303.44]	[154.34; 107.87]	[144.79; 995.86]	[350.61; 1963.46]	[277.17; 1765.12]	[1513.72; 1217.95]	[2458.18; 1332.23]	[1050.00; 2515.62]	[2858.83; 8630.06]
Actual delivered amount [t]	3869.60	7574.81	12311.07	8769.45	30824.92	7942.92	15839.42	28720.36	18747.91	68260.61
Storage level at end of planning horizon[t]	3869.60	7574.81	12311.07	8769.45	30824.92	7942.92	15839.42	28720.36	18747.91	68260.61
	3765.62	7006.94	12058.92	8005.36	29784.44	7437.54	15768.10	27663.52	17247.98	63109.97
	3789.01	7006.94	12064.13	8005.36	29785.34	6966.42	15204.07	26634.19	17181.36	62455.04
Storage level at end of planning horizon[t]	4743.46	9443.96	9984.19	7408.30	8963.43	11679.08	19598.51	27048.65	17586.58	28282.11
	4744.26	9089.26	9970.04	7226.66	6659.67	11659.00	18869.87	27594.41	17585.79	27769.16
	4458.32	8817.64	9364.34	5460.69	6250.62	8029.29	16694.76	17899.72	9150.32	8650.69
	4509.86	8817.64	9344.60	5401.08	5140.75	7725.86	15583.09	15569.88	8203.16	7779.79
CHP heat production [MWt]	11578.24	23099.28	39095.25	27874.72	101894.40	24543.24	50011.41	93342.32	61147.42	231924.69
	11582.85	23263.02	39099.04	27930.90	102709.42	24548.86	50278.72	93357.10	61147.71	232336.81
	11346.67	21399.77	38539.49	25878.22	99192.98	24154.76	50569.49	92727.87	58678.16	220405.81
	11409.61	21440.38	38564.68	25896.65	99626.17	22745.88	49141.38	89844.84	58783.63	218775.78
Auxiliary boiler production [MWt]	396.52	2202.69	8517.12	2798.38	13668.43	276.06	2247.65	4768.95	2046.88	6465.35
	392.31	2037.56	8507.10	2741.10	12851.56	268.89	1979.39	4756.06	2046.70	6053.33
	626.98	3903.68	9072.89	4792.32	16369.86	661.44	1689.57	5383.81	4515.32	17981.81
	564.57	3859.56	9042.73	4773.15	15934.08	2068.05	3115.75	8266.48	4407.42	19610.37

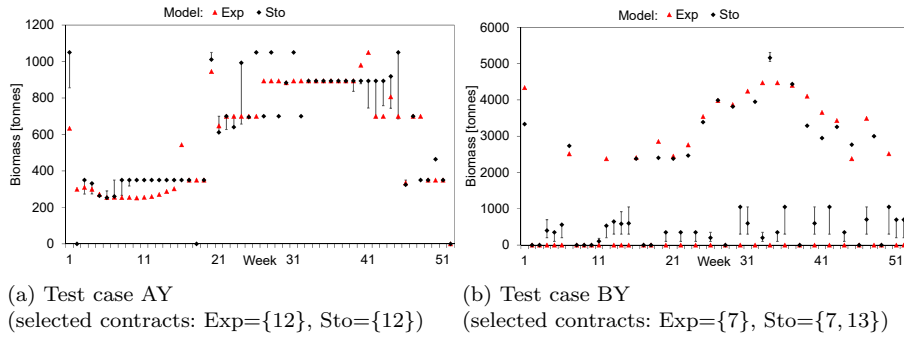


Fig. 6: Biomass contracts selected for yearly instances

Figure 6 shows the selected biomass contracts for municipalities A and B in the stochastic (*Sto*) and expected value solution (*Exp*), respectively. The contracts are valid for all samples. The data points show the contracted biomass amount and the vertical lines that extend from some of the crosses are the amount of upward and downward options bought, i.e., in which range the delivery amount can be changed during the operational planning problem. Because we can decide to buy options for each delivery individually only some of the deliveries have contracted options. We see that only the solutions obtained by the stochastic approach make use of options. As the expected value solution has no scenarios and assumes the expected values of uncertain parameters as deterministic, the contracts are selected in such a way that the solution fits these expected values. Thus, no use of options is reasonable in this case. However, when other biomass amounts are needed in the course of the year, the options contracted in the stochastic solution bring an advantage and reduce the overall cost (see Table 8). From Figure 6 also the difference in the delivery patterns for the two municipalities can be seen. The selected contract for municipality A (contract 12) has smaller amounts but more frequent deliveries. Whereas municipality B has two contracts, one with larger amounts and less deliveries, which relates also to the higher heat demand in municipality B (contract 7). It also uses contract 13 for smaller frequent deliveries to use biomass between the large deliveries.

The actual delivery amounts, i.e., after making use of options, and the biomass storage level are depicted in Figure 7 for the real data starting from 1st June 2016. The amounts are accumulated per week. Furthermore, the contracted delivery amount is depicted to show if the options are actually used in the course of the year. For both municipalities the operational problem uses the options, for example, in week 3 for municipality A and week 4 in municipality B.

The heat production from June 2016 to May 2017 for municipality A and B is shown in Figure 8. In both cases the heat demand was always fulfilled and the production follows similar behavior. At start of the season, the demand can be covered by the biomass-fired CHP. During the winter periods with a

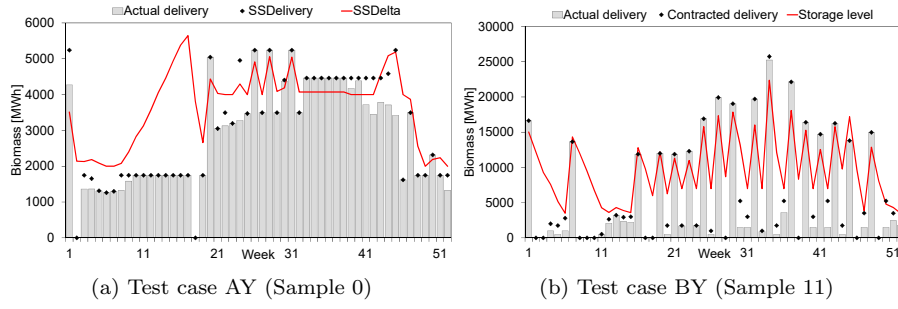


Fig. 7: Biomass storage level and deliveries for the real realization of uncertainties from 1st June 2016

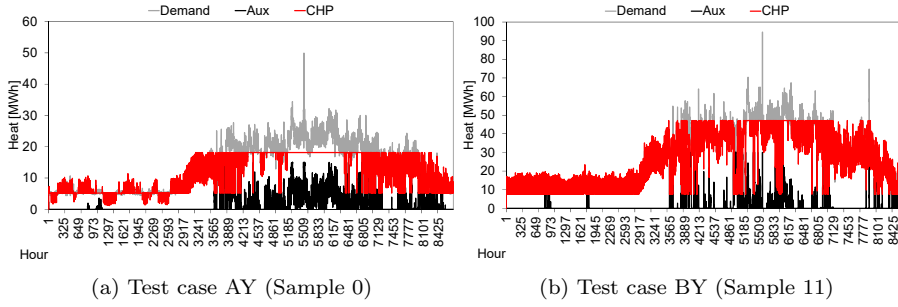


Fig. 8: Heat production for real realization of uncertainties from 1st June 2016

high demand, the auxiliary boiler is used in addition to the CHP to cover the heat demand. Furthermore, at the end of the season the boiler is used more often as in the beginning of the season due to a slightly higher demand and the biomass contract decisions contracting less biomass in the end of the season.

6.3 Hourly vs. weekly biomass contract selection

By using a weekly time scale in the biomass contract selection, some of the technical details of the operation of the plants are lost. For example, capacities are considered on a weekly scale and no commitment decisions for the CHP are included. Furthermore, the thermal storage are excluded from the weekly model.

In this section, the difference of using an hourly model for the biomass contract selection compared to the weekly model are investigated. We perform this analysis only for the quarterly instances, i.e., AQ1, AQ2, AQ3, AQ4, BQ1, BQ2, BQ3 and BQ4. The method used is *StoSto*. Thus, the analysis contains 88 evaluations per method (hourly or weekly).

The model for solving the biomass contract selection model with hourly time periods is a combination of the biomass contract selection in Section 4.1

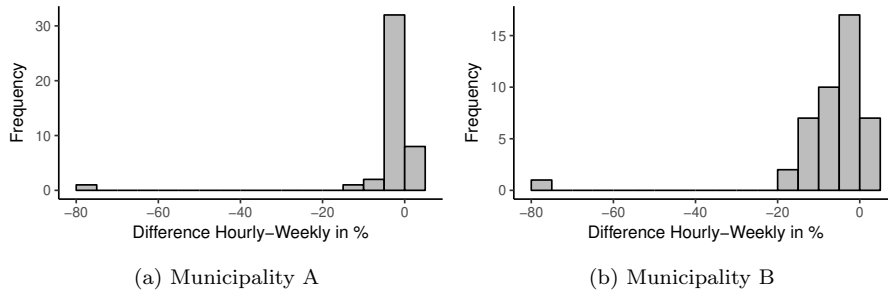


Fig. 9: Difference [%] (Hourly-Weekly) in the rolling horizon setting

using hours instead of weeks and the operational model in section 4.2 without the auxiliary boiler. The objective function and biomass contract constraints are used from the biomass contract selection model (1),(2) and (4)-(9). The technical constraints for the system are taken from the operational model (27),(28), (31)-(46), (48)-(55) and (57) excluding auxiliary boiler related variables. In the out-of-sample evaluation of the biomass contract solutions, we aggregate the solution of the hourly model to weekly contract decisions as it described in Table 3. Thus, deliveries in the hourly model do not need to be planned for a specific hour months in advance, which resembles the approach in the weekly model and in practice.

The results in Figure 9 show that using an hourly model with more technical details does not necessarily perform better in the out-of-sample evaluation. Actually, the weekly model performs better in many cases. In very few cases, the biomass contract decisions of the hourly model even lead to missing heat demand as the large negative difference shows in Figure 9. This observation can also be seen from the average objective values in Table 9. In the rolling horizon setting, the weekly model performs better on average. We can also solve the operational model for the entire sample at once, i.e., no rolling horizon approach. In this case, we assume to know the sample data for the entire planning horizon in the beginning of the quarter (which would not be the case in practice). In this case, the hourly model is more profitable in three of the eight test cases.

To explain why the weekly model often works better, the heat production, thermal storage usage and contract decisions for both the hourly and weekly model are given in Table 10. The table shows that the heat production of the CHP as well as the contracted biomass amounts are similar for both models. Also the total amount of biomass contracted in options is very similar. However, the weekly model contracts upward and downward options, while the hourly model only makes use of upward options. This is due to the representation of the thermal storage in the hourly model. In this model, the production from excess biomass can be stored in the thermal storage for later use making downward options not so valuable. This is not possible in the weekly representation. However, in the out-of-sample evaluation of the contract decisions, this

Table 9: Average objective values [x100,000€] (averaged over all 11 samples) for the out-of-sample evaluation solving the operational model in a *rolling horizon* approach or at once for the *entire sample* using the biomass contract decisions from the hourly (H) and weekly (W) model, respectively.

	Scale	AQ1	AQ2	AQ3	AQ4	BQ1	BQ2	BQ3	BQ4
Rolling horizon	H	10.86	20.40	43.65	24.20	23.34	41.04	93.96	49.44
	W	10.37	19.72	34.29	23.51	20.63	39.33	73.46	47.47
Entire Sample	H	10.66	20.20	34.17	24.08	23.34	41.00	73.49	49.44
	W	10.11	19.35	34.37	24.14	22.18	39.17	73.39	50.38

can lead to a negative effect of the hourly model solution, because the sample can have a heat demand which makes the downward options profitable. Therefore, the weekly model performs better in the out-of-sample evaluation. When knowing the entire sample in advance, the biomass contracts do not require so much flexibility. Which explains, the three cases where the hourly model performs better for the entire sample (see 9).

Finally, Table 10 also shows the computational time and remaining MIP gaps of the hourly and weekly biomass contract selection models. The timeout for the calculation was 24 hours. Compared to the weekly biomass contract selection model, the hourly model takes substantially longer. While the hourly model hits the timeout in 50% of the test cases, the weekly model is solved in less than 2 seconds for all cases. This is also the reason why we could not perform this analysis for the test cases AY and BY, where the solver did not find any solution in the given computational time (72 hours).

We conclude that using a weekly time scale in the biomass contract selection problem not only has significantly shorter computation times, but also adds more flexibility to the operational decisions leading to a better performance in unknown cases.

6.4 Runtime analysis

Figure 10 shows the runtimes for the yearly test cases AY and BY over the 11 samples per municipality. For most of the cases, the runtime to solve the operational model for one week is less than 60 seconds. Also, the biomass contract selection model is solved in less than 10 seconds for both municipalities (see week 0 in Fig. 10). The corresponding model sizes are given in Table 11.

For the few cases with a high runtime the average lies below 400 seconds (see Fig. 10a), which is short enough for a weekly planning problem to be used in practice. The weeks with higher runtime relate to samples where the heat demand is higher than expected in the biomass contract selection phase, which leads to a shortage of biomass in the subsequent weeks (in the beginning of the year in municipality A and in the end of the year in municipality B). Due to this shortage the model tries to avoid penalties for getting below the safety

Table 10: Comparison of biomass contract selection solution features for hourly (H) or weekly (W) time-scale. CHP heat and thermal storage inflow are averaged values over all five scenarios.

	Scale	AQ1	AQ2	AQ3	AQ4	BQ1	BQ2	BQ3	BQ4
CHP Heat [MWh]	H	11968.7	23441.4	38923.6	27997.5	25742.1	51293.2	93385.3	62561.1
	W	11969.1	23541.6	39266.6	28174.6	25761.1	51426.2	94621.7	62844.0
Thermal storage inflow [MWh]	H	330.48	1096.40	346.46	369.86	1060.08	1065.00	1467.88	1744.90
	W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Contracted biomass [t]	H	3917.8	6977.7	11948.5	8566.9	8880.2	16509.9	27432.2	19179.8
	W	3562.8	6720.1	12166.8	8735.4	8311.9	15498.0	27026.0	19399.6
Upward options [t]	H	359.49	1166.87	314.13	1078.40	2128.33	2623.93	3329.83	3460.67
	W	278.27	968.81	154.34	144.79	277.17	1513.72	2458.18	1050.00
Downward options [t]	H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	W	58.89	303.44	107.87	995.86	1765.12	1217.95	1332.23	2515.62
# Deliveries	H	19	31	5	28	28	23	10	17
	W	21	23	5	28	37	6	10	18
Runtime [s]	H	86470.2	51076.9	3532.0	17562.2	86462.3	86469.4	86468.2	8325.9
	W	0.7	5.4	0.4	0.6	0.9	1.5	0.8	1.0
Gap [%]	H	4.5%	0.0%	0.0%	0.0%	4.0%	0.4%	0.0%	0.0%
	W	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

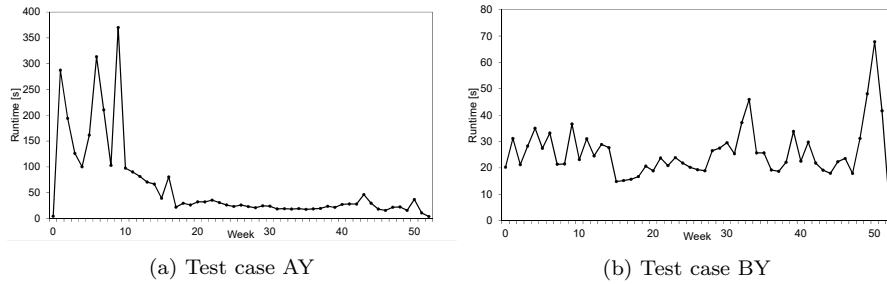


Fig. 10: Average runtimes per week (week 0 corresponds to biomass contract selection, averaged over all samples)

Table 11: Model sizes

	Cont. var.	Int. var. (thereof bin. var.)	Constraints	NZs
Biomass selection	10,878	689	(13)	1,3812
Operational - 4 weeks	60,500	13,440	(13,440)	550,340

storage level while producing as much as possible with the CHP to get income from the electricity market. As the production is not possible in all hours, the model has to select the hours with highest expected electricity prices making it harder for the solver to find the best solution as the electricity prices are close to each other.

7 Summary and outlook

In this work, we propose a solution approach that optimizes the biomass supply planning for a large-scale CHP producer using biomass. The decision-making process is divided into two phases both using two-stage stochastic programs. The first model, named *biomass contract selection*, is solved for a long-term horizon with weekly periods and configures the contracts from a set of biomass suppliers. Those decisions are used in the second model, named *operational planning*, to optimize the heat production. This solutions approach corresponds to the planning process in practice. We evaluate our method on two case studies with 10 test cases and realistic requirements and historical data to create scenarios. We analyze several scenario generation possibilities to create the scenarios based on past data and different forecasting tools. Our analysis investigates the results obtained for 22 samples of realizations of uncertainty.

In practice, biomass contracts are often selected based on one scenario, e.g., expected demand or worst case demand. Our approach shows that by just considering one scenario, a mismatch of needed amounts and ordered amounts can happen. However, our stochastic programming approach can model contractual options to avoid this and add more flexibility for the operator. We show that applying stochastic programming is required to make use of the options, yielding better results than in the expected value case where no options are purchased in most of the cases. Furthermore, the analysis shows that using a weekly timescale for the long-term biomass contract selection is working well and can even lead to better results than using more technical details on an hourly timescale.

We envision four future research directions. First, further uncertainties regarding the delivery of biomass such as amount and quality variations could be included in a supply chain planning model. Second, an economic analysis of the options and different types of contracts should be made to assess their benefit for the producer. That is from both supplier's and producer's points of view. Third, the performance of the method should be confirmed using further test cases. Finally, the comparison of different long-term forecasting tools with the use of data from previous years to create long-term scenarios is another future research direction.

Acknowledgements

The work of Ignacio Blanco, Daniela Guericke and Henrik Madsen is funded by Innovation Fund Denmark through the CITIES research center (no. 1035-00027B). The work of Juan M. Morales is partly funded by the Spanish Research Agency through project ENE2017-83775-P (AEI/FEDER, UE), and by the Research Funding Program for Young Talented Researchers of the University of Malaga through project PPIT-UMA-B1-2017/18. The authors would like to thank Ørsted A/S for their valuable input and comments to this work.

Declarations of interest

None

Appendix A Scenario generation

In this section, we describe the different approaches used for scenario generation in biomass contract selection and operational planning problem, respectively.

A.1 Biomass contract selection

In phase one of the solution approach, scenarios for the heat demand and the expected value for auxiliary boiler costs and electricity prices are part of the model. In this tactical planning problem, we use the heat consumption of the five previous years (i.e. 1st June 2011 - 31st May 2016) from summer to summer of the respective community as heat demand scenarios ($\tilde{D}_{t,\omega}$) resulting in five scenarios. The probability for each scenario is determined based on the year while giving a higher probability to more recent years (first three years: 0.15, last two years: 0.275).

The expected values for electricity and natural gas prices are obtained by calculating a linear combination of the observations of the last five years weighted by the probability ($\hat{x}_t = \sum_{i=1}^5 \Pi_{\omega_i} x_{t,i}$ where x_t is the price for time period $t \in \mathcal{T}$ in year i). Due to the weekly time periods, the values are averaged per week.

A.2 Operational planning problem

In the operational planning more recent information is available for the scenario generation, because we obtain new observations after each week. Furthermore, we are closer to the actual delivery time than in the biomass contract selection problem. Consequently, we can use time series analysis to predict the uncertainties more accurately by updating the models in every week.

There are different possibilities to obtain scenarios for the operational model. We implement and analyze five different types of scenario generation:

Using past data as predictions (P) Data from previous years is used to build scenarios analog to the biomass contract selection scenarios. The scenarios consist of the data from the respective week(s) in previous years.

Combining time series models and past data as predictions (F1) In this method, we use time series models to predict the first week of the rolling horizon and use data from previous years for the remaining weeks of the rolling horizon. The time series model uses the most recent observations to update the forecast for the following week. Heat demand and electricity prices (especially in the short-term horizon) have an autocorrelation and seasonality that can usually be detected using time series models [34]. For this specific case, we use an ARMAX model [22] where the weekly seasonality of prices and consumption is introduced using Fourier series in the form of exogenous parameters. The models have been fitted following the Box-Jenkins methodology [5], where we distinguish between model identification, estimation and diagnosis. Since ARMA models rely on past observations to predict future events, we use past data for the remaining weeks of the rolling horizon because they are further into the future and the risk of inaccurate predictions is higher. To create scenarios from the time series model, we follow the scenario generation process described in Conejo et al. [9]. More specifically, we generate 2500 equiprobable scenarios using Monte Carlo simulation and cluster them using the k-medoid algorithm to obtain five representative scenarios [17]. The forecasted scenarios for the first week have to be combined with data from previous years to get a scenario for the entire rolling horizon. Therefore, we add the data from the most recent year to the scenario with the highest probability.

Using time series models as predictions (F2) This method is similar to *F1*, because it also uses time series models for predictions and uses Monte Carlo simulation and clustering for generating scenarios. However, in this case we make predictions for the entire rolling horizon and do not combine with past data. The time series models, forecasts and scenarios are obtained following the same method as for *F1*.

All three above mentioned methods result in five scenarios for the operational planning problem. As two further possibilities for scenario generation, we use combinations of these methods. Namely, we combine the scenarios obtained from historical data (*P*) with the two time-series-based methods (*F1* and *F2*) resulting in ten scenarios. Note that the probabilities are normalized to result in a sum of one again. These methods are denoted by *P+F1* and *P+F2*.

Note that the above mentioned scenario generation is used for electricity prices and heat demands. For the gas prices in case study A with the gas boiler, we also use an expected value in the operational model. This is due to the fact, that gas prices are daily prices and are not as volatile as, e.g., the electricity price, and we deem the expected value as accurate enough for this model.

Table 12: Objective value [x100,000€] for each mode of scenario generation averaged over 1, 2, 3 or 4 weeks of rolling horizon

S.	Test case AY					S.	Test case BY				
	P	F1	F1+P	F2	F2+P		P	F1	F1+P	F2	F2+P
0	87.59	87.59	87.59	87.64	87.59	11	172.41	172.26	172.22	172.16	172.17
1	88.38	88.42	88.37	88.43	88.41	12	179.84	178.76	179.74	178.86	179.72
2	85.28	85.30	85.28	85.30	85.27	13	168.74	168.78	168.66	168.69	168.64
3	88.06	88.10	88.10	88.13	88.10	14	179.25	178.89	179.10	179.22	179.04
4	85.74	85.76	85.74	85.76	85.73	15	172.41	172.67	172.41	172.61	172.40
5	86.83	86.83	86.99	86.84	87.00	16	173.94	173.17	173.01	173.14	173.00
6	85.74	85.70	85.74	85.75	85.74	17	171.98	172.19	171.95	172.17	171.94
7	90.11	90.11	90.14	90.19	90.15	18	175.68	174.95	175.39	175.19	175.41
8	84.91	84.92	84.91	84.91	84.91	19	168.34	168.42	168.30	168.23	168.29
9	85.91	85.91	85.91	85.95	85.90	20	173.56	173.45	173.36	173.35	173.33
10	86.55	86.56	86.55	86.56	86.49	21	169.40	169.43	169.40	169.78	169.33

Appendix B Analysis of scenario generation methods

In this section, we compare the different methods for scenario generation. The results show the performance of the different scenario generation methods described in Section A of this Appendix for the operational planning problem.

Table 12 shows the results for each sample of both municipalities. The value shown is the average overall costs per scenario generation method averaged over different lengths of the rolling horizon (one, two, three or four weeks). The analysis of different rolling horizon lengths is described in Appendix. Based on Table 12, the best of the implemented scenario generation methods is $F2+P$, i.e., updating the scenarios every week by forecasting the next weeks of heat demand and additionally using previous years data as scenarios. Method $F2+P$ achieves the best result in 4 out of 11 samples for municipality A and in 6 out of 11 cases for municipality B. For the remaining cases, no common favorable method can be determined, as it differs per case.

Based on these results, we conclude for our test cases that it is beneficial to update the scenarios every week and using previous years' data. For application in practice, this should be evaluated individually. Our scenario generation methods can be easily replaced with already existing proved and tested forecasting methods of the operator.

Appendix C Analysis of length of rolling horizon

Table 13 shows the objective value and penalty costs for test cases AY and BY using scenarios generated by method $F2+P$ for different lengths of the rolling horizon, namely one, two, three and four weeks. Note that in no case, penalty costs for exceeding the biomass storage capacity occurred and these are therefore omitted from the table. The most important result is that the objective values are best using four weeks of rolling horizon in most of the cases. This is mainly due to the reduction in penalty costs for not fulfilling the heat demand (see Table 13) and the opportunity of using of options. If the

Table 13: Objective value and penalty costs [x100,000€] for different lengths of rolling horizon

	Sample	Objective				Penalty q^{miss}			
		1	2	3	4	1	2	3	4
Test case AY	0	88.19	87.39	87.38	87.40	1.70	1.38	1.38	1.38
	1	89.19	88.29	88.11	88.05	0.00	0.00	0.00	0.00
	2	86.63	84.86	84.81	84.80	0.24	0.00	0.00	0.00
	3	89.15	87.96	87.65	87.62	0.00	0.00	0.00	0.00
	4	86.84	85.39	85.36	85.34	0.00	0.00	0.00	0.00
	5	88.06	86.65	86.65	86.64	1.55	1.01	1.01	1.01
	6	86.12	85.61	85.62	85.62	0.00	0.00	0.00	0.00
	7	90.98	89.87	89.87	89.87	2.90	2.90	2.90	2.90
	8	84.89	84.93	84.92	84.91	0.00	0.00	0.00	0.00
	9	86.88	85.58	85.58	85.58	0.00	0.00	0.00	0.00
	10	86.99	86.34	86.33	86.29	0.13	0.13	0.13	0.13
Test case BY	11	173.33	171.91	171.73	171.69	1.30	1.30	1.30	1.30
	12	181.96	179.01	178.97	178.93	2.33	0.00	0.00	0.00
	13	170.13	168.44	168.06	167.92	0.00	0.00	0.00	0.00
	14	180.76	179.14	178.17	178.08	0.00	0.00	0.00	0.00
	15	173.24	172.20	172.15	172.03	0.00	0.00	0.00	0.00
	16	173.42	172.90	172.87	172.80	1.33	1.33	1.33	1.33
	17	172.37	171.84	171.81	171.73	0.00	0.00	0.00	0.00
	18	175.80	175.32	175.27	175.24	0.00	0.00	0.00	0.00
	19	168.70	168.26	168.12	168.07	0.00	0.00	0.00	0.00
	20	174.25	173.07	173.04	172.95	0.00	0.00	0.00	0.00
	21	170.07	169.20	169.05	169.00	0.00	0.00	0.00	0.00

rolling horizon already considers scenarios for weeks after the current week, we make use of this information now. If the biomass contract selection (phase 1) scheduled a delivery only in the current week, but not in the next week, having a longer planning horizon can be beneficial. If we only consider the current week, we may not make use of an option, because it is not needed now.

For a rolling horizon of more than one week, the results are quite similar. In a few cases, a longer horizon can lead to slightly poorer results due to the fact, that the heat demand is still uncertain and we may make use of an upward or downward option that corrects the delivery amount according to the uncertain scenarios. If the scenarios show a wrong trend in later weeks, it can be more beneficial to just include a second week (e.g. sample 8, test case AY). The penalty cost for missing the heat demand is $\Phi^{\text{Miss}} = 10000$ [€/MWh], which means we miss at most 29.04 MWh of heat in sample 7 for municipality B in an entire year. In all cases with penalty cost, the missing demand occurs in periods with an exceptionally high demand close to the capacity of the system. Those very high demands are often not covered by the scenarios and therefore wrong planning decisions may cause a shortage of biomass and a penalization for not satisfying the heat demand. Note that in practice a lack of supply in the district heating network would never occur, because the heat producer can gradually decrease the supply temperature or reduce the water

flow to increase the demand covered. However, these cases must be avoided and therefore we penalize them in the objective.

References

1. Aarhus Kommune (2017) Dataadgang. <http://varmeplanaarhus.dk/dataadgang/>, [accessed 08/03/2017]
2. Alipour M, Mohammadi-Ivatloo B, Zare K (2014) Stochastic risk-constrained short-term scheduling of industrial cogeneration systems in the presence of demand response programs. *Appl Energ* 136:393–404
3. Aringhieri R, Malucelli F (2003) Optimal operations management and network planning of a district heating system with a combined heat and power plant. *Ann Oper Res* 120(1-4):173–199
4. Birge JR, Louveaux F (2011) Introduction to stochastic programming. Springer Science & Business Media
5. Box GE, Jenkins GM, Reinsel GC, Ljung GM (2015) Time series analysis: forecasting and control. John Wiley & Sons
6. Bruglieri M, Liberti L (2008) Optimal running and planning of a biomass-based energy production process. *Energy Policy* 36(7):2430–2438
7. Chiarandini M, Kjeldsen NH, Nepomuceno N (2013) Integrated planning of biomass inventory and energy production. *IEEE T Comput* 63(1):102–114
8. Christidis A, Koch C, Pottel L, Tsatsaronis G (2012) The contribution of heat storage to the profitable operation of combined heat and power plants in liberalized electricity markets. *Energy* 41(1):75–82
9. Conejo AJ, Carrión M, Morales JM (2010) Decision making under uncertainty in electricity markets. Springer
10. Danish Energy Agency (2017) Regulation and planning of district heating in Denmark. https://ens.dk/sites/ens.dk/files/Globalcooperation/regulation_and_planning_of_district_heating_in_denmark.pdf, [accessed 07/05/2017]
11. De Meyer A, Cattrysse D, Van Orshoven J (2015) A generic mathematical model to optimise strategic and tactical decisions in biomass-based supply chains (OPTIMASS). *Eur J Oper Res* 245(1):247–264
12. Dimoukas I, Amelin M (2014) Constructing bidding curves for a CHP producer in day-ahead electricity markets. In: 2014 IEEE International Energy Conference (ENERGYCON), pp 487–494
13. Dimoukas I, Amelin M (2015) Probabilistic day-ahead CHP operation scheduling. In: 2015 IEEE Power Energy Society General Meeting, pp 1–5
14. Dvořák M, Havel P (2012) Combined heat and power production planning under liberalized market conditions. *Appl Therm Eng* 43:163–173
15. Energinet (2017) Energy data. <https://en.energinet.dk/Electricity/Energy-data>, [accessed 08/03/2017]

16. Frombo F, Minciardi R, Robba M, Sacile R (2009) A decision support system for planning biomass-based energy production. *Energy* 34(3):362–369
17. Hastie T, Tibshirani R, Friedman J (2009) Unsupervised learning. In: *The elements of statistical learning*, Springer New York, pp 485–585
18. Jensen IG, Münster M, Pisinger D (2017) Optimizing the supply chain of biomass and biogas for a single plant considering mass and energy losses. *Eur J Oper Res* 262(2):744–758
19. Kumbartzky N, Schacht M, Schulz K, Werners B (2017) Optimal operation of a CHP plant participating in the German electricity balancing and day-ahead spot market. *Eur J Oper Res* 261(1):390–404
20. Levitt CJ, Sørensen A (2014) *The Cost of Producing Electricity in Denmark*. The Rockwool Foundation, Denmark
21. Lund H (2007) Renewable energy strategies for sustainable development. *Energy* 32(6):912 – 919
22. Madsen H (2007) *Time series analysis*. CRC Press
23. Maurovich-Horvat L, Rocha P, Siddiqui AS (2016) Optimal operation of combined heat and power under uncertainty and risk aversion. *Energy Buildings* 110:415–425
24. Nielsen MG, Morales JM, Zugno M, Pedersen TE, Madsen H (2016) Economic valuation of heat pumps and electric boilers in the Danish energy system. *Appl Energy* 167:189–200
25. Pirouti M, Wu J, Bagdanavicius A, Ekanayake J, Jenkins N (2011) Optimal operation of biomass combined heat and power in a spot market. In: *2011 IEEE Trondheim PowerTech*, pp 1–7
26. Powernext (2017) Pegas markets - spot market data. <https://www.powernext.com/spot-market-data>, [accessed 08/03/2017]
27. Quddus MA, Chowdhury S, Marufuzzaman M, Yu F, Bian L (2018) A two-stage chance-constrained stochastic programming model for a bio-fuel supply chain network. *Int J Prod Econ* 195:27 – 44
28. Rolfsman B (2004) Combined heat-and-power plants and district heating in a deregulated electricity market. *Appl Energy* 78(1):37–52
29. Rong A, Lahdelma R (2007) Efficient algorithms for combined heat and power production planning under the deregulated electricity market. *Eur J Oper Res* 176(2):1219–1245, DOI 10.1016/j.ejor.2005.09.009
30. Scott J, Ho W, Dey PK, Talluri S (2015) A decision support system for supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments. *Int J Prod Econ* 166(Supplement C):226 – 237
31. Sénéchal S, Grassi G, Nordenmark M, Vivarelli F, Junginger M, Sikkema R (2009) *Logistic management of wood pellets: Data collection on transportation, storage and delivery management*. EUBIA-European Biomass Industry Association, Brussels, Belgium
32. Shabani N, Sowlati T, Ouhimmou M, Rönnqvist M (2014) Tactical supply chain planning for a forest biomass power plant under supply uncertainty. *Energy* 78:346–355

33. Soysal ER, Sneum DM, Skytte K, Olsen OJ, Sandberg E (2016) Electric boilers in district heating systems: A comparative study of the scandinavian market conditions. In: Swedish Association for Energy Economics Conference 2016
34. Weron R (2006) Modeling and forecasting electricity loads and prices: A statistical approach. John Wiley & Sons
35. Yin C, Rosendahl LA, Kær SK (2008) Grate-firing of biomass for heat and power production. *Prog Energ Combust* 34(6):725–754
36. Zugno M, Morales JM, Madsen H (2014) Robust management of combined heat and power systems via linear decision rules. In: 2014 IEEE International Energy Conference (ENERGYCON), pp 479–486