A Utility-Based Power-Control Scheme in Wireless Cellular Systems

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***Abstract*—Distributed power-control algorithms for systems with hard signal-to-interference ratio (SIR) constraints may diverge when infeasibility arises. In this paper, we present a power-control framework called utility-based power control (UBPC) by reformulating the problem using a softened SIR requirement (utility) and adding a penalty on power consumption (cost). Under this framework, the goal is to maximize the net utility, defined as utility minus cost. Although UBPC is still noncooperative and distributed in nature, some degree of cooperation emerges: a user will automatically decrease its target SIR (and may even turn off transmission) when it senses that traffic congestion is building up. This framework enables us to improve system convergence and to satisfy heterogeneous service requirements (such as delay and bit error rate) for integrated networks with both voice users and data users. Fairness, adaptiveness, and a high degree of flexibility can be achieved by properly tuning parameters in UBPC.**

***Index Terms*—Admission control, cellular system, distributed algorithm, fairness, Nash equilibrium, Pareto optimal, power control, robustness, signal-to-interference ratio (SIR), stability, utility function, wireless.**

I. Introduction

WIRELESS networks are characterized by scarce radio spectrum, an unreliable propagation channel (with shadowing, multipath fading, etc.), and user mobility. Hence, in wireless networks, efficiently managing radio resources is a very important problem. In this paper, we focus on power control, which is an important component of the resource management problem in wireless networks.

Power control has been extensively studied in recent years, especially for CDMA systems. It has mainly been used to reduce cochannel interference and to guarantee the signal-to-interference ratio (SIR) of ongoing connections, resulting in a higher utilization and/or better quality of service (QoS). From the viewpoint of practical applications, distributed power-control schemes are of special interest and importance. One of the most well-known distributed algorithms was originally proposed in [1], and has been further studied in several papers, including [2] and [3]. This algorithm is distributed and autonomous because it relies only on local information. It is also "standard" (in the sense defined in [4]), and asynchronously convergent (with geometric rate) to the Pareto optimal power assignment (a power assignment P^* is said to be *Pareto optimal* if it is feasible and any other feasible power assignment P satisfies $P \geq P^*$ componentwise), when the system is feasible [3], [4]. However, if there is no feasible power assignment, this algorithm can diverge, which will be illustrated in Fig. 9. For infeasible systems, the distributed power-control algorithm diverges because of the hard SIR requirements that cannot be achieved in such systems no matter how high the transmitted power is.

In practice, although achieving satisfactory QoS is important for users, they may not be willing to achieve it at arbitrarily high power levels, because power is itself a valuable commodity. Cutting power consumption not only prolongs the life of the battery and alleviates health concerns about electromagnetic emission, but also decreases the interference to other users. In addition, different users may have different views of power consumption. For example, a handset user is more concerned about the power than a user with a vehicle-mounted device. We can capture a user's view of power consumption by also considering the cost of power.

Thus, user satisfaction will depend on both QoS and power consumption. This observation motivates a reformulation of the whole problem using concepts from microeconomics and game theory, as described in [5]. Earlier work applying game theoretic ideas to flow control in telecommunication networks includes [6]–[10]. Our problem naturally fits in this context: the QoS objective can be viewed as a utility function, which represents the degree of user satisfaction with service quality and cost in terms of power consumption. The distributed power-control problem essentially becomes a noncooperative multiplayer game, in which each user (player) tries to maximize its net utility (i.e., utility minus cost). Within this framework, we develop a utility-based distributed power-control algorithm. Our algorithm keeps the major merits of the existing algorithm in [1], but is exempt from the divergence problem because there is no strict SIR threshold requirement. Moreover, the framework provides a unified way to deal with both voice users and data users. In addition to power control, it can also be used for admission control, near–far fairness, and data rate control.

In related work [11], Ji and Huang propose a framework for uplink power control in cellular systems, to obtain better QoS using less power. Their objective function is a concave

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decreasing function of power and a concave increasing function of SIR. The authors provide sufficient conditions for convergence. In this paper, we use a sigmoid-like utility function that does not satisfy the above-mentioned conditions in [11]. The sigmoid-like shape of a utility function is very natural to use because, empirically, a number of important performance measures (e.g., the capture probability) have a sigmoid-like shape (first convex and then concave) as a function of SIR. In [5], Goodman and Mandayam propose a power-control algorithm for wireless data to provide power-efficient transmission for data users in a single-cell system. Their utility function is related to the number of effective bits transmitted per unit of energy. The "natural" form of this utility function turns out to have undesirable properties—the modification proposed in [5] to alleviate this problem leads to a less natural utility function. Moreover, the associated power updating algorithm is complicated and has low Pareto efficiency.

In this paper, the objective is to maximize the net utility (difference between a utility function and a cost function). We will show that the resultant algorithm is flexible and simple to implement in cellular systems. The algorithm does not suffer from divergence, which is achieved by softening the SIR requirement using the notion of utility functions. Our algorithm will automatically decrease the target SIR (and may even turn off transmission) of some users when traffic congestion builds up. This property, which the algorithms in [5] and [11] do not satisfy, facilitates system convergence and admission control. Moreover, our algorithm has several tunable parameters in utility and cost functions to achieve fairness, adaptiveness, and heterogeneous service requirements (such as delay and bit error rate) for integrated networks with both voice users and data users.

The rest of this paper is organized as follows. In Section II, we first present the system model and explain why a hard SIR requirement may result in divergence. In Section III, we reformulate the power-control problem as a noncooperative game maximizing the net utility, which leads to a utility-based power-control algorithm. We also discuss convergence and effects of some parameters in this section. Important extensions and discussions of the utility-based power-control algorithm are made in Section IV. Numerical results are given in Section V. Finally, Section VI concludes the paper.

II. System Model and Related Work

We consider a power-controlled cellular system where the transmitted powers are continuously tunable. Within a cell, every user is associated with a base station (called its home base station). To maintain a reliable connection between the user and its home base station, the SIR at the receiver should be no less than some threshold that corresponds to a QoS requirement such as the bit error rate. We consider only downlink transmissions in this paper because the uplink case can be treated similarly [12], [13].

Assume that there are n users in the system and let P_i be the transmitted power level for the downlink of user i. Let G_{ij} denote the gain from the home base station of user j to user i. Then, the interference power received at user i from the downlink of user j is $G_{ij}P_j$. Let η_i be the background noise received at user i and let γ_i be its desired SIR threshold. Then the SIR for user i is given by

$$\mathrm{SIR}_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ij}P_j + \eta_i}. \tag{1}$$

Note that this model is general enough to represent DS-CDMA systems with matched-filter receivers [14], [15] or TDMA/FDMA systems [2], by giving specific interpretations to the parameters.

For the system considered in [3], [12], and [16], the constraint $\mathrm{SIR}_i \geq \gamma_i$ is enforced for each user i. The objective of a power-control scheme is to find the minimum power satisfying this constraint. To this end, there is a well-known power-control algorithm given by

$$P_i(k+1) = \frac{\gamma_i}{\mathrm{SIR}_i(k)} P_i(k), \qquad \text{for } i = 1, \ldots, n \tag{2}$$

where $P_i(k)$ and $\mathrm{SIR}_i(k)$ correspond to the power level and the SIR for user i at the kth iteration, respectively. This algorithm and its stability have been studied extensively by Foschini and Miljanic [1], Mitra [3], and Bambos, Chen, and Pottie [2], [12]. The algorithm is distributed and autonomous because it relies only on locally available information. It allows each user to have different target SIR values. It has also been shown in [3] to be asynchronously convergent (with geometric rate) to the Pareto optimal power assignment, when the system is feasible (i.e., when there exists a power assignment such that $\mathrm{SIR}_i \geq \gamma_i$ for all i). However, if the system becomes infeasible, this algorithm diverges.

Divergence occurs when the system is infeasible because the SIR requirement is hard (strict), and has to be satisfied at any cost. Intuitively, what user i does through algorithm (2) is to adjust its transmitted power P_i such that its SIR just achieves the threshold γ_i in the next step. In an infeasible system, every user blindly adjusts its power without realizing that it is impossible to satisfy these SIR requirements simultaneously, and consequently transmitted powers build up higher and higher during this procedure. It may appear reasonable to attribute this "blindness" to the fact that the algorithm is distributed and only has local information available. This is not really the case, because a user can recognize that infeasibility would be likely from the extremely high interference received. However, each user still continues to increase its power because its goal is to achieve a SIR value no lower than the threshold.

To avoid introducing infeasible users into the above system, two distributed admission-control schemes were proposed in [16]. The basic idea is that before the new user enters the system and begins to tune its power in full gear according to (2), it tentatively transmits at a fixed level, then decides (in a distributed way) if the system will become infeasible after its joining. Only if the user will not cause infeasibility is it admitted. Although no infeasible user can slip in, there are still some problems that persist. In a wireless cellular system, users can be in constant movement, and an initially feasible user may later become infeasible. Also, due to mobility, the Pareto optimal power assignment achieved by (2) may no longer be optimal and may even be infeasible a moment later, which may result in a high outage probability [17]. We solve these problems by softening the hard SIR requirements.

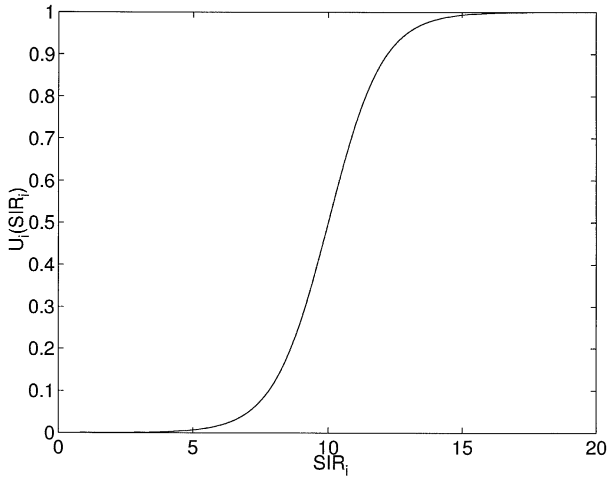

Fig. 1. Sigmoid utility versus SIR for user i.

III. UTILITY-BASED DISTRIBUTED POWER CONTROL

Although achieving satisfactory QoS is important for users, they may not be willing to achieve it at arbitrarily high power levels, because power is itself a valuable commodity. This observation motivates a reformulation of the whole problem using concepts from microeconomics and game theory [5], [11]. In this section, we will use such a reformulation to develop a mechanism for power control where the desire for increased SIR is weighed against the associated cost. We will point out the differences between our work and that in [5] and [11] and discuss some important implications of these differences.

A. Problem Formulation and Basic Algorithm

Instead of enforcing the constraint $\text{SIR}_i \geq \gamma_i$ as in the hard constraint case, we use a utility function U_i to represent the degree of satisfaction of user i to the service quality, and introduce a cost function C_i to measure the cost incurred. The goal is to maximize the net utility NU_i defined as $NU_i = U_i - C_i$ by adjusting the transmitted power P_i. Since each user in the system will try to maximize its own net utility, regardless of what happens to the other users, this problem is a typical noncooperative N-person game [18].

Generally, the QoS depends on SIR, so we let the utility U_i be a function of SIR_i satisfying: $U_i(0) = 0$; $U_i(\infty) = 1$, and that $U_i(\text{SIR}_i)$ increases in SIR_i. This means that a user is more and more satisfied with the service as the quality improves. We will use the utility function given in Fig. 1 (called the Sigmoid function) in this paper, because it captures the value of the service to the user quite naturally. This function is also similar in shape to the capture probability [19]. However, it should be noted that our scheme can be applicable to many other utility functions.

We choose C_i, the cost for user i, as a function of power. As mentioned before, power is itself a valuable commodity. The specific cost function should reflect the expenses of power consumption to the user. There are at least two requirements for the cost function: $C_i(0) = 0$, and that $C_i(P_i)$ increases in power P_i. In this paper, we will use a linear cost function, i.e.,

$$C_i(P_i) = \alpha_i P_i \tag{3}$$

where α_i is the "price" coefficient. Although α_i is a constant independent of P_i, it can be a function of environmental factors such as user location and received interference. In fact, the use of an adaptive price setting can be helpful in achieving fairness and robustness. As is the case of the utility function, other forms of cost functions would also work for our scheme (although, not as simply).

The net utility of user i is $NU_i(\text{SIR}_i, P_i) = U_i(\text{SIR}_i) - C_i(P_i)$. The power-control problem for user i is formulated as

$$\max_{P_i \geq 0} NU_i. \tag{4}$$

Taking the derivative of NU_i in (4), we have

$$\begin{aligned}\frac{d(NU_i(\text{SIR}_i, P_i))}{dP_i} &= \frac{d(U_i(\text{SIR}_i) - C_i(P_i))}{dP_i} \\ &= U_i'(\text{SIR}_i)\frac{d(\text{SIR}_i)}{dP_i} - C_i'(P_i) \\ &= U_i'(\text{SIR}_i)\frac{\text{SIR}_i}{P_i} - C_i'(P_i)\end{aligned} \tag{5}$$

where U_i' and C_i' are the derivatives of U_i and C_i, respectively. In the last step, we use the fact that SIR_i is linear with P_i, as defined by (1). If a *positive* power P_i is a local optimum for problem (4), we require $d(NU_i(\text{SIR}_i, P_i))/dP_i = 0$, i.e.,

$$U_i'(\text{SIR}_i)\text{SIR}_i = C_i'(P_i)P_i. \tag{6}$$

Considering (1) and (3), the above condition becomes

$$U_i'(\text{SIR}_i) = \alpha_i \frac{R_i}{G_{ii}} \tag{7}$$

where $R_i = \sum_{j \neq i} G_{ij} P_j + \eta_i$ is the received interference of user i. Because the right-hand side of (7) is known or locally measurable, we find the solution to be

$$\widehat{\text{SIR}}_i = f_i^{-1}\left(\alpha_i \frac{R_i}{G_{ii}}\right) \tag{8}$$

where $f_i(\text{SIR}_i) = U_i'(\text{SIR}_i)$ in the concave part of U_i where a local maximum is possible. Then one candidate for the optimal power assignment is

$$\hat{P}_i = \widehat{\text{SIR}}_i \frac{R_i}{G_{ii}} = \frac{R_i}{G_{ii}} f_i^{-1}\left(\alpha_i \frac{R_i}{G_{ii}}\right). \tag{9}$$

Clearly, given the coefficient α_i, $\hat{P}_i$ is a function of R_i/G_{ii}. Now the problem is whether $\hat{P}_i$ defined above exists, and whether it is globally optimal (when it exists). Note that $P_i = 0$ is on the constraint boundary of (4) and need not satisfy the above condition even when it is a maximum point. In fact, in some case $P_i = 0$ achieves the global optimum, though it corresponds to zero NU_i. Thus, the optimal power P_i^* is either $\hat{P}_i$ or 0, whichever results in a larger net utility. This leads to a distributed power-control algorithm optimizing net utility. We call the algorithm utility-based power control (UBPC), and next illustrate it graphically.

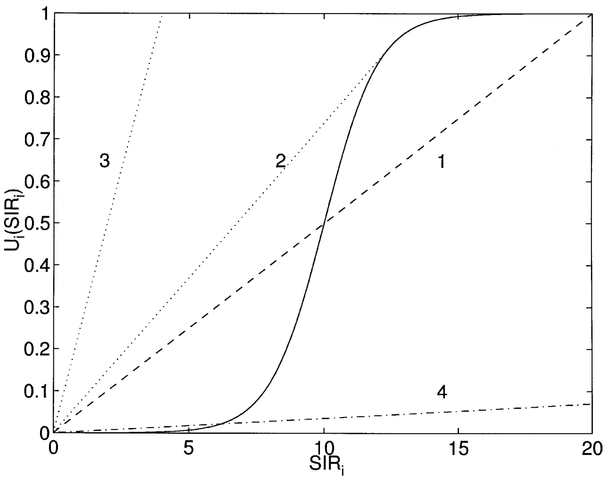

Fig. 2. Sigmoid utility (and cost) versus SIR for user i.

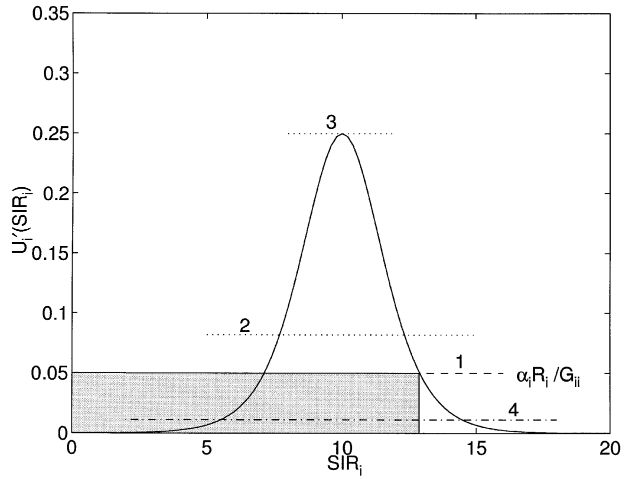

Fig. 3. Derivative function of Sigmoid utility versus SIR for user i.

B. Graphical Illustration

We plot both the utility and the cost versus SIR in Fig. 2. Because $C_i = \alpha_i P_i = \alpha_i (R_i/G_{ii})\mathrm{SIR}_i$, the slope of the cost line in Fig. 2 is $\alpha_i(R_i/G_{ii})$. By changing the slope, we have a different position of C_i relative to U_i. When the slope is small, the cost line has two nonzero intersections with the utility, as shown by line 1. When the slope increases, the two intersections will come closer, and eventually meet on line 2. If we continue to increase the slope until it is equal to the maximum derivative of the utility function, the cost line reaches line 3. Note that $R_i \geq \eta_i$ by definition, so the cost line has a positive lower bound as illustrated by line 4 in Fig. 2. Let $\underline{K}_i$ be the slope of this line, i.e., $\underline{K}_i = \alpha_i(\eta_i/G_{ii})$. This nonzero lower bound prevents infinite $\widehat{\mathrm{SIR}}_i$ in power control.

In Fig. 2, if the cost line (e.g., line 1) lies between line 2 and line 4, then there will be some positive net utility corresponding to $\hat{P}_i$, i.e., $P_i^* = \hat{P}_i$. If the cost line reaches line 2, the maximum net utility is 0, which is achieved at power levels $\hat{P}_i$ and 0. If the cost line (e.g., line 3) is beyond line 2, the best choice is to keep $P_i^* = 0$, because all other powers will result in negative net utility. We will call the SIR associated with cost line 2 the *turnoff SIR* of user i, and denote it by $\underline{\mathrm{SIR}}_i$, which is the lowest SIR value user i will achieve when transmitting. Correspondingly, we denote the slope of line 2 by $\overline{K}_i$. Note that $\overline{K}_i$ and $\underline{\mathrm{SIR}}_i$ depend only on the utility function.

In Fig. 3, we show the derivative $U_i'(\mathrm{SIR}_i)$ of the Sigmoid utility function. The horizontal lines correspond to the cost lines in Fig. 2, respectively. Note that lines 1, 2, and 4 each have two intersections with $U_i'(\mathrm{SIR}_i)$, but only the intersection on the right side qualifies for $\hat{P}_i$ because U_i is concave in this part. In fact, the left-side intersections are the minimum points.

From Figs. 3 and 4, we see that as $\alpha_i(R_i/G_{ii})$ increases, $\widehat{\mathrm{SIR}}_i$ will decrease. This has important implications. If we introduce the iteration index k, then (9) becomes

$$\hat{P}_i(k+1) = \widehat{\mathrm{SIR}}_i(k)\frac{R_i(k)}{G_{ii}(k)} = \frac{\widehat{\mathrm{SIR}}_i(k)}{\mathrm{SIR}_i(k)}P_i(k). \tag{10}$$

At each step, we are trying to maximize the net utility under the given interference. Contrasting (10) with (2), we see that with

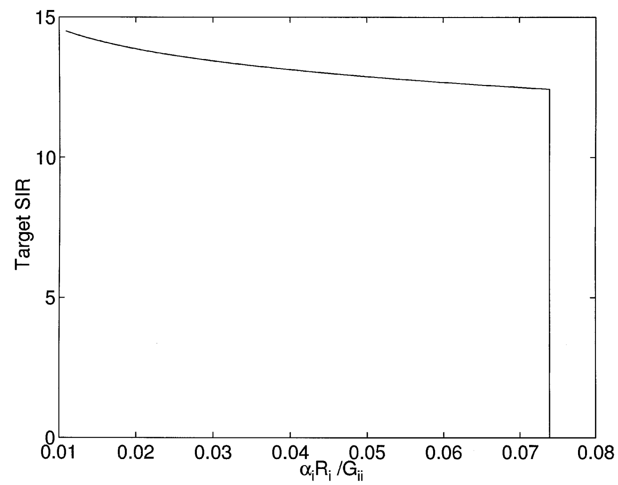

Fig. 4. Target SIR versus transmission environment of user i.

UBPC the target SIR value will decrease automatically for a worse transmission environment (i.e., for larger $R_i(k)/G_{ii}(k)$), while with the power-control algorithm given by (2), it remains the same regardless of the environment. When the transmission environment becomes very hostile, there will be no gain (positive net utility) in transmitting and the transmission will be totally shut off by UBPC. To transmit under UBPC, user i must have its cost line lie between line 2 and line 4, i.e., $\alpha_i R_i/G_{ii} \in [\underline{K}_i, \overline{K}_i]$. To be consistent with the hard SIR requirements, i.e., $\mathrm{SIR}_i \geq \gamma_i$ for user i, from now on we assume $\underline{\mathrm{SIR}}_i = \gamma_i$, because under this setting user i always achieves SIR higher than γ_i when transmitting.

In summary, the iterative procedure of the UBPC for user i is as follows.

Algorithm UBPC

1. Measure the received interference $R_i(k)$, update path gain $G_{ii}(k)$ and price coefficient $\alpha_i(k)$, then calculate $\widehat{\mathrm{SIR}}_i(k)$ using (8). If $\widehat{\mathrm{SIR}}_i(k) < \gamma_i$, turn off transmission (i.e., $P_i(k+1) = 0$), and go to step 3. Otherwise, go to step 2.

2. Calculate $\hat{P}_i(k+1)$ using (10), and set the power level to $\hat{P}_i(k+1)$.
3. Let $k \leftarrow k+1$, and go to step 1.

The information required by UBPC from the utility of user i is the curve of the target SIR, $\widehat{\text{SIR}}_i$ versus $\alpha_i R_i/G_{ii}$, which is shown in Fig. 4. Hence, in implementation, a user only has to provide this curve instead of the utility function. This simplifies implementation, and also enhances flexibility to meet heterogeneous QoS requirements, because the target SIR curve can have other forms, e.g., a staircase form.

In the objective function NU_i, our utility is a function of only SIR, while the cost term depends only on power. This arrangement not only achieves separation (between utility and cost) and simplifies the derivation, but also benefits the system performance and facilitates network resource management. Before elaborating on these points, we will first study a specific example of the Sigmoid utility.

C. Sigmoid Utility

One well-known Sigmoid function is

$$h(x) = 1/1 + e^{-a(x-b)}$$

which has been widely used in the study of neural networks. Clearly, $h(b) = 1/2$, so we call b the center of $h(x)$. It is easy to show that the derivative of $h(x)$ satisfies

$$h'(x) = ah(x)(1 - h(x)). \tag{11}$$

This function is perfect as a utility function for our purposes, except that $h(0) = 1/(1+e^{ab}) \neq 0$. This is not a serious problem, because $h(0) \approx 0$ as e^{ab} is large. Moreover, we can always solve this problem (when desired) by using the linear transformation

$$U(x) = \frac{h(x) - \dfrac{1}{1+e^{ab}}}{1 - \dfrac{1}{1+e^{ab}}}. \tag{12}$$

For simplicity, we will directly use the Sigmoid function $h(x)$ (with $a = a_i$ and $b = \beta_i$) as the utility function for user i, i.e.,

$$U_i(\text{SIR}_i) = \frac{1}{1+e^{-a_i(\text{SIR}_i - \beta_i)}}. \tag{13}$$

There are two tunable parameters in the Sigmoid utility (13): parameters a_i and β_i, which can be used to tune the steepness and the center of the utility, respectively.

In Figs. 5 and 6, we illustrate the utility functions and the corresponding derivative functions for two different values of a_i. When parameter a_i increases, the utility becomes steep, and its derivative function becomes narrow and high. Applying UBPC to two users with utility functions shown in Fig. 5, we can show that the target SIR ($\widehat{\text{SIR}}_i$) of the user with a larger value of a_i decreases more slowly in $\alpha_i R_i/G_{ii}$. Thus, a user having a utility with a large value of a_i is rigid, and is difficult to be turned off (especially when β_i is small). In the limit as a_i approaches infinity, the utility function becomes a step function with a step at $\text{SIR}_i = \beta_i$, and its derivative becomes an impulse function. In this case, the target $\widehat{\text{SIR}}_i$ is fixed at β_i no matter what value

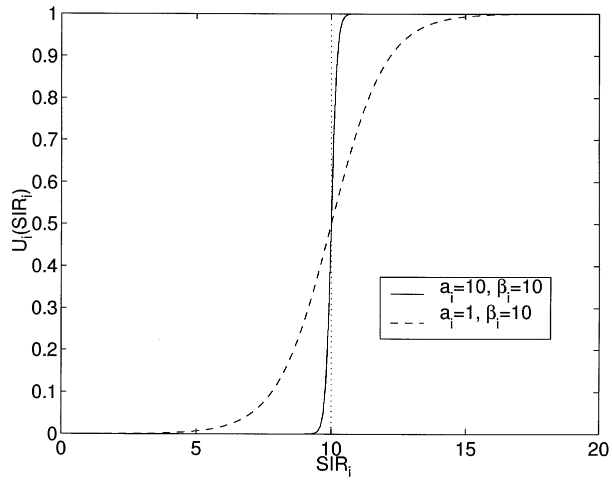

Fig. 5. Sigmoid utility versus SIR with different a_i.

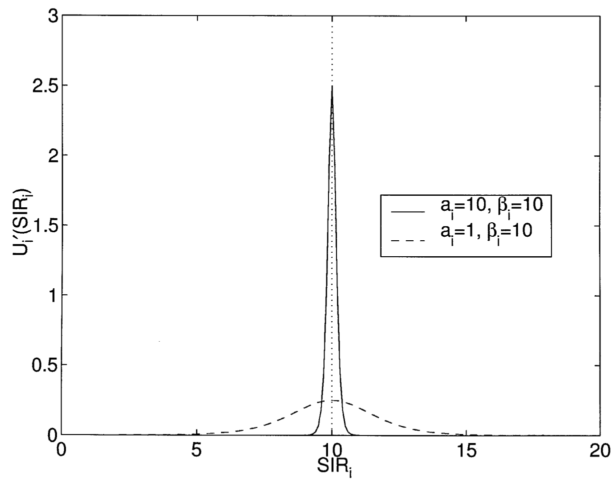

Fig. 6. Derivative funstion of Sigmoid utility versus SIR with different a_i.

$\alpha_i(R_i/G_{ii})$ takes, $\gamma_i = \underline{\text{SIR}}_i = \beta_i$, and $\overline{K}_i$ (the slope of cost line 2) achieves the upper bound $1/\gamma_i$. We should emphasize that a user with a step utility can still be turned off if the slope of its cost line is greater than $\overline{K}_i$. If $\alpha_i = 0$ additionally, then our UBPC for user i just reduces to the power-control algorithm (2), as shown by (10). This type of rigid user can never be turned off by UBPC.

In Fig. 7, we fix a_i and vary β_i. The two utility functions and their derivative functions have the same shape, but with different centers (at β_i). Clearly, a user with a higher β_i in utility will get higher SIR when in service, but is more likely to be turned off by UBPC. When β_i is large enough, $\overline{K}_i$ becomes small and insensitive to the shape of the utility, but a softer utility (with smaller a_i) will show more flexibility and robustness.

Based on the effect of the two parameters on our power control, we make the following important observation: UBPC is suitable for integrated wireless systems with both voice users and data users, important in the third-generation wireless networks. To illustrate this point clearly, we start with the well-known difference between voice and data in their service requirements. For a voice user, the essential objective is low delay, and transmission errors are tolerable up to a relatively high point. Thus, the voice user does not want to be easily

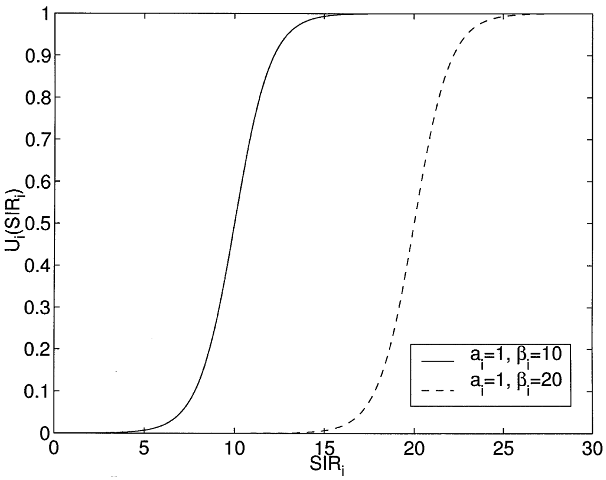

Fig. 7. Sigmoid utility versus SIR with different β_i.

turned off, but its target SIR can be relatively low. This means that for a voice user, the cost should be low, and the utility function should be steep and with low turnoff SIR, i.e., α_i is small, a_i is large, and γ_i is small. Note that a_i must be large enough, but need not be infinite (in contrast to what is used in [5]). On the other hand, a data user can accept some delay but has very low tolerance to errors, which can be satisfied by a utility function with a smaller a_i and larger γ_i. A user with such a utility can achieve a high SIR when transmitting, but it must allow being turned off and delaying its transmission until later. Thus, we can characterize the service requirements of both the voice users and data users by choosing different utility parameters. In this way, these two different types of users have a unified and comparable measure for sharing radio resources. It should also be pointed out that under the framework of UBPC, due to their rigid behavior, voice users can get preemptive priority over data users, which is desirable in this case. Moreover, some factors such as the voice activity factor and data burstiness can be automatically exploited through the received interference. We believe UBPC also works for hybrid systems with both circuit-switched and packet-switched users.

To gain further insight, we derive $\hat{P}_i$ and $\underline{\mathrm{SIR}}_i$ for the Sigmoid utility (13). Combining (7) and (11) yields

$$\hat{P}_i = \frac{R_i\beta_i}{G_{ii}} - \frac{R_i}{G_{ii}a_i} \times \ln\left[\left(\frac{a_i}{2\alpha_i\frac{R_i}{G_{ii}}} - 1 - \sqrt{\left(\frac{a_i}{\alpha_i\frac{R_i}{G_{ii}}} - 1\right)^2 - 1}\right)\right] \tag{14}$$

which is in closed form, but looks quite complex. We observe that it is truly a function of R_i/G_{ii}. One necessary condition that $\hat{P}_i$ exists is $\alpha_i(R_i/G_{ii}) \le a_i/4$, which means that the cost line should lie below line 3 in Fig. 2. If this condition is satisfied, we always have $\hat{P}_i > (R_i/G_{ii})\beta_i$, i.e., $\widehat{\mathrm{SIR}}_i > \beta_i$.

To derive $\underline{\mathrm{SIR}}_i$, we use (11) and the condition that the cost line is the tangent line of U_i through the origin, i.e.,

$$U_i'(\underline{\mathrm{SIR}}_i)\underline{\mathrm{SIR}}_i = U_i(\underline{\mathrm{SIR}}_i). \tag{15}$$

After some algebra, we find that $\underline{\mathrm{SIR}}_i$ satisfies

$$\frac{e^{a_i\underline{\mathrm{SIR}}_i}}{a_i\underline{\mathrm{SIR}}_i - 1} = e^{a_i\beta_i}. \tag{16}$$

We have assumed $\underline{\mathrm{SIR}}_i = \gamma_i$, where the SIR threshold γ_i is predefined. Thus, the center β_i can be calculated using (16) as

$$\beta_i = \gamma_i - \frac{\ln(a_i\gamma_i - 1)}{a_i}. \tag{17}$$

Inserting the above equation into (14) yields

$$\widehat{\mathrm{SIR}}_i = \gamma_i - \frac{1}{a_i}\ln(a_i\gamma_i - 1) - \frac{1}{a_i} \times \ln\left(\frac{a_i}{2\alpha_i\frac{R_i}{G_{ii}}} - 1 - \sqrt{\left(\frac{a_i}{2\alpha_i\frac{R_i}{G_{ii}}} - 1\right)^2 - 1}\right). \tag{18}$$

User i is active in transmission if and only if $\widehat{\mathrm{SIR}}_i \ge \gamma_i$, i.e.,

$$\alpha_i\frac{R_i}{G_{ii}} \le \frac{1}{\gamma_i} - \frac{1}{a_i\gamma_i^2}. \tag{19}$$

Thus, we have $\overline{K}_i = (1/\gamma_i) - (1/a_i\gamma_i^2)$ for the Sigmoid utility U_i in (13). This also verifies the previous result that as a_i goes to infinity (i.e., the utility becomes a step at γ_i), $\overline{K}_i$ achieves its upper bound $1/\gamma_i$.

D. Feasibility and Convergence

1) Feasible Systems Under UBPC: From the above discussions, transmission is automatically turned off when the transmission environment becomes very hostile. This simply implies that UBPC will never result in powers "blowing up." However, because the system cannot accommodate an infinite number of users, we can also define feasibility in the power-controlled system under UBPC. A system is said to be *feasible under UBPC* if there exists a power assignment $\mathbf{P} = [P_1, P_2, \ldots, P_n]^T$, such that

$$\mathrm{SIR}_i \ge \widehat{\mathrm{SIR}}_i \tag{20}$$

and $NU_i > 0$ for all user i. We will call such a power assignment $\mathbf{P}$ feasible under UBPC. Note that in the definition, the condition $NU_i > 0$ is necessary.

To see a counterexample showing that an infeasible system can satisfy the first condition, consider a system feasible under (2) but with an infinite price coefficient. The utility in this case becomes a step function, and the condition (20) reduces to $\mathrm{SIR}_i \ge \gamma_i$. The feasibility under (2) means that there exists a positive power assignment such that the inequality holds for all i [12], [3]. However, any positive power will result in infinite cost and negative net utility, so it is infeasible under UBPC. Next, we study how UBPC performs in a feasible system.

2) Standard Power Control: Remember that a power-control algorithm $\mathbf{P}(k+1) = A(\mathbf{P}(k))$ is said to be *standard* [4] if A satisfies the following properties for all $\mathbf{P} \ge 0$:

- *Positivity*: $A(\mathbf{P}) > 0$;
- *Monotonicity*: If $\mathbf{P}' \ge \mathbf{P}$, then $A(\mathbf{P}') \ge A(\mathbf{P})$;
- *Scalability*: For all $\mu > 1$, $\mu A(\mathbf{P}) > A(\mu\mathbf{P})$.

This framework has been proposed for systems with a hard constraint $\mathbf{P} \geq A(\mathbf{P})$, where $A(\mathbf{P})$ is called the *interference function*. A typical example of a standard power-control algorithm is (2). We next show that UBPC is standard under mild conditions.

Theorem 1: UBPC is standard if $f_i^{-1}(x)(x/\alpha_i)$ is an increasing function on $[\underline{K}_i, \overline{K}_i]$ for all i.

Proof: From (9), the interference function of UBPC is

$$A(\mathbf{P}) = [A_1(\mathbf{P}), A_2(\mathbf{P}), \ldots, A_n(\mathbf{P})]^T$$

where $\mathbf{P} = [P_1, P_2, \ldots, P_n]^T$ and $A_i(\mathbf{P}) = (R_i/G_{ii}) f_i^{-1}(\alpha_i(R_i/G_{ii}))$ with $R_i = \sum_{j \neq i} G_{ij} P_j + \eta_i$. The positivity property is implied by the nonzero background receiver noise η_i.

If $\mathbf{P}' \geq \mathbf{P}$, then $R_i' \geq R_i$, where $R_i' = \sum_{j \neq i} G_{ij} P_j' + \eta_i$. Because the function $f_i^{-1}(x)(x/\alpha_i)$ is an increasing function on $[\underline{K}_i, \overline{K}_i]$, we get $A(\mathbf{P}') \geq A(\mathbf{P})$ for $\mathbf{P}' \geq \mathbf{P}$. (As a reminder, the quantities $\underline{K}_i$ and $\overline{K}_i$ are the slopes of line 2 and line 4 in Fig. 2, respectively.)

For all $\mu > 1$, we have

$$\begin{aligned} A(\mu\mathbf{P}) &= \frac{\sum_{j\neq i} G_{ij}\mu P_j + \eta_i}{G_{ii}} f_i^{-1}\left(\alpha_i \frac{\sum_{j\neq i} G_{ij}\mu P_j + \eta_i}{G_{ii}}\right) \\ &< \frac{\sum_{j\neq i} G_{ij}\mu P_j + \eta_i}{G_{ii}} f_i^{-1}\left(\alpha_i \frac{\sum_{j\neq i} G_{ij} P_j + \eta_i}{G_{ii}}\right) \\ &< \mu \frac{\sum_{j\neq i} G_{ij} P_j + \eta_i}{G_{ii}} f_i^{-1}\left(\alpha_i \frac{\sum_{j\neq i} G_{ij} P_j + \eta_i}{G_{ii}}\right) \\ &= \mu A(\mathbf{P}) \end{aligned}$$

where the first inequality holds because $f_i(x)$ is a decreasing function on $[\underline{K}_i, \overline{K}_i]$. Thus, UBPC is standard if $f_i^{-1}(x)(x/\alpha_i)$ is an increasing function on $[\underline{K}_i, \overline{K}_i]$. □

For a fixed price coefficient, the function $f_i^{-1}(x)(x/\alpha_i)$ is increasing if and only if the shaded area (under line 1 and to the left of its right-side intersection) in Fig. 3 increases in the height of line 1, $(\alpha_i(R_i/G_{ii}))$. This is a very mild condition, because a user in practice usually has a large a_i or a large γ_i (see Section III-C). Hence, UBPC is standard for almost all practical situations of interest.

Like other standard power-control algorithms, UBPC applied to a *feasible* system also has the following properties that can be proved in exactly the same way as in [4].

Properties of UBPC:

1) If there is a unique fixed point $\mathbf{P}^*$, then that point is unique.
2) If the initial power assignment $\mathbf{P}$ is feasible, then UBPC generates a monotone decreasing sequence of feasible power assignments that converges to $\mathbf{P}^*$. This implies that the fixed point $\mathbf{P}^*$ is componentwise minimum in the set of all feasible power assignments.
3) If the initial power assignment is the all-zero vector, then UBPC generates a monotone increasing sequence of power assignments that converges to $\mathbf{P}^*$.
4) UBPC converges from any initial power assignment to the unique fixed point $\mathbf{P}^*$ in both synchronous and asynchronous cases.

The standard power-control framework was originally proposed for systems with a hard constraint (e.g., $\text{SIR}_i \geq \gamma_i$), which directly leads to the feasibility condition and the interference function. However, systems under UBPC do not have a hard requirement, and the feasibility condition in (20) for such systems is implied by the power control instead. In this sense, our work essentially generalizes the standard power-control framework in [4]. Although the feasibility condition in (20) for systems under UBPC is somewhat subtle, we can infer a physical meaning for this feasibility definition from the following theorem.

Theorem 2: A system is feasible under UBPC if and only if no user is turned off when starting from the all-zero power assignment.

Proof: If the system is not feasible under UBPC, then for any power assignment $\mathbf{P}(k)$ we will have either $NU_i \leq 0$ or $\text{SIR}_i(k) < \widehat{\text{SIR}}_i(k)$ for all i. The former condition means that user i has to be turned off. If this happens, we are done with the proof of this part. So we only have to consider the case where $NU_i > 0$ in all the steps, i.e., $\text{SIR}_i(k) < \widehat{\text{SIR}}_i(k)$. By (10), this condition means that UBPC generates an increasing sequence of power assignments. If the increasing sequence is bounded from above, then the sequence will converge [20] to a power assignment $\mathbf{P}^*$, at which $\text{SIR}_i = \widehat{\text{SIR}}_i$ for all i. However, this contradicts the assumption that the system is infeasible. Thus, the increase of the power (and of the cost) could be unlimited. Then there always exists a user j that has $NU_j \leq 0$ at some iteration, because $U_j \leq 1$. This just means that user j is turned off by UBPC at this step.

The other direction of the theorem follows from Property 3 of UBPC. □

In fact, we need not start from the all-zero power assignment at all. If the initial power assignment is arbitrary, UBPC applied to a feasible system may turn off some user temporarily, but eventually all users will be turned on. Theorem 2 implies that an infeasible system always has some user turned off, but this does not exclude the possibility that a user that is switched off retries transmission, which may result in power oscillation. Fortunately, we can avoid such a situation by forbidding immediate retries or by setting a high price coefficient (see Section IV-A). Then, unlike the standard power-control schemes with hard constraints such as (2), UBPC can achieve convergence even when infeasibility arises. Moreover, a system converging under UBPC always reaches the *Nash equilibrium* [18], which is straightforward to check from the definition of Nash equilibrium.

IV. Discussions and Extensions

In this section, we will illustrate that several desirable properties and a high degree of flexibility can be achieved by properly tuning the parameters in UBPC. We also briefly discuss some radio resource management techniques that can be jointly considered with UBPC.

A. Adaptiveness and Fairness Considerations

1) Adaptive Price Setting: It has been shown that $\widehat{\mathrm{SIR}}_i$ decreases as the quantity $\alpha_i(R_i/G_{ii})$ increases, and when this quantity exceeds some critical point $\overline{K}_i$ (the slope of line 2 in Fig. 2), there will be no positive net utility, and user i should stop transmission. Clearly, the price coefficient α_i has the same effect as R_i/G_{ii} does on the power control. In other words, transmission is discouraged by a large value of α_i as well as by the hostile transmission environment (in terms of R_i/G_{ii}). This is intuitive—users are generally unwilling to pay more for the same service. However, as will be explained, a higher price coefficient also means a more robust system, which can facilitate network management in congested situations.

From the network's point of view, it can use the price coefficient as an effective way to manage resources and to maximize revenue. When the transmission environment is desirable, the network should set a low price, allowing users to enjoy good QoS (high SIR or high transmission rate). On the other hand, when congestion builds up, it should set a high price, to improve system robustness. In fact, if the price coefficient is not high enough, under heavy traffic situations, a user may repeat the procedure of being turned off and retrying, resulting in oscillation. Thus, it is desirable to have a price coefficient that is adaptive to the transmission environment. A good measure for the transmission environment experienced by user i is R_i/G_{ii}, so the price coefficient should be set as an increasing function of this quantity. For example, a simple adaptive setting can be

$$\alpha_i = \alpha \frac{R_i}{G_{ii}} \tag{21}$$

where α is a constant (may be provided by the base station). Then, we have a different power-control algorithm

$$\hat{P}_i = \frac{R_i}{G_{ii}} \widehat{\mathrm{SIR}}_i = \frac{R_i}{G_{ii}} f_i^{-1}\left(\alpha\left(\frac{R_i}{G_{ii}}\right)^2\right). \tag{22}$$

Our simulations (in Section V) show that UBPC with the linear adaptive setting works well under a large range of traffic loads.

2) Near–Far Fairness: The basic UBPC scheme exhibits a kind of unfairness, the so-called *near–far unfairness*. A user far from the base station (the "far user") normally has a small path gain, so it is more likely to get a lower SIR or to be turned off. This is beneficial to the network, because the total throughput can be improved in this way. In fact, it has been shown by Oh *et al.* [21] that the optimal scheme to maximize throughput is that near users transmit at the highest power while far users totally turn off. Of course, these arrangements are extremely unfair to users close to cell boundaries, who may never get a chance to transmit. Moreover, the reason to use power control in the first place is to overcome the "near–far effect" in CDMA systems. Thus, we must achieve near–far fairness even at some cost of throughput. To this end, we should set a lower price to the farther user. A simple and natural scheme is

$$\alpha_i = \alpha G_{ii} \tag{23}$$

where α is a constant. Then we have

$$\alpha_i \frac{R_i}{G_{ii}} = \alpha G_{ii} \frac{R_i}{G_{ii}} = \alpha R_i \tag{24}$$

i.e., we perform power control based only on the received interference (see Fig. 4), and have no prejudice against far users. At the same time, we also achieve handoff prioritization. If this weighting is still not enough, we can further decrease the price for handoff users.

Other fairness problems (e.g., deadline fairness) can be solved similarly. Since fairness is often achieved at the cost of throughput, the price to be used should depend on the tradeoff between fairness and throughput.

3) Combined Price Setting: To achieve both adaptiveness and fairness, we only have to combine schemes (21) and (23) to get the following price setting:

$$\alpha_i = \alpha G_{ii} \frac{R_i}{G_{ii}} = \alpha R_i \tag{25}$$

where α is a constant independent of R_i and G_{ii} and can be provided by the base station. In (25), the price coefficient adopted by a user is proportional to its received interference. Now the new power-control algorithm becomes

$$\hat{P}_i = \frac{R_i}{G_{ii}} \widehat{\mathrm{SIR}}_i = \frac{R_i}{G_{ii}} f_i^{-1}\left(\alpha \frac{R_i^2}{G_{ii}}\right). \tag{26}$$

It still depends on G_{ii}, but more on R_i.

B. Integrated Resource Management

It has been shown that power control can greatly improve system capacity [13]. On the other hand, resource management of power-controlled systems often becomes more challenging because of the dynamically variable capacity and limited (locally) available information [22], [16]. In the following, we will illustrate that UBPC can be readily integrated with other resource management techniques and facilitate their implementation.

The task of admission control is to decide whether or not to grant access to a newly arriving user. A good admission-control scheme should admit as many users as possible while maintaining the quality of ongoing users. It is interesting to note that UBPC automatically has some admission-control function. Rejection happens when a new user finds that its cost line is beyond (higher than) line 2 in Fig. 2. If there are enough resources to accommodate a new user, it is accepted to receive service better than its minimum requirement. As a new user gets admitted, the ongoing users may yield a little to make space for it. It is also possible for an infeasible new user to get service by turning an existing user off. Such a situation allows a user of high priority (e.g., voice user) to preempt a user of best-effort service. The user being turned off first is usually the "bottleneck" user, which has the worst transmission environment. As mentioned before, the transmission environment can be changed by tuning the price coefficient. Hence, to provide more protection to ongoing users or handoff users, we only have to lower their prices or set higher initial prices to new users. In short, power-controlled systems under UBPC are highly autonomous, and the behavior of a user depends on its utility, price coefficient, and interactions with other users in the system.

Like the distributed power-control scheme (2), UBPC can also be solved jointly with dynamic base station and channel assignment [14], [23]. The main difference is that we first choose the base station and channel resulting in the best transmission environment (in terms of R_i/G_{ii}). These dynamic assignments, along with the adaptiveness of UBPC, promise good performance in the face of mobility.

Considering the hostile environment encountered in wireless systems, it is desirable for users to have adjustable transmission rates [15]. With rate control, systems under UBPC can take advantage of high SIR in desirable transmission environments, and translate it into high transmission rates to increase throughput.

V. Simulation Results

In this section, we simulate the evolution of power and SIR for different algorithms. The purpose of the simulations is to show that UBPC overcomes the divergence problem and achieves fairness and adaptiveness using different parameters and different price coefficient settings. Capacity improvement of power-controlled systems has been demonstrated in [13], so we will not repeat it here.

By modeling the system as a set of interfering links with individual SIR requirements, we can treat a two-dimensional (2-D) system in the same way as a unidimensional model. Also, from the viewpoint of implementing the algorithm, it makes no difference whether there are just a few users in a carrier or there are many more, because each user adjusts its power in a distributed manner. While the proposed algorithm is applicable to general systems with or without mobility, a simple system is more illustrative in demonstrating our main results. As a result, we consider a one-channel linear cellular system consisting of 20 cells, and simulation results of 2-D systems with or without mobility can be found in [24]. In the system, base stations use omnidirectional antennas and are located at the centers of the cells. The distribution of users is illustrated in Fig. 8. In this figure, each vertical line represents a user, and the height of a line illustrates the SIR threshold of the corresponding user. Each user is numbered according to their order of arrival. Base stations are marked by X's on the x axis, and each user is assigned to the closest base station. The path gain G_{ij} is modeled as $G_{ij} = A_{ij}/d_{ij}^4$, where d_{ij} is the distance between user i and the home base station of user j, and the attenuation factor A_{ij} models the power variation due to shadowing. We assume that all A_{ij} are independent and identically log-normally distributed random variables with 0-dB expectation and 8-dB log-variance as in [2] and [14]. The path gain matrix $\mathbf{G}$ of our simulated system is shown in the equation at the bottom of the page, where $O(10^{-x})$ means a quantity on the order of 10^{-x}. From this matrix, we see that users 2, 5, and 6 are far users, and the latter two are closely coupled. It is easy to check that the Perron–Frobenious eigenvalue of the system is $2.1844 > 1$, i.e., the system is infeasible [2], [16]. Note that the system is feasible after the removal of either user 5 or 6.

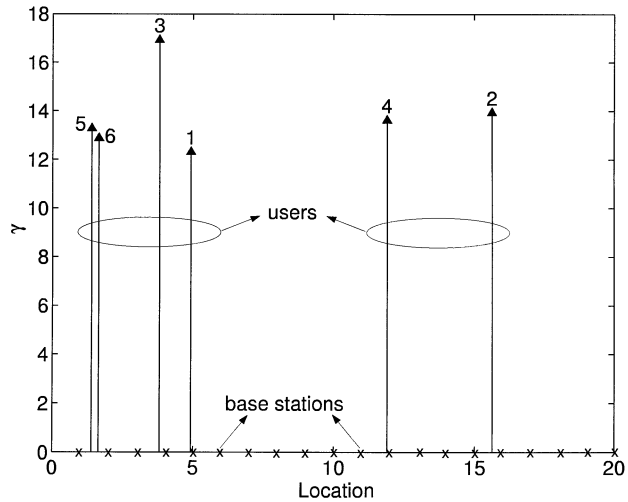

Fig. 8. Distribution of users in the system.

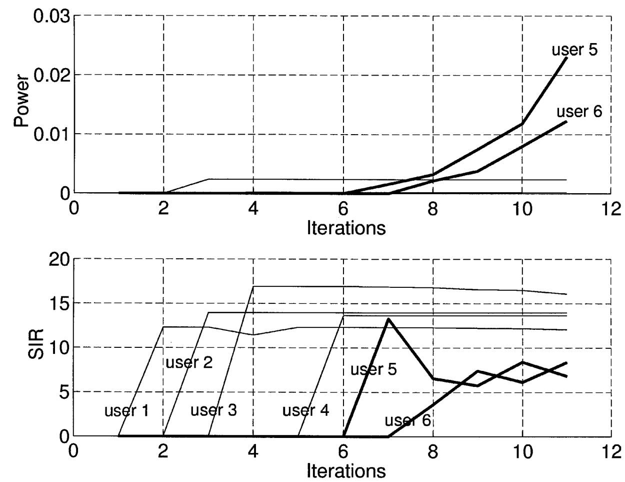

Fig. 9. Evolution of power and SIR, using power control algorithm (2).

We first apply the power-control algorithm (2) without admission control; the results are shown in Fig. 9. Each line in this figure shows the evolution of power or SIR of a user. The leftmost line in the SIR graph corresponds to user 1, the rightmost line corresponds to user 6, and so on (similarly for the other simulation plots in this section). We observe from the figure that algorithm (2) works very well until an infeasible user is admitted, after which the power-control algorithm becomes unstable. This is shown by the power blowup and SIR oscillations of users 5 and 6 in the figure.

$$\begin{bmatrix} 29792 & O(10^{-5}) & 1.37 & O(10^{-4}) & O(10^{-3}) & O(10^{-2}) \\ O(10^{-5}) & 53.67 & O(10^{-5}) & O(10^{-3}) & O(10^{-5}) & O(10^{-5}) \\ 0.49 & O(10^{-5}) & 662.26 & O(10^{-4}) & O(10^{-2}) & O(10^{-2}) \\ O(10^{-4}) & O(10^{-3}) & O(10^{-4}) & 8556.8 & O(10^{-5}) & O(10^{-4}) \\ O(10^{-3}) & O(10^{-5}) & O(10^{-2}) & O(10^{-5}) & 38.43 & 7.80 \\ O(10^{-3}) & O(10^{-5}) & O(10^{-2}) & O(10^{-5}) & 5.43 & 70.75 \end{bmatrix}$$

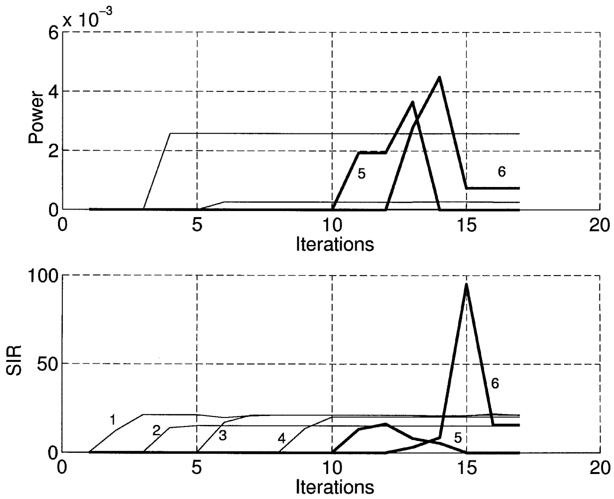

Fig. 10. Evolution of power and SIR, using UBPC with price $\alpha_i = 100$.

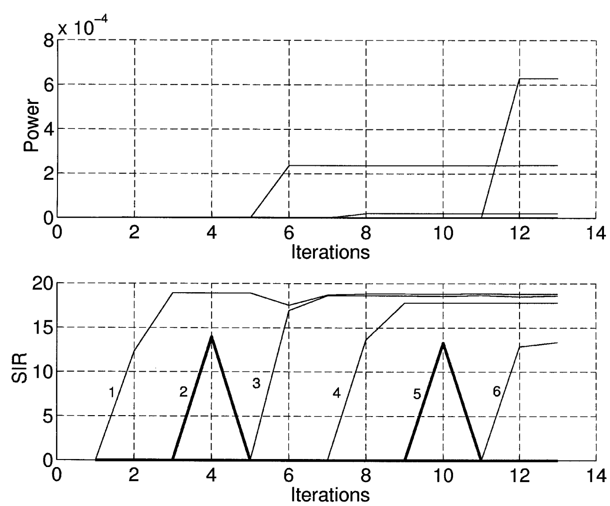

Fig. 11. Evolution of power and SIR, using UBPC with price $\alpha_i = 1000$.

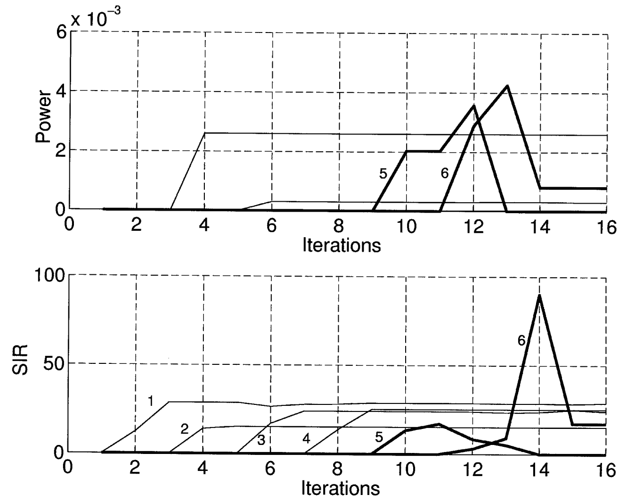

Fig. 12. Evolution of power and SIR, using UBPC with adaptive price α_i.

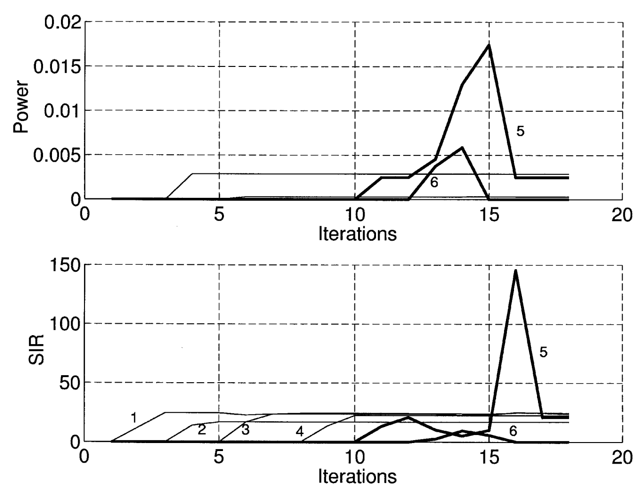

Fig. 13. Evolution of power and SIR, using UBPC with combined price α_i.

To implement UBPC, each user should specify a utility function and a price coefficient. We assume that the six users in the system use Sigmoid utility functions, with turnoff SIRs equal to the corresponding SIR thresholds, and steepness parameters 0.99, 1.35, 0.93, 1.02, 0.66, and 1.04, respectively. Clearly, user 6 is more "rigid" than user 5.

Fig. 10 plots the evolution of power and SIR for UBPC, with price coefficient 100 for all users. It performs similarly to (2) when the system is feasible. However, as the system reaches the point of infeasibility, the admission of user 6 forces ongoing user 5, which is less rigid, to turn off. The turnoff of user 5 allows user 6 to get satisfactory service at lower power.

Now, we increase the the price coefficient to 1000 for all users. The results are shown in Fig. 11, where users 2 and 5 are turned off (at iterations 4 and 10). As pointed out before, users 2 and 5 are far users, which are admissible only when the price is low. Hence, a high price tends to result in conservative admission, leading to high system robustness. However, a high price will also discourage users to transmit at high SIR even when the traffic load is low.

To solve this problem, we use an adaptive price coefficient given by (21), where $\alpha = 50\,000$ for all users. As shown in Fig. 12, user 5 is turned off, and all other users achieve higher SIR. Thus, this pricing scheme achieves transmission environment adaptiveness and robustness at the same time.

All the above schemes exhibit near–far unfairness. To give far users a fairer share of resource, we use UBPC with combined price coefficient given by (25), where $\alpha = 1000$. Fig. 13 shows the results. Under this price setting, neither user 2 nor user 5 is turned off. Instead, user 6 is rejected to maintain the feasibility of the system of existing users, even though user 6 is more rigid. All these results indicate that the combined price setting is superior to all the others above with respect to robustness, fairness, and active link protection.

Thus far, we have not included mobility of users in our simulations. To demonstrate how the algorithm works when users move, we next assume that user 6 comes into the system and moves toward its home base station at a speed that transverses a cell in 100 iteration steps. Fig. 14 shows the mobility case with the same price setting as in Fig. 10 (i.e., price coefficient is 100 for all users). In the figure, the power and SIR corresponding to user 5 are shown in highlighted dashed lines, while those of user 6 are shown in highlighted solid lines. As before, user 6 makes the system infeasible and forces user 5 to turn off.

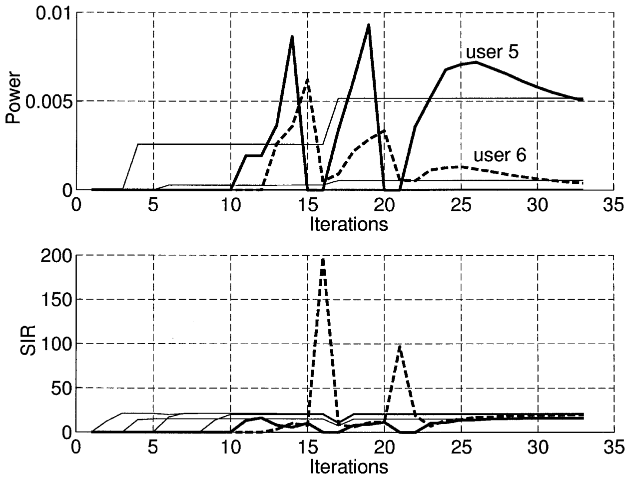

Fig. 14. Evolution of power and SIR of system with mobility $(\alpha_i = 100)$.

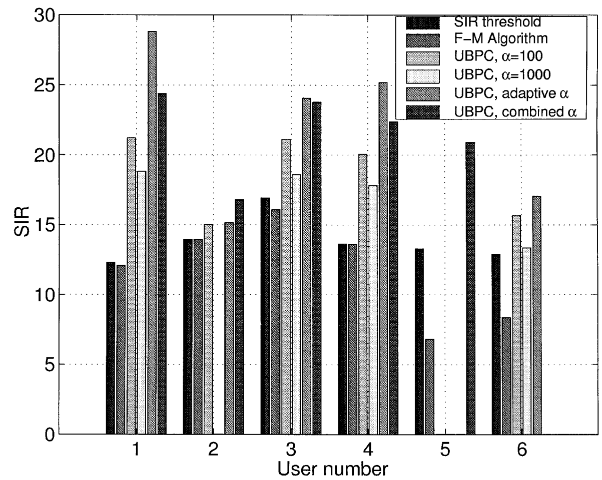

Fig. 16. Comparison of achieved SIR under different power-control schemes.

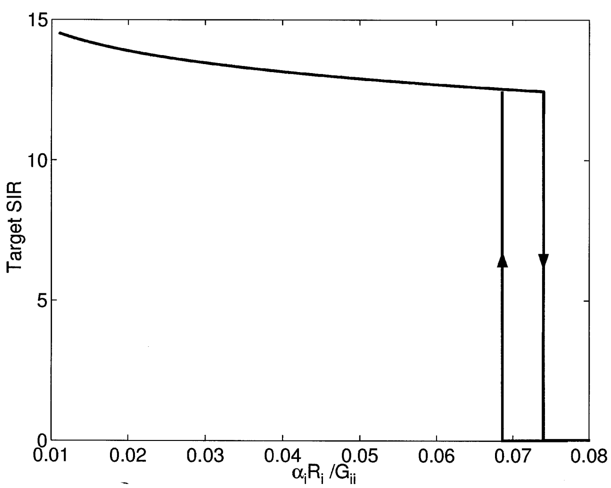

Fig. 15. Target SIR with hysteresis versus transmission environment of user i.

However, as user 6 moves closer to its home base station, the system becomes less infeasible. When user 5 senses that the interference drops below its threshold, it resumes transmission. It is turned off again as shown by the second spike in the line denoting the power, because the system of all six users still remains infeasible. As user 6 moves even closer to its home base station, the situation improves further and user 5 is finally allowed to transmit. Moreover, with the system becoming more feasible, the transmission power levels of users 5 and 6 decrease consistently. This also implies that the algorithm can react appropriately with the movement of user 6. Note that in this simulation, user 5 tries to resume transmission prematurely and incurs some unnecessary disturbance. To avoid this situation, we can introduce hysteresis in the target SIR curve, as shown in Fig. 15, i.e., a user is easier to be turned off than to resume transmission.

Finally, in Fig. 16 we compare the SIRs achieved by the different schemes. There are six groups of bars in the figure, where the ith group corresponds to user i. In each group, the first bar represents the SIR threshold, the minimum SIR required; the remaining bars correspond to the achieved SIR under power-control algorithm (2), UBPC with fixed price 100, with fixed price 1000, with adaptive pricing, and with combined pricing, respectively. The empty slots in some groups are due to turned-off users. This figure illustrates that for infeasible systems, algorithm (2) leads to SIRs lower than their corresponding thresholds, while UBPC guarantees QoS to most users by turning off some bottleneck users. We also verify that a low price encourages transmission, and adaptive price settings generally perform better.

VI. Conclusion

In this paper, we first demonstrate that when infeasibility arises, the well-known distributed power-control algorithm (2) diverges because of the hard constraint of SIR requirements. Using this algorithm, every user tries to achieve its required SIR value, no matter how high the power consumption, ignoring the basic fact that power is itself a valuable commodity. By softening the SIR requirement using utility functions and by introducing a cost function for power, we propose a utility-based power-control framework, called UBPC. Although UBPC is still noncooperative and distributed, some degree of cooperation emerges: a user will automatically decrease its target SIR (and may even turn off transmission) when it senses that traffic congestion is building up. It is this cooperation that prevents the system from blowing up when infeasibility arises. At the same time, under very mild conditions UBPC is standard (in the sense defined in [4]), which implies asynchronous convergence of the algorithm when applied to a feasible system. While UBPC is of a best-effort flavor, several tunable parameters in UBPC enable this scheme to be extremely flexible to satisfy different service requirements (such as fairness, delay, and bit error rate) in integrated networks with both voice and data users. It also allows integration with some network resource management techniques. Significant improvements over the existing algorithm are demonstrated by analysis and simulation.

UBPC provides a promising framework for distributed power control of cellular wireless systems, but several issues remain to be studied. How to translate different QoS requirements into utility and cost functions that lead to a solvable power-control problem, how to achieve system optimality, and how to relate a cost term to practical pricing schemes are all topics requiring further research.

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