A unidirectional conditional proxy re-encryption scheme based on non-monotonic access structure

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Abstract: Recently, Fang et al. [6] introduced an interactive(bidirectional) conditional proxy re-encryption(C-PRE) scheme such that a proxy can only convert ciphertexts that satisfy access policy set by a delegator. Their scheme supports monotonic access policy expressed by "OR" and "AND" gates. In addition, their scheme is called interactive since generation of re-encryption keys requires interaction between the delegator and delegatee. In this paper, we study the problem of constructing a unidirectional(non-interactive) C-PRE scheme supporting non-monotonic access policy expressed by "NOT", "OR" and "AND" gates. A security model for unidirectional C-PRE schemes is also proposed in this paper. To yield a unidirectional C-PRE scheme supporting non-monotonic access policy, we extend the unidirectional PRE scheme presented by Libert et al. [8] by using the ideas from the non-monotonic attributed based encryption (ABE) scheme presented by Ostrovsky et al. [9]. Furthermore, the security of our C-PRE scheme is proved under the modified 3-weak Decision Bilinear Diffie-Hellman Inversion assumption in the standard model.

Keywords: Unidirectional conditional proxy re-encryption, The standard model, Non-monotonic access structure, Chosen ciphertext security, Attributed based encryption

1. Introduction

Encryption is one of the most fundamental cryptographic primitives. The concept of proxy re-encryption (PRE) was introduced by Blaze et al. in 1998 [4]. A proxy in PRE systems can convert a ciphertext encrypted under Alice's public key (delegator) into a ciphertext of the same message under Bob's public key (delegatee). Proxy re-encryption has many applications such as email forwarding, distributed file system [2]. A bidirectional PRE scheme allows a proxy to convert ciphertexts encrypted under Alice into ciphertexts under

Bob via a re-encryption key and the same key can also be used to translate from Bob to Alice. On the other hand, if the re-encryption key only allows one-way conversion (e.g., from Alice to Bob), then the corresponding PRE scheme is called unidirectional.

The PRE scheme in [4] is bidirectional and CPA secure under DDH assumption. In 2005, Ateniese et al. [2] presented several CPA secure unidirectional PRE schemes based on bilinear pairing. Then Canetti and Hohenberger [5] presented an appropriate definition of chosen ciphertext security(CCA) for bidirectional PRE schemes and the first CCA secure bidirectional PRE scheme. The work in [5] left an open problem to come up with a CCA secure unidirectional PRE scheme. Libert and Vergnaud [8] presented a definition of chosen ciphertext security (CCA) for unidirectional PRE schemes and the first unidirectional PRE scheme with CCA security in the standard model.

Normal PRE schemes allow a semi-trusted proxy to translate ciphertexts from Alice to Bob unconditionally. It is desirable that a proxy can only convert ciphertexts under certain constraints set by the delegator. Shao et al. [12] designed a PRE scheme with keyword search property, which allows a proxy equipped with trapdoor information to test whether a ciphertext from Alice contains one specified keyword. However, it is pointed out [13] that the trapdoor still allows the proxy to convert ciphertexts from Alice without any restriction. On the other hand, Weng et al. [14, 15] introduced the notion of conditional proxy re-encryption (C-PRE) such that only ciphertexts whose keywords satisfy certain conditions set by Alice can be converted by a proxy. They also left it as an open problem to construct a C-PRE scheme supporting access policy consisting of “OR” and “AND” gates over keywords.

Wang et al. [13] presented a unidirectional PRE scheme supporting conjunctive keywords search and selective delegation such that a proxy can only re-encrypt ciphertexts that contain a set of keywords specified by the delegator. In other words, their construction supports access policy expressed by “AND” gates. By regarding keywords as attributes, Fang et al. [6] presented an interactive(bidirectional) single-hop C-PRE scheme based on access tree used in the attribute based encryption scheme [7], which supports access policy consisting of “OR” and ‘AND” gates. Their scheme is called interactive since generation of re-encryption keys requires interactions between the delegator and delegatee who take their secret keys as private input. Interactive generation of re-encryption keys is an essential feature of bidirectional C-PRE scheme defined in [5]. CCA security of their C-PRE scheme was proved under the random oracle model. They also left it as an open problem to construct a *non-interactive(unidirectional)* C-PRE scheme with security in *the standard model*.

Although Wang et al. [13] defined their CCA security model for unidirectional PRE schemes supporting conjunctive keywords search, their security model is coupled tightly with the notion of conjunctive keywords search. Hence the model in [13] is not suitable for C-PRE schemes supporting generic access structure. In addition, the work in [6] considered security model for interactive(bidirectional) C-PRE schemes and proved security of their construction under the random oracle model.

Sahai and Waters [11] introduced the concept of attribute based encryption (ABE), in which a ciphertext is associated with a set of attributes, and a user's private key will reflect an access policy over attributes that controls which ciphertexts a user is able to decrypt. The original construction of Sahai and Waters was limited to express threshold access structure. Goyal et al. [7] presented ABE schemes based on access tree in which the private key supports any monotonic access structure. To increase the expressibility of ABE schemes, Ostrovsky et al. [9] designed an ABE construction that supports non-monotonic access structure represented by "NOT", "OR" and "AND" gates over attributes.

Motivated by the above discussion, we aim to design a *unidirectional(non-interactive)* C-PRE scheme supporting *non-monotonic access structure* to enhance the expressibility of C-PRE schemes. The rest of paper is organized as follows. At first, we provide security definitions for unidirectional C-PRE schemes in which a ciphertext is associated with a set of keywords, and a re-encryption key will reflect an access policy that controls which ciphertexts a proxy is able to re-encrypt. Subsequently, we extend the unidirectional PRE scheme [8] to yield a unidirectional C-PRE scheme supporting non-monotonic access structures. Finally our construction is proved to be CCA secure under the standard model.

A challenge in our security proof lies in the fact that a corrupted user in our model is allowed to obtain re-encryption keys from the target user so long as the access structure associated with these re-encryption keys are not satisfied by the challenge set of attributes associated with the challenge ciphertext. On the other hand, in order to support negation by using the techniques in [9], we have to design two types of re-encryption keys, which also affects the structure of a user's secret key in our construction.

2. Preliminaries

2. 1 Bilinear pairing

Given a security parameter λ, an efficient algorithm $PG(1^{\lambda})$ outputs (e, G, G_T, g, p), where G is a cyclic group of a prime order p generated by g, and $2^{\lambda-1} < p < 2^{\lambda}$. G_T is a cyclic group of the same order, and let $e: G \times G \to G_T$ be a efficiently computable bilinear function with the following properties:

1. Bilinear: $e(g^a, g^b) = e(g, g)^{ab}$, for all $a, b \in Z_p$.

2. Non-degenerate: $e(g, g) \neq 1_{G_T}$

2.2 Modified 3-wDBDHI assumption

Given (e, G, G_T, g, p) output by $PG(1^{\lambda})$, we define two experiments in which an adversary A outputs 0 or 1.

Experiment 0: A is given $(g, g^{\frac{1}{a}}, g^a, g^{a^2}, g^b, e(g, g)^{\frac{b}{a^2}}), a, b \leftarrow_R Z_p^*$.

Experiment 1: A is given $(g, g^{\frac{1}{a}}, g^a, g^{a^2}, g^b, T)$, $a, b \leftarrow_R Z_p^*, T \leftarrow_R G_T$.

The modified 3-**weak Decision Bilinear Diffie-Hellman Inversion** assumption [8] claims for any polynomial time algorithm A, the probability $|\Pr[W_0] - \Pr[W_1]|$ is negligible, where W_i is the event that A outputs 1 in experiment i.

2.3 One-time signature

A digital signature scheme $Sig = (\text{Gen}, \text{S}, \text{V})$ consists of the following algorithms:

1. **Gen**(λ): Outputs a secret/public key pair (sk, pk).

2. **S** (sk, m): Given a secret key sk and a message m, then outputs a signature σ.

3. **V**(pk, m, σ): Takes as input a public key pk, a message m and a signature σ, then outputs either 1 or 0 to denote "accept" or "reject".

We review the definition of strong existential unforgeability for a signature scheme denoted by $Sig = (\mathrm{Gen,S,V})$ in experiment $Exp^{Sig}_{1^\lambda,SCMA}(A)$.

$Exp^{Sig}_{1^k,SCMA}(A)$

The challenger C runs $(pk, sk) \leftarrow \mathrm{Gen}(1^\lambda)$ and sets $S_\sigma \leftarrow \emptyset$.

$(m^*, \sigma^*) \leftarrow A^{\text{O-Sig}}(pk)$.

The adversary A wins if $(m^*, \sigma^*) \notin S_\sigma$ and $\mathbf{V}(pk, m^*, \sigma^*)$ =1.

Advantage of A in experiment $Exp^{Sig}_{1^\lambda,SCMA}(A)$ is defined to be the probability that A wins in the experiment.

The oracle O-Sig is defined as follows:

O-Sig(m)

Returns $\sigma = S(sk, m)$ and updates $S_\sigma = S_\sigma \bigcup \{(m, \sigma)\}$.

A strongly unforgeable one-time signature scheme Sig requires that for any PPT adversary A who can access the oracle O-Sig only once, its advantage Adv^{OTS} in experiment $Exp^{Sig}_{1^k,SCMA}(A)$ is negligible.

3. Security definitions and model

3.1 Syntax of unidirectional C-PRE schemes

A *unidirectional* single-hop C-PRE scheme consists of the following algorithms. A ciphertext is associated with a set of keywords, and a re-encryption key will reflect an access policy over keywords that controls which ciphertexts a proxy is able to re-encrypt.

$\mathrm{Setup}(\lambda)$: Given the security parameter λ, this algorithm produces a set par of global public parameters.

$\mathrm{Keygen}(par)$: Given par, this algorithm generates a secret/public key pair (sk, pk).

$\mathrm{ReKeygen}(par, sk_i, pk_j, \widetilde{A})$: Given par, the secret key sk_i of user i, the public key pk_j of user $j \neq i$ and an access structure $\widetilde{A}$, this algorithm generates a re-encryption key $R_{i\to j,\widetilde{A}}$. We use an algorithm rather than an interactive protocol to implicitly assume that the process of generating re-encryption keys is non-interactive.

$\mathrm{Enc}_1(par, pk_i, m)$: Given par, a public key pk_i and a message m, this algorithm outputs a first level ciphertext CT_1 that cannot be re-encrypted for another party.

$\mathrm{Enc}_2(par, pk_i, m, \gamma)$: Given par, a public key pk_i, a message m and a set γ of keywords(attributes), this algorithm outputs a second level ciphertext CT_2 that can be re-encrypted into a first level ciphertext.

$\mathrm{ReEnc}(par, CT_2, \gamma, pk_i, R_{i\to j,\widetilde{A}})$: Given par, a re-encryption key $R_{i\to j,\widetilde{A}}$ and a second level ciphertext CT_2 encrypted under pk_i and a set γ of keywords, this algorithm outputs a first level ciphertext CT_1 encrypted under pk_j when γ satisfies access structure $\widetilde{A}$;otherwise a message “invalid” is returned.

$\mathrm{Dec}_1(par, sk_i, CT_1)$: Given par, a secret key sk_i and a first level ciphertext CT_1, this algorithm outputs a message m or a message “invalid”.

$\mathrm{Dec}_2(par, sk_i, CT_2)$: Given par, a secret key sk_i and a second level ciphertext CT_2, this algorithm outputs a message m or a message “invalid”.

In the following, we will take par as implicit input for simplicity. For any message m, any couple of secret/public key pair (sk_i, pk_i), (sk_j, pk_j), the following conditions of correctness should be satisfied:

(1) $\mathrm{Dec}_1(sk_i, \mathrm{Enc}_1(pk_i, m)) = m$; $\mathrm{Dec}_2(sk_i, \mathrm{Enc}_2(pk_i, m, \gamma)) = m$;

(2) If γ satisfies the access structure $\widetilde{A}$, the following should hold:

$$CT_1 = \mathrm{ReEnc}(\mathrm{Enc}_2(pk_i, m, \gamma), \gamma, pk_i, \mathrm{ReKeygen}(sk_i, pk_j, \widetilde{A})),$$

$$\mathrm{Dec}_1(sk_j, CT_1) = m.$$

3.2 Security of second level ciphertexts

Init: As in [8], the adversary Ad determines the target user i^*, the corrupted users and declares a set γ^* of keywords that he wishes to be challenged upon at this stage.

Setup: The challenger C runs $\mathrm{Setup}(\lambda)$ to produce the global public parameters par and generates key pairs as follows:

$$\mathrm{KeyGen}(\cdot) \to (pk^*, sk^*),\ \mathrm{KeyGen}(\cdot) \to (pk_x, sk_x),\ \mathrm{KeyGen}(\cdot) \to (pk_h, sk_h).$$

(pk^*, sk^*) is the key pair for the honest target user i^*. Key pairs subscripted by h or h' represents honest parties and corrupted key pairs are subscripted by x or x'.

Phase 1: Ad takes pk^*, $\{pk_h\}, \{pk_x, sk_x\}$ as input and issue queries to oracles O_{rekey}, O_{renc} and O_{dec-1}.

Challenge: Ad outputs two equal-length messages (m_0, m_1). The challenger C flips a random bit b and returns $CT_2^* = \mathrm{Enc}_2(pk^*, m_b, \gamma^*)$.

Phase 2: Ad still issues queries to oracles O_{rekey}, O_{renc} and O_{dec-1}.

Guess: Ad outputs a bit b'.

The advantage of the adversary in this game is $\varepsilon = |\Pr[b' = b] - 0.5|$. A C-PRE scheme is CCA secure at level 2 if ε is negligible.

The re-encryption key oracle O_{rekey}

Given a tuple $(pk_i, pk_j, \widetilde{A})$, this oracle proceeds as follows:

(1) If both pk_i and pk_j are honest, returns $R_{i \to j, \widetilde{A}} \leftarrow \mathrm{ReKeygen}(sk_i, pk_j, \widetilde{A})$;

(2) If the honest $pk_i = pk^*$, pk_j is corrupted and γ^* does not satisfy $\widetilde{A}$, returns $R_{i^* \to j, \widetilde{A}} \leftarrow \mathrm{ReKeygen}(sk^*, pk_j, \widetilde{A})$;

The re-encryption oracle O_{renc}

Given a tuple $((CT_2, \gamma, pk_i), pk_j, \widetilde{A})$, where CT_2 is a second level ciphertext encrypted under $(pk_i,\ \gamma)$, and pk_i, pk_j are public keys produced by Keygen, this oracle proceeds as follows:

(1) If $pk_i = pk^*$, pk_j is corrupted and $\mathrm{Dec}_2(sk^*, CT_2) \in \{m_0, m_1\}$, returns a message "invalid" since re-encryption may leak information about the challenge bit b in this case.

(2) If γ satisfies the access structure $\widetilde{A}$, computes $R_{i\to j,\widetilde{A}} \leftarrow \mathrm{ReKeygen}(sk_i, pk_j, \widetilde{A})$ and returns the first level ciphertext $CT_1 \leftarrow \mathrm{ReEnc}((CT_2, pk_i, \gamma), R_{i\to j,\widetilde{A}})$. Otherwise, outputs a message "invalid".

First level decryption oracle O_{dec-1}

Given a tuple (pk_i, CT_1), where CT_1 is a first level ciphertext encrypted under the public key pk_i, this oracle proceeds as follows:

(1) If (pk_i, CT_1) is a **derivative** of the challenge pair (pk^*, CT_2^*), returns a message "invalid".

(2) Otherwise, returns $m \leftarrow \mathrm{Dec}_1(sk_i, CT_1)$.

A **Derivative** (pk_i, CT_1) of the challenge pair (pk^*, CT_2^*) in this game is defined as follows:

If CT_1 is a first level ciphertext and $pk_i = pk^*$, or pk_i belongs to a honest user, (pk_i, CT_1) is a **derivative** of the challenge pair if $\mathrm{Dec}_1(sk_i, CT_1) \in \{m_0, m_1\}$.

3.3 Security of first level ciphertexts

Init: The adversary Ad determines the target user i^* and the corrupted users at this stage.

Setup: The challenger C runs $\mathrm{Setup}(\lambda)$ to produce the global public parameters par and generates key pairs in the same way as described previously:

$$\mathrm{KeyGen}(\cdot) \to (pk^*, sk^*),\ \mathrm{KeyGen}(\cdot) \to (pk_x, sk_x),\ \mathrm{KeyGen}(\cdot) \to (pk_h, sk_h).$$

Phase 1: The adversary Ad who takes as input pk^*, $\{pk_h\}, \{pk_x, sk_x\}$ can issue queries to oracles O_{rekey}, O_{dec-1}.

Challenge: Ad outputs two equal-length message (m_0, m_1). The challenger C flips a random bit b and returns $CT_1^* = \mathrm{Enc}_1(pk^*, m_b)$.

Phase 2: Ad still issues queries to the oracle O_{dec-1}.

Guess: The adversary outputs a bit $b^/$.

The advantage of the adversary in this game is $\varepsilon = |\Pr[b^/ = b] - 0.5|$. A C-PRE scheme is CCA secure at level 1 if ε is negligible.

The re-encryption key oracle O_{rekey}

Given a tuple $(pk_i, pk_j, \widetilde{A})$, this oracle returns $R_{i \to j, \widetilde{A}} \leftarrow \mathrm{ReKeygen}(sk_i, pk_j, \widetilde{A})$. This means that the adversary is allowed access to all re-encryption keys without any restriction.

First level decryption oracle O_{dec-1}

Given a tuple (pk_i, CT_1), where CT_1 is a first level ciphertext encrypted under the public key pk_i, this oracle proceeds as follows:

If (pk_i, CT_1) is a **derivative** of the challenge pair (pk^*, CT_1^*), returns a message "invalid". Otherwise, returns $m \leftarrow \mathrm{Dec}_1(sk_i, CT_1)$.

A **Derivative** (pk_i, CT_1) of the challenge pair (pk^*, CT_1^*) in this game is defined as follows:

If CT_1 is a first level ciphertext and $pk_i = pk^*$, (pk_i, CT_1) is a **derivative** of the challenge pair if $\mathrm{Dec}_1(sk_i, CT_1) \in \{m_0, m_1\}$.

Ateniese et al. [2] defined a security notion called *master secret security* for unidirectional PRE schemes. This notion requires that no coalition of dishonest delegatees be able to pool their re-encryption keys in order to expose the secret key of their common delegator. It is discussed in [6, 8] that CCA security at level 1 implies master secret security for single-hop PRE schemes.

4. Our C-PRE scheme

$\mathrm{Setup}(\lambda)$: Given (e, G, G_T, g, p) output by $PG(1^\lambda)$, picks generators $(g_1, u, v) \leftarrow G, g_2 = g^w, w \leftarrow Z_p^*$ and a strongly unforgeable one-time signature scheme $Sig = (Gen, S, V)$. Let parameter d specifies the exact number of keywords that every second level ciphertext has. We associate each keyword with a unique element in Z_p^*.

Then chooses two random polynomials $h(x)$ and $q(x)$ of degree d subject to the constraint $q(0) = w^{-1} \bmod p$. We also define two functions $T(x) = g_1^{x^d} \cdot g_2^{h(x)}$ and $V(x) = g_2^{q(x)}$ that are publicly computable by interpolation. The set par of public parameters is $(g, u, v, g_1, g_2, g_2^{q(0)} = g, \cdots, g_2^{q(d)}, g_2^{h(0)}, \cdots, g_2^{h(d)}, Sig)$.

Keygen : Picks $(x_{i1}, x_{i2}) \leftarrow Z_p^*$ and sets a secret/public key pair for user i as $sk_i = (x_{i1}, x_{i2}), pk_i = (X_{i1} = g^{x_{i1}}, X_{i2} = g^{x_{i2}})$.

$\mathrm{ReKeygen}(sk_i, pk_j, \widetilde{A})$: Given the secret key sk_i of user i, the public key pk_j of user j and a non-monotonic access structure $\widetilde{A}$, user i generates a re-encryption key $R_{i \to j, \widetilde{A}}$ as follows:

When dealing with a non-monotonic access structure $\widetilde{A}$ over a set of (unprimed)keywords $\widetilde{P}$, we proceed similarly as in [9]. For each unprimed keyword $p \in \widetilde{P}$, we define another primed keyword $p^{/}$. Let $P^{/}=\{p^{/} \mid p \in \widetilde{P}\}$. Then define a monotonic access structure A over $P=P^{/} \bigcup \widetilde{P}$ in such a way that $S \in \widetilde{A}$ if and only if $N(S) \in A$, where $N(\cdot)$ is an operator defined as $N(S)=S \bigcup \{p^{/} \in P^{/} \mid p \in P \setminus S\}$. That is, $N(S)$ consists of all the keywords in S plus the primed part of all the keywords that are not in S.

Let A be associated with a linear secret sharing mechanism $\prod$. Then user i applies $\prod$ over the set P to obtain shares $\{\lambda_k\}$ of the secret x_{i2}^{-1}. For each keyword $\widetilde{p_k} \in P$ (the underlying unprimed keyword is p_k), a random $r_k \leftarrow Z_p$ is chosen:

If $\widetilde{p_k} = p_k$ is unprimed, $D_k = (D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i2}^{r_k})$.

If $\widetilde{p_k} = p_k^{/}$ is primed, $D_k = (D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i2}^{r_k})$.

The re-encryption key $R_{i \to j, \widetilde{A}} = \{D_k\}_{\widetilde{p_k} \in P}$.

$\mathrm{Enc}_1(pk_i, m)$: Given a public key pk_i and a message m, this algorithm proceeds as follows:

(1) Chooses $r \leftarrow Z_p$ and generates a fresh one-time signature key pair $(ssk, svk) \leftarrow Gen(\lambda)$;

(2) $C_1 = svk, C_2^{/} = e(g, X_{i1})^r, C_3 = e(g,g)^r \cdot m$, $C_4 = (u^{svk} v)^r$;

(3) Generates a one-time signature $\sigma = S(ssk, m \| C_3 \| C_4)$;

The first level ciphertext $CT_1 = (C_1, C_2^{/}, C_3, C_4, \sigma)$.

$\mathrm{Enc}_2(pk_i, m, \gamma)$: Given the public key pk_i, a message m and a set γ of d keywords, outputs a second level ciphertext CT_2 that can be re-encrypted into a first level ciphertext as follows:

(1) Chooses $r \leftarrow Z_p$ and generates a fresh one-time signature key pair (ssk, svk);

(2) $C_1 = svk, C_2 = X_{i2}^{r}, C_3 = e(g,g)^r \cdot m, C_4 = (u^{svk} v)^r$

$C_5^p = \{T(p)^r\}_{p\in\gamma}, C_6^p = \{V(p)^r\}_{p\in\gamma}$;

(3) Generate a one-time signature $\sigma = S(ssk, m \| C_3 \| C_4)$.

The second level ciphertext $CT_2 = (C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$.

$\mathrm{ReEnc}((CT_2, \gamma, pk_i), R_{i\to j,\widetilde{A}})$: Given a re-encryption key $R_{i\to j,\widetilde{A}}$ and a second level ciphertext CT_2 encrypted under (pk_i, γ), if γ satisfies access structure $\widetilde{A}$, this algorithm outputs a first level ciphertext CT_1 encrypted under pk_j as follows:

(1) Parses CT_2 as $C_1 = svk, C_2 = X_{i2}{}^r, C_3 = e(g,g)^r \cdot m, C_4 = (u^{svk}v)^r$

$$C_5^p = \{T(p)^r\}_{p\in\gamma}, C_6^p = \{V(p)^r\}_{p\in\gamma},\ \sigma$$

(2) Recall that $\widetilde{A}$ induces a monotonic access structure A. Denote $\gamma^/ = N(\gamma)$. As γ satisfies access structure $\widetilde{A}$, $\gamma^/$ is authorized in A by previous definition of the operator $N(\cdot)$. Let $I = \{k : \widetilde{p_k} \in \gamma^/\}$. A set of coefficients $\{\omega_k\}_{k\in I}$ can be efficiently computed such that $\sum_{k\in I}\omega_k\lambda_k = x_{i2}{}^{-1}$ [3].

For every unprimed attribute $\widetilde{p_k} = p_k \in \gamma^/$, (so $p_k \in \gamma$ by definition of the operator $N(\cdot)$), we proceeds as follows:

(2.1) Extracts $D_k =$ $(D_k^{(1)} = X_{j1}{}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i2}{}^{r_k})$ from the re-encryption key;

(2.2) Computes $Z_k = e(D_k^{(1)}, C_2) / e(D_k^{(2)}, C_5^{p_k})$

$$= e(g^{x_{j1}\lambda_k} \cdot T(p_k)^{r_k}, g^{x_{i2}r}) / e(g^{x_{i2}r_k}, T(p_k)^r) = e(g,g)^{x_{j1}x_{i2}\lambda_k r}$$

For every primed attributed $\widetilde{p_k} = p_k^/ \in \gamma^/$ (so $p_k \notin \gamma$ by definition), let $\gamma_k = \gamma \bigcup \{p_k\}$. Note that $|\gamma_k| = d+1$ and recall that the degree of the polynomial $q(\cdot)$ is d. Using the keywords in γ_k as an interpolation set, we compute lagrangian coefficients $\{\sigma_p\}_{p\in\gamma_k}$ such that $\sum_{p\in\gamma_k}\sigma_p q(p) = q(0) = (\log_g g_2)^{-1}$. Then we proceeds as follows:

(2.3) Extracts D_k = $(D_k^{(3)} = X_{j1}{}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i2}{}^{r_k})$ from the re-encryption key and computes:

$$Z_k = \frac{e(D_k^{(3)}, C_2)}{e(D_k^{(5)}, \prod_{p\in\gamma}(C_6^p)^{\sigma_p})e(D_k^{(4)}, C_2)^{\sigma_{p_k}}} = \frac{e(g^{\lambda_k x_{j1}} g^{r_k}, g^{x_{i2} r})}{e(g^{x_{i2} r_k}, \prod_{p\in\gamma}(V(p)^r)^{\sigma_p})e(V(p_k)^{r_k}, g^{x_{i2} r})^{\sigma_{p_k}}}$$

$$= \frac{e(g^{x_{j1}\lambda_k}, g^{x_{i2} r})e(g^{r_k}, g^{x_{i2} r})}{e(g^{x_{i2} r_k}, \prod_{p\in\gamma}(g_2^{q(p)\cdot r})^{\sigma_p})e(g_2^{q(p_k) r_k}, g^{x_{i2} r})^{\sigma_{p_k}}}$$

$$= \frac{e(g^{x_{i2} x_{j1}\lambda_k}, g^r)e(g^{r_k}, g^{x_{i2} r})}{e(g, g_2)^{x_{i2} r_k r \sum_{p\in\gamma_k}\sigma_p q(p)}} = e(g, g)^{x_{j1} x_{i2}\lambda_k r}$$

Finally we have $\prod_{k\in I} Z_k^{\omega_k} = e(g, g)^{x_{j1} x_{i2} r \cdot (\sum_{k\in I}\omega_k \lambda_k)} = e(g, g)^{\frac{r x_{j1} x_{i2}}{x_{i2}}} = e(g, g)^{r x_{j1}}$.

The first level ciphertext $CT_1 = (C_1, C_2^{/} = e(g, X_{j1})^r, C_3, C_4, \sigma)$.

$\mathrm{Dec}_1(sk_i, CT_1)$: Given a secret key sk_i and a first level ciphertext CT_1, this algorithm proceeds as follows:

(1) Parses CT_1 as $(C_1, C_2^{/}, C_3, C_4, \sigma)$;

(2) Computes $C_2^{/\frac{1}{x_{i1}}} = e(g, g^{x_{i1}})^{\frac{r}{x_{i1}}} = e(g, g)^r$ and $m = C_3 / e(g, g)^r$;

(3) Tests $V(C_1, m \| C_3 \| C_4) = 1$ (V1)

If relation V1 does not hold, outputs a message “invalid”; otherwise outputs m.

$\mathrm{Dec}_2(sk_i, (CT_2, \gamma))$: Given a secret key sk_i and a second level ciphertext CT_2, this algorithm proceeds as follows:

(1) Parses CT_2 as $(C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$;

(2) Tests $e(C_2, (u^{C_1} v)) = e(X_{i2}, C_4)$ (V2)

If relation V2 does not hold, outputs a message “invalid”.

(3) Otherwise, computes $m = \frac{C_3}{e(g, C_2)^{\frac{1}{x_{i2}}}} = \frac{e(g, g)^r \cdot m}{e(g, g^{x_{i2}\cdot r})^{\frac{1}{x_{i2}}}}$;

(4) If relation V1 does not hold, outputs a message “invalid” ; otherwise outputs m.

Remark: Although our construction requires that every second level ciphertext has exactly

d keywords, this restriction can be mitigated by using the method proposed in [9].

Theorem 1: Assume that the one-time signature scheme is strongly unforgeable. Our scheme is CCA secure at level 2 under the modified 3-wDBDHI assumption.

Proof: Let $(g, A_{-1} = g^{\frac{1}{a}}, A_1 = g^a, A_2 = g^{a^2}, B = g^b, T)$ be a modified 3-wDBDHI instance. We build an algorithm B_A deciding whether $T = e(g,g)^{\frac{b}{a^2}}$ from a successful CCA adversary Ad at level 2 with advantage ε.

Init: The adversary Ad determines the target user i^*, the corrupted users and declares a set γ^* of d keywords to be challenged upon.

Setup: B_A picks a one-time signature scheme $Sig = (Gen, S, V)$ such that the maximal probability δ that any public key can be selected should be less than $2^{-\lambda}$ as in [8]. B_A generates a fresh one-time signature key pair (ssk^*, svk^*) and sets $u = A_1^{\alpha_1}$, $v = A_1^{-\alpha_1 \cdot svk^*} \cdot A_2^{\alpha_2}$, $g_1 = (A_1)^{\mu}, g_2 = A_2, \alpha_1, \alpha_2, \mu \leftarrow Z_p^*$.

Having chosen a random degree d polynomial $f(x)$, two random degree d polynomials $u(x)$ and $h(x)$ are defined as follows:

Let $\gamma^* = \{p_1^*, \cdots, p_d^*\}$. B_A sets $u(x) = -x^d$ for all $x \in \gamma^*$ and $u(x) \neq -x^d$ for some(arbitrary) $x \notin \gamma^*$. This ensures that $u(x) = -x^d$ if and only if $x \in \gamma^*$. Let $h(x) = (a^{-1} \cdot \mu \cdot u(x) + f(x))$. Hence $T(x) = g_1^{x^d} \cdot g_2^{h(x)} = g_1^{x^d + u(x)} \cdot g_2^{f(x)}$ can be publicly computed for arbitrary x.

Then B_A picks $\{\theta_1, \cdots, \theta_d\} \leftarrow Z_p^*$ and implicitly defines a random degree d polynomial $q(x)$ such that $q(0) = (a^2)^{-1}$, $q(p_i^*) = \theta_i$, $1 \le i \le d$. We have $g_2^{q(0)} = g$ and $V(x) = g_2^{q(x)}$ can be computed for arbitrary x by interpolation. These parameters are distributed identically to that in the real scheme.

Key generation: Public key of an honest user $i \in HU \setminus \{i^*\}$ is defined as $X_{i1} = g^{ax_{i1}} = A_1^{x_{i1}}$, $X_{i2} = g^{x_{i2}}$ for randomly chosen $x_{i1}, x_{i2} \leftarrow Z_p^*$. The target user's public key is set as $X_{i^*1} = g^{a \cdot x_{i^*1}} = A_1^{x_{i^*1}}$, $X_{i^*2} = g^{a^2 \cdot x_{i^*2}} = A_2^{x_{i^*2}}$ for randomly chosen $x_{i^*1}, x_{i^*2} \leftarrow Z_p^*$. Key pair of a corrupted user j is set as $X_{j1} = g^{x_{j1}}$, $X_{j2} = g^{x_{j2}}$ for randomly chosen $x_{j1}, x_{j2} \leftarrow Z_p^*$.

Given a tuple $(pk_i, pk_j, \widetilde{A})$ chosen by Ad, the re-encryption key oracle O_{rekey} is simulated by B_A as follows:

Let $\prod$ be the linear secret sharing mechanism associated with the monotonic access structure A induced by $\widetilde{A}$ over a set P. Let M be the share-generating matrix for $\prod$ with l rows and n columns. Each row M_k of M is labeled by a keyword named $\widetilde{p_k} \in P$ and p_k is the unprimed keyword underlying $\widetilde{p_k}$. We list the following propositions:

Proposition 1 [3]**:** Assume Q is not an authorized set in the access structure A. $(\underbrace{1, 0, \cdots, 0}_{n})$ is linearly independent of the rows M_Q, where M_Q is the sub-matrix of M containing those rows labeled by keywords in Q.

Proposition 2 [1, 10]**:** A vector π is linearly independent of a matrix N if and only if there exist a vector θ which can be efficiently computed such that $N \cdot \theta = \vec{0}$ while $\pi \cdot \theta = 1$.

Then we consider the following cases:

(1) $i = i^*$, $j \in CU$ and γ^* does not satisfy $\widetilde{A}$:

When $\widetilde{A}$ is not satisfied by γ^*, $\gamma^{*/} = N(\gamma^*)$ is not an authorized set in A. According to **Proposition** 1 and 2, there exists a column vector $\vec{\theta} = (\theta_1, \cdots, \theta_n)^T$ such that $M_{\gamma^{*/}} \cdot \vec{\theta} = \vec{0}$ and $(\underbrace{1, 0, \cdots, 0}_{n}) \cdot \vec{\theta} = \theta_1 = 1$.

Given a row vector $R = (r_1, \cdots, r_n) \leftarrow Z_p$, let $S = R + (s - r_1) \cdot \vec{\theta}$, where

$s = (a^2 \cdot x_{i^*2})^{-1}$. Note that S is uniformly distributed subject to the constraint that the first component is s. Let $M \cdot S^T$ be the vector of l shares for the secret s. Let M_k be the row labeled by $\widetilde{p_k} \in \gamma^{*/}$, we have that $M_k \cdot \vec{\theta} = 0$ by **Proposition** 2. Hence the share $\lambda_k = M_k \cdot S^T = M_k \cdot R^T$, which has no dependency on s.

(1.1) For a unprimed keyword $\widetilde{p_k} = p_k \in \gamma^* \subseteq \gamma^{*/}$, B_A picks a random $r_k \leftarrow Z_p$ and outputs the following:

$$D_k = (D_k^{(1)} = X_{j1}{}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i^*2}{}^{r_k})$$

The above is computable since the share λ_k is independent of $(a^2 \cdot x_{i^*2})^{-1}$ by the above-mentioned discussion.

(1.2) For a unprimed keyword $\widetilde{p_k} = p_k \notin \gamma^*$, λ_k is linearly dependent on $(a^2 \cdot x_{i^*2})^{-1}$. B_A picks a random $r_k^{/} \leftarrow Z_p$, implicitly defines $r_k = r_k^{/} - \dfrac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}$ and outputs $D_k = (D_k^{(1)}, D_k^{(2)})$ as follows:

$$D_k^{(1)} = X_{j1}{}^{\lambda_k} \cdot T(p_k)^{r_k} = (g^{x_{j1}})^{\lambda_k} \cdot (g_1{}^{(p_k)^d + u(p_k)} \cdot g_2{}^{f(p_k)})^{r_k}$$

$$= (g^{x_{j1}})^{\lambda_k} \cdot (g_1{}^{(p_k)^d + u(p_k)} \cdot g_2{}^{f(p_k)})^{r_k^{/} - \frac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}}$$

$$= (g_1{}^{(p_k)^d + u(p_k)} \cdot g_2{}^{f(p_k)})^{r_k^{/}} \; g^{-\frac{f(p_k) \cdot x_{j1} \cdot a \cdot \mu^{-1} \cdot \lambda_k}{(p_k)^d + u(p_k)}}$$

$$= (g_1{}^{(p_k)^d + u(p_k)} \cdot g_2{}^{f(p_k)})^{r_k^{/}} (g^{a \cdot \lambda_k})^{-\frac{f(p_k) \cdot x_{j1} \cdot \mu^{-1}}{(p_k)^d + u(p_k)}}$$

$$D_k^{(2)} = X_{i^*2}{}^{r_k} = (A_2{}^{x_{i^*2}})^{r_k^{/} - \frac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}} = (A_2{}^{x_{i^*2} r_k^{/}})(g^{a \cdot \lambda_k})^{-\frac{\mu^{-1} \cdot x_{j1} \cdot x_{i^*2}}{(p_k)^d + u(p_k)}}$$

As λ_k is linearly dependent on $(a^2 \cdot x_{i^*2})^{-1}$, $g^{a \cdot \lambda_k}$ can be computed via A_{-1} and A_1.

(1.3) For a primed keyword $\widetilde{p_k} = p_k^{/} \notin \gamma^{*/}$ (the underlying unprimed keyword $p_k \in \gamma^*$),

λ_k is linearly dependent on $(a^2 \cdot x_{i^*2})^{-1}$. B_A picks a random $r_k^{/} \leftarrow Z_p$, and implicitly defines $r_k = r_k^{/} - \lambda_k \cdot x_{j1}$. Then outputs $D_k = (D_k^{(3)}, D_k^{(4)}, D_k^{(5)})$ as follows:

$$D_k^{(3)} = {X_{j1}}^{\lambda_k} g^{r_k} = g^{r_k^{/}}$$

$$D_k^{(4)} = V(p_k)^{r_k} = (g_2)^{\theta_{p_k} \cdot (r_k^{/} - \lambda_k \cdot x_{j1})} = (A_2)^{r_k^{/} \cdot \theta_{p_k}} \cdot ({A_2}^{\lambda_k})^{-x_{j1} \cdot \theta_{p_k}}$$

$${X_{i^*2}}^{r_k} = (g^{a^2 \cdot x_{i^*2}})^{(r_k^{/} - \lambda_k \cdot x_{j1})} = ({A_2}^{r_k^{/} \cdot x_{i^*2}}) \cdot (g^{a^2 \cdot \lambda_k})^{-x_{j1} \cdot x_{i^*2}}$$

$g^{a^2 \cdot \lambda_k}$ can be computed via A_2 since λ_k is linearly dependent on $(a^2 \cdot x_{i^*2})^{-1}$.

(1.4) For a primed keyword $\widetilde{p_k} = p_k^{/} \in \gamma^{*/}$ (the underlying unprimed keyword $p_k \notin \gamma^*$), λ_k is independent of $(a^2 \cdot x_{i^*2})^{-1}$. B_A picks a random $r_k \leftarrow Z_p$, Then outputs the following:

$$D_k = (D_k^{(3)} = {X_{j1}}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = {X_{i^*2}}^{r_k})$$

(2) $i = i^*$ and $j \in HU \setminus \{i^*\}$:

(2.1) For a primed keyword $\widetilde{p_k} = p_k^{/}$, B_A picks a random $r_k \leftarrow Z_p$ and outputs $D_k = (D_k^{(3)}, D_k^{(4)}, D_k^{(5)})$ as follows:

$$D_k^{(3)} = {X_{j1}}^{\lambda_k} g^{r_k} = g^{a \cdot x_{j1} \lambda_k} g^{r_k} = (g^{a \cdot \lambda_k})^{x_{j1}} g^{r_k}$$

$$D_k^{(4)} = V(p_k)^{r_k}$$

$$D_k^{(5)} = {X_{i^*2}}^{r_k} = {A_2}^{r_k \cdot x_{i^*2}}$$

As λ_k may be linearly dependent on $(a^2 \cdot x_{i^*2})^{-1}$, $g^{a \cdot \lambda_k}$ can be computed via A_{-1} and A_1.

(2.2) For a unprimed keyword $\widetilde{p_k} = p_k$, B_A picks a random $r_k \leftarrow Z_p$ and outputs $D_k = (D_k^{(1)}, D_k^{(2)})$ as follows:

$$D_k^{(1)} = {X_{j1}}^{\lambda_k} \cdot T(p_k)^{r_k} = (g^{a \cdot \lambda_k})^{x_{j1}} \cdot T(p_k)^{r_k}$$

$$D_k^{(2)} = {X_{i^*2}}^{r_k} = {A_2}^{r_k \cdot x_{i^*2}}$$

Similarly, $g^{a \cdot \lambda_k}$ can be computed via A_{-1} and A_1.

(3) If $i \in HU \setminus \{i^*\}$: This can be handled easily since we know the shared secret x_{i2} in this case.

Finally, returns the re-encryption key $R_{i \to j, \widetilde{A}} = \{D_k\}$.

Let the challenge second level ciphertext $CT_2^* = (svk^*, C_2^*, C_3^*, C_4^*, C_5^{p*}, C_6^{p*}, \sigma^*)$ and m_b be the encrypted challenge message. Then we define an event F_{OTS} as follows and bound its probability to occur in a way similar to [8].

(1) The adversary issues a re-encryption query in which $CT_2 = (svk^*, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$ and m is the implicit message embedded in CT_2 such that $(m \| C_3 \| C_4, \sigma) \neq (m_b \| C_3^* \| C_4^*, \sigma^*)$ and $V(svk^*, m \| C_3 \| C_4, \sigma) = 1$.

(2) The adversary issues a first level decryption query in which $CT_1 = (svk^*, C_2^{/}, C_3, C_4, \sigma)$ and m is the implicit message embedded in CT_1 such that $(m \| C_3 \| C_4, \sigma) \neq (m_b \| C_3^* \| C_4^*, \sigma^*)$ and $V(svk^*, m \| C_3 \| C_4, \sigma) = 1$.

As the adversary has no information on svk^* before the challenge phase, the probability of occurrence of F_{OTS} in phase 1 can be bounded by $q_O \cdot \delta \leq \frac{q_O}{2^\lambda}$, where q_O is the total number of oracle queries made by the adversary and δ is the maximal probability that any one-time public key can be selected.

During the guess phase, the event F_{OTS} could be used to construct an algorithm breaking strong unforgeability of the one-time signature scheme. Therefore $\Pr[F_{OTS}] \leq \frac{q_O}{2^\lambda} + Adv^{OTS}$.

The re-encryption oracle O_{renc} is simulated as follows:

Given a tuple $((CT_2, \gamma), pk_i, pk_j, \widetilde{A})$ chosen by the adversary, B_A proceeds as follows:

(1) Parses CT_2 as $(C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$;

(2) If $\widetilde{A}$ is not satisfied by γ or relation V2 does not hold, outputs a message “invalid”;

(3) If $i \in HU \setminus \{i^*\}$ or $j \notin CU$, B_A makes a query to the oracle O_{rekey} and re-encrypts by using the returned re-encryption key.

(4) If $i = i^*$, $j \in CU$, we consider the following sub-cases:

(4.1) $C_1 = svk^*$: If $(m, C_3, C_4, \sigma) = (m_b, C_3^*, C_4^*, \sigma^*)$ in this situation, we have $m = m_b$. But we should return “invalid” by our convention for the re-encryption oracle presented in section 3.2. In addition, $C_1 = svk^* \wedge (m, C_3, C_4, \sigma) \neq (m_b, C_3^*, C_4^*, \sigma^*)$ implies an occurrence of F_{OTS}. Hence B_A halts when $C_1 = svk^*$ at step (4.1).

(4.2) $C_1 \neq svk^*$: Assuming $C_4 = (u^{svk} v)^r$, relation V2 guarantees that $C_2 = (X_{i^*2})^r$. So $C_2^{1/x_{i^*2}} = (A_2^{r \cdot x_{i^*2}})^{1/x_{i^*2}} = A_2^r$, $C_4 = (u^{svk} v)^r = (A_1^{\alpha_1(svk - svk^*)} \cdot A_2^{\alpha_2})^r$, B_A computes $\left(\frac{C_4}{C_2^{\alpha_2/x_{i^*2}}}\right)^{\frac{1}{\alpha_1(svk - svk^*)}} = A_1^r$, $e(A_1^r, A_{-1}^{x_{j1}}) = e(g, X_{j1})^r = C_2^{/}$. Let $CT_1 = (C_1, C_2^{/}, C_3, C_4, \sigma)$. If $\mathrm{Dec}_1(sk_j, CT_1) \in \{m_0, m_1\}$, returns a message “invalid”. Otherwise, returns CT_1.

The first-level decryption oracle O_{dec-1} is simulated as follows:

Given a tuple (pk_i, CT_1), where CT_1 is a first level ciphertext encrypted under a public key pk_i, B_A proceeds as follows:

(1) Parses CT_1 as $(C_1, C_2^{/}, C_3, C_4, \sigma)$;

(2) If $i \in CU$, decrypts the ciphertext by the known secret key;

(3) $i \in HU$, $C_1 = svk^*$: Assuming $(m, C_3, C_4, \sigma) = (m_b, C_3^*, C_4^*, \sigma^*)$, we should output a message “invalid” to indicate that CT_1 is a Derivative of the challenge (pk^*, CT_2^*). Otherwise, $(m, C_3, C_4, \sigma) \neq (m_b, C_3^*, C_4^*, \sigma^*)$ means that we either face with an occurrence of the event F_{OTS} or B_A should output a message “invalid” to indicate that relation V1 does not hold. Hence B_A halts at step (3).

(4) $i \in HU$ and $C_1 \neq svk^*$: As $C_4 = (u^{svk}v)^r = (A_1^{\alpha_1(svk-svk^*)} \cdot A_2^{\alpha_2})^r$, B_A computes:

$$e(A_{-1}, C_4) = e(A_{-1}, A_1^{\alpha_1(svk-svk^*)r})e(A_{-1}, A_2^{\alpha_2 r})$$

$$(C_2^{/})^{\frac{\alpha_2}{x_i}} = e(g, X_{i1})^{r \cdot \frac{\alpha_2}{x_{i1}}} = e(g, g^{a \cdot x_{i1}})^{r \cdot \frac{\alpha_2}{x_{i1}}} = e(g, g^a)^{\alpha_2 r}$$

$$\left(\frac{e(A_{-1}, C_4)}{(C_2^{/})^{\frac{\alpha_2}{x_i}}}\right)^{\frac{1}{\alpha_1(svk-svk^*)}} = e(g,g)^r, \quad m = C_3 / e(g,g)^r$$

(5) If relation V1 does not hold or $m \in \{m_0, m_1\}$, outputs a message “invalid”; Otherwise outputs m.

Challenge: The adversary outputs two equal-length message (m_0, m_1). B_A flips a random bit b and outputs the challenge second level ciphertext CT_2^* as follows:

$$C_1^* = svk^*, C_2^* = B^{x_{i^*2}}, C_3^* = T \cdot m_b, C_4^* = B^{\alpha_2}, \quad C_5^{p*} = \{B^{f(p_i^*)}\}_{p_i^* \in \gamma^*} \quad C_6^{p*} = \{B^{\theta_i^*}\}_{p_i^* \in \gamma^*},$$

$$\sigma^* = S(ssk^*, m_b \| C_3^* \| C_4^*)$$

When Ad outputs $b^/ = b$, B_A outputs 1 to indicate that $T = e(g,g)^{\frac{b}{a^2}}$. Otherwise B_A outputs 0 to indicate that T is random.

If $T = e(g,g)^{\frac{b}{a^2}}$, we have $r^* = \frac{b}{a^2}$ and the following equations:

$$C_2^* = X_{i^*2}^{\ r^*} = (g^{a^2 \cdot x_{i^*2}})^{\frac{b}{a^2}} = B^{x_{i^*2}}$$

$$C_4^* = (u^{svk^*}v)^{r^*} = (A_2^{\alpha_2})^{r^*} = B^{\alpha_2}$$

$$C_5^{p*} = \{T(p_i^*)^{r^*}\}_{p_i^* \in \gamma^*} = \{A_2^{f(p_i^*) \cdot r^*}\}_{p_i^* \in \gamma^*} = \{B^{f(p_i^*)}\}_{p_i^* \in \gamma^*}$$

$$C_6^{p*} = \{V(p_i^*)^{r^*}\}_{p_i^* \in \gamma^*} = \{A_2^{q(p_i^*) \cdot r^*}\}_{p_i^* \in \gamma^*} = \{B^{\theta_i^*}\}_{p_i^* \in \gamma^*}$$

So CT_2^* is a valid encryption of m_b if $T = e(g,g)^{\frac{b}{a^2}}$. In contrast, if T is random, CT_2^* perfectly hides m_b and Ad guesses b with probability 0.5. Hence the overall advantage of B_A is $\varepsilon - \Pr[F_{OTS}]$.

Theorem 2: Assume that the one-time signature scheme is strongly unforgeable, our scheme is CCA secure at level 1 under the modified 3-wDBDHI assumption.

Proof: Let $(g, A_{-1} = g^{\frac{1}{a}}, A_1 = g^a, A_2 = g^{a^2}, B = g^b, T)$ be a modified 3-wDBDHI instance. We build an algorithm B_A deciding whether $T = e(g,g)^{\frac{b}{a^2}}$ from a successful CCA adversary Ad at level 1 with advantage ε.

Init: The adversary Ad determines the target user i^* and the corrupted users at this stage.

Setup: B_A picks a one-time signature scheme $Sig = (Gen, S, V)$ and generates a fresh one-time signature key pair (ssk^*, svk^*).

B_A sets $u = g^{\alpha_1}$, $v = g^{-\alpha_1 \cdot svk^*} \cdot X_{i^*1}^{\alpha_2}, \alpha_1, \alpha_2 \leftarrow Z_p^*$, $g_1 \leftarrow G, g_2 = g^w, w \leftarrow Z_p^*$. Then chooses two random polynomials $h(x)$ and $q(x)$ of degree d subject to the constraint $q(0) = w^{-1}$. Subsequently B_A defines two publicly computable functions $T(x) = g_1^{x^d} \cdot g_2^{h(x)}$ and $V(x) = g_2^{q(x)}$.

Key generation: Key pair of an honest users $i \in HU \setminus \{i^*\}$ is defined as $X_{i1} = g^{x_{i1}}$, $X_{i2} = g^{x_{i2}}$ for randomly chosen $x_{i1}, x_{i2} \leftarrow Z_p^*$. The target user's public key is set as $X_{i^*1} = g^{a^2 \cdot x_{i^*1}} = A_2^{x_{i^*1}}$, $X_{i^*2} = g^{a \cdot x_{i^*2}}$ for randomly chosen $x_{i^*1}, x_{i^*2} \leftarrow Z_p^*$. Key pair of a corrupted user j is set as $X_{j1} = g^{x_{j1}}$, $X_{j2} = g^{x_{j2}}$ for randomly chosen $x_{j1}, x_{j2} \leftarrow Z_p^*$.

Given a tuple $(pk_i, pk_j, \widetilde{A})$ chosen by the adversary, the re-encryption key oracle O_{rekey} is simulated by B_A as follows:

Let $\prod$ be the linear secret sharing mechanism associated with the monotonic access structure A induced by $\widetilde{A}$ over a set P. As the secret keys of corrupt users and honest users $i \neq i^*$ are known to B_A, we only consider how to handle the following case:

(1) $i = i^*, j \neq i^*$:

Let M be the share-generating matrix for $\prod$ with l rows and n columns. Each row M_k of M is labeled by a keyword named $\widetilde{p_k} \in P$ and p_k is the unprimed keyword underlying $\widetilde{p_k}$. Given a vector $R = ((a \cdot x_{i^*2})^{-1}, r_2, \cdots, r_n)$, where $(r_2, \cdots, r_n)$ are randomly chosen from Z_p, $M \cdot R^T$ is the vector of l shares for the secret $(a \cdot x_{i^*2})^{-1}$. For each keyword $\widetilde{p_k} \in P$ (the underlying unprimed keyword is p_k), a random $r_k \leftarrow Z_p$ is chosen by B_A.

If $\widetilde{p_k}$ is unprimed, $D_k =$ $(D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i2}^{r_k})$.

If $\widetilde{p_k}$ is primed, $D_k = (D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i2}^{r_k})$

A key point in the above expressions is to compute $X_{j1}^{\lambda_k} = (g^{\lambda_k})^{x_{j1}}$. As $\lambda_k = M_k \cdot R^T$ may be linearly dependent on the secret $(a \cdot x_{i^*2})^{-1}$, g^{λ_k} can be computed via A_{-1}.

The first level decryption oracle O_{dec-1} is simulated as follows:

As the secret keys of corrupt users and honest users $i \neq i^*$ are known to B_A, given a first level ciphertext CT_1 and a public key pk_i, we only consider how to handle the case $i = i^*$:

(1) Parses CT_1 as $(C_1, C_2^/, C_3, C_4, \sigma)$;

(2) $C_1 = svk^*$: Assume $(m \| C_3 \| C_4, \sigma) = (m_b \| C_3^* \| C_4^*, \sigma^*)$, which implies the randomness $r = r^*$. So $C_2^/ = C_2^{/*}$ and we have $CT_1 = CT_1^*$. But the challenge ciphertext CT_1^* is not allowed to be decrypted by our security definition. On the other hand, $(m \| C_3 \| C_4, \sigma) \neq (m_b \| C_3^* \| C_4^*, \sigma^*)$ means that we either face with an occurrence of the event F_{OTS} or B_A should output a message "invalid" to indicate that relation V1 does not hold. Hence B_A halts at step (2).

(3) $C_1 \neq svk^*$: As $C_4 = (u^{svk}v)^r = (g^{\alpha_1(svk-svk^*)} \cdot X_{i^*1}{}^{\alpha_2})^r$, B_A computes:

$$e(g, C_4) = e(g, g^{\alpha_1(svk-svk^*)r})e(g, X_{i^*1}{}^{\alpha_2 \cdot r})$$

$$(C_2^{/})^{\alpha_2} = e(g, X_{i^*1})^{\alpha_2 \cdot r}$$

$$\left(\frac{e(g, C_4)}{(C_2^{/})^{\alpha_2}}\right)^{\frac{1}{\alpha_1(svk-svk^*)}} = e(g,g)^r, \quad m = C_3 / e(g,g)^r$$

(4) If relation V1 does not hold or $m \in \{m_0, m_1\}$, outputs a message "invalid"; Otherwise outputs m.

Challenge: The adversary outputs two equal-length message (m_0, m_1). B_A flips a random bit b and outputs the challenge first level ciphertext CT_1^* as follows:

$$C_1^* = svk^*, C_2^{/*} = e(g, B^{x_{i^*1}}), C_3^* = T \cdot m_b, C_4^* = B^{\alpha_2 \cdot x_{i^*1}}, \quad \sigma^* = S(ssk^*, m_b \| C_3^* \| C_4^*)$$

If $T = e(g,g)^{\frac{b}{a^2}}$, we have $r^* = \frac{b}{a^2}$ and the following equations:

$$C_2^{/*} = e(g, X_{i^*1})^{r^*} = e(g, g^{a^2 \cdot x_{i^*1}})^{\frac{b}{a^2}} = e(g, B^{x_{i^*1}})$$

$$C_4^* = (u^{svk^*}v)^{r^*} = X_{i^*1}{}^{\alpha_2 \cdot r^*} = B^{\alpha_2 \cdot x_{i^*1}}$$

So CT_1^* is a valid encryption of m_b if $T = e(g,g)^{\frac{b}{a^2}}$. In contrast, if T is random, CT_1^* perfectly hides m_b and Ad guesses b with probability 0.5. Hence the overall advantage of B_A is $\varepsilon - \Pr[F_{OTS}]$.

5. Conclusion

Fang et al. [6] presented an interactive bidirectional single-hop C-PRE scheme, which supports access policy consisting of "OR" and 'AND" gates. They also left it as an open problem to construct a *non-interactive* C-PRE scheme with security in *the standard model*. In this paper, we present a security model for unidirectional(non-interactive) C-PRE schemes. To yield a unidirectional C-PRE scheme supporting non-monotonic access policy expressed by "NOT", "OR" and "AND" gates, we extend the unidirectional PRE scheme [8] by using the ideas from the non-monotonic attributed based encryption(ABE) [9]. Hence our C-PRE scheme enables more flexible access policy set by delegator in comparison with previous works. Non-interactive feature of our unidirectional scheme also simplifies generation of re-encryption keys. Finally we prove our scheme to be CCA secure under the modified 3-weak Decision Bilinear Diffie-Hellman Inversion assumption in the standard model.

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