Synchronization A universal concept in nonlinear sciences

Arkady Pikovsky, Michael Rosenblum and Jürgen Kurths University of Potsdam, Germany



PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS The Edinburgh Building, Cambridge CB2 2RU, UK 40 West 20th Street, New York, NY 10011-4211, USA 477 Williamstown Road, Port Melbourne, VIC 3207, Australia Ruiz de Alarcón 13, 28014, Madrid, Spain Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

© A. Pikovsky, M. Rosenblum and J. Kurths 2001

This book is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreementatio-Hahn-Bihlinthel no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2001 First paperback edition 2003 Reprinted 2003

B 3/105 WAX-PLANCK-IN

Koil-Friedrich-Bonhoeff

für biophysikalische Chemie 2007 10

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10.25/13.5pt. System IATEX 2. [DBD]

A catalogue record of this book is available from the British Library

Library of Congress Cataloguing in Publication data

Pikovsky, Arkady, 1956-

Synchronization: a universal concept in nonlinear sciences / Arkady Pikovsky, Michael Rosenblum, Jürgen Kurths.

p. cm. – (The Cambridge nonlinear science series; 12)

Includes bibliographical references and index.

ISBN 0 521 59285 2

1. Synchronization. 2. Nonlinear theories. I. Rosenblum, Michael, 1958-II. Kurths, J. (Jürgen), 1953- III. Title. IV. Series.

O172.5.S96 P54 2001. 003'.75-dc21 2001018104

ISBN 0 521 59285 2 hardback ISBN 0 521 53352 X paperback

Contents

Preface xvii

Chapter I Introduction 1

| 1.1 | Synchronization in historical perspective 1 |
|-------|--|
| 1.2 | Synchronization: just a description 7 |
| 1.2.1 | What is synchronization? 8 |
| 1.2.2 | What is NOT synchronization? 14 |
| 1.3 | Synchronization: an overview of different cases 18 |
| 1.3.1 | Terminological remarks 22 |
| 1.4 | Main bibliography 23 |

Part I: Synchronization without formulae

Chapter 2 Basic notions: the self-sustained oscillator and its phase 27

- 2.1 Self-sustained oscillators: mathematical models of natural systems 28
- 2.1.1 Self-sustained oscillations are typical in nature 28
- 2.1.2 Geometrical image of periodic self-sustained oscillations: limit cycle 29
- 2.2 Phase: definition and properties 31
- 2.2.1 Phase and amplitude of a quasilinear oscillator 31
- 2.2.2 Amplitude is stable, phase is free 32
- 2.2.3 General case: limit cycle of arbitrary shape 33
- 2.3 Self-sustained oscillators: main features 35
- 2.3.1 Dissipation, stability and nonlinearity 35
- 2.3.2 Autonomous and forced systems: phase of a forced system is not free! 38
- 2.4 Self-sustained oscillators: further examples and discussion 40
- 2.4.1 Typical self-sustained system: internal feedback loop 40

2.4.2 Relaxation oscillators 41

Chapter 3 Synchronization of a periodic oscillator by external force 45

3.1 Weakly forced quasilinear oscillators 46 3.1.1 The autonomous oscillator and the force in the rotating reference frame 46 3.1.2 Phase and frequency locking 49 3.1.3 Synchronization transition 53 3.1.4 An example: entrainment of respiration by a mechanical ventilator 56 3.2 Synchronization by external force: extended discussion 59 3.2.1Stroboscopic observation 59 3.2.2 An example: periodically stimulated firefly 61 3.2.3 Entrainment by a pulse train 62 3.2.4 Synchronization of higher order. Arnold tongues 65 3.2.5 An example: periodic stimulation of atrial pacemaker cells 67 3.2.6 Phase and frequency locking: general formulation 67 3.2.7 An example: synchronization of a laser 69 3.3 Synchronization of relaxation oscillators: special features 71 3.3.1 Resetting by external pulses. An example: the cardiac pacemaker 71 3.3.2 Electrical model of the heart by van der Pol and van der Mark 72 3.3.3 Variation of the threshold. An example: the electronic relaxation oscillator 73 3.3.4 Variation of the natural frequency 76 3.3.5 Modulation vs. synchronization 77 3.3.6 An example: synchronization of the songs of snowy tree crickets 78 3.4 Synchronization in the presence of noise 79 3.4.1 Phase diffusion in a noisy oscillator 80 3.4.2 Forced noisy oscillators. Phase slips 81 3.4.3 An example: entrainment of respiration by mechanical ventilation 85 An example: entrainment of the cardiac rhythm by weak external stimuli 85 3.4.4 3.5 Diverse examples 86 3.5.1 Circadian rhvthms 86 The menstrual cycle 88 3.5.2 Entrainment of pulsatile insulin secretion by oscillatory glucose infusion 89 3.5.3 Synchronization in protoplasmic strands of Physarum 90 3.5.4 Phenomena around synchronization 90 3.6 Related effects at strong external forcing 91 3.6.1 Stimulation of excitable systems 93 3.6.2 Stochastic resonance from the synchronization viewpoint 94 3.6.3 Entrainment of several oscillators by a common drive 98 3.6.4

Contents

| Chapter 4 | Synchronization of two and many oscillators 102 |
|-----------|--|
| 4.1 | Mutual synchronization of self-sustained oscillators 102 |
| 4.1.1 | Two interacting oscillators 103 |
| 4.1.2 | An example: synchronization of triode generators 105 |
| 4.1.3 | An example: respiratory and wing beat frequency of free-flying barnacle |
| | geese 107 |
| 4.1.4 | An example: transition between in-phase and anti-phase motion 108 |
| 4.1.5 | Concluding remarks and related effects 110 |
| 4.1.6 | Relaxation oscillators. An example: true and latent pacemaker cells in the |
| | sino-atrial node 111 |
| 4.1.7 | Synchronization of noisy systems. An example: brain and muscle activity of a |
| | Parkinsonian patient 112 |
| 4.1.8 | Synchronization of rotators. An example: Josephson junctions 114 |
| 4.1.9 | Several oscillators 117 |
| 4.2 | Chains, lattices and oscillatory media 119 |
| 4.2.1 | Synchronization in a lattice. An example: laser arrays 119 |
| 4.2.2 | Formation of clusters. An example: electrical activity of mammalian |
| | intestine 121 |
| 4.2.3 | Clusters and beats in a medium: extended discussion 122 |
| 4.2.4 | Periodically forced oscillatory medium. An example: forced |
| | Belousov–Zhabotinsky reaction 124 |
| 4.3 | Globally coupled oscillators 126 |
| 4.3.1 | Kuramoto self-synchronization transition 126 |
| 4.3.2 | An example: synchronization of menstrual cycles 129 |
| 4.3.3 | An example: synchronization of glycolytic oscillations in a population of |
| | yeast cells 130 |
| 4.3.4 | Experimental study of rhythmic hand clapping 131 |
| 4.4 | Diverse examples 131 |
| 4.4.1 | Running and breathing in mammals 131 |
| 4.4.2 | Synchronization of two salt-water oscillators 133 |
| 4.4.3 | Entrainment of tubular pressure oscillations in nephrons 133 |
| 4.4.4 | Populations of cells 133 |
| 4.4.5 | Synchronization of predator-prey cycles 134 |
| 4.4.6 | Synchronization in neuronal systems 134 |
| | |

Chapter 5 Synchronization of chaotic systems 137

| 5.1 Chaotic oscillators 1 | 37 |
|---------------------------|----|
|---------------------------|----|

- 5.1.1 An exemplar: the Lorenz model 138
- 5.1.2 Sensitive dependence on initial conditions 140

| 5.2 | Phase synchronization of chaotic oscillators | 141 |
|-----|--|-----|
|-----|--|-----|

- 5.2.1 Phase and average frequency of a chaotic oscillator 142
- 5.2.2 Entrainment by a periodic force. An example: forced chaotic plasma
 - discharge 144
- 5.3 Complete synchronization of chaotic oscillators 147
- 5.3.1 *Complete synchronization of identical systems. An example:*
- synchronization of two lasers 148
- 5.3.2 Synchronization of nonidentical systems 149
- 5.3.3 *Complete synchronization in a general context. An example: synchronization and clustering of globally coupled electrochemical oscillators* 150
- 5.3.4 Chaos-destroying synchronization 152

Chapter 6 Detecting synchronization in experiments 153

- 6.1 Estimating phases and frequencies from data 153
- 6.1.1 Phase of a spike train. An example: electrocardiogram 154
- 6.1.2 Phase of a narrow-band signal. An example: respiration 155
- 6.1.3 Several practical remarks 155
- 6.2 Data analysis in "active" and "passive" experiments 156
- 6.2.1 "Active" experiment 156
- 6.2.2 "Passive" experiment 157
- 6.3 Analyzing relations between the phases 160
- 6.3.1 Straightforward analysis of the phase difference. An example: posture control in humans 160
- 6.3.2 High level of noise 163
- 6.3.3 Stroboscopic technique 163
- 6.3.4 Phase stroboscope in the case $n\Omega_1 \approx m\Omega_2$. An example: cardiorespiratory interaction 164
- 6.3.5 Phase relations in the case of strong modulation. An example: spiking of electroreceptors of a paddlefish 166
- 6.4 Concluding remarks and bibliographic notes 168
- 6.4.1 Several remarks on "passive" experiments 168
- 6.4.2 Quantification and significance of phase relation analysis 170
- 6.4.3 Some related references 171

Part II: Phase locking and frequency entrainment

| Chapter 7 | Synchronization of periodic oscillators by periodic externa | |
|-----------|---|--|
| | action 175 | |

- 7.1 Phase dynamics 176
- 7.1.1 A limit cycle and the phase of oscillations 176

Contents

| 7.1.2 | Small perturbations and isochrones 177 |
|-------|---|
| 7.1.3 | An example: complex amplitude equation 179 |
| 7.1.4 | The equation for the phase dynamics 180 |
| 7.1.5 | An example: forced complex amplitude equations 181 |
| 7.1.6 | Slow phase dynamics 182 |
| 7.1.7 | Slow phase dynamics: phase locking and synchronization region |
| 7.1.8 | Summary of the phase dynamics 187 |
| 7.2 | Weakly nonlinear oscillator 189 |
| 7.2.1 | The amplitude equation 189 |
| 7.2.2 | Synchronization properties: isochronous case 192 |
| 7.2.3 | Synchronization properties: nonisochronous case 198 |
| 7.3 | The circle and annulus map 199 |
| 7.3.1 | The circle map: derivation and examples 201 |
| 7.3.2 | The circle map: properties 204 |
| 7.3.3 | The annulus map 210 |
| 7.3.4 | Large force and transition to chaos 213 |
| 7.4 | Synchronization of rotators and Josephson junctions 215 |
| 7.4.1 | Dynamics of rotators and Josephson junctions 215 |
| 7.4.2 | Overdamped rotator in an external field 217 |
| 7.5 | Phase locked loops 218 |
| 7.6 | Bibliographic notes 221 |

Chapter 8 Mutual synchronization of two interacting periodic

oscillators 222

| 8.1 | Phase dynamics 222 |
|-------|--------------------|
| 0 1 1 | |

- 8.1.1 *Averaged equations for the phase* 224
- 8.1.2 *Circle map* 226
- 8.2 Weakly nonlinear oscillators 227
- 8.2.1 General equations 227
- 8.2.2 Oscillation death, or quenching 229
- 8.2.3 Attractive and repulsive interaction 230
- 8.3 Relaxation oscillators 232
- 8.4 Bibliographic notes 235

Chapter 9 Synchronization in the presence of noise 236

- 9.1 Self-sustained oscillator in the presence of noise 236
- 9.2 Synchronization in the presence of noise 237
- 9.2.1 Qualitative picture of the Langevin dynamics 237
- 9.2.2 Quantitative description for white noise 240

184

- 9.2.3 Synchronization by a quasiharmonic fluctuating force 244
- 9.2.4 Mutual synchronization of noisy oscillators 245
- 9.3 Bibliographic notes 246

Chapter 10 Phase synchronization of chaotic systems 247

- 10.1 Phase of a chaotic oscillator 248
- 10.1.1 Notion of the phase 248
- 10.1.2 Phase dynamics of chaotic oscillators 254
- 10.2 Synchronization of chaotic oscillators 255
- 10.2.1 Phase synchronization by external force 256
- 10.2.2 Indirect characterization of synchronization 258
- 10.2.3 Synchronization in terms of unstable periodic orbits 260
- 10.2.4 Mutual synchronization of two coupled oscillators 262
- 10.3 Bibliographic notes 263

Chapter 11 Synchronization in oscillatory media 266

11.1 Oscillator lattices 266 11.2 Spatially continuous phase profiles 269 11.2.1 Plane waves and targets 269 11.2.2 Effect of noise: roughening vs. synchronization 271 11.3 Weakly nonlinear oscillatory medium 273 11.3.1 Complex Ginzburg-Landau equation 273 11.3.2 Forcing oscillatory media 276 11.4 Bibliographic notes 278

Chapter 12 Populations of globally coupled oscillators 279

- 12.1 The Kuramoto transition 279
- 12.2 Noisy oscillators 283
- 12.3 Generalizations 286
- 12.3.1 Models based on phase approximation 286
- 12.3.2 Globally coupled weakly nonlinear oscillators 289
- 12.3.3 Coupled relaxation oscillators 290
- 12.3.4 Coupled Josephson junctions 291
- 12.3.5 Finite-size effects 294
- 12.3.6 Ensemble of chaotic oscillators 294
- 12.4 Bibliographic notes 296

Part III: Synchronization of chaotic systems

Chapter 13 Complete synchronization I: basic concepts 301

- 13.1 The simplest model: two coupled maps 302
- 13.2 Stability of the synchronous state 304
- 13.3 Onset of synchronization: statistical theory 307
- 13.3.1 *Perturbation is a random walk process* 307
- 13.3.2 The statistics of finite-time Lyapunov exponents determine diffusion 308
- 13.3.3 Modulational intermittency: power-law distributions 310
- 13.3.4 Modulational intermittency: correlation properties 316
- 13.4 Onset of synchronization: topological aspects 318
- 13.4.1 Transverse bifurcations of periodic orbits 318
- 13.4.2 Weak vs. strong synchronization 319
- 13.4.3 Local and global riddling 322
- 13.5 Bibliographic notes 323

Chapter 14 Complete synchronization II: generalizations and complex

systems 324

- 14.1 Identical maps, general coupling operator 324
- 14.1.1 Unidirectional coupling 325
- 14.1.2 Asymmetric local coupling 327
- 14.1.3 Global (mean field) coupling 328
- 14.2 Continuous-time systems 329
- 14.3 Spatially distributed systems 331
- 14.3.1 Spatially homogeneous chaos 331
- 14.3.2 Transverse synchronization of space-time chaos 332
- 14.3.3 Synchronization of coupled cellular automata 334
- 14.4 Synchronization as a general symmetric state 335
- 14.4.1 Replica-symmetric systems 336
- 14.5 Bibliographic notes 337

Chapter 15 Synchronization of complex dynamics by external forces 340

15.1 Synchronization by periodic forcing 341
15.2 Synchronization by noisy forcing 341
15.2.1 Noisy forced periodic oscillations 343
15.2.2 Synchronization of chaotic oscillations by noisy forcing 345
15.3 Synchronization of chaotic oscillations by chaotic forcing 346
15.3.1 Complete synchronization 346
15.3.2 Generalized synchronization 347

- 15.3.3 Generalized synchronization by quasiperiodic driving 352
- 15.4 Bibliographic notes 353

Appendices

Appendix A1: Discovery of synchronization by Christiaan Huygens 357

- A1.1 A letter from Christiaan Huygens to his father, Constantyn Huygens 357
- A1.2 Sea clocks (sympathy of clocks). Part V 358

Appendix A2: Instantaneous phase and frequency of a signal 362

- A2.1 Analytic signal and the Hilbert transform 362
- A2.2 Examples 363
- A2.3 Numerics: practical hints and know-hows 366
- A2.4 Computation of the instantaneous frequency 369

References 371 Index 405