

A Two-Stage Fuzzy Chance-Constrained Model for Solid Waste Allocation Planning

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ABSTRACT. A two-stage double-sided fuzzy version of the chance-constrained mixed-integer programming (TDFCCMP) model was developed in this study for supporting municipal solid waste management under multiple uncertainties. TDFCCMP integrates the double-sided fuzzy version of the chance-constrained programming (DFCCP) model and the mixed-integer programming (MIP) model within a two-stage stochastic programming (TSP) framework. It could deal with possibilistic or probabilistic uncertainties and tackle complexities derived from capacity-expansion issues. A hypothetical long-term solid waste management problem was used to demonstrate the applicability of the proposed method. The results indicated that TDFCCMP was useful in assisting the decision makers analyze policy scenarios that were associated with economic penalties within a multi-stage and multi-period context. The model also allowed violation of system constraints at specified confidence-levels under two reliability conditions, leading to solutions with lower costs under acceptable magnitudes of system-failure risk. The generated solutions could help decision makers establish various waste-flow allocation patterns and capacity-expansion plans under complex uncertainties, and gain in-depth insights into the trade-offs between system economy and reliability.

Keywords: fuzzy version of the chance-constrained programming, mixed-integer programming, two-stage stochastic programming, solid waste management, uncertainty

1. Introduction

Municipal solid waste (MSW) management continues to be a challenging topic for many urban communities (Huang and Chang, 2003; Xi et al., 2008). In a typical MSW management system, many system parameters, such as waste-generation rate and treatment capacities, as well as their inter-relationships, might be uncertain. Therefore, incorporation of various uncertainties and complexities within a general optimization framework is desired to help evaluate the effects of various solid waste management policies (Li et al., 2008; Lv et al., 2010; Qin, 2011).

Over the past decades, many inexact simulation and optimization techniques have been developed for management of environmental and water resources systems. The majority of optimization methods were related to stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP) and interval linear programming (ILP) (Huang et al., 1993, 1995; Chang and Wang, 1997; Huang and Chang, 2003; Chang and Davila, 2007, 2008; Li et al., 2008; Liu et al., 2009; Xu et al., 2009; Zhu et al., 2009; Xu and Qin, 2010;

Sun and Huang, 2010; Xu and Qin, 2013, 2014). Among these techniques, the two-stage stochastic programming (TSP), as one of the SMP methods, was useful for situations where the analysis of policy scenarios is necessary and the associated uncertainties are expressed as probability distributions functions (PDFs). Previously, many integrated approaches based on TSP were applied in the solid waste management field (Maqsood et al., 2004; Li et al., 2008). As the related results demonstrated, TSP is effective in tackling the uncertainties presented as PDFs and taking corrective actions after a random event has taken place. However, it might be limited in model formulation, since the available information in real-world applications might not be of sufficient quality to be presented as PDFs. In addition, TSP cannot reflect the risk of constraint violations.

Recently, the fuzzy version of the chance-constrained programming (i.e. FCCP) has been presented as an innovative FMP method. The fundamental idea of FCCP is that it incorporates predefined confidence levels of constraints satisfaction into optimization models (Liu and Iwamura, 1998). According to the differences in the model constraints, FCCP is partitioned into the two categories: (i) single-sided FCCP (i.e. SFCCP), where the right-hand side coefficients of constraints were expressed as fuzzy numbers; (ii) double-sided FCCP (i.e. DFCCP), where both left-hand and right-hand side coefficients of constraints were presented as fuzzy formats simultaneously. Currently, applications of SFCCP models

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have been reported in many fields (Rong and Lahdelma, 2008; Cao et al., 2009). However, the applications of DFCCP were relatively limited. The main advantages of the DFCCP are that it could deal with the fuzzy uncertainties at both the left-hand and the right-hand side coefficients in model constraints, and allow system constraints be satisfied at specific confidence levels. However, it was incapable of providing a linkage between the pre-regulated policies and the associated economic implications. In addition, the DFCCP method can hardly handle binary-decision (i.e. yes/no decision) problems which may be important for seeking solutions to capacity-expansion or operation-scheduling issues (Li et al., 2008).

With various strengths and limitations, these methods are potentially combined into a general framework for tackling more complicated problems. Therefore, this study aims to develop an integrated model, namely two-stage double-sided fuzzy version of the chance-constrained mixed-integer programming (TDFCCMP) model, and apply it to a MSW management system. The objective entails: (i) formulation of a TDFCCMP model based on TSP, DFCCP and MIP approaches; (ii) application of the proposed model to a MSW management case; (iii) analysis of results and discussion of model applicability.

2. Overview of Solid Waste Management System

In a typical MSW management system, the manager is responsible for designing rational waste-flow allocation patterns with low cost and environmental damage. In addition, the manager also needs to decide whether the capacity-expansion planning of the treatment facilities should be implemented for meeting the waste-disposal demands. Optimization models can be used for dealing with such a problem. Many parameters within MSW system, such as waste-generation rate and disposal capacities, may appear uncertain; Meanwhile, these uncertainties could be further complicated by the dynamic features of the system and the interrelationships among various system components. Generally, these uncertainties could be addressed by different optimization approaches such as SMP, FMP and ILP.

Among them, TSP is effective for handling optimization problems where an analysis of policy scenarios is desired and the model's right-hand side uncertainties are expressed as probability distributions (Huang and Loucks, 2000). The waste-generation rate is directly associated with the decision alternatives and attract more attentions from the experts and public, and thus own complete record for generating the PDFs. In order to reflect the policies established by the local authority, the allowable waste-generation amounts from each district should also be incorporated into the management framework. The above characteristics of the system can be described by TSP. DFCCP improves upon the FMP by allowing fuzzy constraints to be satisfied at specified confidence levels with two reliability conditions. In the MSW system, the design safety coefficients and capacities of the incinerator and composting plant are subjected to human judgments, and should be represented by fuzzy formats. Along with the

socio-economic development, the waste-generation rates may be increased continuously and lead to high system costs while wastes are completely treated. DFCCP is especially useful for the above situations when a low system cost with a compromise of environmental protection is desired. In addition, in order to alleviate the contradiction between increased waste generation rates and decreased waste treatment/disposal capacity, the rational expansion scheme of capacities is necessary. Mixed integer programming (MIP) is a useful tool through using 0-1 integer variables to indicate whether a facility development or expansion option needs to be undertaken (Huang et al., 1995). Therefore, in this study, a TDFCCMP model is considered more advantageous and flexible in dealing with MSW management issues under multiple uncertainties.

3. General Methodology

3.1. Double-Sided Fuzzy Version of the Chance-Constrained Programming

Double-sided fuzzy version of the chance-constrained programming (DFCCP) was firstly proposed by Fiedler et al. (2006). In a DFCCP model, both left-hand and right-hand side coefficients in some constraints are represented by triangular fuzzy numbers. In addition, the ability to satisfy fuzzy constraints is expressed as a series of confidence levels with two reliability scenarios. According to Fiedler et al. (2006), a general DFCCP model could be written as:

$$\text{Minimize } f = \sum_{j=1}^J c_j x_j \tag{1a}$$

Subject to:

$$\text{Pos} \left\{ \tilde{a}_{ij}, \tilde{b}_i \left| \sum_{j=1}^J \tilde{a}_{ij} x_j \leq \tilde{b}_i \right. \right\} \geq \alpha_s, \quad \forall i \tag{1b}$$

$$\sum_{j=1}^J t_{jk} x_j \leq e_k, \quad \forall k \tag{1c}$$

$$x_j \geq 0, \quad \forall j \tag{1d}$$

$$c_j, a_{ij}, t_{jk} \neq 0, \quad \forall i, j, k. \tag{1e}$$

where j ($j = 1, 2, \dots, J$) is the index of decision variables; i is the index of fuzzy constraints, where $i = 1, 2, \dots, I$; α_s is the predefined confidence level where $s = 1, 2, \dots, S$, and S is the total number of the confidence levels; k is the index of deterministic constraints, where $k = 1, 2, \dots, K$; x_j are deterministic decision variables; \tilde{a}_{ij} and \tilde{b}_i are assumed to be triangular fuzzy numbers with fuzzy membership functions $\mu(\tilde{a}_{ij})$ and $\mu(\tilde{b}_i)$, respectively; c_j , t_{jk} and e_i are fixed coefficients.

Referring to Fiedler et al. (2006), each confidence level consists of two scenarios (i.e. the minimum and maximum reliabilities):

$$\begin{aligned}
 & Pos\{\tilde{a} \lesssim^{\min} \tilde{b}\} \\
 & = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)) \mid x, y \in \mathfrak{R}, x \leq y\} \geq \alpha_s \quad (2a) \\
 & \Leftrightarrow a^L(\alpha_s) \leq b^R(\alpha_s)
 \end{aligned}$$

$$\begin{aligned}
 & Pos\{\tilde{a} \lesssim^{\max} \tilde{b}\} = \\
 & \inf\{\max(1 - \mu_{\tilde{a}}(x), 1 - \mu_{\tilde{b}}(y)) \mid x, y \in \mathfrak{R}, x \leq y\} \geq \alpha_s \quad (2b) \\
 & \Leftrightarrow a^R(1 - \alpha_s) \leq b^L(1 - \alpha_s)
 \end{aligned}$$

$$b^R(\alpha_s) = \sup\{b \mid b = \mu^{-1}(\alpha_s)\} \quad (2c)$$

$$a^L(\alpha_s) = \inf\{a \mid a = \mu^{-1}(\alpha_s)\} \quad (2d)$$

where $\tilde{a} \lesssim^{\min} \tilde{b}$ and $\tilde{a} \lesssim^{\max} \tilde{b}$ present that the equation $\tilde{a} \leq \tilde{b}$ be satisfied at the minimum and maximum reliability, respectively. The item μ^{-1} is the inverse function of μ . According to Zadeh (1978), a triangular fuzzy set can be transformed into two deterministic values at each α -cut level (except for the peak point). Under the maximum reliability, the left-hand side parameters should achieve their maximum values, and the right-hand side parameters reach their minimum ones. This could guarantee that the “less than or equal to” constraints be more reliably satisfied under the impact of uncertainties. Conversely, under the minimum reliability, the left-hand side parameters reach their minimum values, and the right-hand side parameters would be maximum; the solutions from such a loose-restricted condition would easily lead to violation of constraints when system is deviated by influences of uncertainties. Based on Eqs (2a) to (2d), the constraints (1b) can be converted to two crisp equivalents, respectively. The two transformed models can be formulated as follows (Fiedler et al., 2006).

(1) Confidence levels under the minimum reliability:

$$Minimize \ f = \sum_{j=1}^J c_j x_j \quad (3a)$$

Subject to:

$$\sum_{j=1}^J a_{ij}^L(\alpha_s) x_j \leq b_i^R(\alpha_s), \quad \forall i \quad (3b)$$

$$\sum_{j=1}^J t_{jk} x_j \leq e_k, \quad \forall k \quad (3c)$$

$$x_j \geq 0, \quad \forall j \quad (3d)$$

$$c_j, a_{ij}, t_{jk} \neq 0, \quad \forall i, j, k. \quad (3e)$$

(2) Confidence levels under the maximum reliability:

$$Minimize \ f = \sum_{j=1}^J c_j x_j \quad (4a)$$

Subject to:

$$\sum_{j=1}^J a_{ij}^R(1 - \alpha_s) x_j \leq b_i^L(1 - \alpha_s), \quad \forall i \quad (4b)$$

$$\sum_{j=1}^J t_{jk} x_j \leq e_k, \quad \forall k \quad (4c)$$

$$x_j \geq 0, \quad \forall j \quad (4d)$$

$$c_j, a_{ij}, t_{jk} \neq 0, \quad \forall i, j, k. \quad (4e)$$

Finally, two groups of deterministic solutions, including the objective function values and decision variables (i.e. f_{opt} and $x_{j,opt}$), are obtained through solving models (3) and (4), respectively.

3.2. Two-Stage Stochastic Programming

The fundamental idea behind the TSP is the concept of recourse, which defines the ability to take corrective actions based on the identified results of the random event. A TSP model can be formulated as (Huang and Loucks, 2000):

$$Minimize \ f = \sum_{j=1}^J c_j x_j + E \left[\sum_{j=1}^J \sum_{h=1}^H p_h q_j y_{jh} \right] \quad (5a)$$

Subject to:

$$\sum_{j=1}^J a_{ij} x_j + \sum_{j=1}^J g_{ij} y_{jh} \leq b_i, \quad \forall i, h \quad (5b)$$

$$\sum_{j=1}^J d_{ij} x_j + \sum_{j=1}^J e_{ij} y_{jh} \leq z_{ih}, \quad \forall i, h \quad (5c)$$

$$x_j \geq 0, \quad \forall j \quad (5d)$$

$$y_{jh} \geq 0, \quad \forall j, h \quad (5e)$$

where h is the probabilistic level, where $h = 1, 2, \dots, H$, and H is the total number of probabilistic levels; x_j and y_{jh} are the first-stage and second-stage decision variables, respectively; z_{ih} is a discrete random variable described as a deterministic value at the probabilistic level h ; $E[\cdot]$ is the expected value of a random variable. To solve the model (5), each value of i and h in constraints (5b) and (5c), it is necessary to repeat the basic constraint set that relates z_{ih} to x_j and y_{jh} . Finally, the solutions of the objective function value, first-stage and second-stage decision variables will be obtained (i.e. f_{opt} , $x_{j,opt}$ and $y_{jh,opt}$). Thus, the optimal scheme is formulated as $W_{jh,opt} = [X_{j,opt} + y_{jh,opt}]$, $\forall j, h$.

3.3. Two-stage Double-Sided Fuzzy Version of the Chance-Constrained Programming

In real-world applications, it is difficult to find such variables (i.e. a_{ij} , g_{ij} , and b_i in Eq. 5b), which are completely deterministic. It is also allowable that some constraints (i.e. Eq. 5b) are satisfied in acceptable confidence levels in order to obtain cost-effective alternatives. In addition, some parameters may be described by many kinds of uncertain properties rather than single characteristic. To address these problems, an integrated two-stage double-sided fuzzy version of the chance-constrained programming (TDFCCP) model is formulated as follows:

$$\text{Minimize } f = \sum_{j=1}^J c_j x_j + E \left[\sum_{j=1}^J \sum_{h=1}^H p_h q_j y_{jh} \right] \quad (6a)$$

Subject to:

$$\text{Pos} \left\{ \tilde{a}_{ij}, \tilde{b}_i \left[\sum_{j=1}^J \tilde{a}_{ij} x_j + \sum_{j=1}^J \tilde{g}_{ij} y_{jh} \leq \tilde{b}_i \right] \right\} \geq \alpha_s, \quad \forall i, h \quad (6b)$$

$$\sum_{j=1}^J d_{ij} x_j + \sum_{j=1}^J e_{ij} y_{jh} \leq z_{ih}, \quad \forall i, h \quad (6c)$$

$$x_j \geq 0, \quad \forall j \quad (6d)$$

$$y_{jh} \geq 0, \quad \forall j, h \quad (6e)$$

Based on models (3) to (5), two groups of objective values, first-stage and second-stage decision variables at various confidence levels would be obtained (i.e. f_{opt} , $x_{j,opt}$ and $y_{jh,opt}$). Thus, the optimal schemes at various confidence levels are calculated as $W_{jh,opt} = [X_{j,opt} + y_{jh,opt}]$, $\forall j, h$. The optimization model in this study is coded in LINGO 12.0, which is a standard optimization software platform. Previously, many applications have demonstrated that LINGO provides an easy-to-use programming language and friendly user interface, and is mostly suitable for tackling large-scale real-world problems.

4. Case Study

4.1. Overview of the Study Case

In this study, a MSW management case is used for demonstrating the applicability of the proposed method (as shown in Figure 1). A typical MSW management system involves a number of processes, such as waste generation, storage, collection, transportation and treatment (Wilson, 1985). The municipal solid wastes typically include paper, yard waste, food waste, plastics, metals, glass, wood and others (Li et al., 2008). It is assumed that the above types of wastes are classified and pre-treated in the storage process, such that different types of wastes are allocated to the respective treatment plants according to their properties and facility capacities. In the studied system, three treatment options are available for disposing of solid wastes, including landfill, incineration and composting. A landfill is used to meet the demand of waste disposal or to receive residues from incinerators or composting plants; it typically has a cumulative capacity limit. In order to encourage residents to reduce the amount of wastes that end up at the landfill, the incinerator and composting plant are also used to dispose of wastes, and generate revenue returns under the limitation of daily operating capacities (Nie et al., 2006). From a long-term planning point of view, in the next fifteen years (with three five-year stages), waste-generation rates in the city will be increased continuously. This will lead to the fact that, the facilities might have insufficient capacities to meet the city's waste disposal demand in the future. Therefore, how to generate an integrated waste management scheme, which consists of the waste allocation pattern and the capacity-expansion plan, is highly concerned by decision makers.

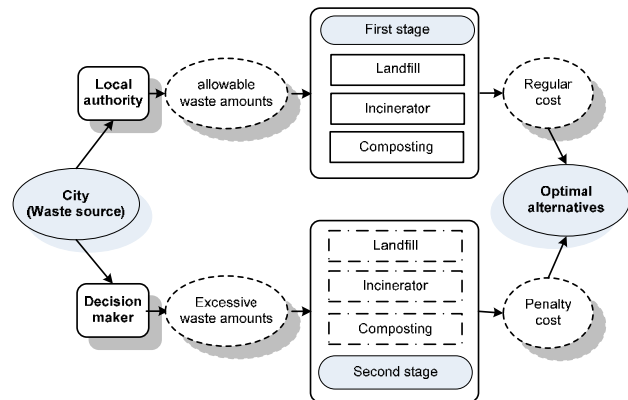


Figure 1. The study system.

According to the practical investigation, in this study, the waste-generation rates are assumed to be of random natures. Over the three periods, the low waste-generation rates at a probability of 0.2 are 250, 310 and 375 t/d, respectively. The medium values at a probability of 0.6 are 300, 370 and 435, respectively. The high rates at a probability of 0.2 are 350, 430 and 495 t/d, respectively (Li et al., 2008). Because of the increase of waste-generation rates, the capacity-expansion plans for the facilities would be considered. The related para-

Table 1. Capacity-expansion Options and Capital Costs for Different Facilities

Treatment/disposal facility	Expansion Option (u)	Time period		
		k = 1	k = 2	k = 3
Landfill				
Expansion capacity (10 ⁶ t)	u = 1	0.35	0.35	0.35
	u = 2	0.50	0.50	0.50
	u = 3	0.65	0.65	0.65
Capital cost (10 ⁶ \$)	u = 1	3.60	3.10	2.60
	u = 2	4.10	3.70	3.10
	u = 3	4.60	4.20	3.60
Incinerator				
Expansion capacity (t/d)	u = 1	25	25	25
	u = 2	40	40	40
	u = 3	55	55	55
Capital cost (10 ⁶ \$)	u = 1	2.5	2.2	1.9
	u = 2	3.0	2.7	2.4
	u = 3	3.4	3.1	2.8
Composting plant				
Expansion capacity (t/d)	u = 1	15	15	15
	u = 2	30	30	30
	u = 3	45	45	45
Capital cost (10 ⁶ \$)	u = 1	1.7	1.4	1.1
	u = 2	2.2	1.9	1.6
	u = 3	2.6	2.3	2.0

* The related data are referred to Li et al. (2008).

meters associated with the expansion options are listed in the Table 1. Other economic parameters, such as the regular costs for waste collection, transportation and treatment, are estimated based on practical investigations in the research areas and facilities. The penalty costs for excess waste flows are expressed in terms of raised collection, transportation, and operation costs, significantly higher than the regular ones (Li et al., 2008). Tables 2 and 3 provide transportation costs for allowable and excess waste flows, operating costs of facilities, penalty costs for surplus waste flows, and revenues.

Among various facilities, the landfill has a stable cumulative capacity and will be presented as a fixed value. As for other facilities (e.g. incinerator and composting plant), the random arrival and service times of waste delivery vehicles could result in waste buildup, leading to a risk of contingent insufficiency in the receiving facility (Nie et al., 2006). The introduction of the safety coefficient would be used to tackle this problem. Usually, this coefficient is estimated empirically and is thus of fuzzy nature. The design safety coefficients for incinerators and composting plants are assumed to be (1.45, 1.6, 1.8) and (1.35, 1.5, 1.7), respectively. In addition, the capacities of the incinerator and composting plant could be influenced by many factors such as the service time of facilities, property of receiving wastes, and operation manner of workers. These parameters are difficult to be accurately defined and, in most cases, their values are subject to human judgment. Fuzzy set theory is capable of handling the vague feature of such problems. Potential techniques to identify the fuzzy possibility distribution information may

Table 2. Regular Costs for the Allowable Waste Flows

Regular costs	Planning period		
	k = 1	k = 2	k = 3
Transportation cost for allowable waste (\$/t)			
To landfill	16	17.6	19.4
To incinerator	12	13.2	14.6
To composting plant	15.1	16.6	18.2
Operational cost for allowable waste (\$/t)			
Landfill	37.5	50	65
Incinerator	62.5	75	85
Composting plant	67.5	80	87.5
Transportation cost for allowable residue (\$/t)			
To incinerator	5.9	6.5	7.2
To composting plant	11.0	12.1	13.2
Revenue generated by allowable waste (\$/t)			
Incinerator	15	20	25
Composting plant	20	25	30

* The related data are referred to Li et al. (2008).

Table 3. Penalty Costs for the Excess Waste Flows

Penalty costs	Planning period		
	k = 1	k = 2	k = 3
Transportation cost for excess waste (\$/t)			
To landfill	24	26.4	29
To incinerator	19	20.8	22.8
To composting plant	22.6	24.8	27.2
Operational cost for excess waste (\$/t)			
Landfill	52.5	75	97.5
Incinerator	90	107.5	122.5
Composting plant	127.5	137.5	147.5
Transportation cost for excess waste residue (\$/t)			
To incinerator	8.8	9.8	10.7
To composting plant	16.4	17.9	19.7
Revenue generated by excess waste (\$/t)			
Incinerator	15	20	25
Composting plant	20	25	30

* The related data are referred to Li et al. (2008).

include expert consultation, public survey, and stakeholders meeting. For simplicity in demonstrating the proposed method, the triangular fuzzy sets are used to describe the related parameters. Therefore, the existing capacities of the facilities are expressed in terms of triangular fuzzy numbers over the three periods. The capacities of the incinerator are (85, 90, 97), (98, 103, 110) and (110, 115, 122) t/d, respectively. For the composting plant, they are (62, 65, 70), (75, 78, 83) and (87, 90, 95) t/d, respectively.

4.2. Formulation of a TDFCCMP model

The MIP would be incorporated into a general TDFCCP framework for handling capacity-expansion issues, such that a TDFCCMP model for the MSW management can be formulated as follows (Li et al., 2008).

Objective function:

$$\begin{aligned}
 \text{Min } f = & \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K L_k X_{ijk} (TR_{ijk} + OP_{ik}) + \\
 & \sum_{i=2}^I \sum_{j=1}^J \sum_{k=1}^K L_k X_{ijk} FE_i (FT_{ik} + OP_{1k}) + \\
 & \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H L_k P_{jh} M_{ijkh} (DR_{ijk} + DP_{ik}) + \\
 & \sum_{i=2}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H L_k P_{jh} M_{ijkh} FE_i (DT_{ik} + DP_{1k}) + \\
 & \sum_{m=1}^M \sum_{k=1}^K FLC_{mk} Y_{mk} + \sum_{i=2}^I \sum_{m=1}^M \sum_{k=1}^K FTC_{imk} Z_{imk} - \\
 & \sum_{i=2}^I \sum_{j=1}^J \sum_{k=1}^K L_k X_{ijk} RE_{ik} - \sum_{i=2}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{h=1}^H L_k P_{jh} M_{ijkh} RM_{ik}
 \end{aligned} \tag{7a}$$

Subject to,

(1) Constraints of treatment/disposal capacities:

$$\sum_{j=1}^J \sum_{k=1}^{k'} L_k \left[(X_{1jk} + M_{1jkh}) + \sum_{i=2}^I FE_i (X_{ijk} + M_{ijkh}) \right] \tag{7b}$$

$$\leq LC + \sum_{m=1}^M \sum_{k=1}^{k'} \Delta LC_{mk} Y_{mk} \quad \forall h \text{ and } k' = 1, 2, \dots, K$$

$$\begin{aligned}
 \tilde{\alpha} \sum_{j=1}^J (X_{ijk} + M_{ijkh}) & \leq \tilde{TC}_i + \\
 \sum_{m=1}^M \sum_{k=1}^{k'} \Delta TC_{imk} Z_{imk} & , \quad \forall k, \tag{7c}
 \end{aligned}$$

(2) Waste disposal demand constraints:

$$\sum_{i=1}^I (X_{ijk} + M_{ijkh}) \geq W_{jkh}, \quad \forall j, k, h, \tag{7d}$$

(3) Non-negativity and technical constraints:

$$X_{ijk \max} \geq X_{ijk} \geq M_{ijkh} \geq 0, \quad \forall i, j, k, h, \tag{7e}$$

(4) Constraints of capacity-expansion options:

$$0 \leq Y_{mk} \leq 1, 0 \leq Z_{imk} \leq 1, \quad \forall m, k \text{ and } i = 2, 3, \dots, I. \tag{7f}$$

$$\sum_{m=1}^M \sum_{k=1}^K Y_{mk} \leq 1, \sum_{m=1}^M Z_{imk} \leq 1, \quad \forall k \text{ and } i = 2, 3, \dots, I. \tag{7g}$$

Y_{mk} and Z_{imk} are integers;

where f is the net system cost (\$); k ($k = 1, 2, \dots, K$) is the index of time periods where K is the number of time periods;

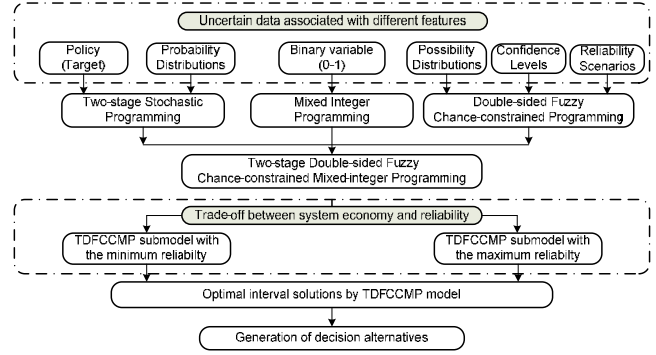


Figure 2. Framework of the TDFCCMP model.

k' is an intermediate index satisfying $I \leq k' \leq K$; L_k is length of time period k (d); i is type of waste management facility, where $i = 1$ for landfill, and $i = 2, 3, \dots, I$ for other waste treatment facilities; j is name of district, $j = 1, 2, \dots, J$; h denotes level of waste-generation rate in district j , and $h = 1, 2, \dots, H$; DP_{ik} and OP_{ik} are operating cost of facility i for allowable and excess waste flow during period k (\$/t); DR_{ijk} and TR_{ijk} are transportation cost for allowable and excess waste flow from district j to facility i during period k (\$/t); DT_{ik} and FT_{ik} are transportation cost of allowable and excess waste residue from waste treatment facility i to the landfill during period k (\$/t); FE_i is residue flow rate from facility i to the landfill; FLC_k and FTC_{imk} are capital cost of facilities expansion in period k (\$); LC is existing landfill capacity (t); ΔLC_{mk} is the amounts of landfill capacity expansion with option m in period k (t); M_{ijkh} is amount by which the allowable waste flow level (X_{ijk}) is exceeded when the waste-generation rate is w_{jkh} ; p_{jh} is probability of random variable; w_{jkh} is amount of waste generated in district j with waste-generation rate h in period k ; RE_{ik} and RM_{ik} are revenue from facility i during period k (\$/t); TC_i is existing capacity of facility i (t/d); ΔTC_{imk} is level of capacity expansion option m for facility i at the start of period k (t/d); X_{ijk} and $X_{ijk \max}$ are allowable and maximum allowable waste flow from district j to facility i during period k (t/d); Y_{mk} and Z_{imk} are binary decision variable for facilities expansion with option m at the start of period k ; $\tilde{\alpha}$ is design safety factor assuring the waste flows can be treated during period k .

Figure 2 shows the general framework of the TDFCCMP model. As an integration of TSP, DFCCP and MIP, the advantages of these techniques are combined together in TDFCCMP. For example, the probability distributions and policy implications were handled by the TSP; the uncertainties of fuzzy nature and constraints violation at various confidence levels were reflected due to the existence of the DFCCP; the capacity-expansion planning was described by the MIP. The detailed procedures of solving TDFCCMP model are summarized as follows:

Step 1: Identify all uncertain variables and acquire the related uncertain information in a MSW management system;

Step 2: Formulate a TDFCCMP model;

Step 3: Convert the fuzzy chance-constraints to their re-

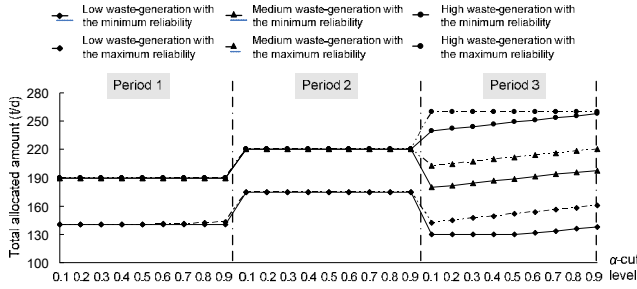


Figure 3. Total waste flows to the landfill under two reliability scenarios.

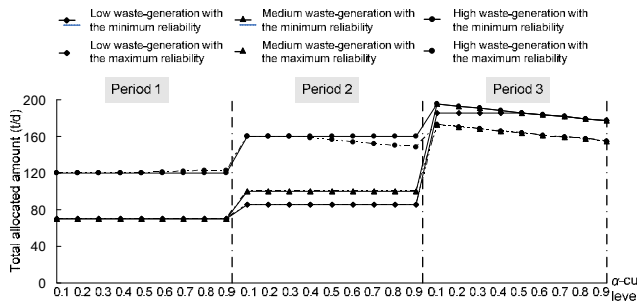


Figure 4. Total waste flows to the incinerator under two reliability scenarios.

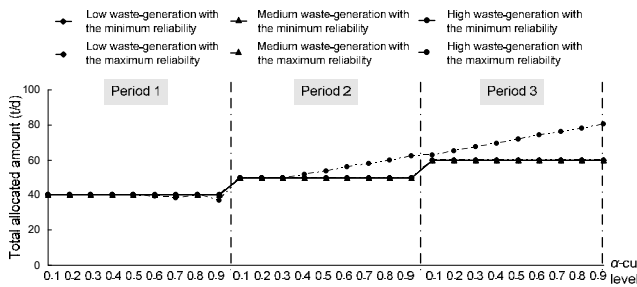


Figure 5. Total waste flows to the composting plant under two reliability scenarios.

spective crisp equivalents at various confidence-levels under two reliability scenarios;

Step 4: Solve the two sub models, and obtain f_{opt} , $X_{ijk,opt}$ and $M_{ijkh,opt}$;

Step 5: Calculate and obtain the final solutions $A_{ijkh,opt} = [X_{ijk,opt} + M_{ijkh,opt}]$, $\forall i, j, k, h$.

5. Result Analysis

Figures 3 to 5 show the obtained solutions of continuous decision variables through TDFCCMP model at various confidence levels with the two reliability scenarios. Table 4 shows the solutions of continuous decision variables at specific confidence levels. It is demonstrated that TDFCCMP owns characteristics of the DFCCP, MIP and TSP approaches.

From Figures 3 to 5, it is demonstrated that the temporal and spatial variations of waste-generation rates may result in varied waste allocation patterns. Generally, the treated

amounts of the facilities over the three periods would increase. For example, at a confidence level of 0.3 with the minimum reliability, over the three periods, the waste flows allocated to the incinerator under low waste-generation rates were 70, 85 and 185 t/d, respectively. Under the high waste-generation rate, the treated amounts of the landfill facility were 190, 220 and 244.43 t/d, respectively. This is because the waste-generation rate will increase during the three periods; meanwhile, the waste-generation rates will increase at the three probabilistic levels. This also indicates that the waste-generation rate is a main factor in the MSW management system and poses considerable influence on generating the management alternatives.

Table 4 also indicated that the allocated amounts among the three facilities have considerable differences. In aspect of excess waste, the allocated amounts to the landfill were the highest, that to the incinerator ranks in the middle, and that to the composting plant is the lowest. For example, at a confidence level of 0.5 with the maximum reliability, over the three periods, the excess waste flows to the landfill under high waste-generation rate were 95, 110 and 130 t/d, respectively. The excess amounts to the incinerator were 50.31, 71.18 and 58.24 t/d, respectively. Those transferred to the composting plant were 0, 3.82 and 11.76 t/d, respectively. This is due to the fact that, to minimize total cost is the objective of TSP; meanwhile, the sum of regular and penalty costs of the landfill are lowest. This also demonstrates that TSP is able to take corrective actions after a random event have taken place, and thus facilitates the generation of the cost-effective management strategies.

Figures 3 to 5 also indicated that the solutions of decision variables significantly vary with confidence levels (from 0.1 to 0.9). Generally, the waste flows allocated to the landfill would increase. Conversely, the allocated amounts to the incinerator would decrease while it remains unchanged for the composting plant. For example, at the period 3, the total waste flows allocated to the landfill under the high waste generation rate would be 239.57, 242.03, 244.43, 246.79, 249.10, 251.36, 253.59, 255.76 and 257.90 t/d, respectively. Correspondingly, the allocated amounts to the incinerator would be 195.43, 192.97, 190.57, 185.90, 183.64, 181.41, 179.24 and 177.10 t/d, respectively. The allocated amounts to the composting plant would be 60 t/d. The reason of the above facts is that, based on DFCCP, as the increase of the confidence levels, the capacity constraints of the incinerator and composting plant would be strict, leading to the decrease of the disposal amounts by the incinerator. Meanwhile, the disposal amounts of the landfill would increase in order to meet the total disposal requirement. As the increase of the confidence levels, the possibility of system-failure risk (i.e. risk of insufficiency in the receiving facilities) would be low; meanwhile, the total system cost may be higher. This situation also reflects the trade-off between the system economy and reliability.

From Table 4, it is also indicated that, at the same confidence level, the allocated amounts to the landfill and composting plant at the maximum reliability are higher than that at the minimum reliability. The variation trend of the disposal

Table 4. Solutions of TDFCCMP Model

Facilities (j)	Probability (h)	Period (k)	$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$		
			Min	Max	Min	Max	Min	Max	Min	Max	
To landfill (t day ⁻¹)	p = 1	k = 1	140	140	140	140	140	140.99	140	142.29	
		k = 2	175	175	175	175	175	175	175	175	
		k = 3	130	144.88	130	149.52	131.36	153.95	135.76	158.18	
	p = 2	k = 1	190	190	190	190	190	190	190	190	
		k = 2	220	220	220	220	220	220	220	220	
		k = 3	182.03	204.88	186.79	209.52	191.36	213.95	195.76	218.18	
	p = 3	k = 3	242.03	260	246.79	260	251.36	260	255.76	260	
	To incinerator (t day ⁻¹)	p = 1	k = 1	70	70	70	70	70	70	70	70
			k = 2	85	85	85	85	85	85	85	85
k = 3			185	170.12	185	165.48	183.64	161.05	179.24	156.82	
p = 2		k = 2	100	100	100	100	100	100	100	100	
		k = 3	192.97	170.12	188.21	165.48	183.64	161.05	179.24	156.82	
		k = 1	120	120	120	120	120	120.99	120	122.29	
p = 3		k = 2	160	160	160	158.33	160	154.07	160	150	
		k = 3	192.97	170.12	188.21	165.48	183.64	161.05	179.24	156.82	
		k = 1	40	40	40	40	40	40	40	40	
To composting plant (t day ⁻¹)	p = 1	k = 2	50	50	50	50	50	50	50	50	
		k = 3	60	60	60	60	60	60	60	60	
		k = 1	40	40	40	40	40	39.01	40	38.35	
	p = 3	k = 2	50	50	50	51.67	50	55.93	50	60	
		k = 3	60	64.88	60	69.52	60	73.95	60	78.18	
		k = 1	40	40	40	40	40	40	40	40	

* The treated amounts by the landfill with the high generation rates during the periods 1 and 2 are same with those with the medium ones. The allocated amounts to the composting plant with medium rates are same as those with low generation rates.

amounts by the incinerator is opposite. For example, in period 3, corresponding to confidence levels (0.2, 0.4, 0.6 and 0.8) with the maximum reliability, the waste flows allocated to landfill under medium waste-generation rates would be 204.88, 209.52, 213.95 and 218.18 t/d, respectively. The amounts under the minimum reliability would be 182.03, 186.79, 191.36 and 195.76 t/d, respectively. Similarly, the amounts allocated to the composting plant under high waste-generation rate would be 64.88, 69.52, 73.95 and 78.18 t/d, respectively. The amounts under the minimum reliability are constants, being 60 t/d, respectively. As for the incinerator, at the maximum reliability, the amounts to the incinerator under high waste-generation rates would be 170.12, 165.48, 161.05 and 156.82 t/d, respectively. The amounts at the minimum reliability are 192.97, 188.21, 183.64 and 179.24 t/d, respectively. To calculate the variation amounts of the incinerator and composting plants, the total allocated amounts to the incinerator and composting plant at the maximum reliability are lower than those at the minimum reliability. This is due to the fact that, the facility capacities under the maximum reliability would be lower than those under the minimum one, leading to the decrease of the total allocated amounts to the incinerator and composting plant; meanwhile, the amounts allocated to the landfill would increase. In addition, the total costs of the incinerator are lower than that of the composting plant, such that the excess waste would be allocated to incinerator firstly and then to composting plant. This also can be used to explain that the amounts transferred to the incinerator at the two reliability scenarios are higher than those to the composting plant.

In addition, the variation in waste-flow allocated patterns also results in the changes of capacity-expansion plans. For example, at a confidence level of 0.1 with the maximum reliability, the composting plant would be expanded during period 3, with an increment of 15 t/d, respectively. At a confidence level of 0.9, the composting plant would be expanded at the period 2, with each having an increment of 30 t/d, respectively. This is because as the increases of the confidence levels, the amounts allocated to the composting plant would increase. Figure 6 presents the variation of total costs at various confidence levels with the minimum and maximum reliabilities. The system costs would increase as the increase of confidence levels; meanwhile, system costs under the minimum reliability would be lower than those under the maximum reliability. For example, at various confidence levels (from 0.1 to 0.9) under the minimum reliability, the system costs are 166.476, 166.499, 166.520, 166.542, 166.563, 166.586, 166.611, 166.636 and 166.660 ($\times 10^6$ \$), respectively. Under the maximum reliability, the system costs are 167.828, 167.871, 168.214, 168.273, 168.352, 168.448, 168.541, 168.633 and 169.222 ($\times 10^6$ \$), respectively. This is due to the fact that, as the increases of the confidence level, the capacities of the facility would decrease. Moreover, the facility capacities also would decrease from the minimum to the maximum reliability. This implies that a low system cost could lead to a high system failure risk. The trade-off between the total system cost and the reliability of satisfying model constraints needs to be analyzed in order to gain an in-depth insight into the characteristics of solid waste management systems.

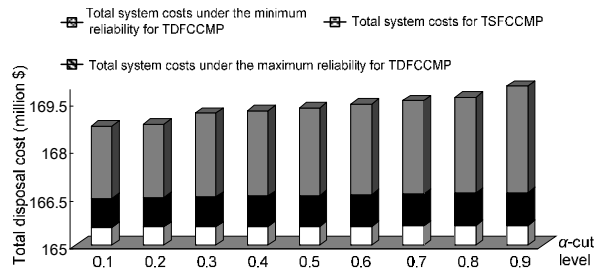


Figure 6. Comparison of system cost between TDFCCMP and TSFCCMP

The above results demonstrated that the TDFCCMP has advantages of TSP, DFCCP and MIP models, and is capable of handling MSW management problems under multiple uncertainties. In detail, TDFCCMP could: (i) address uncertainties as probability, possibility distribution and 0-1 integer variables; (ii) examine the pre-regulated waste-generation management policies associated with economic implications; (iii) incorporate various confidence levels and two reliability scenarios of constraints satisfaction into the framework; (iv) provide supports for decision makers to identify cost-effective solid waste management strategies with both costs and risk information being considered under complex uncertainties.

6. Discussion

In order to demonstrate the advantages of TDFCCMP, a two-stage single-sided fuzzy version of the chance-constrained programming (TSFCCMP) model is also applied to resolve the same MSW management problem. TDFCCMP can be converted into TSFCCMP by ignoring the influence of safety coefficients. Figure 6 also shows the varying trend of the system costs from TSFCCMP model. The system costs would increase as the increase of confidence levels, being 165.568, 165.575, 165.581, 165.587, 165.594, 165.600, 165.606, 165.613 and 165.619 ($\times 10^6$ \$), respectively. Obviously, the system costs from TSFCCMP are lower than those from TDFCCMP. This is due to the fact that the safety coefficients in TDFCCMP would lead to strict limitations in the capacities of the incinerator and composting plant. In such a condition, the system failure risk would be reduced. The comparison results demonstrated that: (i) TSFCCMP is incapable of handling fuzzy uncertainties associated with the left-hand side parameters and examining reliability scenarios, leading to the oversimplification in dealing with uncertainties and restrictions of the decision-making; (ii) TSFCCMP ignored risk of insufficiency and might result in the high environmental pollution risk.

TDFCCMP also has much space for improvement. Firstly, TDFCCMP could hardly tackle the uncertainties of cost coefficients in the objective function. This could be solved through incorporating other uncertain-optimization methods, such as interval linear programming (ILP). Secondly, the general environmental planning involves many issues related to socio-economic development, environmental protection, and resources conservation. Therefore, the single objective

function may not be sufficient to reflect the characteristics of the studied system and examine the balance among various system components. How to incorporate multi-objective programming (MOP) techniques into the model framework is important and deserves an in-depth study.

7. Conclusions

A two-stage double-sided fuzzy chance-constrained mixed-integer programming (TDFCCMP) model was developed in this study for supporting municipal solid waste management under multiple uncertainties. The model integrated the DFCCP, MIP and TSP models into a general framework and could be used to deal with uncertainties expressed as not only possibilistic distributions associated with both left-and right-hand-side components of constraints but also probabilistic distributions associated with the right-hand side components of constraints. The binary 0-1 integer variables were used to reflect the capacity expansion issues.

A long-term waste management case was used to demonstrate the applicability of the proposed method. The study results indicated that TDFCCMP allowed violation of system constraints at specified confidence-levels with two reliability scenarios. This could lead to model solutions with low system costs under acceptable risk magnitudes. Moreover, it could facilitate analyses of the policy scenarios that were associated with economic penalties when the predefined waste-flow allocation policies were violated. The generated solutions could help decision makers establish various waste-flow allocation patterns and capacity-expansion plans under complex uncertainties, and gain in-depth insights into the trade-offs between system economy and reliability.

Although this study was the first attempt in applying TDFCCMP to solid waste management problems, the results demonstrated that it is also applicable to many other environmental problems where complex uncertainties exist. In addition, many other uncertainty analysis methods, such as fuzzy robust programming (FRP) and interval linear programming (ILP), have potentials to be further integrated into a TDFCCMP framework for dealing with more complicated problems.

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