A two-stage stochastic programming framework for transportation planning in disaster response

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This study proposes a two-stage stochastic programming model to plan the transportation of vital first-aid commodities to disaster-affected areas during emergency response. A multi-commodity, multi-modal network flow formulation is developed to describe the flow of material over an urban transportation network. Since it is difficult to predict the timing and magnitude of any disaster and its impact on the urban system, resource mobilization is treated in a random manner, and the resource requirements are represented as random variables. Furthermore, uncertainty arising from the vulnerability of the transportation system leads to random arc capacities and supply amounts. Randomness is represented by a finite sample of scenarios for capacity, supply and demand triplet. The two stages are defined with respect to information asymmetry, which discloses uncertainty during the progress of the response. The approach is validated by quantifying the expected value of perfect and stochastic information in problem instances generated out of actual data.

Introduction

The World Health Organization defines a disaster as any occurrence that causes damage, destruction, ecological disruption, loss of human life, human suffering, deterioration of health and health services on a scale sufficient to warrant an extraordinary response from outside the affected community or area. Earthquakes, hurricanes, tornadoes, volcanic eruptions, fire, floods, blizzard, drought, terrorism, chemical spills, nuclear accidents are included among the causes of disasters, and all have significant devastating effects in terms of human injuries and property damage. Response is defined as the set of actions conducted during the initial impact of these emergency situations, including those to save lives and prevent further property damage providing emergency relief to victims of natural or man-made disasters. Naturally, the response planners should possess robust and generic decision tools and models to enhance their disaster relief and response capability and should be proactively prepared for effective response. Since this is a situation where the decision-makers generally have random and imprecise information about the scope, timing and resource requirements of the disaster prior to the event, the development of quick response and efficient disaster relief plans poses itself as a complex stochastic decision problem.

This study addresses the issue of planning the transportation of vital first-aid commodities (medicine, food, clothing, machinery, etc) and emergency personnel to disaster-affected areas by developing a generic modelling framework to be used in case of earthquakes. The physical transportation network of a densely populated urban area is represented as a large network with many arcs and nodes, and the resource-mobilization system is modelled as a probabilistic, multi-commodity, multi-modal network flow problem. In the presence of multiple source and destination nodes for each commodity, a node–arc formulation is proposed for the flow problem that details all the paths between the origin and destination nodes of each commodity and makes the model directly implementable in a real-world situation.

Since it is almost impossible to know the timing and the intensity of any earthquake, it is very difficult to estimate the impact, damage and resource needs exactly in advance; thus, the planning problem should be naturally treated as a stochastic problem where randomness arises not only from demand but also from supply and route capacity perspectives as well. Obviously, the decision process must be responsive to the variations in these random parameters. The survivability of the routes and the vulnerability of the supply nodes is one main issue that further complicates the problem. The probable collapse of certain arcs on the transportation network that may prevent the flow of commodities to specific disaster areas leads to random arc capacities. Moreover, the damage of the supply and service providers that are also directly subjected to the effects of earthquake naturally randomizes the availability and

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usability of commodities; consequently, it should be kept in mind that the whole transportation system is vulnerable and may be totally unoperational.

Many techniques have been developed for dealing with uncertainty in mathematical programming models, among which stochastic programming (SP) with recourse is cited as a general-purpose technique that can deal with uncertainty in any one of the model parameters. Stochastic programs with recourse are employed to find nonanticipative decisions that must be taken prior to knowing the realizations of some random variables such that the total expected costs of possible recourse actions are minimized. The first formulation of stochastic problems with recourse was given by Dantzig,[1] while later Birge and Louveaux[2] and Kall and Wallace[3] reviewed the basic concepts, solution procedures and application areas of SP. A proactive approach is introduced with the notion of robust optimization by Mulvey and Vanderbei.[4] Different aspects of SP have been studied by Vladimirou and Zenios,[5] Mulvey and Vladimirou,[6] Escudero,[7] Escudero et al,[8] Shapiro and Homem-de-Mello,[9] Wallace,[10] Powell and Cheung,[11–13] Frantzeskakis and Powell[14,15] and Glockner and Nemhauser.[16]

Here, the transportation problem is formulated as a scenario-based, two-stage SP linear model to represent the randomness arising from earthquake magnitude and impact. The main reason for choosing this approach is the flexibility it offers in modelling the logistic decision process and defining the large number of earthquake scenarios (ESs). The proposed stochastic model is validated by using the actual data of the August 1999, $M = 7.4$, Marmara earthquake in Turkey, and the benefit of using it is quantified by measuring the value of stochastic information. It is proposed that this model be used effectively within a decision-aid tool by public and nonpublic response agencies that are obscured by the variability of impact estimations under a large number of different ESs.

The paper is organized as follows: The second section discusses the representation of the earthquake response problem as a two-stage stochastic problem. The third section describes the two-stage SP model developed for the multi-modal, multi-commodity network flow problem. The penultimate section discusses the generation of ESs and the computational results, while the final section provides some concluding remarks.

Representation of the earthquake response problem as a two-stage stochastic problem

The inevitability of the occurrence of earthquakes in earthquake-prone urban centres makes it imperative that certain preparedness and emergency procedures be contrived in the event of and prior to an earthquake disaster. In urban centres, the impact of disastrous earthquakes is best portrayed and quantified through the preparation of earth-quake-damage scenarios. In fact, realistic earthquake-hazard scenarios constitute the prerequisite elements for developing robust and efficient disaster response and management plans. The first ingredient of such scenarios is the assessment of the earthquake hazard that is usually depicted as annual probabilities of exceedance for given ground motion levels. It requires the compilation and evaluation of all topological, geological, geo-tectonic, seismological, geophysical and ground motion data. In order to calculate the earthquake renewal probability, one needs to deduce earthquake magnitudes, the mean inter-event time of similar events and the elapsed time since the last shock on each fault. Then, the epicentre location and the magnitude are inferred through an empirical attenuation relation. Factors that determine the ground motion impact, on the other hand, include the geometry of fault rupture, mechanical interaction between faults, site–response characteristics and the expected performance of the building stock and infrastructure. Thus, the vulnerabilities and the damage statistics of lives, structures, systems and the socio-economic structure constitute the second ingredient. Vulnerability analysis involves the elements at risk (physical, social and economic) and the type of the associated risk (damage to structures, systems and human casualties). It basically consists of compiling demographic information, lifeline, infrastructure and building stock in the form of a GIS database. Earthquake-damage scenarios are based on the intelligent consideration and combination of uncertainties in these physical and social parameters of hazard and vulnerabilities. This study presumes that randomness inherent in the scenarios is twofold and can be divided into two components: the first component deals with the determination of the epicentre and the magnitude and will be called ES assessment. The second component of randomness is basically related to the estimation of the impact scenarios (ISs).

In the early postevent period very shortly after the receipt of an earthquake signal, accurate information about the epicentre and the magnitude of the earthquake becomes readily available through rapid communication channels such as remote sensors, conventional and Doppler radar, satellite imagery systems. This is indeed the event perception point where the degree of uncertainty is basically diminished to amplification, soil effects and site–response characteristics, and response and resource mobilization is initiated without exactly knowing the scope of the induced damage. At this point, initial response will be solely based upon adequate ISs developed prior to the emergency event, and the effectiveness of initial response is highly dependent on the accuracy of ISs developed for each ES. As precise information about the kinematics of the rupture and the impact of the disaster and relief needs is acquired over time, ongoing response activities should be monitored to meet the actual needs. Here, the decision-makers are expected to make their response planning based on both ESs and conditional ISs; consequently, the pre-emergency phase
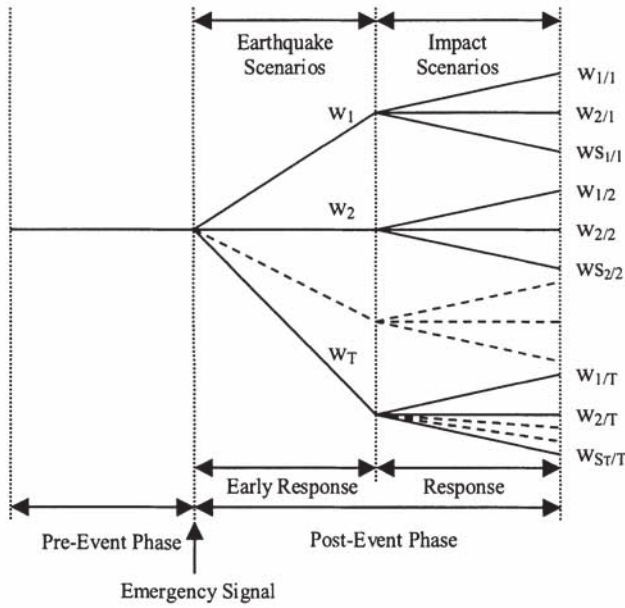
Figure 1   Stochastic programming structure.

should divide the response phase into two stages according to the asymmetry of information and should consider all Es and conditional ISs. The SP structure of the resulting two-stage stochastic problem is shown in Figure 1.

The random parameters of the first-stage problem are defined over the probability space (Ω_1, P_1), where $\Omega_1 = \{w_1, w_2, \ldots, w_T\}$ is the sample space of the random quantities with $T = |\Omega_1|$ and w_t an ES in Ω_1 for all $t = 1, 2, \ldots, T$. The associated probability of each ES is defined as $p(w_t)$ such that $\sum_{t=1}^{T} p(w_t) = 1$. The random parameters of the second-stage problem are defined over the conditional probability space $(\Omega_{2/t}, P_{2/t})$ of each ES w_t, where $\Omega_{2/t} = \{w_{1/t}, w_{2/t}, \ldots, w_{S_t/t}\}$ is the sample space of the random quantities with $S_t = |\Omega_{2/t}|$ and $w_{s/t}$ an IS in $\Omega_{2/t}$ for all $s = 1, 2, \ldots, S_t$. The conditional probability of each IS is defined as $p(w_s/w_t)$ such that $\sum_{s=1}^{S_t} p(w_s/w_t) = 1$. Furthermore, the random vectors $\varepsilon_1(w_t) = \{K_{ij}^v(1, w_t), U_i^k(1, w_t)\}$ and $\varepsilon_2(w_{s/t}) = \{K_{ij}^v(2, w_{s/t}), D_i^k(2, w_{s/t})\}$ are defined as the joint realizations of the random parameters.

In general, two-stage stochastic linear programs with recourse consist of two distinct components: a structural component (first-stage) that is fixed and free of any uncertainty, and a control component (second-stage) that is affected by the uncertainty in input data. The first-stage variables are subject to fixed, structural constraints, and represent decisions that must be made before the values of uncertain parameters are observed. The optimal values of these design variables should be independent of the realization of the uncertain parameters. Subsequently, based on these decisions, the second-stage *control variables* represent recourse actions that can be taken after a specific realization of the uncertain parameters is observed. In the

literature, these variables are also called *recourse variables*. Their optimal value depends both on the realization of the uncertain parameters, and on the optimal value of the first-stage variables to which they are linked through scenario-contingent constraints. Although the stochastic structure in Figure 1 appears different from the conventional two-stage stochastic program because of the existence of uncertainty in the first stage, it can be easily observed that it is the superimposition of T two-stage stochastic models. Since the acquisition of first-stage data is instantaneous and uncertainty for the early response stage will diminish before any response action is mobilized, the response planner will need to solve T independent two-stage stochastic programs, one for each ES, during the pre-event phase, and upon event perception will implement the plan corresponding to the realized ES. Here, the resource mobilization plan for each ES will be a compromise solution integrating all possible ISs before accurate information can be obtained about the impact. Indeed, without such a modelling tool the decision-makers would be blindly staging the relief and might need to carry out significant modifications on their original plans during the recourse; this would mean loss of time and effort in saving human life and property.

Description of the two-stage stochastic programming model for the multi-commodity, multi-modal network flow problem (SP-MCM)

The earthquake disaster master plan for an urban area must address the issue of responding to the emergency situation in an efficient manner to minimize the loss of life and maximize the efficiency of search and rescue operations. The basic underlying logistical problem in the later situation is to move a number of different commodities using different modes of transportation as soon as possible to the disaster area. Haghani and Oh[17] can be cited among the few researchers who have addressed the logistical issues in disaster-relief management by employing a deterministic approach. This study, on the other hand, is a pioneering effort to include uncertainties that exist in estimating resource requirements of first-aid commodities, vulnerability of resource provider facilities and survivability of the connecting routes in the disaster area. Thus, the estimation of routing capacity and commodity supplies that will survive the earthquake impact should be explicitly treated in any modelling effort.

Here, the resource mobilization during response to a disaster is modelled as a multi-commodity, multi-modal network flow problem with random arc capacity, supply and demand requirements. The objective is to transport the commodities from one location to another over a network $G(N, A)$, where N is the set of nodes and A is the set of arcs with finite and random capacity, to satisfy requirements with minimum cost. In the stated problem context, nodes in the

network may either represent the resource provider facilities in a region or a disaster node with a random service demand. Each node may be a supply or demand point for one or more commodities, or both for different commodities. In addition to the presence of some pure transhipment nodes, a supply or demand node of a commodity also acts as a transhipment node for the other commodities. The arcs of the base network represent the connecting routes between the physical facilities.

The model includes K commodities that are to be transported along the network with multiple source and destination nodes. The presence of multiple sources and destinations with multiple paths for each commodity further complicates the problem. Furthermore, different modes of transportation are assumed available to facilitate the accessibility to each node. The random capacities are defined for each mode on each arc, and a variable transportation cost is defined as a linear function of the quantity carried by each mode along each arc. Inter-modal shifts at nodes are allowed to enhance node accessibility, but at an additional cost that is assumed to be fixed. Mode-commodity compatibility is defined by specifying a set of possible modes for each commodity and assuming that certain commodities are captive to a mode or a subset of modes.

The definition of the flow variables gives the model its special structure. In this study, a path (l, m) is defined as the set of arcs included in the route from supply node l to demand node m and the flow decision variables are defined for each arc (i, j) in each path (l, m) of each commodity k using mode v. The model (SP-MCM) is different from the general multi-commodity, multi-modal network flow problem designed with the decision variable X_{ij}^{kv}. Although the latter gives the same objective function value with the model (SP-MCM), the general model only provides the flow amounts between any two nodes without specifying the destined (l, m) path of the flow. However, since this study aims to generate the detailed flow information, the flow quantities are represented by X_{lmij}^{kv}, and thus the model (SP-MCM) provides all the paths for each origin–destination pair. This additional path information makes the solution of the model (SP-MCM) directly implementable in a real-world system.

The two-stage stochastic programming model (SP-MCM) with full recourse represents a situation where both the first- and second-stage problems are transportation systems that arise in different time phases on the same base network. Although the first-stage supplies and arc capacities are only probabilistically known in the pre-emergency phase, as soon as the earthquake signal is received with the magnitude and epicentre, the first-stage information about usable supplies and operational arc capacities is extracted from the associated Es, and becomes deterministically known for the first stage, while the demand and second-stage arc capacities are still only probabilistically known. In the first stage, initial supply amounts must be allocated from supply

nodes to other nodes prior to realizing demand in the second stage. Here, flows in the first stage create supplies at the beginning of the second stage where additional external supplies are not allowed. In the second stage, for the current supply mobilization plan, a second transportation problem must be solved for a given realization of the demands and the arc capacities. Thus, a state variable that summarizes the state of the system after stage one is defined to communicate the decisions in stage one to the decisions in stage two. One difficulty is that the first-stage decisions may not be feasible for a given realization. This situation is handled by allowing excess and shortage amounts in the second-stage problem within a goal-programming framework. The objective function consists of the first-stage decision costs and the expected value (EV) of the second-stage recourse costs that will also include the penalty cost of not satisfying demand requirements.

Deterministic data are defined below:

G	(N, A)
N	set of nodes
A	set of arcs
K	set of commodities
V	set of modes
SM_{ij}^k	set of available modes for commodity k over arc (i, j)
SO^k	set of origin nodes for commodity k
SD^k	set of destination nodes for commodity k
S^k	$SO^k \cup SD^k$
C_v	inventory holding cost
C_w	shortage cost
C_{ms}	fixed cost of mode-shifting one unit of each commodity
C_{ij}^{kv}	cost of carrying one unit of commodity k from node i to node j by mode v

Random data used in the models are defined below:

$\tilde{U}_i^k(1)$	random supply amount of commodity k at node i in stage one
$U_i^k(1, w_t)$	a realization of $\tilde{U}_i^k(1)$
$\tilde{K}_{ij}^v(1)$	random capacity of mode v of arc (i, j) in stage one
$K_{ij}^v(1, w_t)$	a realization of $\tilde{K}_{ij}^v(1)$
$\tilde{K}_{ij}^v(2)$	random capacity of mode v of arc (i, j) in stage two
$K_{ij}^v(2, w_{s/t})$	a realization of $\tilde{K}_{ij}^v(2)$
$\tilde{D}_i^k(2)$	random demand of commodity k at node i in stage two
$D_i^k(2, w_{s/t})$	a realization of $\tilde{D}_i^k(2)$

Decision variables are defined below:

$R_i^k(1, w_t)$	internal supply amount of commodity k at node i in stage two resulting from the decisions made in stage one according to ES w_t (state variable)

$X_{lmij}^{kv}(1, w_t)$ — amount of commodity k sent over arc (i, j) by mode v from source node l to destination node m in stage one in ES w_t

$X_{lmij}^{kv}(2, w_{s/t})$ — amount of commodity k sent over arc (i, j) by mode v from source node l to destination node m in stage two in ground motion scenario $w_{s/t}$

$P_{lmi}^{kv}(1, w_t)$ — amount of commodity k in path (l, m) shifted from any other mode to mode v at node i in stage one in ES w_t

$P_{lmi}^{kv}(2, w_{s/t})$ — amount of commodity k in path (l, m) shifted from any other mode to mode v at node i in stage two in ground motion scenario $w_{s/t}$

$Q_{lmi}^{kv}(1, w_t)$ — amount of commodity k in path (l, m) shifted from mode v to another mode at node i in stage one in ES w_t

$Q_{lmi}^{kv}(2, w_{s/t})$ — amount of commodity k in path (l, m) shifted from mode v to another mode at node i in stage two in ground motion scenario $w_{s/t}$

$V_i^k(2, w_{s/t})$ — excess amount of commodity k in demand node i in ground motion scenario $w_{s/t}$

$W_i^k(2, w_{s/t})$ — shortage amount of commodity k in demand node i in ground motion scenario $w_{s/t}$

In the pre-event phase, the objection function is defined as

$$\min E_{\varepsilon_l}[Q_1(\varepsilon_1(w_t))] = \min \sum_{t=1}^{T} p(w_t) Q_1(\varepsilon_1(w_t)) \quad (1)$$

Under the ES w_t, a node–arc formulation of the first-stage problem can be given as follows:

$$Q_1(\varepsilon_l(w_t)) = \min \sum_{k \in K} \sum_{v \in V} \sum_{l \in SO^k} \sum_{m \in S^k} \sum_{\substack{i \in N \\ i \neq j}} \sum_{j \in N} [C_{ij}^{kv} X_{lmij}^{kv}(1, w_t)$$
$$+ C_{ms}(P_{lmi}^{kv}(1, w_t) + Q_{lmi}^{kv}(1, w_t))/2] \quad (2)$$
$$+ \bar{Q}_2(R(1, w_t))$$

subject to

$$\sum_{k \in K} \sum_{l \in SO^k} \sum_{m \in S^k} X_{lmij}^{kv}(1, w_t) \leq K_{ij}^v(1, w_t)$$
$$\forall v \in SM_{ij}^k, \ (i, j) \in A \quad (3)$$

$$\sum_{v \in SM_{ij}^k} \sum_{\substack{j \in N \\ j \neq i}} X_{lmij}^{kv}(1, w_t) - \sum_{v \in SM_{ij}^k} \sum_{\substack{j \in N \\ j \neq i}} X_{lmji}^{kv}(1, w_t) = 0$$
$$\forall k \in K, \ l \in SO^k, \ m \in S^k, \ i \in N, \ \text{and} \ l \neq i, \ m \neq i \quad (4)$$

$$\sum_{\substack{j \in N \\ j \neq i}} X_{lmij}^{kv}(1, w_t) - \sum_{\substack{j \in N \\ j \neq i}} X_{lmji}^{kv}(1, w_t) = P_{lmi}^{kv}(1, w_t) - Q_{lmi}^{kv}(1, w_t)$$
$$\forall k \in K, \ v \in SM_{ij}^k, \ l \in SO^k, \ m \in S^k, \ i \in N,$$
$$\text{and} \ l \neq i, \ m \neq i \quad (5)$$

$$\sum_{v \in SM_{ij}^k} \sum_{m \in S^k} \sum_{j \in N} X_{lmij}^{kv}(1, w_t) = U_i^k(1, w_t)$$
$$\forall k \in K, \ i \in SO^k, \ \text{and} \ i = l \quad (6)$$

$$\sum_{v \in SM_{ij}^k} \sum_{l \in SO^k} \sum_{j \in N} X_{lmji}^{kv}(1, w_t) = R_i^k(1, w_t)$$
$$\forall k \in K, \ i \in S^k, \ \text{and} \ i = m \quad (7)$$

$$X_{lmij}^{kv}(1, w_t) \geq 0$$
$$\forall k \in K, \ v \in SM_{ij}^k, \ l \in SO^k, \ m \in S^k, \ (i, j) \in A \quad (8)$$

$$P_{lmi}^{kv}(1, w_t) \geq 0, \quad Q_{lmi}^{kv}(1, w_t) \geq 0$$
$$\forall k \in K, \ v \in SM_{ij}^k, \ l \in SO^k, \ m \in S^k, \ i \in N \quad (9)$$

where the expected recourse function is defined as:

$$\bar{Q}_2(R(1, w_t)) = E_{\varepsilon_2}[Q_2(R(1, w_t), \varepsilon_2(w_{s/t}))]$$
$$= \sum_{s=1}^{S_t} p(w_s/w_t) Q_2(R(1, w_t), \varepsilon_2(w_{s/t})) \quad (10)$$

The objective function (2) is the minimization of the total first-stage transportation cost and the expected recourse cost for ES w_t. Constraints (3), (4) and (5) are the capacity, flow conservation and mode shift control constraints, respectively. Constraints (6) and (7) together force the available supplies at each supply node of each commodity to be shipped to the other supply or demand nodes of that commodity or allow to be reserved in the source node, where the state variable $R_i^k(1, w_t)$ stores flow shipment information for stage two. Since this is the only variable that communicates information to the second stage, it must also store the amounts that are reserved in the source nodes. Here, pure transhipment nodes are not allowed to reserve commodities. Constraints (8) and (9) are the non-negativity constraints. The expected recourse function (10), namely the expectation of individual recourse costs $Q_2(R(1, w_t), \varepsilon_2(w_{s/t}))$, is determined by solving the second-stage problem for each scenario $w_{s/t}$ according to the second-stage supplies $R_i^k(1, w_t)$ that are determined in the first stage and the joint realization of the random parameters $\varepsilon_2(w_{s/t}) = \{K_{ij}^v(2, w_{s/t}), D_i^k(2, w_{s/t})\}$. The second-stage problem for a specific scenario $w_{s/t}$ is as follows:

$$Q_2(R(1, w_t), \varepsilon_2(w_{s/t}))$$
$$= \min \sum_{k \in K} \sum_{v \in V} \sum_{l \in S^k} \sum_{m \in S^k} \sum_{\substack{i \in N \\ i \neq j}} \sum_{j \in N} [C_{ij}^{kv} X_{lmij}^{kv}(2, w_{s/t})$$
$$+ C_{ms}(P_{lmi}^{kv}(2, w_{s/t}) + Q_{lmi}^{kv}(2, w_{s/t}))/2$$
$$+ C_V V_i^k(2, w_{s/t}) + C_W W_i^k(2, w_{s/t})] \quad (11)$$

subject to

$$\sum_{k \in K} \sum_{l \in S^k} \sum_{m \in S^k} X_{lmij}^{kv}(2, w_{s/t}) \leqslant K_{ij}^{v}(2, w_{s/t})$$

$$\forall v \in \mathrm{SM}_{ij}^{k}, \ (i,j) \in A \tag{12}$$

$$\sum_{v \in \mathrm{SM}_{ij}^{k}} \sum_{\substack{j \in N \\ j \neq i}} X_{lmij}^{kv}(2, w_{s/t}) - \sum_{v \in \mathrm{SM}_{ij}^{k}} \sum_{\substack{j \in N \\ j \neq i}} X_{lmji}^{kv}(2, w_{s/t}) = 0$$

$$\forall k \in K, \ l \in S^k, \quad m \in S^k, \ i \in N, \ \text{and} \ l \neq i, \ m \neq i \tag{13}$$

$$\sum_{\substack{j \in N \\ j \neq i}} X_{lmij}^{kv}(2, w_{s/t}) - \sum_{\substack{j \in N \\ j \neq i}} X_{lmji}^{kv}(2, w_{s/t})$$

$$= P_{lmi}^{kv}(2, w_{s/t}) - Q_{lmi}^{kv}(2, w_{s/t}) \tag{14}$$

$$\forall k \in K, \ v \in \mathrm{SM}_{ij}^{k}, \ l \in S^k, \ m \in S^k, \ i \in N,$$

$$\text{and} \ l \neq i, \ m \neq i$$

$$\sum_{v \in \mathrm{SM}_{ij}^{k}} \sum_{m \in S^k} \sum_{j \in N} X_{lmij}^{kv}(2, w_{s/t}) \leqslant R_i^k(1, w_t) \quad \forall k \in K, \ i \in S^k,$$

$$\text{and} \ i = l \tag{15}$$

$$\sum_{v \in \mathrm{SM}_{ij}^{k}} \sum_{l \in S^k} \sum_{j \in N} X_{lmji}^{kv}(2, w_{s/t}) - D_i^k(2, w_{s/t}) = V_i^k(2, w_{s/t})$$

$$- W_i^k(2, w_{s/t}) \quad \forall k \in K, \ i \in S^k, \ \text{and} \ i = m \tag{16}$$

$$X_{lmij}^{kv}(2, w_{s/t}) \geqslant 0$$

$$\forall k \in K, \ v \in \mathrm{SM}_{ij}^{k}, \ l \in S^k, \ m \in S^k, \ (i,j) \in A \tag{17}$$

$$P_{lmi}^{kv}(2, w_{s/t}) \geqslant 0, \quad Q_{lmi}^{kv}(2, w_{s/t}) \geqslant 0$$

$$\forall k \in K, \ v \in \mathrm{SM}_{ij}^{k}, \ l \in S^k, \ m \in S^k, i \in N \tag{18}$$

$$V_i^k(2, w_{s/t}) \geqslant 0, \quad W_i^k(2, w_{s/t}) \geqslant 0 \quad \forall k \in K, \ i \in N \tag{19}$$

The recourse function (11) is the minimization of the total flow costs, mode shift costs and the penalty costs of inventory holding and shortage in the second stage. Constraints (12), (13) and (14) are the capacity, flow conservation and mode shift control constraints of the second-stage problem, respectively. Since service demand is now known, constraints (15) and (16) together allow the supply deliveries that are already determined in the first stage and communicated through the variable $R_i^k(1, w_t)$ to be shipped to the realized demand nodes to satisfy demand or to be held at the source node as inventory. Constraint (16) determines the excess and shortage amounts of demands, while constraints (17)–(19) are the non-negativity constraints. The first-stage and second-stage problems together

form the model (SP-MCM) that simply reallocates the initial supplies between the nodes of the base network to facilitate the demand satisfaction once demand is realized. Since the model also gives a second-stage solution for each scenario, the decision-maker can react quickly and efficiently to the realized uncertainties.

Computational results

The model (SP-MCM) is validated by using the actual data from August 1999, $M = 7.4$, Marmara earthquake in Turkey. The city of Istanbul situated astride the Bosphorus in both Europe and Asia has experienced numerous earthquakes in history, and in recent decades the earthquake disaster risks in Istanbul have increased due to overcrowding, faulty land-use planning, poor construction quality, inadequate infrastructure and environmental degradation. Although the isoseismic map of 1999 Marmara earthquake shows that the general intensity was VI in Istanbul, a limited region in Avcilar, which is a borough to the west of the city with a population of 214 621, experienced an intensity VII and suffered 981 casualties and 41 180 seriously/moderately damaged buildings. Being a very vulnerable urban settlement location due to its soil condition, Avcilar data are used here to show the application of the (SP-MCM) methodology.

A network representation of the Avcilar region is given in Figure 2 where six demand nodes (D_1, D_2, D_3, D_4, D_5, D_6) correspond to designated evacuation sites in different neighbourhoods of Avcilar, five supply nodes (S_1, S_2, S_3, S_4, S_5) correspond to resource provider facilities, and three pure transshipment nodes (N_1, N_2, N_3) are determined from the existing transportation network of the area. The connecting roads are represented as arcs of mode 1 where truck transportation is used, while the nodes that can be reached by the helicopters are designated with arcs of mode
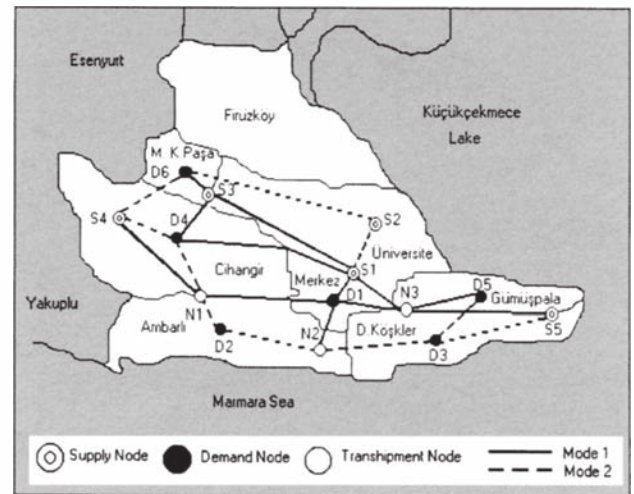
Figure 2 Network representation of Avcilar.

2. Here, mode shifts are allowed only at the supply nodes that have both road connection and helicopter landing facilities, namely at nodes S_1, S_4 and S_5.

Since vital first-aid commodities are generally delivered in the form of standard-configured containers, this problem instance includes only one type of commodity that can be transported with each of the two modes to the disaster areas. The transportation costs (TCs) are assumed to be a linear function of distance, while the actual distance data are used for the roads, and the Euclidean distances are used for helicopter transportation. Without loss of generality, transportation with mode 1 is assumed to be cheaper than helicopter transportation, and the cost of mode 2 is taken to be twice as high as the cost of mode 1 per unit distance. Mode shift and shortage costs are defined as 35 and 50 per unit, respectively, rather large compared with the TCs, and the inventory holding cost is not included in the objective function.

The model (SP-MCM) is solved for eight ESs, each branching into nine ISs. Thus, $T = 8$ and $S_t = 9$ for all t. There are numerous studies that have been recently conducted after the August 1999 Marmara ($M_w = 7.4$) earthquake to study the faults of the Marmara plate, and they all indicate that the rupture of the tectonic elements along the North Anatolian fault zone, which passes through the northern half of the Marmara Sea, forming a series of discontinuous pull-apart basins and ridges, is estimated to produce earthquakes of 7.5+ magnitude. The ESs used in the study are generated from Parsons et al,[18] who concur about an average probability of 65% for the occurrence of a $M_w \geqslant 7.0$ magnitude earthquake affecting Istanbul.

In the second-stage problem that deals with the ISs, data for random arc capacities and random requirements are generated by using the damage scenarios developed in Erdik et al.[19] However, supply data for all scenario combinations are generated using the actual response and service plans already developed by local authorities projecting the actual requirements realized in the August 1999 Marmara earthquake and assumed to be constant for all ESs. These actual supply amounts and expected relief requirements, which will be shortly called demand, under the most possible IS developed in Erdik et al[19] are provided in Table 1, and the data of existing arc capacities and TCs can be obtained in www.ie.boun.edu.tr/etm/x_html/disaster.htm.

The ISs are constructed in the same manner for all ESs, and different realizations of the random quantities, namely $K_{ij}^v(2, w_{s/t})$ and $D_i(2, w_{s/t})$ for each IS $s = 1, 2, \ldots, 9$ are generated by perturbing the existing arc capacities and the expected demand values in Table 1 with certain percentages. From now on, the best IS is defined as the scenario with the highest arc capacities and lowest demand amounts, and the worst-case scenario as the scenario with the lowest arc capacities and highest demand amounts. As can be seen in Table 2, the best IS is scenario $s = 1$ with arc capacities 70% and demand amounts 130% of the values mentioned above,

Table 1   Actual supply and demand amounts

N	Demand amount	N	Supply amount
D_1	10 370	S_1	13 500
D_2	5920	S_2	9000
D_3	7300	S_3	11 700
D_4	3570	S_4	12 300
D_5	11 470	S_5	12 800
D_6	1720		
Total	40 350	Total	59 300

while scenario $s = 9$ is the worst-case scenario with 10% arc capacities and 190% demand amounts. Then, different probabilities of occurrence are assigned to respective ISs under different ESs as shown in Table 2. Here, the ESs are designed by assigning increasing probabilities to the ISs with higher arc capacities and lower demands and labelled in increasing order; consequently, ES $t = 8$ can be defined as the best-case scenario, while scenario $t = 1$ is the worst-case scenario. For each ES $t = 1, 2, \ldots, 8$ the realizations of random supply amounts, namely $U_i(1, w_t)$, are defined as the actual supplies given in Table 1.

One disadvantage of scenario-based SP is that the resulting mathematical models can be very large, therefore requiring special solution algorithms. However, the dimensionality of the real-case models developed in this study has not prohibited the possibility of solving these models by commercial optimizers and permitted the implementation by local response planners for the Istanbul case.

The models are solved using GAMS/OSL[20] and SLP-IOR.[21] The models were initially coded in GAMS and solved as a single large-scale linear program using GAMS/OSL. The problem instance provided in this section consists of 874 605 columns and 255 491 rows, and the results reported below were obtained by using GAMS/OSL on Pentium IV 1.80 GHz–512 MB Ram in 15–17 min (20 000–22 000 iterations). Then, smaller problems are solved easily by implementing SLP-IOR, the version of which consists of two-stage recourse modelling, since the general-purpose LP solvers available within GAMS are automatically also connected to SLP-IOR.

Since stochastic programs have the reputation of being computationally difficult, solving simpler versions like wait-and-see (WS) and EV problems is a natural temptation when faced with real-world problems. Under the assumption that perfect information about future realizations is available and each particular scenario can be optimized independently, the EV of the optimal solutions of the scenarios can be computed, and is known as the WS solution. Although this gives a lower objective function value for each individual scenario when compared with the SP solutions, finding WS solution may be impossible if perfect information is just not available at any price, and also it is impractical since it

Table 2 Scenario data

Impact scenarios (ISs)	Capacity (%)	Demand (%)	Probabilities for earthquake scenarios (ES)							
			1	2	3	4	5	6	7	8
1	70	130	0.03	0.02	0.01	0.025	0.04	0.075	0.25	0.45
2	50	140	0.045	0.05	0.025	0.03	0.06	0.06	0.23	0.225
3	50	150	0.03	0.08	0.05	0.08	0.3	0.3	0.04	0.04
4	30	150	0.025	0.03	0.03	0.02	0.045	0.2	0.225	0.075
5	40	160	0.08	0.075	0.25	0.4	0.4	0.25	0.075	0.08
6	30	170	0.075	0.225	0.2	0.045	0.02	0.03	0.03	0.025
7	20	170	0.04	0.04	0.3	0.3	0.08	0.05	0.08	0.03
8	20	180	0.225	0.23	0.06	0.06	0.03	0.025	0.05	0.045
9	10	190	0.45	0.25	0.075	0.04	0.025	0.01	0.02	0.03

delivers a set of solutions instead of one implementable solution as provided by SP. The expected value of perfect information (EVPI) is used to compare the WS and SP solutions and defined as EVPI = SP−WS. EVPI simply measures the maximum amount a decision-maker would be ready to pay in return for complete and accurate information about the future.

Another attempt may be to solve the EV problem that is obtained by replacing all random variables by their EV. Since EV gives a unique and implementable first-stage decision, this solution does not consider all the scenarios and generally higher objective function values are obtained. The value of stochastic solution (VSS) is the concept that precisely measures how good or bad a decision obtained by EV is in terms of SP and defined as VSS = EEV−SP where EEV is the expected result of using the EV solution. This discussion reveals that the higher values of EVPI and VSS would require and justify the use of SP more.

In order to compare (SP-MCM) with WS and EV problems, WS and EEV are calculated for each ES. The WS solution for an ES w_t is calculated by solving a two-stage problem with the vectors $\{K_{ij}^v(1, w_t), U_i^k(1, w_t)\}$ and $\{K_{ij}^v(2, w_{s/t}), D_i^k(2, w_{s/t})\}$ for all $s = 1, 2, ..., S_t$ and then taking the expectation of the obtained objective function values. The EV problem, on the other hand, is solved by solving a two-stage problem with the vectors $\{K_{ij}^v(1, w_t), U_i^k(1, w_t)\}$ and $\{E[\tilde{K}_{ij}^v(2)], E[\tilde{D}_i^k(2)]\}$ and then the obtained $R_i^k(1, w_t)$ values are used to optimize each IS $w_{s/t}$ with random vector $\{K_{ij}^v(2, w_{s/t}), D_i^k(2, w_{s/t})\}$. Then, EEV is calculated by taking the expectation of the obtained objective function values.

These three approaches are applied to eight ESs. Since WS problem optimizes each IS independently without taking into account the IS probabilities, the same WS solution is obtained for each ES as summarized in Table 3 where unsatisfied demand (UD) amounts, the first-stage transportation costs (FSTC), the second-stage transportation costs (SSTC) and the original costs (OC), which includes the first-

Table 3 Results for the WS problem

IS	UD	FSTC	SSTC	OC
1	6922	502 814	257 570	1 106 484
2	13 859	334 855	331 023	1 358 828
3	17 365	511 118	159 139	1 538 506
4	20 815	472 665	142 943	1 656 358
5	22 596	464 386	183 462	1 777 648
6	27 827	480 551	146 588	2 018 489
7	29 552	530 276	70 388	2 078 264
8	33 058	508 704	97 725	2 259 329
9	38 289	539 332	46 388	2 500 170

and second-stage transportation costs and the shortage costs of UDs are given for each IS. Owing to the nature of the WS problem, these results are the minimum possible values that can be obtained. However, WS gives a different FSTC value for each IS that makes it impossible to implement in a stochastic nature. Also, it can be observed that UD and OC values are increasing consistently as the severity of the scenarios increases.

The results for the (SP-MCM) and EV problems are given in Table 4 for each ES. (SP-MCM) and EV problems give higher UD amounts than WS in each ES, but with a unique first-stage solution that is implementable. Since EV takes into account the EVs of the random variables, the UD amounts given by EV are higher than SP-MCM and they increase as the severity of the ESs decreases since the effect of scenarios with higher expectations of demands and lower expectations of arc capacities is reduced. However, this trend cannot be observed in SP-MCM.

The overall results are reported in Table 5 where the expectations of the optimum OCs and the total TCs for the SP-MCM, WS and EV problems under each ES are given together with the EVPI and VSS values. It can be observed that OCs are decreasing as the severity of the scenarios decreases. Since the UD amounts

Table 4 Results for the SP-MCM and EV problems

		SP-MCM			EV					SP-MCM			E		
ES	IS	UD	FSTC	SSTC	UD	FSTC	SSTC	ES	IS	UD	FSTC	SSTC	UD	FSTC	SSTC
1	1	9740	512283	170014	9922	455251	201450	4	1	9922	498152	159216	9922	399773	252780
	2	13859	512283	158387	14041	455251	209558		2	14041	498152	163176	14041	399773	263778
	3	17365	512283	159282	17365	455251	217541		3	17365	498152	172104	17365	399773	271760
	4	20815	512283	104906	20815	455251	162892		4	20815	498152	117456	21298	399773	208418
	5	22596	512283	136201	22596	455251	194187		5	22596	498152	149696	22596	399773	248406
	6	27827	512283	115128	27827	455251	172531		6	27827	498152	128987	28310	399773	219781
	7	29552	512283	88653	29863	455251	140467		7	29552	498152	102512	31760	399773	164694
	8	33058	512283	94419	33369	455251	146314		8	33058	498152	108277	35266	399773	171352
	9	38289	512283	74561	40746	455251	86261		9	38289	498152	93233	44130	399773	87557
2	1	9740	504426	171526	9922	432146	221211	5	1	9740	497479	165677	9922	373029	278862
	2	13859	504426	163706	14041	432146	232209		2	13859	497479	168399	14398	373029	280935
	3	17365	504426	167139	17365	432146	240192		3	17365	497479	172777	17365	373029	297842
	4	20815	504426	112763	20815	432146	185543		4	20815	497479	118402	22747	373029	208418
	5	22596	504426	144058	22596	432146	216838		5	22596	497479	150642	22803	373029	271093
	6	27827	504426	122985	27827	432146	195182		6	27827	497479	129933	29759	373029	220444
	7	29552	504426	96510	30656	432146	150217		7	29552	497479	103458	33209	373029	165356
	8	33058	504426	102276	34162	432146	156685		8	33058	497479	109223	36715	373029	172014
	9	38289	504426	83123	42160	432146	86715		9	38289	497479	95297	45579	373029	89002
3	1	9922	503087	158477	9922	416326	236539	6	1	9052	483891	197683	9922	371155	280414
	2	14041	503087	159186	14041	416326	247537		2	13859	483891	181987	14484	371155	280331
	3	17365	503087	167169	17365	416326	255520		3	17365	483891	186366	17365	371155	299395
	4	20815	503087	112520	20815	416326	200871		4	20815	483891	133022	22833	371155	208418
	5	22596	503087	144761	22596	416326	232165		5	22596	483891	165263	22889	371155	271415
	6	27827	503087	124051	27827	416326	210928		6	27827	483891	144553	29845	371155	220765
	7	29552	503087	97576	31156	416326	158014		7	29552	483891	118813	33295	371155	165678
	8	33058	503087	103342	34662	416326	164672		8	33058	483891	125471	36801	371155	172336
	9	38289	503087	86441	43128	416326	87207		9	39072	483891	94472	45665	371155	89704
4	1	9922	498152	159216	9922	399773	252780	7	1	7647	483658	249901	9398	363065	307392
	2	14041	498152	163176	14041	399773	263778		2	13859	483658	182577	14426	363065	289379
	3	17365	498152	172104	17365	399773	271760		3	17365	483658	188511	17365	363065	307191
	4	20815	498152	117456	21298	399773	208418		4	20815	483658	135362	23299	363065	208616
	5	22596	498152	149696	22596	399773	248406		5	22596	483658	167603	23355	363065	271907
	6	27827	498152	128987	28310	399773	219781		6	27827	483658	146893	30311	363065	221257
	7	29552	498152	102512	31760	399773	164694		7	29552	483658	121153	33761	363065	166170
	8	33058	498152	108277	35266	399773	171352		8	33058	483658	127811	37267	363065	172828
	9	38289	498152	93233	44130	399773	87557		9	39202	483658	94472	46131	363065	90777
5	1	9740	497479	165677	9922	373029	278862	8	1	6922	483216	277168	7853	340089	387112
	2	13859	497479	168399	14398	373029	280935		2	13859	483216	192705	14325	340089	317870
	3	17365	497479	172777	17365	373029	297842		3	17365	483216	190011	17831	340089	323358
	4	20815	497479	118402	22747	373029	208418		4	20815	483216	145761	24731	340089	208283
	5	22596	497479	150642	22803	373029	271093		5	22596	483216	169301	24787	340089	271574
	6	27827	497479	129933	29759	373029	220444		6	27827	483216	148592	31743	340089	220780
	7	29552	497479	103458	33209	373029	165356		7	29552	483216	122683	35193	340089	165692
	8	33058	497479	109223	36715	373029	172014		8	33058	483216	129341	38699	340089	172350
	9	38289	497479	95297	45579	373029	89002		9	39280	483216	94472	47505	340089	91747

are fluctuating in (SP-MCM) and increasing in EV, the consistent decrease in the OC can be explained by the IS probabilities.

These results are consistent with Birge and Louveaux[2] who have proven that $WS \leqslant SP \leqslant EV$ and $EVPI \geqslant 0$ and $VSS \geqslant 0$ for stochastic problems with fixed recourse matrix and fixed objective coefficients. Of course, these results are valid when one considers the original objective function. The TC under WS is higher than that of SP for all ESs, while the TC of EV is lower than that of SP. This indicates that less transportation is achieved under EV, and the actual delivery might have been overestimated in WS. It is attempted to analyse the behaviour of EVPI and VSS as the variances of the random variables increase. It was intuitively expected

Table 5   Overall results

ES	SP-MCM		WS		EEV		EVPI	VSS
	OC	TC	OC	TC	OC	TC		
1	2 194 889	609 786	2 192 060	611 184	2 232 342	587 154	2829	37 454
2	2 080 179	619 437	2 078 144	620 220	2 121 532	596 866	2034	41 353
3	1 967 248	623 343	1 966 313	624 135	1 998 662	607 734	935	31 415
4	1 887 109	629 807	1 885 571	632 293	1 922 657	612 359	1538	35 548
5	1 711 931	648 162	1 709 851	651 718	1 738 626	633 231	2080	26 695
6	1 650 900	647 824	1 647 704	653 007	1 680 897	628 474	3196	29 997
7	1 551 467	661 072	1 546 481	666 061	1 604 091	616 478	4986	52 624
8	1 426 667	699 426	1 421 118	695 363	1 473 273	657 090	5549	46 606

that as the randomness in the problem increases, EVPI and VSS would increase by increasing variance; however, this could not be shown due to the complex inter-relationship between random variables in the problem. This is again consistent with the findings of Birge and Louveaux.[2] Since a general pattern could not be observed for the behaviour of EVPI and VSS, it was intuitively observed that EVPI and VSS values increase when the scenarios with higher arc capacities and lower demands are given relatively higher probabilities.

The (SP-MCM), WS and EV problems are also applied to the same eight ESs by assigning the shortage costs as penalties (500 000 per unit), and it is observed that (SP-MCM) gives the same UD amounts with WS, but with higher TCs and higher objective function values. It is also observed that EV results have higher UD amounts and objective function values when compared with the (SP-MCM) and WS problems. The stated observations for the EVPI and VSS values are also validated.

Conclusion

In this study, a scenario-based SP model is developed to represent a multi-commodity, multi-modal network flow problem in a general context, and its applicability in disaster relief operations is validated by using the actual data of the August 1999, $M = 7.4$, Marmara earthquake in Turkey. Since disasters present the biggest threat for the survival of human and life support systems, utmost effort should be directed towards developing decision-making capability and improving disaster response planning. If ISs are accurately estimated by earth scientists and earthquake engineers, the model developed in this study will provide the best plan that compromises diverse response actions to a large number of random expectations. This will in turn enhance early warning and quick response performance of all disaster management authorities. Furthermore, this study not only proposes a model that can be incorporated into any such decision-support tool but it also reveals the value of

information on instances where uncertainty discloses itself only at the moment of emergency.

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