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A Two-Port Framework for the Design of Unconditionally Stable Haptic Interfaces

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Abstract

A haptic interface is a kinesthetic link between a human operator and a virtual environment. This paper addresses stability and performance issues associated with haptic interaction. It generalizes and extends the concept of a virtual coupling network, an artificial connection between a haptic display and a virtual world, to include both the impedance and admittance models of haptic interaction. A benchmark example exposes an important duality between these two cases. Linear circuit theory is used to develop necessary and sufficient conditions for the stability of a haptic simulation, assuming the human operator and virtual environment are passive. These equations lead to an explicit design procedure for virtual coupling networks which give maximum performance while guaranteeing stability. By decoupling the haptic display control problem from the design of virtual environments, the use of a virtual coupling network frees the developer of haptic-enabled virtual reality models from issues of mechanical stability.

I. INTRODUCTION

A haptic interface conveys a kinesthetic sense of presence to a human operator interacting with a computer generated environment. Historically, human-computer interaction has taken place through one-directional channels of information. Haptic interaction is fundamentally different in that kinesthetic energy flows bi-directionally, from *and to* the human operator. The human grasp may be responsible for stabilizing or destabilizing the overall system. Since the haptic display actively generates physical energy, instabilities can damage hardware and even pose a physical threat to the human.

A number of authors have proposed an artificial coupling between a haptic display and a virtual environment to create stable interaction. Colgate et. al. [1] introduced the idea of a virtual coupling which guarantees stability for arbitrary passive human operators and environments. Zilles and Salisbury [2] presented a heuristically motivated "god-object" approach which greatly simplifies control law design. These implementations can be grouped together as special cases of a virtual coupling network, a two-port interface between a haptic display and a virtual environment. This network can play the important role of making the stability of a haptic simulation independent of human grasp impedance and of the details of virtual environment design. The above-mentioned work focuses exclusively on impedance-type haptic displays. No similar work on virtual couplings has appeared for admittance displays and very little exists in explicit criteria for the design of virtual coupling networks.

This paper extends the concept of a virtual coupling to admittance displays and attempts to treat the problem of stable haptic interaction in a more general framework. Llewelyn's criteria for *unconditional stability* is introduced as a tool in the design and evaluation of virtual coupling networks. A benchmark example illustrates some fundamental stability and performance tradeoffs for different classes of haptic displays. The example also brings to light an important duality between the impedance and admittance models of haptic interaction.

II. PRELIMINARIES

A. Terminology

The following terms are used throughout this paper.

- *haptic display*- a mechanical device configured to convey kinesthetic cues to a human operator.

Haptic displays vary greatly in kinematic structure, workspace, and force output. They can be broadly classified into two categories: impedance displays and admittance displays. Impedance displays generate forces in response to measured displacements. They typically have low inertia and are highly back-drivable. The well known PHANToM [3] haptic displays fall into this class, along with many others. Admittance displays generate displacements in response to measured forces. These are often high-inertia, non back-drivable manipulators fitted with force sensors and driven by a position or velocity control loop. An example is Carnegie Mellon University's WYSIWYF Display [4], based upon a PUMA 560 industrial robot.

- *haptic interface*- a link between the human operator and a virtual environment, includes a haptic display and any software required to ensure stable interaction.

• *virtual environment* - a computer generated model of some physically motivated scene.

The virtual world may be as elaborate as a high-fidelity walk-through simulation of a new aircraft design, or as simple as a computer air hockey game. Regardless of its complexity, there are two fundamentally different ways in which a physically based model can interact with the haptic interface. The environment can act as an impedance, accepting velocities (or positions) and generating forces according to some physical model. This class includes all so-called penalty based approaches and to-date has been the most prevalent [3], [5]. The other possibility is for the virtual environment to act as an admittance, accepting forces and returning velocities (or positions). Included here are constraint based techniques. These approaches, already common in the computer science community, are now seeing application in haptic simulations [2], [6].

• *haptic architecture* - the choice of haptic display type (impedance or admittance) and virtual environment type (impedance or admittance), of which there are four possibilities.

• *haptic simulation* - the synthesis of human operator, haptic interface, and virtual environment which creates a kinesthetically immersive experience.

B. Two-port Characterizations

Two-port models, common in circuit theory, are a natural way of describing stability and performance in bilateral teleoperation [7], [8]. They have similar utility in haptic simulation for characterizing the exchange of energy between human operator, haptic interface, and virtual environment. A general two-port is a "black-box" which captures the relationship between *efforts* (forces F_1, F_2) and *flows* (velocities v_1, $-v_2$) at the two accessible terminals. The negative sign on velocity, seen here and throughout the paper, is necessary to maintain consistency with the network formalism.

The relationship between efforts and flows is commonly described in terms of an *immittance matrix*. We will refer to a mapping between two vectors, $y = Pu$, as an *immittance mapping* if $y^T u = F_1 v_1 + F_2(-v_2)$. The matrix P is then considered an immittance matrix. Possible immittance matrices are the impedance matrix, Z, the admittance matrix, Y, the hybrid matrix, H, and the alternate hybrid matrix, G.

C. Stability Concepts

For the following development, we represent a haptic interface as a linear two-port with terminals for a human operator (F_h, v_h) and a virtual environment (F_e, $-v_e$). The haptic interface may, or may not, include a virtual coupling network. Fig. 1 shows the components of a haptic simulation.

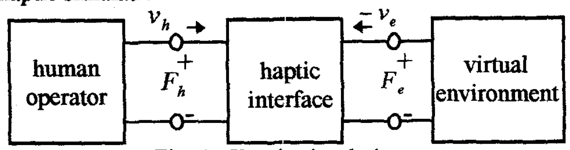

Fig. 1. Haptic simulation

We will base stability arguments upon the assumption that the human operator and virtual environment are passive operators. There is reasonable precedence for treating human interaction with a robotic manipulator as passive [9]. The design of virtual environments which presents a passive port to the haptic interface is the subject of ongoing research [6].

Definition: A linear two-port is *unconditionally stable* if and only if there exists no set of passive terminating one-port immittances for which the system is unstable [10].

For the problem at hand, unconditional stability means that the haptic interface must be stable for any set of passive human operators and virtual environments. In other words, the haptic interface will remain stable whether the operator holds it with a steel grip, or breaks contact completely. Simultaneously, the environment may simulate free or rigidly constrained motion.

For linear two-ports, Llewellyn's stability criteria provide both necessary and sufficient conditions for unconditional stability [11],

$$\mathrm{Re}(p_{11}) \ge 0,$$

$$2\,\mathrm{Re}(p_{11})\,\mathrm{Re}(p_{22}) \ge |p_{12}p_{21}| + \mathrm{Re}(p_{12}p_{21}) \qquad (1)$$

Together, these two inequalities imply $\mathrm{Re}(p_{22}) \ge 0$.

D. Performance Concepts

The performance of a haptic interface can be described in terms of transparency, the quality in which velocities and forces are passed between the human operator and the virtual environment. A haptic interface with perfect transparency has the hybrid mapping [7],

$$\begin{bmatrix} F_h \\ -v_e \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_h \\ F_e \end{bmatrix} \qquad (2)$$

Colgate and Brown [12] proposed using the *Z*-width as a measure of performance. The *Z*-width is defined as the achievable range of impedances which the haptic interface can stably present to the operator. This range is delimited by frequency dependent lower and upper bounds. The ideal haptic interface could simulate free motion without inertia or friction, as well as infinitely rigid and massive objects.

III. A BENCHMARK EXAMPLE

The following example, while simple, encompasses many of the most important factors which affect the stability and performance of haptic interfaces. These include open-loop device impedance, sample-hold effects, and, in the case of admittance displays, the gains of the inner servo loop. This benchmark problem reveals a number of fundamental issues in designing stable haptic interfaces and uncovers an important duality between the impedance and admittance models of haptic interaction. We consider a one degree-of-freedom, rigid manipulator with mass m and damping b, shown in Fig. 2. This device is governed by the equations of motion,

$$m\dot{v}_d + bv_d = F_h - F_d, \quad v_d = v_h \tag{3}$$

The velocity of the human operator at the point of contact with the device is v_h. The velocity of the device at the point of actuation is v_d. The force, F_h, is applied to/by human operator at the point of contact. F_d is the force applied by/to the device at the point of actuation.

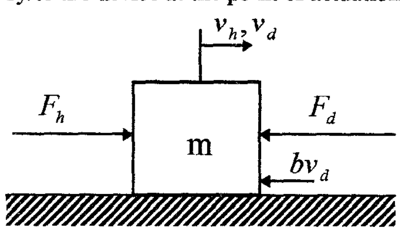

Fig. 2. A benchmark device.

A. Impedance Display

In the impedance model of haptic interaction, forces are applied to the human operator in response to measured displacements. The continuous-time transfer function from F_h to v_h and v_d can be found by taking the Laplace transform of (3). This function is discretized using Tustin's method, which preserves the passivity of the mapping. The discrete-form admittance function is,

$$Y_{d_I}(z) = \left(\frac{1}{ms+b}\right)\Bigg|_{s\to\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \tag{4}$$

Commands to the actuator must go through digital-to-analog conversion. The transfer function from F_d to v_h and v_d is formed by putting (4) in series with a zero-order hold. The effect of the zero-order hold is approximated by a low-pass filter with unity steady-state gain and 90 degrees phase lag at the Nyquist frequency.

$$ZOH(z) = \frac{(z+1)}{2z} \tag{5}$$

Defining the open loop impedance of the device as,

$$Z_{d_I}(z) = 1/Y_{d_I}(z), \tag{6}$$

we can form the discrete hybrid matrix of the impedance display,

$$\begin{bmatrix} F_h \\ -v_d \end{bmatrix} = \begin{bmatrix} Z_{d_I}(z) & ZOH(z) \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_h \\ F_d \end{bmatrix} \tag{7}$$

We now combine the haptic display with a virtual coupling network to form the haptic interface. Our goal is to design the virtual coupling network such that the combined system is unconditionally stable. In other words, no combination of passive human operator and virtual environment will destabilize the system. Fig. 3 illustrates the concept.

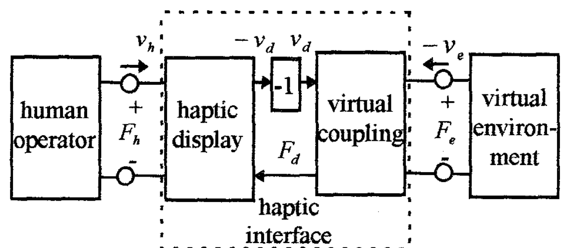

Fig. 3. The haptic interface for impedance display case

In general, the virtual coupling network can have arbitrary structure. A physically motivated example is a spring-damper with stiffness, k_c, and damping, b_c, linking the haptic display to the virtual environment. Fig. 4 shows the mechanical analog of this coupling.

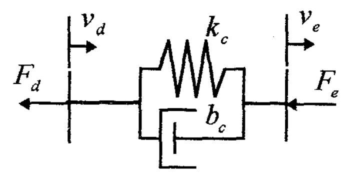

Fig. 4. Virtual coupling network, impedance display case

If we simulate an infinitely stiff environmental constraint, the stiffness perceived by the human operator is not infinite, but that of the virtual coupling. An optimal stability-performance trade-off is achieved when virtual coupling stiffness is maximized, while preserving the unconditional stability of the combined two-port.

For ease of implementation, discretization of virtual coupling impedance can be performed using a rectangular integration approximation,

$$Z_{c_I}(z) = \left(b_c + \frac{k_c}{s}\right)\Bigg|_{s\to\left(\frac{z-1}{Tz}\right)} \tag{8}$$

The hybrid mapping for the haptic interface is then the cascade connection of the impedance display with the virtual coupling network,

$$\begin{bmatrix} F_h \\ -v_e \end{bmatrix} = \begin{bmatrix} Z_{d_I}(z) & ZOH(z) \\ -1 & \dfrac{1}{Z_{c_I}(z)} \end{bmatrix} \begin{bmatrix} v_h \\ F_e \end{bmatrix} \tag{9}$$

Note that the only change from (7) to (9) is the addition of $1/Z_{c_I}(z)$ in the lower-right block.

Directly applying (1), the necessary and sufficient conditions for unconditional stability are,

$$\mathrm{Re}\left(Z_{d_I}(z)\right) \geq 0, \ \mathrm{Re}\left(1/Z_{c_I}(z)\right) \geq 0, \tag{10}$$

$$\cos(\angle ZOH(z)) + \frac{2\,\mathrm{Re}\left(Z_{d_I}(z)\right)\mathrm{Re}\left(1/Z_{c_I}(z)\right)}{|ZOH(z)|} \geq 1$$

$$z = e^{j\omega T} \tag{11}$$

We can make the following observations about (10) and (11),

- $\mathrm{Re}\left(Z_{d_I}(z)\right)$ can be interpreted as the physical damping of the impedance display. It must be non-zero and positive for unconditional stability to be possible. This is the level of damping the human operator feels when the virtual environment simulates free motion.
- $\mathrm{Re}\left(1/Z_{c_I}(z)\right)$ can be interpreted as the conductance of the virtual coupling. This function dictates the amount of "give" the human operator perceives in the haptic display when the virtual environment simulates a rigid constraint. Some minimum positive value of this "give" is necessary to achieve unconditional stability.
- Larger values of $\mathrm{Re}\left(Z_{d_I}(z)\right)$ permit smaller values of $\mathrm{Re}\left(1/Z_{c_I}(z)\right)$. This means that increasing device damping increases the maximum impedance that can be presented to the human operator. If we want to simulate rigid contact, significant physical damping in the haptic display is required. This observation is consistent with those made by Brown and Colgate [12].
- $\angle ZOH(z)$ is the phase loss due to sample-hold effects. Reducing the sampling frequency will cause an increase in this phase loss and require an augmentation in either device damping or virtual coupling conductance to maintain unconditional stability.

Manipulating (11) gives us the following condition for unconditional stability.

$$\mathrm{Re}\left(1/Z_{c_I}(z)\right) \geq \frac{1-\cos(\angle ZOH(z))}{2\,\mathrm{Re}\left(Z_{d_I}(z)\right)}|ZOH(z)|$$

$$z = e^{j\omega T} \tag{12}$$

Both sides of this inequality are functions of frequency. We now have a design procedure for the virtual coupling network. Plot the right-hand side of (12) versus frequency, then synthesize $1/Z_{c_I}(z)$ so that its real part is positive and exceeds this lower-bound.

Note that if the inequality (12) holds, unconditional stability is satisfied, regardless of whether an impedance or admittance type virtual environment is used. We can therefore design the haptic interface without considering the virtual environment implementation, as long as it is passive.

The hybrid matrix of the combined haptic interface network, (9), illustrates that to best approximate perfect transparency, (2), $Z_{c_I}(z)$ should be as large as possible. This means for performance, we want high virtual stiffness and virtual damping. The best virtual coupling is therefore one that drives (12) to equality, providing the minimum level of conductance for unconditional stability.

B. Admittance Display

In the admittance model of haptic interaction, the display generates displacements in response to measured forces. We can derive such a display by adding a proportional-plus-integral (PI) velocity control loop and measuring force at the point of device-human contact.

$$F_d = K_{PI}(v_d - v_{com}), \ F_{meas} = F_h \tag{13}$$

Note that PI feedback of velocity is equivalent to proportional-plus-derivative feedback of position. The former is used here for consistency of notation. v_{com} is the commanded velocity and F_{meas} is the measured force.

Using (7) and (13), we can define the complementary sensitivity tracking function

$$T(z) = \frac{ZOH(z)K_{PI}(z)}{Z_{d_I}(z) + ZOH(z)K_{PI}(z)}, \tag{14}$$

and the driving-point impedance function,

$$Z_{d_A}(z) = Z_{d_I}(z) + K_{PI}(z)ZOH(z) \tag{15}$$

The resulting alternate hybrid mapping for the admittance display is,

$$\begin{bmatrix} v_h \\ F_{meas} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{Z_{d_A}(z)} & -T(z) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_h \\ -v_{com} \end{bmatrix} \tag{16}$$

We can now make a very important observation. The network representation of the admittance display, (16), has a dual relationship to the network form of the impedance display, (7). Forces map to velocities, velocities map to forces, impedance functions map to admittance functions, and force transfer functions map to velocity transfer functions. This duality is useful when considering system stability and the design of virtual coupling networks.

Our goal is again to design a virtual coupling network such that the combined haptic interface network is unconditionally stable. Fig. 5 shows the combined system.

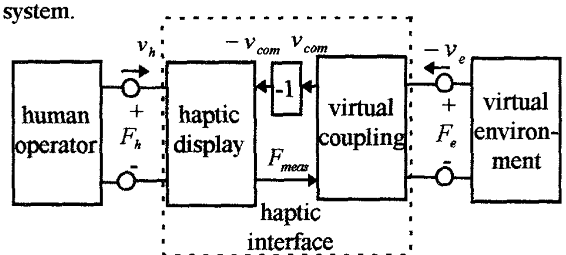

Fig. 5. The haptic interface for admittance display case

The choice of a virtual coupling function is not intuitive in this case. We know that in a network sense, the admittance display is the dual of the impedance display. It follows that the coupling for the admittance display should be the dual of the impedance display virtual coupling network. The mechanical dual of the parallel spring-damper in Fig. 4 is a series mass-damper combination. Fig. 6 shows a free-body-diagram of this coupling scheme.

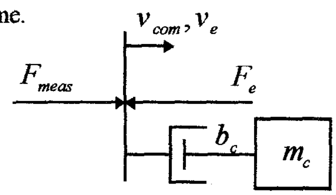

Fig. 6. Virtual coupling network for admittance display

In this case, the virtual coupling aims to provide some minimum level of impedance for the virtual environment. It limits the degree to which the haptic interface can simulate free motion. The chosen coupling can be thought of as a frequency-dependent damper. It has zero steady-state impedance. At high-frequencies the mass acts like a rigid constraint, giving an effective impedance of b_c. The admittance function of the virtual coupling is

$$Y_{c_A}(z) = \left(\frac{1}{b_c} + \frac{1}{m_c s} \right) \Bigg|_{s \to \left(\frac{z-1}{Tz} \right)} \tag{17}$$

The corresponding impedance function is $Z_{c_A}(z) = 1/Y_{c_A}(z)$. With the coupling in place, the human operator will always feel some level of viscosity and inertia in the haptic interface. The best stability/performance trade-off is achieved when coupling impedance is set to the minimum level which makes the combined two-port unconditionally stable.

The alternate hybrid mapping for the combined haptic interface network is,

$$\begin{bmatrix} v_h \\ F_e \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_{d_A}(z)} & -T(z) \\ 1 & Z_{c_A}(z) \end{bmatrix} \begin{bmatrix} F_h \\ -v_e \end{bmatrix} \tag{18}$$

With the virtual coupling in place, only the lower-right term has changed in the alternate hybrid matrix from (16) to (18). For unconditional stability, the necessary and sufficient conditions are,

$$\mathrm{Re}\left(Z_{c_A}(z)\right) \geq 0, \ \mathrm{Re}\left(1/Z_{d_A}(z)\right) \geq 0, \tag{19}$$

and

$$\cos(\angle T(z)) + \frac{2\,\mathrm{Re}(Z_{c_A}(z))\,\mathrm{Re}(1/Z_{d_A}(z))}{|T(z)|} \geq 1$$

$$z = e^{j\omega T} \tag{20}$$

We can make the following observations about (19) and (20),

- $\mathrm{Re}\left(Z_{c_A}(z)\right)$ can be interpreted as the damping of the virtual coupling. It must be non-zero and positive for unconditional stability to be possible. This damping is what the human operator feels when the virtual environment simulates free motion.
- $\mathrm{Re}\left(1/Z_{d_A}(z)\right)$ can be interpreted as the conductance of the admittance display. This function dictates the amount of "give" the human operator perceives in the haptic display when the virtual environment simulates a rigid constraint. Some minimum positive value is necessary to achieve unconditional stability.
- Larger values of $\mathrm{Re}\left(1/Z_{d_A}(z)\right)$ permit smaller values of $\mathrm{Re}\left(Z_{c_A}(z)\right)$. This means that reducing the inner loop gains, $K_{PI}(z)$, improves the ability of the haptic interface to simulate free motion. At the same time, high values of $K_{PI}(z)$ are desirable to simulate rigid constraints. The inner loop control must be chosen to strike a trade-off between these conflicting requirements.

Manipulating (20) gives us the following condition for unconditional stability.

$$\mathrm{Re}\left(Z_{c_A}(z)\right) \geq \frac{1-\cos(\angle T(z))}{2\,\mathrm{Re}\left(1/Z_{d_A}(z)\right)} |T(z)|,$$

$$z = e^{j\omega T} \tag{21}$$

A design procedure for the virtual coupling network is to plot the right-hand side of (21) versus frequency, then synthesize $Z_{c_A}(z)$ so that its real part is positive and exceeds this lower-bound. As before, unconditional stability is satisfied as long as (21) holds, regardless of whether an impedance or admittance type environment is used.

The alternate hybrid matrix of the combined haptic interface network, (18), shows that to maximize transparency, $Z_{c_A}(z)$ should be as small as possible. In other words, for performance, we want small b_c and m_c. The best virtual coupling network is one that minimally exceeds the lower-bound for unconditional stability.

IV. DISCUSSION

The virtual coupling impedance functions for the cases of impedance and admittance display, $Z_{c_I}(z)$ and $Z_{c_A}(z)$, restrict the impedance range which the haptic interface can present to the human operator and, in doing so, guarantee unconditional stability. $Z_{c_I}(z)$ generates an upper-bound on the maximum impedance of the impedance display, while $Z_{c_A}(z)$ creates a lower-bound on the minimum impedance of the admittance display. The physically motivated virtual couplings discussed in this paper represent only particular choices among infinite solutions. More complex network structures, including ones without mechanical analogs, will likely provide increased performance.

V. CONCLUSIONS

The two-port mapping of network theory provides a framework for the unification of different models of haptic interaction. Four possible haptic architectures can be formed by selecting either an impedance or admittance display and an impedance or admittance virtual environment model. The introduction of a virtual coupling network between the haptic display and the virtual environment guarantees the stability of the combined haptic interface for arbitrary passive human operator and environmental immittances. Necessary and sufficient conditions, based on Llewellyn's stability criteria, lead to an explicit procedure for the design of such couplings. We find that if the virtual environment is passive, the virtual coupling network design is independent of the impedance or admittance causality of the virtual environment model. In addition, the two-port network which arises in admittance display implementation and that which arises in impedance display implementation are dual. The unification of these different cases creates important insights into stability and performance for kinesthetic interaction with virtual worlds.

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