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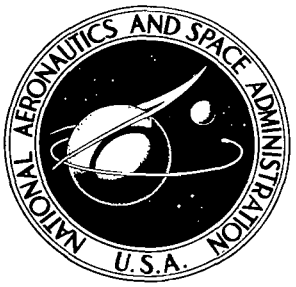

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A UNIFIED FORM OF LAMBERT'S THEOREM

by E. R. Lancaster and R. C. Blanchard

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1969

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1. Report No. NASA TN D-5368	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A Unified Form of Lambert's Theorem		5. Report Date September 1969 6. Performing Organization Code
7. Author(s) E. R. Lancaster and R. C. Blanchard		8. Performing Organization Report No.
9. Performing Organization Name and Address Goddard Space Flight Center Greenbelt, Maryland 20771		10. Work Unit No. 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Note 14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

A unified form of Lambert's theorem is presented which is valid for elliptic, hyperbolic, and parabolic orbits. The key idea involves the selection of an independent variable x and a parameter q such that the normalized time of flight T is a single-valued function of x for each value of q. The parameter q depends only upon known quantities. For less than one revolution, T is a monotonic function of x for each q, making possible the construction of a simple algorithm for finding x, given T and q. Detailed sketches are given for $T(x, q)$ and formulas developed for the velocity vectors at the initial and final times. Also included is a careful derivation of the classical form of Lambert's equations, including the multirevolution cases.

17. Key Words Suggested by Author Lambert's theorem orbit determination rendezvous targeting	18. Distribution Statement Unclassified—Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 14	22. Price $3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

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A UNIFIED FORM OF LAMBERT'S THEOREM

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INTRODUCTION

Lambert's problem, as it arises in most applications, is concerned with the determination of an orbit from two position vectors and the time of flight. It has important applications in the areas of rendezvous, targeting, and preliminary orbit determination. In this paper a unified form of Lambert's theorem will be presented which is valid for elliptic, hyperbolic, and parabolic orbits.

The key idea involves the selection of an independent variable x and a parameter q such that the normalized time of flight T is a single-valued function of x for each value of q. The parameter q depends only upon known quantities. The problem then is to find x for given values of q and the time of flight. For less than one revolution, T is a monotonic function of x for each value of q. Thus it is an easy task to design an algorithm for finding x. For multirevolution cases, $T(x)$ has a single minimum for each q.

This idea was presented in a previous paper (Reference 1), where a unified formula was given for the computation of T from x and q. Detailed sketches were given for $T(x, q)$, and a simple formula developed for the time derivative of the magnitude of the radius vector at the initial time in terms of x and given quantities.

The present paper expands upon the previous one by:

1. giving complete derivations which were only sketched before;
2. giving a careful derivation of the classical form of Lambert's equations, including the multirevolution cases;
3. deriving a number of useful auxiliary formulas, e.g., for the semilatus rectum and for the velocity vectors at the initial and final times.

THE PROBLEM

Suppose a particle in a gravitational inverse-square central force field has distances r_1 and r_2 from the center of attraction at times t_1 and t_2. Let c be the distance and θ the central angle between the positions of the particle at the two times, where $0 \leq \theta \leq 2\pi$.

Lambert's problem is that of finding the semimajor axis or some related quantity for the orbit of the particle, given t_1, r_1, t_2, r_2, and θ. When Lambert's problem has been solved, other quantities associated with the orbit are easily found, as will be later discussed. Using the law of cosines, we can express c in terms of r_1, r_2, and θ:

$$c^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta \ . \tag{1}$$

We define

G = universal gravitational constant,

M = mass of attracting body,

μ = GM,

a = semimajor axis of transfer orbit,

e = eccentricity of transfer orbit.

We will follow the common sign convention for a, i.e., $a > 0$ for elliptic orbits and $a < 0$ for hyperbolic orbits.

Definitions of other symbols will be given as they are introduced.

THE CLASSICAL FORM OF LAMBERT'S EQUATIONS

The path of a particle in an inverse-square central force field is an ellipse, parabola, or hyperbola. With origin at the center of attraction, we have, for elliptic motion,

$$r_1 = a\left(1 - e\cos\phi_1\right) \ , \tag{2}$$

$$r_2 = a\left(1 - e\cos\phi_2\right) \ , \tag{3}$$

$$n\left(t_1 - t_p\right) = \phi_1 - e\sin\phi_1 \ , \tag{4}$$

$$n\left(t_2 - t_p\right) = \phi_2 - e\sin\phi_2 \ , \tag{5}$$

where ϕ_1 and ϕ_2 are the eccentric anomalies at times t_1 and t_2, t_p is the time at pericenter, and

$$n = \left(\mu/a^3\right)^{1/2} \ .$$

If $\vec{\epsilon}_1$ is a unit vector pointing towards periapsis and $\vec{\epsilon}_2$ is a unit vector in the plane of motion 90° ahead of $\vec{\epsilon}_1$ in the direction of motion, then for the position vectors $\vec{r}_1$ and $\vec{r}_2$ at times t_1 and t_2

$$\vec{r}_1 = a(\cos\phi_1 - e)\vec{\epsilon}_1 + a(1-e^2)^{1/2}(\sin\phi_1)\vec{\epsilon}_2 ,$$

$$\vec{r}_2 = a(\cos\phi_2 - e)\vec{\epsilon}_1 + a(1-e^2)^{1/2}(\sin\phi_2)\vec{\epsilon}_2 .$$

Substituting these equations in

$$c^2 = r_1^2 + r_2^2 - 2\vec{r}_1 \cdot \vec{r}_2 ,$$

where the dot indicates scalar product, we have

$$c^2 = a^2(\cos\phi_2 - \cos\phi_1)^2 + a^2(1-e^2)(\sin\phi_2 - \sin\phi_1)^2$$

$$= 4a^2\left[1 - e^2\cos^2\tfrac{1}{2}(\phi_1+\phi_2)\right]\sin^2\tfrac{1}{2}(\phi_2-\phi_1) . \qquad (6)$$

Adding Equations 2 and 3,

$$r_1 + r_2 = 2a\left[1 - e\cos\tfrac{1}{2}(\phi_1+\phi_2)\cos\tfrac{1}{2}(\phi_2-\phi_1)\right] . \qquad (7)$$

Subtracting Equation 4 from Equation 5,

$$n(t_2 - t_1) = \phi_2 - \phi_1 - 2e\cos\tfrac{1}{2}(\phi_1+\phi_2)\sin\tfrac{1}{2}(\phi_2-\phi_1) . \qquad (8)$$

Equations 6, 7, and 8 determine the three unknowns a, $\phi_2 - \phi_1$, and $e\cos(1/2)(\phi_1+\phi_2)$. Let

$$\cos\tfrac{1}{2}(\alpha+\beta) = e\cos\tfrac{1}{2}(\phi_1+\phi_2) , \qquad 0 \le \alpha+\beta < 2\pi , \qquad (9)$$

$$\alpha - \beta = \phi_2 - \phi_1 - 2m\pi , \qquad 0 \le \alpha-\beta < 2\pi , \qquad (10)$$

where m is the number of complete circuits made by the particle between times t_1 and t_2.

Equations 6, 7, and 8 become

$$c/2a = \sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\alpha-\beta) , \qquad (11)$$

$$(r_1 + r_2)/2a = 1 - \cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta) , \tag{12}$$

$$n(t_2 - t_1) = 2m\pi + \alpha - \beta - 2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta) . \tag{13}$$

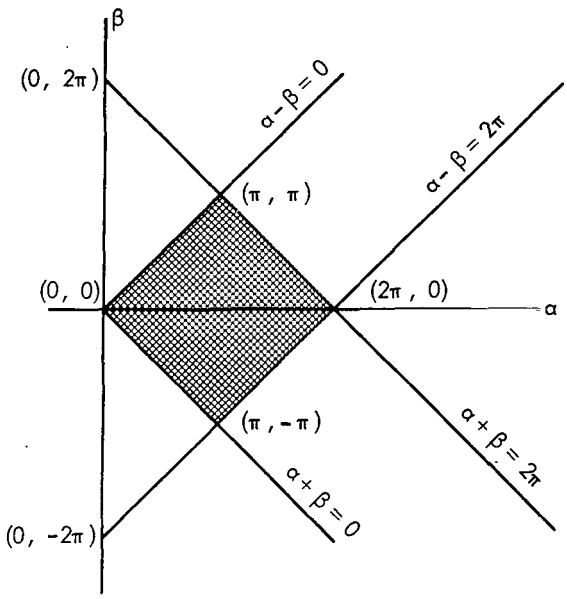

Figure 1.

The two inequalities for Equations 9 and 10 are geometrically equivalent to the shaded region of Figure 1, from which it is evident that $0 \le \alpha < 2\pi$ and $-\pi \le \beta < \pi$. We can also obtain $0 \le \alpha < 2\pi$ by adding the inequalities for Equations 9 and 10; and, if we add $\beta - \alpha$ to each part of the inequality for Equation 9 and divide the result by 2, we obtain $-(\alpha-\beta)/2 \le \beta < \pi + (\alpha-\beta)/2$, or $-\pi \le \beta < \pi$.

With appropriate trigonometric identities, Equations 11 and 12 become

$$\cos\beta - \cos\alpha = c/a ,$$

$$\cos\beta + \cos\alpha = 2 - (r_1 + r_2)/a .$$

Solving these two equations,

$$\cos\alpha = 1 - s/a = 1 + 2E , \tag{14}$$

$$\cos\beta = 1 + 2KE , \tag{15}$$

where we have defined

$$s = (r_1 + r_2 + c)/2 ,$$

$$E = -s/2a ,$$

$$K = 1 - c/s .$$

Since $\cos\alpha = 1 - 2\sin^2(\alpha/2)$, Equation 14 can be changed to

$$E = -\sin^2(\alpha/2) , \qquad 0 \le \alpha < 2\pi . \tag{16}$$

Similarly, Equation 15 becomes

$$KE = -\sin^2(\beta/2) , \qquad -\pi \le \beta < \pi , \tag{17}$$

$$K = (s-c)/s = (r_1 + r_2 - c)/2s$$

$$= \left[(r_1 + r_2)^2 - c^2\right]/4s^2 .$$

Introducing Equation 1, we have

$$K = \left(r_1 r_2/2s^2\right)(1 + \cos\theta)$$

$$= \left(r_1 r_2/s^2\right)\cos^2(\theta/2) .$$

Substituting Equation 16 in Equation 17,

$$\sin(\beta/2) = q \sin(\alpha/2) , \qquad -\pi \leq \beta < \pi , \tag{18}$$

where

$$q = \pm\sqrt{K} = \left[\left(r_1 r_2\right)^{1/2}/s\right]\cos(\theta/2) . \tag{19}$$

Note that the sign of q is taken care of by the angle θ:

$$1 \geq q \geq 0 \quad \text{if} \quad 0 \leq \theta \leq \pi ,$$

$$0 \geq q \geq -1 \quad \text{if} \quad \pi \leq \theta \leq 2\pi .$$

We can introduce E into Equation 13, since

$$n\left(t_2 - t_1\right) = \left(\mu/a^3\right)^{1/2}\left(t_2 - t_1\right) = (-E)^{3/2} T ,$$

where

$$T = (8\mu/s)^{1/2}\left(t_2 - t_1\right)/s . \tag{20}$$

$$T = (-E)^{-3/2}\left[2m\pi + \alpha - \beta - 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)\right] . \tag{21}$$

This can also be written as

$$T = (-E)^{-3/2}\left[2m\pi + \alpha - \beta - (\sin\alpha - \sin\beta)\right] . \tag{22}$$

Substituting Equation 16 into Equation 22,

$$T \sin^3(\alpha/2) = 2m\pi + \alpha - \beta - \sin\alpha + \sin\beta . \tag{23}$$

Equations 18 and 23 with $0 \leq \alpha < 2\pi$ are Lambert's equations for elliptic motion. Given T and q, they are to be solved for α and β, after which it is a simple matter to find all other quantities associated with the orbit.

It is customary in the literature (e.g., Reference 2) to consider T as a function of E (or a) and break the elliptic case of Lambert's theorem into four cases, depending upon the sign of q and whether $\alpha/2$ is taken in the first or second quadrant in Equation 16. The choice of E as the independent variable makes T a double-valued function. This problem can be avoided by choosing α as the independent variable. However, an even better choice will be discussed in the next section.

By a derivation very similar to that for the elliptic case, one finds for the hyperbolic case

$$T = -E^{-3/2}\left[\gamma - \delta - (\sinh\gamma - \sinh\delta)\right] , \tag{24}$$

$$E = \sinh^2(\gamma/2) , \tag{25}$$

$$\sinh(\delta/2) = q\sinh(\gamma/2) . \tag{26}$$

When $m = 0$, Equations 22 and 24 break down for $E = 0$ and suffer from a critical loss of significant digits in the neighborhood of $E = 0$. To remedy this, Equation 22 is written in the form

$$T = \sigma(-E) - qK\sigma(-KE) , \tag{27}$$

where

$$\sigma(u) = 2\left[\arcsin u^{1/2} - u^{1/2}(1-u)^{1/2}\right]/u^{3/2} .$$

Replacing $\arcsin u^{1/2}$ and $(1-u)^{1/2}$ by series (Reference 3) with $0 \leq \alpha < \pi$,

$$\sigma(u) = 4/3 + \sum_{n=1}^{\infty} a_n u^n , \qquad |u| < 1 ,$$

$$a_n = 1 \cdot 3 \cdot 5 \cdots (2n-1)/2^{n-2}(2n+3)n!$$

A similar procedure produces the same series for the hyperbolic case. For the parabolic case we have $E = 0$, in which case the series gives

$$T = (4/3)\left(1 - q^3\right) .$$

Thus with $m = 0$ we have a series which is valid for elliptic, hyperbolic, and parabolic transfer provided $|E| < 1$ (with $0 \leq \alpha < \pi$ for the elliptic case).

A UNIFIED FORM OF LAMBERT'S EQUATIONS

As mentioned in the previous section, T is a single-valued function of α. However, a better-behaved function is obtained if we choose as the independent variable

$$x = \cos(\alpha/2) , \qquad -1 \leq x \leq 1 ,$$

$$= \cosh(\gamma/2) , \qquad x > 1 .$$

We then have, for elliptic, hyperbolic, and parabolic transfer,

$$E = x^2 - 1 ;$$

if for parabolic transfer, we let $x = 1$.

For the elliptic case, let

$$y = \sin(\alpha/2) = (-E)^{1/2} ,$$

$$z = \cos(\beta/2) = (1 + KE)^{1/2} ,$$

$$f = \sin\frac{1}{2}(\alpha - \beta) = y(z - qx) ,$$

$$g = \cos\frac{1}{2}(\alpha - \beta) = xz - qE ,$$

$$h = \frac{1}{2}(\sin\alpha - \sin\beta) = y(x - qz) ,$$

$$\lambda = \arctan(f/g) , \qquad 0 \leq \lambda \leq \pi .$$

It then follows from Equation 21 for the elliptic case that

$$T = 2(m\pi + \lambda - h)/y^3 .$$

For the hyperbolic case, let

$$y = \sinh(\gamma/2) = E^{1/2} ,$$

$$z = \cosh(\gamma/2) = (1 + KE)^{1/2} ,$$

$$f = \sinh \frac{1}{2}(\gamma - \delta) = y(z - qx) ,$$

$$g = \cosh \frac{1}{2}(\gamma - \delta) = xz - qE .$$

Note that $0 \leq \gamma - \delta < \infty$, since $0 \leq f < \infty$. Let

$$h = \frac{1}{2}(\sinh \gamma - \sinh \delta) = y(x - qz) .$$

It follows that

$$\frac{1}{2}(\gamma - \delta) = \operatorname{arctanh}(f/g) ,$$

$$= \frac{1}{2} \ell n \left[(f + g)/(g - f)\right] ,$$

$$= \frac{1}{2} \ell n \left[(f + g)^2/(g^2 - f^2)\right] ,$$

$$= \ell n (f + g) .$$

Thus, for the hyperbolic case,

$$T = 2\left[h - \ell n (f + g)\right]/y^3 .$$

It is now apparent that, given q and x, the following steps produce T for all cases:

1. $K = q^2$
2. $E = x^2 - 1$
3. If x is near 1, compute T from Equation 27
4. $y = |E|^{1/2}$
5. $z = (1 + KE)^{1/2}$
6. $f = y(z - qx)$
7. $g = xz - qE$

8. If $E < 0$, $\lambda = \arctan(f/g)$, $d = m\pi + \lambda$, $0 \le \lambda \le \pi$
   If $E > 0$, $d = \ell n(f + g)$

9. $T = 2(x - qz - d/y)/E$.

The following formula for the derivative holds for all cases except for $x = 0$ with $K = 1$, and for $x = 1$:

$$dT/dx = (4 - 4qKx/z - 3xT)/E .$$

If x is near 1, differentiate Equation 27 to obtain

$$dT/dx = 2x\left[qK^2\sigma'(-KE) - \sigma'(-E)\right] ,$$

$$\sigma'(u) = d\sigma/du = \sum_{n=1}^{\infty} n a_n u^{n-1} .$$

The derivative in the case of $x = 0$ with $k = 1$ will be discussed in the next section.

AUXILIARY FORMULAS

In this section we will show how to obtain a number of useful quantities associated with the two-body orbit, assuming Lambert's problem has been solved for x.

In the derivation of Lambert's equation for the elliptic case, α and β are defined in such a way that

$$\phi_2 - \phi_1 = \alpha - \beta + 2m\pi = 2(\lambda + m\pi) .$$

From Equation 22, the eccentric anomaly difference can also be written in the form

$$\phi_2 - \phi_1 = (-E)^{3/2} T + \sin\alpha - \sin\beta$$

$$= y^3 T + 2y(x - qz) . \qquad (28)$$

From Equation 28 we have

$$\sin(\phi_2 - \phi_1) = 2y(z - qx)(xz - qE)$$

$$= 2y(x - qz) + 4y^3 q(z - qx) ,$$

$$\phi_2 - \phi_1 - \sin(\phi_2 - \phi_1) = y^3 T - 4y^3 q(z - qx) , \tag{29}$$

$$1 - \cos(\phi_2 - \phi_1) = 2y^2 (z - qx)^2 . \tag{30}$$

We now obtain a formula for the time derivative $\dot{r}$ at time t_1. Kepler's equation in the elliptic case can be written in the form (Reference 4)

$$(\mu/a^3)^{3/2}(t_2 - t_1) = \phi_2 - \phi_1 + r_1 \dot{r}_1 \left[1 - \cos(\phi_2 - \phi_1)\right]/(\mu a)^{1/2} - (1 - r_1/a)\sin(\phi_2 - \phi_1) .$$

Substituting $1/a = 2y^2/s$, $t_2 - t_1 = s^{3/2} T/(8\mu)^{1/2}$, and making use of Equations 29 and 30 gives

$$(2/\mu s)^{1/2}(z - qx)\, r_1 \dot{r}_1 = 2q - (2r_1/s)(xz - qE) .$$

Multiplying through by $z + qx$, we have, since $(z - qx)(z + qx) = 1 - K = c/s$ and $(z + qx)(xz - qE) = x + qz$,

$$(2/\mu s)^{1/2} c\, r_1 \dot{r}_1 = 2qs(z + qx) - 2r_1 (x + qz)$$

$$= 2qz(s - r_1) + 2x(Ks - r_1) ;$$

$$Ks - r_1 = (1 - c/s)\, s - r_1 = s - c - r_1 = r_2 - s .$$

Thus we have finally that

$$\dot{r}_1 = (2\mu s)^{1/2}\left[qz(s - r_1) - x(s - r_2)\right]/cr_1 . \tag{31}$$

At time t_2 we find

$$\dot{r}_2 = (2\mu s)^{1/2}\left[x(s - r_1) - qz(s - r_2)\right]/cr_2 . \tag{32}$$

By a similar procedure we can show that Equations 31 and 32 hold also for the hyperbolic and parabolic cases.

Having x, we can find the semimajor axis a, or its reciprocal,

$$1/a = 2y^2/s .$$

We know that

$$e\cos\phi = 1 - r/a ,$$

$$e\sin\phi = r\dot{r}/(\mu a)^{1/2} ,$$

where r is the magnitude of the position vector, and ϕ is the eccentric anomaly, at time t.

Thus for the eccentricity we have

$$e^2 = (1 - r/a)^2 + (r\dot{r})^2/\mu a .$$

For the semilatus rectum p we have

$$p = a\left(1 - e^2\right) = 2r - r^2/a - (r\dot{r})^2/\mu ,$$

and for the value of r at the point of closest approach to the center of attraction we have

$$r_p = \frac{p}{1 + e} .$$

For the speed we have

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) .$$

For the component of velocity perpendicular to a radius vector,

$$v_\theta = (\mu p)^{1/2}/r .$$

The above formulas hold for any type of two-body motion. Either of the subscripts 1 or 2 can be placed on r, $\dot{r}$, and v.

If $\dot{r}_1 < 0$ and $\dot{r}_2 > 0$, or if $\dot{r}_1$ and $\dot{r}_2$ have the same sign with $\theta > \pi$, periapsis passage will occur between times t_1 and t_2, in which case it may be of interest to compute r_p.

If the plane of motion is known, the velocity $\vec{v}$ at either time t_1 or time t_2 can be written in any convenient coordinate system, since the components $\dot{r}$ and v_θ are known. The plane of motion can be found from the position vectors $\vec{r}_1$ and $\vec{r}_2$ at times t_1 and t_2, provided they are not parallel. Since this is a common case, we will express $\vec{v}_1$ and $\vec{v}_2$ (the velocities at t_1 and t_2) in terms of $\vec{r}_1$ and $\vec{r}_2$.

For the velocity $\vec{v}_1$ we have

$$\vec{v}_1 = \vec{v}_{r1} + \vec{v}_{\theta 1} ,$$

where $\vec{v}_{r1}$ is along $\vec{r}_1$ and $\vec{v}_{\theta 1}$ is in the plane of motion perpendicular to $\vec{r}_1$ and in the direction of motion, i.e., in the direction of increasing true anomaly. We have

$$\vec{v}_{r1} = (\dot{r}_1/r_1)\,\vec{r}_1 ,$$

$$\vec{v}_{\theta 1} = c_1 \vec{r}_1 + c_2 \vec{r}_2 ,$$

where c_1 and c_2 are to be determined.

$$\vec{r}_1 \cdot \vec{v}_{\theta 1} = 0 = c_1 r_1^2 + c_2 \vec{r}_1 \cdot \vec{r}_2$$

$$\vec{r}_2 \cdot \vec{v}_{\theta 1} = r_2 v_{\theta 1} \sin\theta = c_1 \vec{r}_1 \cdot \vec{r}_2 + c_2 r_2^2 .$$

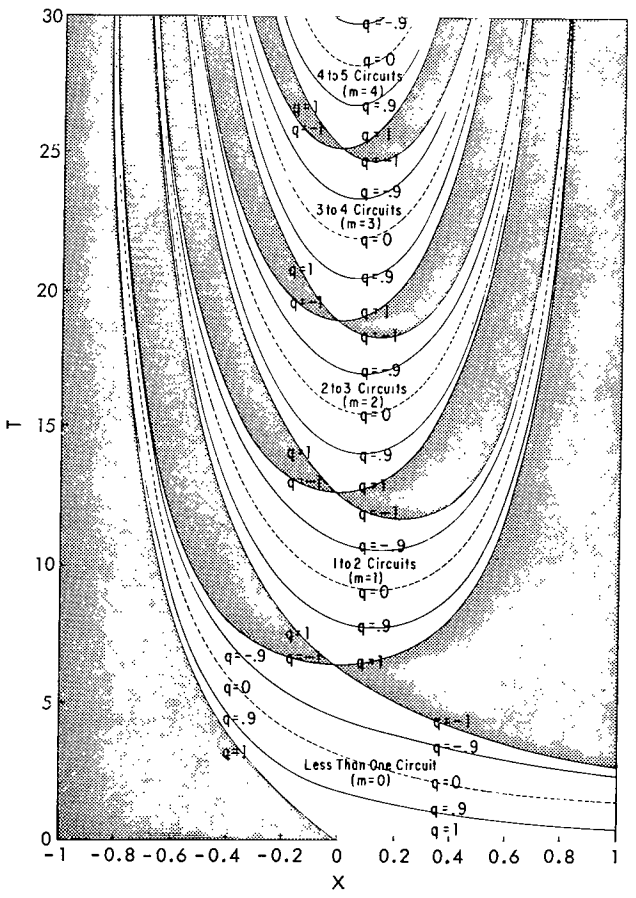

Figure 2.

Since $\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos\theta$, we have

$$r_1 c_1 + (r_2 \cos\theta) c_2 = 0 ,$$

$$(r_1 \cos\theta) c_1 + r_2 c_2 = v_\theta \sin\theta .$$

Solving for c_1 and c_2, we have

$$\vec{v}_1 = (\dot{r}_1 - v_{\theta 1} \cot\theta)(\vec{r}_1/r_1) + (v_{\theta 1} \csc\theta)(\vec{r}_2/r_2) .$$

In a similar way, we find

$$\vec{v}_2 = -(v_{\theta 2} \csc\theta)(\vec{r}_1/r_1) + (\dot{r}_2 + v_{\theta 2} \cot\theta)(\vec{r}_2/r_2) .$$

Figure 2 shows T as a function of x. Note the discontinuity in the slope for $x = 0$ with $K = 1$. For $K = 1$, $z = |x|$. Thus we are led to consider four cases: $q = \pm 1$ with $x \geq 0$, $q = \pm 1$ with $x \leq 0$. Examination of the formulas for dT/dx in these cases reveals that if $q = 1$ we have a left-hand derivative of -8 and a right-hand derivative of 0 at $x = 0$. If $q = -1$ we have a left-hand derivative of 0 and a right-hand derivative of -8 at $x = 0$.

With further analysis we find that the cases ($q = 1, x \leq 0$) and ($q = -1, x \geq 0$) represent rectilinear orbits.

For $m = 0$, T is a monotone function of x, making possible a simple numerical procedure for solving Lambert's problem. Figure 2 is for the elliptic case, Figure 3 for the hyperbolic case, the parabolic case occurring at $x = 1$ in both figures. Figure 4 shows a small region of Figure 2

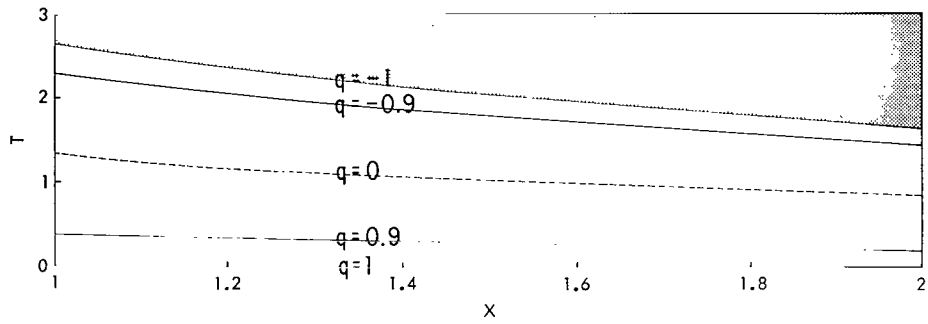

Figure 3.

where d^2T/dx^2 is negative. If the Newton-Raphson method is being used to find x, a switch should be made in this region to the secant (regula falsi) method.

No solutions of Lambert's problem exist in the shaded regions of Figures 2 and 3. $x = 1$ ($m > 0$) and $x = -1$ are vertical asymptotes. $T \to 0$ as $x \to \infty$.

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, May 28, 1969
188-48-01-05-51

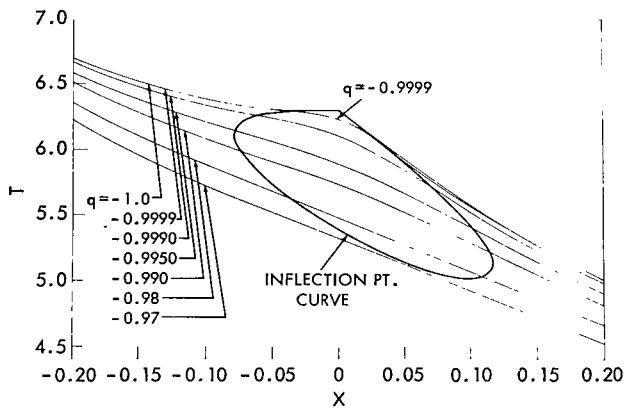

Figure 4.

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