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A TWO-STATE ANALYSIS OF FIXED-INTERVAL RESPONDING IN THE PIGEON[1]

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The behavior of pigeons on six geometrically spaced fixed-interval schedules ranging from 16 to 512 sec is described as a two-state process. In the first state, which begins immediately after reinforcement, response rate is low and constant. At some variable time after reinforcement there is an abrupt transition to a high and approximately constant rate. The point of rapid transition occurs, on the average, at about two-thirds of the way through the interval. Response rate in the second state is an increasing, negatively accelerated function of rate of reinforcement in the second state.

Skinner (1938, p. 125), in the first parametric study of fixed-interval (FI) behavior in rats, pointed out that after prolonged exposure, certain intervals show signs of a temporal discrimination, in that there appears to be a pause after reinforcement, followed by a constant rate of response. Cumming and Schoenfeld (1958) showed that pigeons under extended exposure to an FI 30-min schedule developed what they called break-and-run performance; that is, after an initial pause, the subject started responding at a constant terminal rate. Sherman (1959) also reported that rats on FIs ranging from 10 sec to 4 min developed break-and-run behavior after extended training.

These studies characterize fixed-interval responding after extended training as a discrete two-state process, that is, as an extended pause after reinforcement with little or no responding (state 1), followed by a rapid transition to a high and constant response rate (state 2). To document their case, these investigators presented selected cumulative records for individual animals demonstrating break-and-run performance. These essentially qualitative descriptions of behavior, however, do not permit an exact characterization of the response parameters of the typical interval, *i.e.*, the length of the pause, the variation in pause length, the growth of rate after pause, *etc.* The estimation of such response parameters is of crucial importance in interpreting certain overall measures of fixed-interval performance. For instance, if fixed-interval performance involves a temporal discrimination, the average rate over an entire interval involves the averaging of two entirely different kinds of behavior: the first-state behavior, which includes little or no responding; and the second-state behavior, which includes a high steady rate of responding. Such an averaging technique may obscure the real relationship between reinforcement frequency and response rate on a fixed-interval schedule.

The present study attempted to develop a quantitative method for describing break-and-run performance to determine the role of temporal discriminations in fixed-interval performance and the interaction between temporal discriminations and measures of response strength. This method involved collecting and analyzing all interresponse times emitted during the terminal sessions at each FI value in order to permit a detailed analysis of each interval.

METHOD

Subjects

Six male adult White Carneaux pigeons were maintained at 80% of their free-feeding

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weight. They had previously experienced repeated sessions of continuous reinforcement followed by extinction. The birds were run seven days a week and fed enough grain after each session to maintain them at the desired weight.

Apparatus

An experimental chamber of the type described by Ferster and Skinner (1957) was used. The dimensions of the subject's chamber were 27.9 by 27.9 by 27.9 cm. A single Gerbrands response key was mounted on one wall behind a 1.9-cm diameter hole at a height of 20.6 cm and transilluminated by a white light. Each effective response produced a feedback click. Eleven and four-tenths cm beneath the key was a 5.1- by 5.1-cm opening in the panel through which a hopper filled with mixed grain could be presented. The reinforcer was a 2.9-sec period of access to grain. During the reinforcement cycle, the key light was extinguished and the grain hopper was illuminated by two 6-w bulbs directly above. Stimulus and reinforcement events were scheduled using relay circuitry. White noise was continuously present.

In addition to the standard recording devices, such as counters and a cumulative recorder, Sodeco-Geneve type 1Tpb3 print-out counters were used to record interresponse times (IRTs). To record very short IRTs it was necessary to use three print-out counters operating in sequence. At the beginning of each interval, counter one started to count. The first response terminated counting on counter one and initiated its print cycle. The first response also switched the output of the pulsing device to counter two. Hence, counter two counted the IRT of the second response while counter one was printing. Counter three was added in the same fashion to insure that no response would be lost due to the print time (200 msec) of the Sodeco counter. The counters were not reset until reinforcement occurred.

Data Processing Technique

The data from the last three days on each FI value were punched onto IBM cards by the experimenter and three professional keypunchers. Since the data on each tape were cumulative over an interreinforcement interval, IRTs were determined by subtracting the n-1th number from the nth number in an interval. If any of these numbers was negative (indicating a key-punch error), the entire interreinforcement interval was rejected.

Also, the omission or addition of a response to the interreinforcement interval by a keypuncher was determined by counting the number of responses per interval on each tape. These numbers had to satisfy one of the following restrictions or the interval was rejected. These were: (1) $N_1 = N_2 + 1 = N_3 + 1$; (2) $N_1 = N_2 = N_3 + 1$; (3) $N_1 = N_2 = N_3$; where N_i = the number of responses on tape i. These restrictions follow directly from the recording technique.

These two error-rejection procedures could conceivably introduce a systematic bias. If keypunch errors are assumed to occur in a random fashion, more errors are likely to occur in intervals containing a greater number of responses. To attempt to determine the maximum systematic error committed in this way, the data from one bird with the highest interval rejection rate (33%) and one of the highest average number of responses per interval (150) were analyzed further. The mean number of responses per interval was computed for the entire set of intervals and compared with the mean number of responses for only the accepted intervals. A mean difference of six responses in the predicted direction was found. This difference represents only a 4% error rate. Since this is the error rate for the bird with the most intervals rejected, the average rate is certain to be considerably lower. The effect of this systematic error then, should be small enough in relation to the major effects to be insignificant. The remaining data from the three tapes were then merged to yield the fixed intervals upon which subsequent analysis was based.

Table 1

Experimental design. The number in the cell is the bird number.

Order	Sessions	*Fixed Intervals (in sec)* 16	32	64	128	256	512
1	(124)	451	452	253	254	325	255
2	(39)	253	254	451	255	452	325
3	(87)	255	451	254	325	253	452
4	(75)	325	253	255	452	451	254
5	(47)	254	325	452	253	255	451
6	(53)	452	255	325	451	254	253

Procedure

The six subjects experienced each of six geometrically spaced fixed-interval schedules ranging from 16 to 512 sec according to the Latin Square design of Table 1. The second column gives the number of sessions at each point. The sessions lasted until 55 reinforcements were produced. All FI 512-sec schedules were run overnight.

RESULTS

Figure 1 presents 10 cumulative response intervals for Bird 451 on each value of FI. (The cumulative records were reconstructed from the IRTs of the eleventh through twentieth processed intervals from the first terminal session.) The fifth interval in each set of 10 has been enlarged and presented to the right of the 10 cumulative intervals. An examination of these records indicates break-and-run performance. Immediately after reinforcement there appears to be little if any responding, although occasional responses occur. At some point after reinforcement, the behavior changes abruptly to a high and approximately constant rate of response. Occasional deviations from this pattern can be noted. For instance, the second interval on FI 512-sec exhibits a scallop pattern, *i.e.*, the response rate increases continuously over the interval. The fifth interval on FI 128-sec, on the other hand, shows a fairly abrupt transition to a high rate which gradually declines toward the end of the interval. The majority of intervals, however, indicate a two-state process, an extended pause after reinforcement with little or no responding, followed by a rapid acceleration to a high and approximately constant rate of response.

To characterize the parameters of break-and-run responding some way must be found to determine the breakpoint, the rate before the breakpoint, and the rate after it. The time to the first response would seem to be a likely candidate for estimating the point of maximum acceleration (the breakpoint). Visual inspection of the cumulative records, however, shows that the first response does not always indicate the point where the change in rate is most rapid. Often, one or more responses occur in a seemingly random fashion before the rate accelerates to its terminal value. The subsequent averaging of rate before and after the first response would result in a smoothing of the individual function.

A graphical procedure which suggests itself is (1) to determine the point of maximum acceleration (the breakpoint) by eye, (2) fit one straight line to the part of the cumulative response interval which is before the breakpoint (state 1), and (3) fit another straight line to the portion of the interval after the breakpoint (state 2) so that these two straight lines intersect at the breakpoint. Figure 2 shows how this graphical procedure can be applied to a

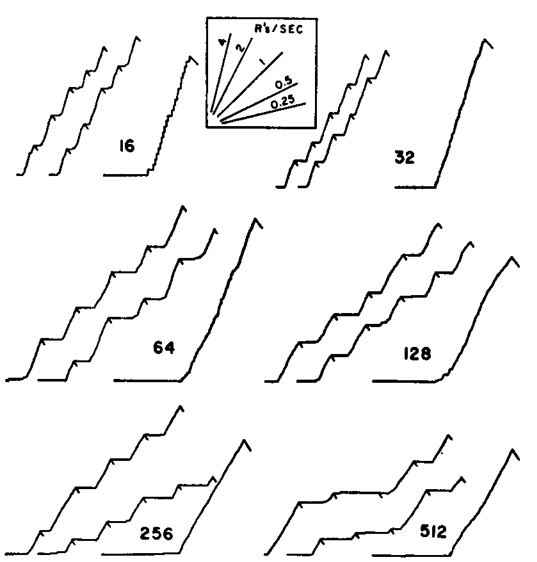

Fig. 1. Selected cumulative response records for Bird 451 on six different fixed-interval schedules. The fifth interval in each set of 10 has been enlarged and presented to the right.

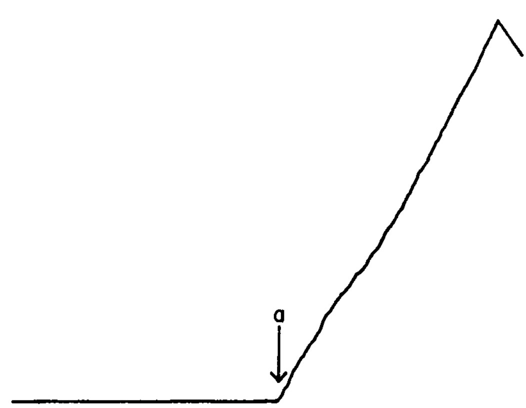

Fig. 2. The separation of a typical cumulative response interval into a first and second state. The point of maximum acceleration is located at "a" and divides the interval into two states (see text).

typical cumulative response interval (the fifth interval of Bird 451 on FI 512-sec). The point of maximum acceleration (the second derivative) is estimated by eye and is marked "a". A straight line through the origin can be fitted to the part of the record before the breakpoint. A second straight line intersecting the first at the breakpoint can be fitted to the remainder of the cumulative record. If this procedure were carried out for all the interreinforcement intervals, an average cumulative response record could be obtained for a group of FIs after first superimposing their breakpoints. The numerical analogue to this graphical procedure is to use a least-squares procedure to determine the best fitting two straight lines to each cumulative response interval. Since the breakpoint is one of the parameters to be determined, an iterative process is employed in which the point of intersection of these two lines is systematically varied until the sum of squared deviations is minimized. The point of intersection that minimizes the sum of squared deviations from the lines is taken as the breakpoint. The details of this procedure are outlined in the Appendix.

Once the breakpoint for each interval has been determined, the average momentary rate before and after the breakpoint can be determined in the following way. First, the number of responses in the 4-sec period following the breakpoint for each interval is determined. Second, this number is summed over all intervals in question. Third, the sum is divided by four times the number of intervals to obtain the average momentary rate (responses/sec) for the 4-sec period following the breakpoint. These three steps are then repeated for the second 4-sec period following the breakpoint, and so on, until no more instances are encountered. This procedure is then repeated for the 4-sec periods preceding the breakpoint. One drawback of this technique is that the number of intervals represented in each 4-sec time period decreases as distance from the breakpoint increases in either direction. For example, if on FI 64-sec the breakpoint occurs at 12 sec, there are only three 4-sec periods before the breakpoint and thirteen 4-sec periods after it. If for a second interval the breakpoint occurs at 48 sec, there are 12 intervals before the breakpoint and only four after it. Hence, an average cumulative record based on these two hypothetical intervals would have twenty-five 4-sec periods, *i.e.*, would be 100 sec long. This would mean that the estimate of rate for the first nine and the last nine 4-sec periods would be based on a single interval only, while the estimate of rate for the intermediate 4-sec periods would be based on an average of two intervals. And, in general, the number of intervals represented in each 4-sec time period would decrease as distance from the breakpoint increased. To eliminate unreliable estimates of rate in the subsequent analysis, only those time segments for which there were 20 or more occurrences are included.

Figure 3 (top) shows the results of such calculations for Bird 451 on FI 256-sec. Rate

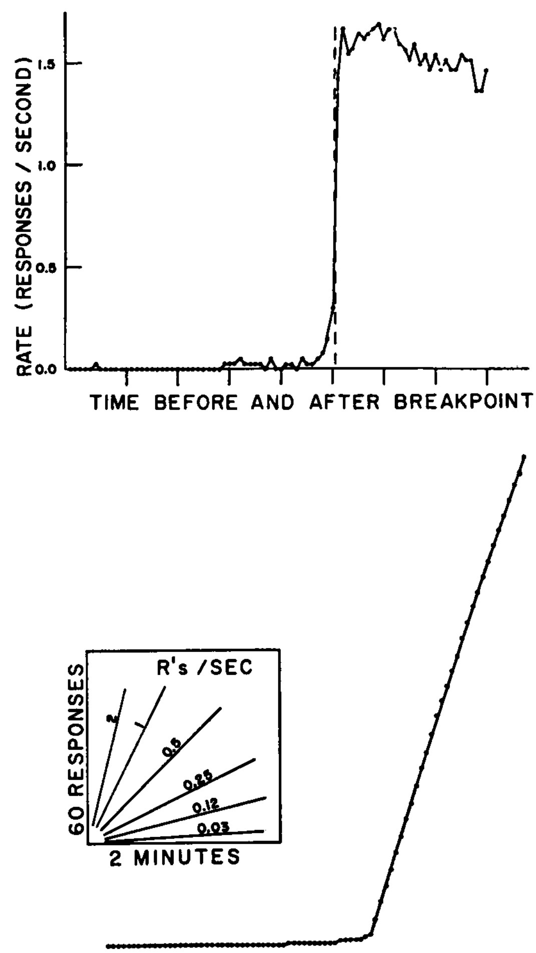

Fig. 3. Top. Response rate before and after breakpoint (dotted line) for Subject 451 on FI 256-sec. Four-sec time segments (see text) were used. Hence, one unit on the abscissa (comprising 10 time segments) represents 40 sec. Bottom. Average cumulative response record for Bird 451 on FI 256-sec when records are averaged after superimposing breakpoints (see text).

before the breakpoint (dotted line) is low and approximately constant. In the vicinity of the breakpoint there is a rapid acceleration to a high and approximately constant rate. In the bottom half of Fig. 3, the number of responses in each 4-sec time segment was cumulated to present an "average" cumulative record when rate is averaged after first superimposing the breakpoints. (Note that the length of this average cumulative record is greater than 256 sec due to the fact that records are averaged after superimposing the breakpoints.) A comparison of this average cumulative record with individual intervals for the same bird (Fig. 1) shows that this method of averaging reflects the pattern of responding of the individual interval.

Figure 4 presents average cumulative records for Bird 451 on all values of FI using the breakpoint averaging technique. Figure 5 presents average cumulative records averaged over all birds for all values of FI using the same technique. (Four-sec time intervals were used for FI 64-sec and above. One- and two-sec time segments were used for FI 16-sec and 32-sec respectively to permit finer resolution of these intervals.) When rate is averaged after superimposing breakpoints, the average records reflect the break-and-run, *i.e.*, two-state nature of fixed-interval responding. Response rate at the beginning of the interval is low and approximately constant. At some point in the interval there is a rapid acceleration to a high and constant rate. This transition is more abrupt for shorter FIs. The second-state rate also appears to decrease as FI values increase.

Hence, the asymptotic behavior on an FI schedule may be described, to a first approximation, as consisting of two states. In state one, which begins immediately after reinforcement, there is a low and approximately constant rate of response. At some point during the interval there is a rapid acceleration to a high and approximately constant rate. This point of rapid acceleration varies from interval to interval.

Figure 6 presents the distribution of breakpoints for Bird 451 on all values of FI. An

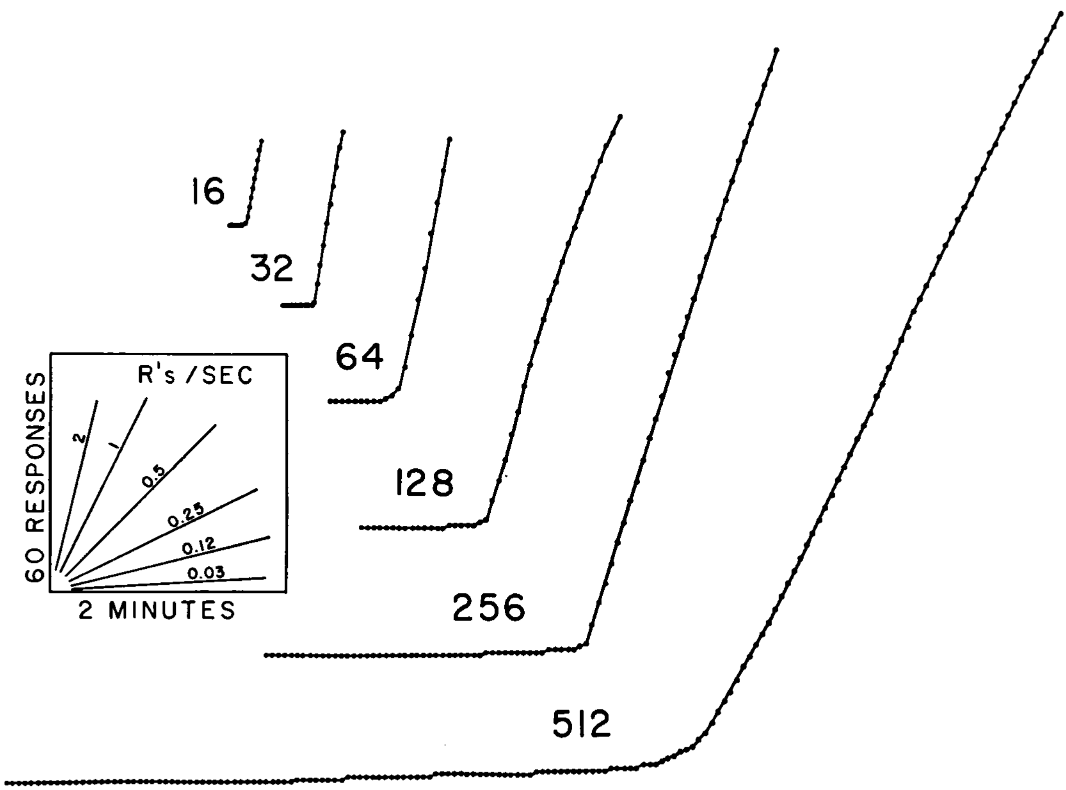

Fig. 4. Average cumulative response records for Bird 451 on six different fixed-interval schedules when records are averaged after superimposing breakpoints (see text). Four-sec time segments were used for FI 64-, 128-, 256-, and 512-sec. One- and 2-sec time segments were used for FI 16- and 32-sec respectively.

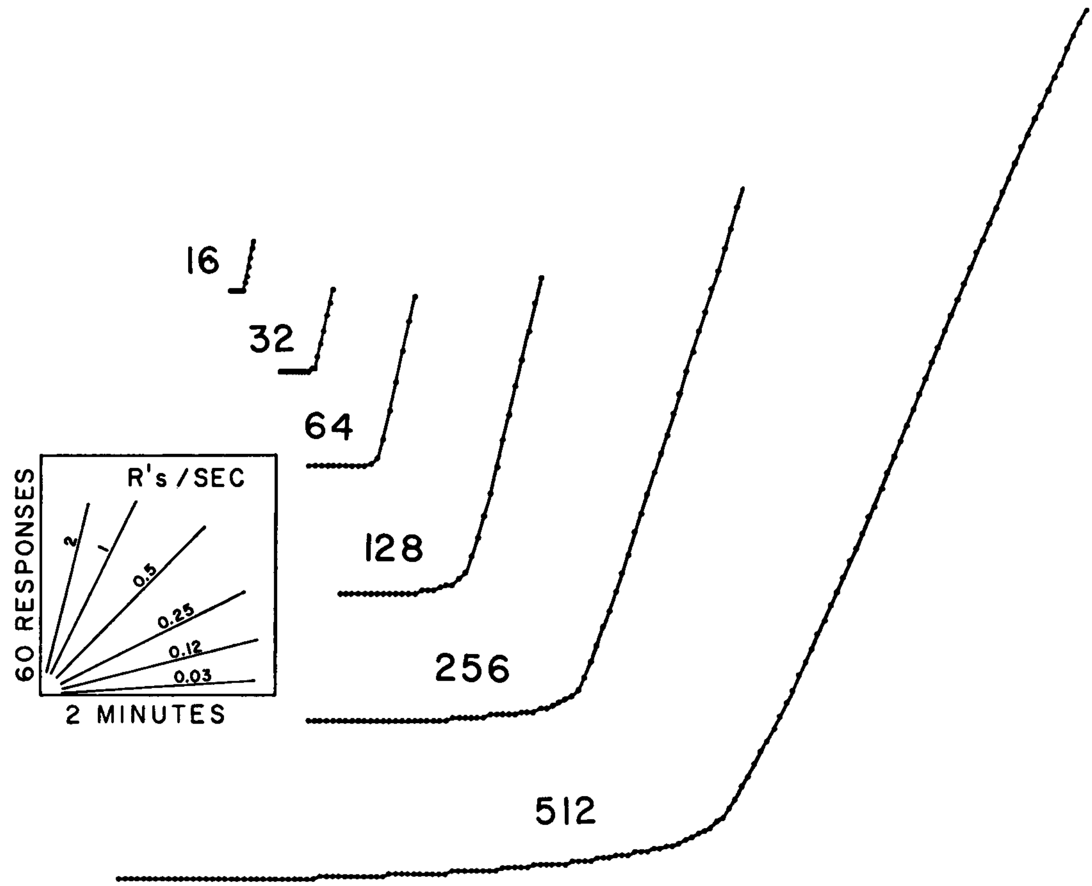

Fig. 5. Average cumulative response records averaged over all birds on six different fixed-interval schedules when records are averaged after superimposing breakpoints (see text). Four-sec time segments were used for FI 64-, 128-, 256-, and 512-sec. One- and 2-sec time segments were used for FI 16- and 32-sec respectively.

examination of this figure shows that the breakpoint distributions are highly variable and approximately normal in form. There is, however, a slight tendency for the longer intervals (FI 256-sec and 512-sec) to be more variable and negatively skewed. The breakpoint distributions for the other birds were quite similar to the one shown.

Figure 7 presents the breakpoint averaged over all birds as a function of fixed-interval length. The mean breakpoint appears to be linear with a slope of 0.67 (least-squares estimate). Table 2 presents the results of an analysis of variance on log breakpoint. The log transformation was used because (1) the standard deviation of the breakpoint was an approximately linear function of the mean breakpoint, and (2) the distributions were approximately normal in form. Both fixed intervals and subjects are significant. There is no effect due to the order in which the birds experienced the schedules. However, fixed-interval effects account for 99% of the total variance, *i.e.*, the effect due to subjects is relatively unimportant. This means that the value of the FI almost exclusively determines the breakpoint and there is relatively little intersubject variability.

Table 2

Log Breakpoint (Analysis of Variance)

Source	*Sum of Squares*	*d.f.*	*F*	*P*	*Tot. % S.S.*
FIs	52.56	5	1302.5	.005	99%
Subjects	0.20	5	4.849	.005	0%
Order	0.04	5	1.089	n.s.	0%
Residual	0.16	20			
S.S. Total	52.96	35			

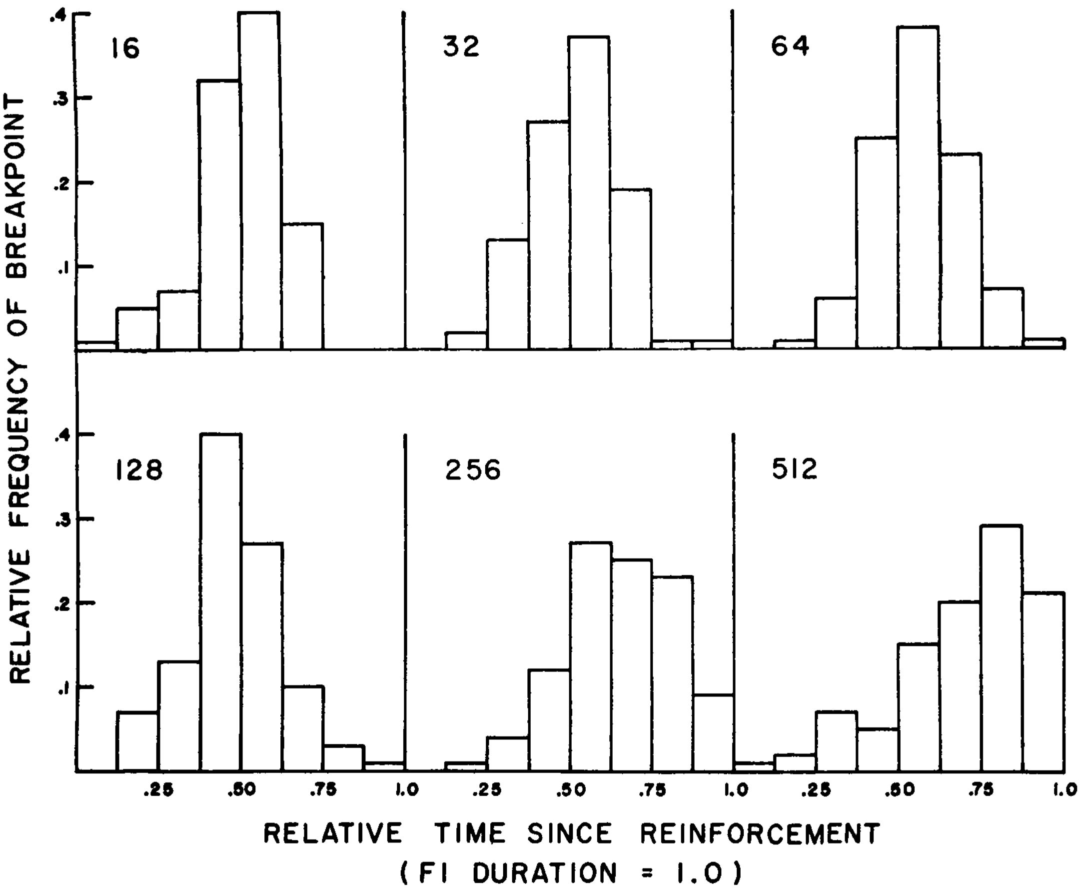

Fig. 6. Breakpoint distributions for Bird 451 on six different fixed-interval schedules. Each distribution was normalized so that the area summed to 1.0.

Figure 6 showed that the time from the beginning of the interval to the breakpoint was variable. This implies that the time from the breakpoint to reinforcement also varied. For

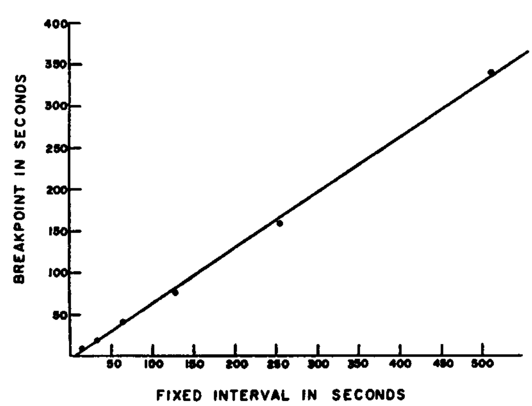

Fig. 7. Breakpoint averaged over six birds as a function of fixed-interval length. The slope of the line (method of least squares) is 0.67.

example, on FI 256-sec the breakpoints ranged from 32 to 256 sec with a mean of 170 sec. Therefore, the time from the breakpoint to reinforcement averaged 86 sec with values ranging from 224 sec to 0 sec. In a sense, then, the bird is on a VI schedule in the second state, where the interreinforcement intervals are determined by the bird's breakpoint distribution. Responding in the second state, then, would be expected to be similar to responding on a VI, and in particular, it would be expected that rate of response in the second state would be an increasing function of rate of reinforcement in the second state. (Second-state rate was obtained by averaging rate in consecutive time segments after the breakpoint for each bird and then taking the average over birds.) Figure 8 (unfilled circles) shows that rate of response in the second state is, with the exception of FI 32-sec, an increasing negatively accelerated function of rate of

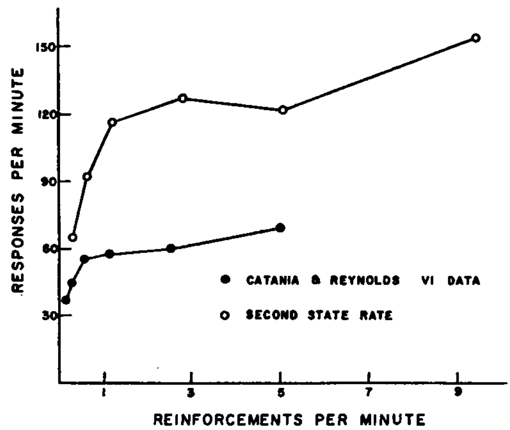

Fig. 8. Response rate in state 2 (unfilled circles) as a function of rate of reinforcement in state 2. The filled circles are the data obtained by Catania and Reynolds (1968) for variable-interval performance.

reinforcement in the second state. These data are similar in form to those (filled circles) obtained by Catania and Reynolds (1968) for variable-interval performance. First-state rate, on the other hand, was approximately constant over FI values (4.8, 4.2, 4.2, 4.2, 3.6, and 3.0 responses/min for FI 16-, 32-, 64-, 128-, 256-, and 512-sec respectively).

Figure 9 presents the overall rate of response (solid circles) as a function of the frequency of reinforcement. Response rate seems to increase, level off, and then increase again as the rate of reinforcement increases. A comparison of Fig. 8 and 9 indicates a discrepancy between these two measures. To the extent that the two-state analysis is correct, the overall rate should be predictable from the rate in the second state, the rate in the first state, and the average breakpoint. Accordingly, overall rate was predicted using the following formula.

$$R_o = \frac{(R_1 \times B + (FI\text{-}B) \times R_2)}{FI}$$

where,

R_o = overall rate
FI = fixed interval
B = Breakpoint
R_1 = first state rate
R_2 = second state rate

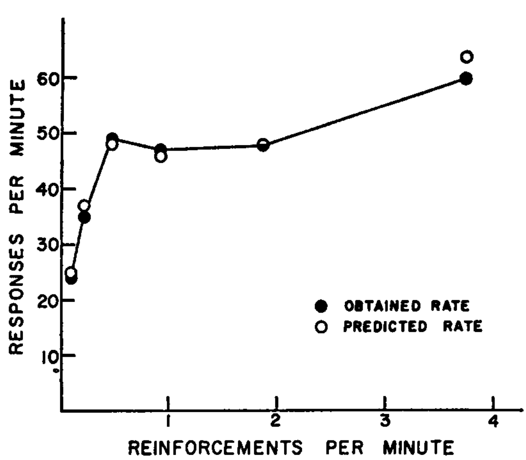

Fig. 9. Overall rate of responding (filled circles) as a function of scheduled rate of reinforcement. The unfilled circles represent the predictions of the two-state model (see text).

The open circles in Fig. 9 are the predicted rates. The agreement is quite close. The "bend" in the overall rate function can be accounted for in terms of the length of the breakpoint and the rates of response in the first and second states.

Table 3 presents analyses of variance for

Table 3

Overall Rate (Analysis of Variance)

Source	*Sum of Squares*	*d.f.*	*F*	*P*	*% S.S. Tot.*
FIs	1.313	5	4.80	<0.005	31%
Subjects	1.464	5	5.36	<0.005	34%
Orders	0.389	5	1.42	n.s.	
Residual	1.093	20			
S.S. Total	4.258	35			

Second-State Rate (Analysis of Variance)

Source	*Sum of Squares*	*d.f.*	*F*	*P*	*% S.S. Tot.*
FIs	7.82	5	8.10	0.001	42%
Subjects	4.58	5	4.75	0.01	25%
Orders	2.37	5	2.46	n.s.	13%
Residual	3.86	20			
S.S. Total	18.64	35			

second-state rate and overall rate to determine the relative contributions of fixed-interval values, subjects, and order of running. As Table 3 shows, both FIs and subjects have a significant effect on the two rate measures while order of running has no significant effect. However, FI value accounts for a greater proportion of the variance in second-state rate (42%) than it does for overall rate (31%). This suggests that FI value primarily influences second-state rate and, that as a result of averaging two different kinds of behavior (first and second state), the influence of FI value on overall rate is diminished.

DISCUSSION

When responding is not uniform over time, overall measures of response rate often obscure or ignore the pattern of responding. For instance, Weiss and Moore (1956) averaged rate over consecutive 18-sec periods of FI 180-sec. They found that average rate was approximately a linear function of time since reinforcement. However, such results may be obtained even if fixed-interval performance is break-and-run, as long as the breakpoint is variable. Sidman (1952) showed that a continuous curve can be obtained from averaging several discontinuous ones, providing that the discontinuities do not overlap. As a result, such averaging techniques may misrepresent the pattern of behavior on a fixed-interval schedule.

This problem was handled in the present experiment by using a least-squares technique to estimate the point of maximum acceleration (the breakpoint) for each interreinforcement interval, and subsequently averaging rate after superimposing the breakpoints. The obtained averages indeed appear by inspection to resemble a majority of the individual records (see Fig. 1 and 4). Furthermore, if the breakpoint averaging technique is used to obtain average records for several "scalloping" patterns, the obtained break-and-run pattern is demonstrably not an artifact of the averaging technique.

Figure 10 presents four theoretical response patterns on a fixed-interval schedule. In Fig. 10a, shallow scallops were generated, assuming that rate increased linearly over the interval. (Specifically, $R = A \times T$ where T is the time since reinforcement and R is the response rate. The values of A were chosen so that the mean number of responses per interval and its range were representative of the values obtained for FI 256-sec.) In Fig. 10b, sharp scallops were generated, assuming that rate increased exponentially over the interval. (Specifically, $R = A \times (e^{0.025T} - 1.0)$, where T is the time since reinforcement and R is the response rate. The values of A were chosen so that the mean number of responses per interval and its range were representative of the values obtained for FI 256-sec.) Figure 10c presents cumulative records composed of mixed sharp scallop and break-and-run performance, while Fig. 10d presents samples of pure break-and-run per-

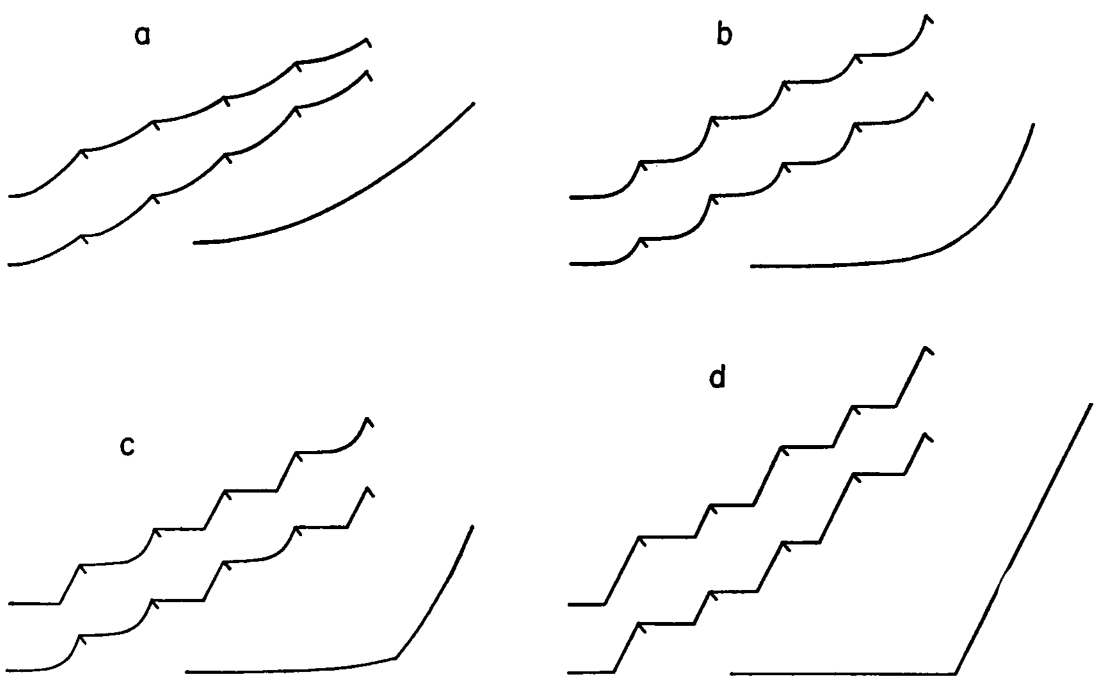

Fig. 10. Four hypothetical patterns of responding on a fixed-interval schedule. The figure to the right of each pattern is the average cumulative response record obtained by using the breakpoint averaging technique.

formance. (The breakpoints were chosen so that their mean was approximately two-thirds of the interval value and their range was representative of the breakpoints occurring on FI 256-sec.) The average cumulative record obtained using the breakpoint averaging technique is presented to the right of the theoretical records for all four patterns of responding. In each case, the breakpoint averaging technique accurately reproduces the form of the original curves. A comparison of the average cumulative records from the present experiment (Fig. 4 and 5) with Fig. 10 shows that the actual pattern of responding more closely approximates the pure break-and-run performance than any of the other theoretical types of performance.

In addition, several features of this break-and-run performance suggest that a fixed-interval schedule after extended training can be considered a multiple extinction variable-interval schedule. That is, the first state can be considered as a temporally discriminated post-reinforcement extinction period followed by a second component (state 2) that terminates with reinforcement. Since the breakpoint or point of transition from one component to the next is variable (see Fig. 6), this means that the time from the breakpoint to reinforcement will also vary from interval to interval. Hence, responding in the second state will be reinforced on a variable-interval schedule, the parameters of which are determined by the parameters of the breakpoint distribution.

Several factors suggest the correctness of this analysis. First, the response rate in the first state (extinction component) is low and approximately constant. Second, the response rate in the second state or VI component is high and approximately constant, *i.e.*, behavior characteristic of VI schedules. Third, the function relating response rate and reinforcement frequency in the second state is similar in form to the equivalent function on straightforward VI schedules (see Fig. 8). The major difference between these two functions is that response rate in the second state is considerably higher than on equivalent VI schedules. This might possibly be due to a "contrast" effect. Catania and Gill (1964) showed that interruption of responding on FI with extinction led to an increase in FI rate following the extinction component. Perhaps the "contrast" between the temporally discriminated post-reinforcement component and the second-state VI component is responsible for the high rates in state 2.

If the present analysis is correct, it suggests that fixed-interval responding, after extended training, consists of two components: a temporally discriminated post-reinforcement interval, followed by a rapid transition to a high response rate component with the point of transition occurring approximately two-thirds of the way through the interval. Several investigators (Cumming and Schoenfeld, 1958; Mechner, Guevrekian, and Mechner, 1963; and Sherman, 1959) have reported such break-and-run behavior on fixed-interval schedules after extended training. Furthermore, Berryman and Nevin (1962), Chung and Neuringer (1967), and Sherman (1959) have obtained linear relations between post-reinforcement pause and average interreinforcement interval with slopes close to two-thirds.

On the other hand, many investigators (Keller and Schoenfeld, 1950; Skinner, 1953; Ferster and Skinner, 1957; and Dews, 1962) have characteristically found a different pattern to fixed-interval responding. According to these investigators, rate is a smoothly increasing function of time since reinforcement, *i.e.*, there are no discontinuities in the performance. Since this so-called scallop pattern of responding is at least as well documented as the break-and-run pattern, it is quite likely that both may appear at different stages in the development of fixed-interval responding. Indeed, Cumming and Schoenfeld (1958) and Sherman (1959) argued that the scallop pattern is characteristic of earlier performance, and that the break-and-run pattern appears only after extended training. Sherman (1959) also suggested that the break-and-run pattern applies better to shorter intervals and may not appear for FIs of 9 min or greater. The present data (see Fig. 4 and 5) tend to support this suggestion. There may be other variables, not determined by the present work, that determine when and if a break-and-run pattern will occur.

APPENDIX

The problem consists of fitting a straight line to each leg of the cumulative record so that the sum of squared deviations from these lines is minimized. The major complication is that the intersection of the lines is one of the parameters to be determined. The method used here is an iterative one in which the point of intersection is systematically varied until the sum of squared deviations is minimized.

The FI is divided into n time periods of z-sec duration. This has the effect of quantizing the x axis of the cumulative record. The cumulative number of responses up to and including (i)th time segment is designated by y_i. Its corresponding time segment is designated as x_i. Assume that the breakpoint occurs between time segment m and time segment m + 1. According to the model, the relation between y_i and x_i is

$$\text{I.}\quad y_i = ax_i \qquad \text{for } x_i \leq x_m$$
$$y_i = b(x_i - x_m) + ax_m \qquad \text{for } x_i > x_m$$

If new constants are defined $a' = a(z)$ and $b' = b(z)$, equation I can be rewritten

$$\text{II.}\quad y_i = a'(i) \qquad \text{for } i \leq m$$
$$y_i = a'(m) + b'(i - m) \qquad \text{for } i > m$$

These equations can be written in matrix form as follows

$$\text{III.}\quad Y = XB \qquad \text{where}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ m & 0 \\ m & 1 \\ m & 2 \\ \cdot & \cdot \\ \cdot & \cdot \\ m & n\text{-}m \end{bmatrix} \begin{array}{l} \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} m \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} n\text{-}m \end{array} \quad B = \begin{bmatrix} a' \\ b' \end{bmatrix}$$

Least-squares theory offers a unique solution to these equations which minimizes the sum of squared deviations if, and only if, the determinant of $X'X \neq 0$. This solution is given by equation IV (Greybill, 1961).

$$\text{IV.}\quad B = (X'X)^{-1}X'Y$$

It can easily be shown that the determinant (D) of $X'X \neq 0$ for $1 \leq m \leq n - 1$.

Least-squares theory shows also that the sum of squared deviations is equal to $Y'Y - B'X'Y$. The value of m that minimizes this function is found by solving the equations for each integral value of m between 1 and n. (In the case where the rate was identical throughout the interval, it was assumed that the breakpoint occurred at the beginning of the interval. If the reinforced response was the only response in the interval, it was assumed that the breakpoint occurred at the end of the interval.) The value m′ that minimizes the sum of squares is multiplied by z to obtain the breakpoint. The first-state rate is given by a'/z and the second-state rate by b'/z.

Obviously, the larger the size of the time segment z, the more inaccurate the estimate of the breakpoint. Preliminary runs indicated that time segments from 0.5 to 4 sec do not seem to affect the estimate of breakpoint for FIs of greater than 64 sec. In order to minimize computer time, 4-sec segments were used for FI 64-, 128-, 256-, and 512-sec. One- and 2-sec segments were used for FI 16 and 32-sec respectively.

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