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ABSTRACT

This paper develops and applies a novel test of the Holt, et al. (1961) linear quadratic inventory model. It is shown that a central property of the model is that a certain weighted sum of variances and covariances of production, sales and inventories must be nonnegative. The weights are the basic structural parameters of the model. The model may be tested by seeing whether this sum in fact is nonnegative. When the test is applied to some non-durables data aggregated to the two-digit SIC code level, it almost always rejects the model, even though the model does well by traditional criteria.

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The linear quadratic inventory model, originated by Holt et al. (1961), has been the basis of much theoretical and empirical work on manufacturers' inventories of finished goods. The model argues that the basic reason firms hold finished goods inventories is to smooth production in the face of randomly fluctuating sales. In some versions of the model a desire to avoid sales backlogs provides an additional motive for holding inventories.[1] That firms might hold inventories for these reasons seems theoretically compelling (Blinder (1983)), and much empirical work has been interpreted as being supportive of the model (e.g., Blanchard (1983)).

Some basic facts about finished goods inventories, however, seem to contradict the spirit if not the letter of this model. The model suggests that firms will smooth production by building up inventory stocks when sales are low and drawing down stocks when sales are high (Summers (1983)). As is well known, however, manufacturers generally do precisely the opposite. Stocks tend to be decumulated in cyclical downturns and accumulated in cyclical upturns (Blinder (1981a)). In addition, it has been suggested that the fact that production has a larger variance than sales in many industries is inconsistent with the model (Blanchard (1983), Blinder (1981b)). The argument presumably is that firms could always make production exactly as variable as sales by holding no inventories. So if firms are holding

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inventories to smooth production, they appear not to be doing so very successfully.

It is, however, somewhat difficult to evaluate this seemingly unfavorable evidence, and to balance such evidence against the favorable results found in recent econometric studies such as Blanchard (1983). None of the authors cited in the previous paragraph formally establish any implications of the production smoothing model for variances and covariances of inventories, sales and production. Still less do any try to quantify the economic or statistical significance of the aspects of inventory behavior apparently inconsistent with the production smoothing model. Whether these aspects provide no or considerable evidence against the model therefore has not yet been established.

This paper formally establishes an inequality summarizing the implications of the production smoothing model for the variances and covariances of inventories, sales and production, and then uses some aggregate data to test the inequality statistically. It turns out that the model <u>is</u> consistent both with accumulation of inventories in cyclical upturns and with production being more variable than sales, at least when a desire to avoid sales backlogs provides a motive for holding inventories (Blanchard (1983)). But even the model that allows for such a desire restricts the movements of inventories, sales and production, so that only a certain amount of excess variability of production is consistent with the model. The inequality that this paper derives summarizes these restrictions.

The inequality is derived by comparing how much better off the firm would have expected to have been by ignoring random sales fluctuations and simply letting inventories increase from period to period at their trend rate of growth. This may be calculated as the difference between expected costs under this static policy and the policy that is optimal according to the model. This difference, which should be nonnegative if the model is correct, may be expressed as a simple weighted sum of certain variances and covariances of inventories, sales and production. The weighted sum includes in particular the excess of production over sales variability. The weights are the basic structural parameters of the model, obtainable in standard fashion from an Euler equation. Even if all the estimates of parameters are right signed and significant, the estimate of this difference in principle may be insignificantly positive, or even negative.

If the difference is negative for a given set of data, it seems unlikely that inventories truly are chosen in accordance with the supposedly optimal policy and therefore unlikely that the model is correct. The inequality quantifies the cost savings produced by the optimal inventory policy--that is, it quantifies the extent to which firms cut costs by adjusting inventories in response to random sales fluctuations. If the model is correct, a violation of the inequality indicates nonsensically that firms adjusted inventories to increase costs. Such violation would therefore mean that there is no evidence that production smoothing provided the motive for holding inventories.

And in fact, for almost all of the aggregate non-durables industries studied here, the inequality _is_ violated--that is, the allegedly optimal policy could for almost all the industries have been expected to increase costs relative to the static one. The increase is statistically significant about half the time. Moreover, it is economically large, with expected deviations of costs from trend that are up to 50 percent higher than under the static policy. This strongly suggests that in these industries production smoothing does not provide the only motive for holding inventories.

The conclusion that the model does not adequately explain the data considered here seems particularly compelling since the test performed here requires relatively few economic or statistical assumptions. The test, for example, is consistent with but does not require the assumptions about market structure, causality and demand made in the recent studies of Blanchard (1983) and Eichenbaum (1982). Also, and again in contrast to Blanchard (1983) and Eichenbaum (1982), it is computationally straightforward, requiring only linear estimation. In fact, in some cases, it could be concluded that the static inventory policy would be expected to cost less than the supposedly optimal policy without even calculating any of the model's parameters. All that was required was the calculation of certain variances and covariances. Since the test easily extends to cover other linear quadratic models, and perhaps some non-linear models as well, it may be of general interest.

This is especially so since the test appears to be economically more informative than the usual test of cross equation restrictions, at least in the present case. The significance of a rejection or acceptance of the variance bounds test can be measured not only in statistical but also in economic terms, by the calculation of the increase in expected costs mentioned above. In addition, the test itself suggests a reason for any rejection that occurs: some unexplained factors are making production too volatile. This indicates that model needs to be modified to account for such excess volatility, and the concluding section to this paper briefly discusses some possible modifications. In contrast, statistical rejections of tests of cross equation restrictions appear to be difficult to interpret in economic terms (e.g., Blanchard (1983, p387)).

To prevent misunderstanding, it should be emphasized at the outset that the innovation in the present paper is not in the model used but in the test performed. Two general formulations of the model are studied, both drawn from the existing literature on the linear quadratic inventory model. The two are motivated only briefly and uncritically. A critical evaluation of the model may be found in

West (1983a) and Blinder (1983). The two were chosen because they are representative of the many versions of the model that have been formulated. Both are not only quite similar to most versions studied but are even identical to or strictly more general than some (e.g., Holt et al. (1961), Belsley (1969)).

But the two of course do not incorporate all aspects of all formulations of the model. It is worth mentioning in particular that both follow the mainstream of work in the model and assume that inventories are held to cut production and possibly backlog costs in the face of randomly fluctuating sales. Some recent formulations of the model such as Blinder (1983) allow inventories to also serve to cut production costs in the face of randomly varying production costs. Extensions of the present paper to cover this and other major extensions to the linear quadratic model are left for future work.

The paper is organized as follows. Part II develops the test, part III contains empirical results, and part IV contains conclusions. An appendix contains econometric details.

II. THE TEST

This section first describes the model and then derives an inequality that is central to the test.

A. The Model

The model under consideration is intended for finished goods inventories in so-called "production to stock" industries (Abramowitz (1951), Rowley and Trevedi (1975)). Its precise formulation varies from author to author, and this paper's empirical work tests two versions. Both may be derived from the following general model. Firms producing a single homogeneous good maximize expected discounted real profits:

$$(1) \qquad \max E_0 \sum_{t=0}^{\infty} d_1^t([p_t S_t] - d_2^t[a_0(\Delta Q_t)^2 + a_1(Q_t^2) + a_2(H_t - a_3 S_{t+1})^2])$$

$$\text{s.t. } Q_t = S_t + H_t - H_{t-1}$$

where

E_0 mathematical expectations, conditional on information available at time 0

d_1 fixed real discount rate, $0 < d_1 < 1$

d_2 fixed rate of technological progress, $0 < d_2 < 1$

p_t real price in period t

S_t units sold in period t

Q_t units produced in period t

H_t units of finished goods inventories at end of period t

a_i strictly positive parameters

Two general comments on (1) will be made, before the individual terms of the equation are briefly discussed. First, the firm's choice variables have intentionally been left unspecified. The estimation here is consistent with any of the standard ones: output only (Belsley (1969)) or inventories only (Blanchard (1982)) in models in which sales are exogenous: output, inventories and sales in models in which the firm is a perfect competitor (Blanchard and Melino (1981), Eichenbaum (1982)) 2/; output, price and inventories in models in which the firm is a monopolist (Blinder (1982)). The firm's information set has been left unspecified for the same reason.

Second, for the present, all variables should be assumed to be deviations from trend (where trend should be understood to encompass all deterministic components, seasonal as well as secular). This assumption is made for algebraic simplicity and will be relaxed shortly. What we wish to derive are some restrictions that are implied for arbitrary trend, and the algebra is less cluttered when trend terms are set to zero.

The first term in brackets in equation (1) is revenue, the second is costs. Although the revenue function will play no role in the bulk of this paper, it is worth pointing out some of the implications of its presence at this initial state to emphasize the generality of the tests performed here. The market may be perfect (Eichenbaum (1982))or imperfect (Blinder (1982)). Price speculation on the supply side (Eichenbaum (1982)) or perhaps even on the demand side may be present. Pricing and production decisions may be made simultaneously (Eichenbaum (1982), Blinder (1982)) or separately (Holt, et al., (1961)). In short, Summers' (1981) criticisms of inventory models that ignore interactions between firms and their customers are not relevant here.

The second term in brackets is costs. These are the focus of the model, and, here as elsewhere, are central. Total per period costs are the sum of three terms.

The first is the cost of changing production, which is quadratic in the period to period change in the number of units produced. This represents, for example hiring and firing costs.

The second is the cost of production, which is quadratic in the number of units produced. This approximates an arbitrary concave cost function that results as usual from a decreasing returns to scale technology.

The third and final term embodies inventory and backlog costs, and is quadratic in how far inventories are from a target level. A brief explanation of its rationale is as follows (see West (1983a) for a lengthier discussion and critique). Inventory holding costs (e.g., storage and handling charges) are reflected in a_2. The parameter a_3 is the inventory to expected sales ratio that would be set in the absence of both types of production costs ($a_0 = a_1 = 0$). all authors agree that this ratio should be anything but zero, and the two major variations in (1) accommodated in the tests here turn on whether a_3 is allowed to be non-zero. Those who do so (Blanchard (1983), Eichenbaum (1982), Holt, et al., (1961)) argue that sales sometimes exceed inventories on hand, forcing firms to backlog orders. Firms face costs when such a backlog develops, perhaps because of loss of future sales. Thus, <u>ceteris paribus</u>, when expected sales are higher, inventories should be higher as well. The target level for inventories, $a_3 E_t S_{t+1}$, trades off backlog and inventory costs. In this model <u>with a target level</u>, inventories can serve two functions.[3/] They can buffer production, allowing it to be smoothed in the presence of fluctuating demand. And they can cut backlog costs. Optimal inventories balance production, holding and backlog costs.

Some other authors, however, insist that in the absence of production costs, the target level for inventories would be zero (Auerbach and Green (1980), Belsley (1969), Blinder (1982)). They impose $a_3 = 0$. Inventories are then held purely to smooth production. In this model without a target level, optimal inventories balance savings in production costs against the costs of carrying inventories.

The tests performed here will thus accommodate equation (1) both with and without a target level for inventories.

B. An Inequality

We now derive an inequality that compactly expresses the production smoothing motive for holding inventories, by calculating the effect inventories have on expected costs.[4/] (The algebra carries along a_3. The effect in models without a target level is obtained simply by setting $a_3 = 0$ in the manipulations that follow.) According to the model, firms solve (1), subject to transversality and market equilibrium conditions to select optimal H_t^* and/or Q_t^* (and, as noted above, possibly p_t^* and S_t^* as well) (Eichenbaum (1982), Hansen and Sargent (1981), Sargent (1981)). In this optimal closed loop policy, the endogenous control variables are set by a feedback rule, with their optimal period t values a function of their own past values and past and present values of forcing variables.

Let us assume that the sequences $(H_t^*)(Q_t^*)$, and (S_t^*) are covariance stationary. Methods for calculating this stationary solution in particular cases may be found in Eichenbaum (1982), Holt, et al., (1961) and Blanchard (1983). Let $E_0V_0^*$ be the expectation at time t of the value of the objective function that results from this policy:

(2) $$E_0 \sum_{t=0}^{\infty} d_1^t([p_t^* S_t^*] - d_2^t[a_0(\Delta Q_t^*)^2 + a_1(Q_t^*)^2 + a_2(H_t^* - a_3 S_{t+1}^*)^2])$$

Let $E_0V_0^A$ be the expectation at time t of the value of the objective function that would result from the alternative policy of setting $H_t^A = 0$ in every period, $Q_t^A = S_t^A = S_t^*$. Price $p_t^A = p_t^*$ will in general still be consistent with buyers demanding $S_t^A = S_t^*$.[5/] The value of the objective function under this alternative policy is then

(3) $$E_0 \sum_{t=0}^{\infty} d_1^t([p_t^* S_t^*] - d_2^t[a_0(\Delta S_t^*)^2 + a_1(S_t^*)^2 + a_2(-a_3 S_{t+1}^*)^2])$$

This alternative decision rule in general is feasible. (The only apparent circumstance under which the policy is not feasible is when production takes place with a lag and inventories absorb sales expectational errors, as in Blinder (1982). Even here the inequality about to be developed may be considered approximately correct if those errors are small relative to the size of the inventory stock, as seems reasonable.) By assumption, then, since V_0^* is optimal, $E_0V_0^* \geq E_0V_0^A$. Now, $E_0V_0^*$ and $E_0V_0^A$ are random with respect to unconditional information and $E_0V_0^* - E_0V_0^A$ is a well-defined random variable with respect to this information set. Since it is nonnegative it has a nonnegative expectation. Thus $E(E_0V_0^* - E_0V_0^A) \geq 0$. By the law of iterated expectations, then

(4) $$EV_0^* \geq EV_0^A$$

$$==> \quad E \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta Q_t^*)^2 + a_1(Q_t^*)^2 + a_2(H_t^* - a_3 S_{t+1}^*)^2] \Big)$$

$$\geq$$

$$E \sum_{t=0}^{\infty} d_1^t \Big([p_t^* S_t^*] - d_2^t [a_0(\Delta S_t^*)^2 + a_1(S_t^*)^2 + a_2(-a_3 S_{t+1}^*)^2] \Big)$$

Let $var(Q^*) = E(Q_t^*)^2$ denote the variance of production and $cov(Q,Q_{-1}) = E(Q_t^* Q_{t-1}^*)$ its first autocovariance, with analogous notation for other variables. (No time subscripts are necessary by the assumption of covariance stationarity.) Also define $d = d_1 d_2$. With this notation (4) becomes

$$(5) \quad \sum_{t=0}^{\infty} d_1^t \; E[p_t^* S_t^*] - \sum_{t=0}^{\infty} d^t \; [(a_0 var(\Delta Q^*) + a_1 var(Q^*) + a_2 var(H^* - a_3 S_{+1}^*)]$$

$$\geq$$

$$\sum_{t=0}^{\infty} d_1^t \; E[p_t^* S_t^*] - \sum_{t=0}^{\infty} d^t \; [(a_0 var(\Delta S^*) + a_1 var(S^*) + a_2 var(-a_3 S_{+1}^*)]$$

Using $Q_t = S_t + H_t - H_{t-1}$ where convenient, expanding $var(H^* - a_3 S_{+1}^*) = var(H^*) - 2a_3 cov(H^*, S_{+1}^*) + a_3^2 var(S^*)$, moving all terms to the left hand side of the inequality, and then applying the standard formula for a geometric sum transforms (5) into

$$(6) \; 0 < (1-d)^{-1} \; [a_0(var(\Delta S^*) - var(\Delta Q^*)) + a_1(var(S^*) - var(Q^*)) - a_2 var(H^*) + 2a_2 a_3 cov(H^*, S_{+1}^*)]$$

It is the two versions of this inequality -- with and without a target level -- that will be tested:

$$(7.1)\ 0 < (1-d)^{-1}\left[a_0(\mathrm{var}(\Delta S)-\mathrm{var}(\Delta Q)) + a_1(\mathrm{var}(S)-\mathrm{var}(Q)) - a_2\mathrm{var}(H)\right]$$

$$(7.2)\ 0 < (1-d)^{-1}\left[a_0(\mathrm{var}(\Delta S)-\mathrm{var}(\Delta Q)) + a_1(\mathrm{var}(S)-\mathrm{var}(Q)) - a_2\mathrm{var}(H) + 2\, a_2a_3\mathrm{cov}(H,S_{+1})\right]$$

The "*" superscripts have been dropped in accordance with the null hypothesis that observed H, S and Q accord with the optimal solution to (1).

(7.1) and (7.2) have been derived assuming that all variables have zero unconditional expectations. These inequalities still hold even when such expectations are non-zero and firms account for them when maximizing expected discounted profits. For let the variables in (1) include deterministic components -- constant, time trends, seasonal dummies, etc. -- and add linear terms such as $a_{10}(\Delta Q_t)$ to the cost function in equation (1). It is then easily verified (see West (1983a)) that if the alternative policy is the no-feedback, open loop one that sets inventories equal to their unconditional expectation each period ($H_t^A=EH_t^*$, $p_t^A=p_t$, $S_t^A=S_t^*$, $Q_t^A=S_t^*+E(H_t^*-H_{t-1}^*)$), the inequalities in (7) still result.[6/] (Note that this alternative policy entails varying inventories from period to period if inventories display a time trend and/or seasonal variation.) For the remainder of the paper, (7.1) and (7.2) will be understood to apply to just such a model with deterministic terms. It should be noted again that for expositional convenience all such terms will be referred to as "trend," even though the word "trend" is perhaps somewhat misleading if deterministic seasonal fluctuations are present or if secular growth is not.

In this light, let us interpret (7.1) and (7.2). The right-hand sides of these two equations describe the cost savings that could be (unconditionally) expected to result from setting inventories optimally rather than without feedback. The first two terms express differences of production costs, the third that of inventory costs, and the fourth, in (7.2), that of costs of inventories that deviate from their target level. The expected difference in inventory holding costs, $-a_2\text{var}(H_t)$, is always negative. Therefore, according to the model, these expected cost increases are more than offset by savings elsewhere (otherwise the optimal policy would not be optimal). Inequality (7.1), applicable when there is no target level, says that the firm must expect to save either on costs of changing production ($\text{var}(\Delta Q) < \text{var}(\Delta S)$), or on costs of production ($\text{var}(Q) < \text{var}(S)$), or both, and the expected savings must be large enough that overall expected costs are lower, i.e., (7.1) holds. Similarly, (7.2), applicable when there is a target level, says that the optimal policy must be expected to more than offset increases in expected inventory holding costs with expected savings in production and/or target level costs.

Thus it would seem to be a minimal economic requirement that (7.1) and (7.2) be satisfied by data that are to be explained by the model. The inequalities merely ask that the optimal policy be expected to cost less than the static one. The static policy is the one that would be optimal in the absence of any random fluctuations in sales. The inequalities therefore summarize how production, sales and inventories are expected to interact as they are dynamically adjusted in response to sales shocks. And this is precisely what the model purports to explain. It is perhaps reasonable, therefore, to ask that the data not only satisfy (7.1) and (7.2), but do so to an extent that is significant in economic or statistical terms.

The next section sees how well some aggregate nondurables data satisfy these inequalities. Given that (7.1) and (7.2) have been derived for a single firm, however, it is appropriate to make a remark on aggregation before examining these empirical results. The inequalities do still hold at an aggregate level, provided that all the parameters representing technology (e.g., the a_i's) and the stochastic characteristics of forcing variables (i.e., their ARMA parameters) are the same for each individual firm. As is explained in detail in West (1983a) under these sufficient though perhaps not necessary conditions each firm's behavior is summarized by a set of linear regressions with identical coefficients on the regressors. As usual, therefore, the model aggregates exactly, and aggregate behavior is characterized by the same set of regressions. It is no surprise, then, that aggregate production, sales and inventories satisfy (7.1) and (7.2), for arbitrary correlations of production, sales and inventories across firms.

III. Empirical results

Data and estimation are described briefly before the basic and some additional empirical results are presented.

A. Data

The data were real (1972 dollars) and monthly. Both seasonally adjusted and unadjusted data were used. Seasonally adjusted data were available for 1959 to 1980 for aggregate non-durables and for all six two-digit industries that Belsley (1969) identified as operating in production to stock markets: food (SIC 20), tobacco (SIC 21), apparel (SIC 23), chemicals (SIC 28), rubber (SIC 30), and petroleum (SIC 29). Seasonally unadjusted data were available for aggregate non-durables and three two digit industries (chemicals, petroleum, and rubber). (Again, durable goods and the remaining non-durable goods industries were excluded because the model is intended to apply only to industries that produce to stock, and, according to Belsley (1969), none of these other industries produce to stock.)

Sales were obtained by using the appropriate wholesale price index to deflate the Bureau of the Census nominal figures for sales (all figures found in the Citibank Economic Database, in the Bureau of the Census's (1978,1982) Manufacturer's Shipments, Inventories and Orders or obtained directly from the Bureau of the Census). The seasonally adjusted inventory figures were obtained by converting the Bureau's receutly calculated constant dollar seasonally adjusted finished goods inventory series (Hinrichs and Eckman (1981)) from "cost" to "market" so that one dollar of inventories represented the same physical units as one dollar of sales (see West (1983b) for a definition of "cost" and "market" and an explanation of why a conversion was necessary). As in Reagan and Sheehan (1982) the seasonally unadjusted constant dollar inventory figures were obtained by multiplying the adjusted figures by the corresponding unadjusted to adjusted ratio for book value (nominal) finished goods inventories. (This procedure was adopted since no unadjusted constant dollar data appear to be available. It makes the plausible assumption that the "seasonal deflator" is the same for book value and constant dollar inventories.[7/]) Production was obtained from the identity $Q_t = S_t+H_t-H_{t-1}$.

B. Estimation

The sample period covered 1959:5 to 1980:10, with 1980:11 and 1980:12 used for leads and 1959:2 to 1959:4 used for lags. All regressions included deterministic terms: a constant and a time trend, and, for seasonally unadjusted data, seasonal dummies as well.[8/]

Three specific aspects of estimation will be briefly discussed. These are estimation of the a_i, of the second moments of inventories, sales and production, and, finally, of the standard error of (7). (Throughout this section, references to "(7)" should be understood to be shorthand for "(7.1) and (7.2)"). Additional details will be found in the appendix and in West (1983a).

The a_i's in the model with a target level were obtained as follows. (The same procedure was applied to the model without a target level, except that $a_3=0$ was imposed.) A necessary first order condition to solve (1) at time $t \geq t_0$ is obtained by differentiating (1) with respect to H_t and setting the result equal to zero:[9/]

$$(8)\ E_t\ 2[d^2a_0H_{t+2} - (2d^2a_0+2da_0+da_1)H_{t+1} + (d^2a_0+4da_0+a_0+da_1+a_1+a_2)H_t - (2a_0+2da_0+a_1)H_{t-1} + a_0H_{t-2} + d^2a_0S_{t+2} - (d^2a_0+2da_0+da_1+a_2a_3)S_{t+1} + (2da_0+a_0+a_1)S_t - a_0S_{t-1} + \text{deterministic terms}\ ]= 0$$

After defining lower case $q_t = dQ_t - Q_{t-1}$ and dividing this first order condition by two, the Euler equation (9) results

$$(9)\ E_t[\ a_0dq_{t+2}-(a_1+a_0(1+d))q_{t+1}+a_0q_t+a_2H_t-a_2a_3S_{t+1} + \text{deterministic terms}\ ]= 0$$

A normalization is required to estimate the a_i's. The normalization chosen is arbitrary since changing the a_i by a scale factor does not change inequality (7). The normalization used was $a_1 + (1+d)a_0 = 1$, so (9) becomes

$$(10)\ q_{t+1}= a_0(dq_{t+2}+q_t) + a_2H_t - a_2a_3S_{t+1} + u_{1t} + \text{deterministic terms}$$

where the disturbance u_{1t} has a moving average component.[10/] With a monthly discount rate imposed (10) can be estimated by instrumental variables. The results here report d = .995 (corresponding annual discount rate is about six per cent); results with d = .990 and d = .999 were virtually identical. The six instruments used apart from the deterministic terms in (10) were three lags each of inventories and sales. The estimation required two steps, as described in Hansen and Singleton (1982). The first step calculated the variance-covariance matrix of the u_{1t} and the second obtained the optimal instrumental variables estimator. See the appendix and West (1983a) for further details. Since the equation is overidentified -- the model without a target level has four fewer right-hand side variables than instruments, and that with has three -- Hansen's (1982) test of over-identifying restrictions was calculated.

Variances and covariances were calculated from a bivariate (inventories, sales) autoregression of order three:[11/]

(11) H_t = deterministic terms $+ \phi_{11}H_{t-1} + \phi_{12}H_{t-2} + \phi_{13}H_{t-3} + \phi_{14}S_{t-1} + \phi_{15}S_{t-2} + \phi_{16}S_{t-3} + u_{2t}$

S_t = deterministic terms $+ \phi_{21}H_{t-1} + \phi_{22}H_{t-2} + \phi_{23}H_{t-3} + \phi_{24}S_{t-1} + \phi_{25}S_{t-2} + \phi_{26}S_{t-3} + u_{3t}$

The Yule-Walker equation using the estimated ϕ_{ij} was then used in the standard way (Anderson (1971, p. 182)) to obtain the needed second monents of sales and inventories. The second moments of production were derived from the identity $Q_t = S_t + H_t - H_{t-1}$, e.g. $var(Q) = var(S) + 2cov(S,H) - 2cov(S,H_{-1}) + 2var(H) - 2cov(H,H_{-1})$.

Finally, the standard error of the statistic (7) was derived as follows. Let θ be the parameter vector needed to calculate (7). θ consists of the coefficients on the right-hand side variables in the three equation system consisting of (10) and (11) and the three elements of the covariance matrix of the error terms in (11). Thus, θ is (1 x 24) for seasonally adjusted data (24 = 15 RHS variables explicitly listed in (10) and (11) + 6 constant and trend terms + 3 elements of variance-covariance matrix of the residuals in (11)). Similarly, θ is (1 x 57) for seasonally unadjusted data. The estimated $\hat{\theta}$ is asymptotically normal with a covariance matrix V defined in the appendix. The statistic (7) is a function of θ, say, $g(\theta)$, and thus is asymptotically normal with covariance matrix $(dg/d\theta)V(dg/d\theta)'$. The standard error of (7) is the square root of $(dg/d\theta)V(dg/d\theta)'$. The derivatives $dg/d\theta$ were calculated numerically.

It is to be noted that this procedure takes into account not only the uncertainty in the estimates at the a_i but also in the estimates of the first and second moments. The procedure also accounts for the covariance between the estimates of the a_i and of moments. Again, for details see the appendix and West (1983a).

C. Results

We will shortly present estimates of the size and the standard errors of the right hand sides of (7.1) and (7.2) for the data described above. This will require estimates not only of the appropriate variances and covariances of inventories, sales and production, but of the a_i parameters as well. First, however, let us consider whether these data are qualitatively consistent with the inequalities, by examining the appropriate second moments. Tables I and II have these, for seasonally adjusted and unadjusted data respectively.

It follows immediately from the trivial calculations underlying the entries in Tables I and II that for both seasonally adjusted and unadjusted data, the model without a target level violates (7.1) for almost all industries! (The only possible exception is chemicals.) Columns (5)-(7) indicate that for all but the chemical industry, $var(\Delta S_t)-var(\Delta Q_t)<0$, and, of course, $var(H_t)>0$. Since the a_i are known a priori to be positive it follows that for all but chemicals, $0>a_0(var)(\Delta S)-var(\Delta Q)) + a_1(var(S)-var(Q)) - a_2var(H)$. In other words, according to the model itself, the static, no-feedback policy of letting inventories grow at their trend rate would have been expected to be preferable to the optimal policy that the model claims actually was followed:

lower costs of changing production, lower costs of production, and lower inventory costs. From these simple calculations we can conclude that with the possible exception of the chemical industry, the data studied here are inconsistent with the model without a target level. This suggests that backlog costs, whose existence is used to rationalize a non-zero target level, are of crucial importance to this model.

It also follows from Tables I and II that even the model with a target level is inconsistent with the seasonally unadjusted behavior of the petroleum industry, since inventories here covary negatively with next period's sales. Relative to the static policy, the optimal policy that supposedly was followed would have been expected to increase all the costs just noted, and the cost of being away from a target level as well. Thus, this data set is incompatible with the model, with or without a target level. For the remaining industries, (7.1) and (7.2) cannot be signed without the a_i's. Let us therefore turn to precise calculation of the inequalities.

In Tables III and IV are the a_i's for the models with and without a target level, respectively. Almost all of the parameter estimates are indeed positive. Consider the model without a target level first. With seasonally adjusted data 11 of 14 free signs on the a_i are correct, and with unadjusted the figure is 5 of 8. (The number of free signs is 14 and 8 rather than 21 and 12 because the normalization rule $a_1+(1+d)a_0=1$ constrains either a_0 or a_1 to be positive in each equation.) The comparable figures for the model with a target level are 19 of 21 and 9 of 12. Only two of the wrong-signed coefficients are significant at the .05 level (a_0 in the model with a target level, for both seasonally adjusted rubber and seasonally unadjusted aggregate non-durables). In most equations the production cost a_1 and the cost of changing production a_0 are significant. Somewhat puzzling is the imprecision of the estimates of the inventory holding cost a_2 and the target level parameter a_3, which are rarely significant at the .05 level. They are, however, almost always positive and stand here in about the same ratio to the other a_i and to each other as they did in Blanchard's (1983) estimates for the automobile industry.

However, these parameters, though positive and often significant, are not enough to make the model plausible. Results of the variance bounds test for the model without a target level are shown in Table V, and for the model with a target level in Table VI. It was noted above what would result for all data sets except possibly chemicals for the model without a target level, and for the seasonally unadjusted petroleum industry in the model with a target level. Thus it is no surprise that Tables V and VI indicate that (7.1) and (7.2) were violated for all of these. However, the inequality for the model without a target level was violated for seasonally unadjusted chemicals as well, as was the inequality for the model with a target level for most of the data sets. Thus, the inequalities were violated in seventeen out of twenty-two instances, and nine of these were significant at the .05 level. The four data sets that did satisfy (7.2) did so insignificantly, with standard errors uniformly larger than the sizes of the inequality. Also, two of these four produced the only significantly wrong-signed parameter (a_0 for adjusted rubber and unadjusted aggregate non-durables). It therefore appears that the model does not well explain any of the data studied here.

Moreover, the increase in deviations of costs from trend attributable to the optimal policy would appear to be economically as well as statistically noticeable. Column (2) in Tables V and VI contain total deviations of costs from trend (again, in "normalized" dollars, $a_1+(1+d)a_0=1$):

$$(12) \quad (1-d)^{-1}\left[a_0\mathrm{var}(\Delta Q)+a_1\mathrm{var}(Q)+a_2\mathrm{var}(H) - 2a_2a_3\mathrm{cov}(H,S_{+1})+a_2a_3\mathrm{var}(S)\right]$$

When (7.1) or (7.2) is divided by (12) (possibly with $a_3=0$ imposed in (12)) the result is a dimensionless measure of the extent to which the optimal policy increases or decreases deviations of costs from trend relative to the static policy. This is shown in column 3 of Tables V and VI. The optimal policy <u>increases</u> expected cost deviations by up to 56 percent. If this increase were to be believed it would mean that deviations of profit margins from trend, and therefore presumably profit margins themselves, are substantially reduced.

It is of some interest to compare the results of the inequality tests with those of a common test of specification, the Hansen (1982) test of over-identifying restrictions that is reported in the columns labelled *J*. This was accepted at the .05 level for about two thirds of the data sets (food, tobacco, apparel, petroleum, rubber) and was rejected at the .05 but accepted at the .005 level for the two other data sets. This compares favorably with the tests of the overidentifying restrictions in other recent studies (Blanchard (1983), Eichenbaum (1982)). Thus it is perhaps fair to say that this traditional test is supportive of the model. It would appear, then, that the variance bounds test was an essential element in assessing the reasonableness of this model for these data.

Additional empirical results

The robustness of the conclusions of the previous subsection was checked by calculating two additional sets of estimates. The first related to some variance inequalities applied to deterministic seasonal components, the second to quarterly (instead of monthly) data.

(I) Let a "j" superscript denote the deterministic seasonal component of a variable in month j; $\bar{X}^s$ the mean deterministic seasonal component of variable X, $\bar{X}^s = \frac{1}{12}\sum_{j=1}^{12} X^j$; var($X^s$) the "variance" of the deterministic seasonal component, $\mathrm{var}(X^s) = \frac{1}{12}\sum_{j=1}^{12}(X^j - \bar{X}^s)^2$, with var ($\Delta X^s$) and cov($X^s$,$Y^s$) defined in the obvious way.

Consider comparing costs under the optimal policy with costs that result under the alternative policy that suppresses all deterministic seasonal variation in inventories but otherwise allows inventories to grow at their trend rate, $H_t^A = EH_t + (\bar{H}^s - H^j)$, where j is the month corresponding to time period t. It may be shown by an argument analogous to that in Section II (details available on request) that the model (1) implies that

$$(13.1) \quad 0 < (1-d)^{-1}\{[a_0(\mathrm{var}(\Delta S)-\mathrm{var}(\Delta Q))+a_1(\mathrm{var}(S)-\mathrm{var}(Q))-a_2\mathrm{var}(H)] + [a_0(\mathrm{var}(\Delta S^s)-\mathrm{var}(\Delta Q^s))+a_1(\mathrm{var}(S^s)-\mathrm{var}(Q^s))-a_2\mathrm{var}(H^s)]\}$$

$$(13.2) \quad 0 < (1-d)^{-1}\{[a_0(\mathrm{var}(\Delta S)-\mathrm{var}(\Delta Q))+a_1(\mathrm{var}(S)-\mathrm{var}(Q))-a_2\mathrm{var}(H) + 2a_2a_3\mathrm{cov}(H,S_{+1}) + [a_0(\mathrm{var}(\Delta S^s)-\mathrm{var}(\Delta Q^s))+a_1(\mathrm{var}(S^s)-\mathrm{var}(Q^s))-a_2\mathrm{var}(H^s) + 2a_2a_3\mathrm{cov}(H^s,S^s_{+1})]\}$$

(13.1) applies to a model without a target level, (13.2) to a model with a target level. Inequality (13.1) in conjunction with inequality (7.1) says that when firms allow deterministic inventory seasonals to depart from their mean level, this must not increase costs to such an extent that the cost savings detailed in (7.1) are more than offset. Further, these departures will cut costs only insofar as they make $var(Q^S)$ and $var(\Delta Q^S)$, the deterministic seasonal costs of production and changing production, smaller than var (S^S) and $var(\Delta S^S)$, the deterministic seasonal cost that obtains when there are no departures of inventory seasonals from their mean levels. Inequality (13.2) in conjunction with inequality(7.2) has a comparable interpretation.

It is of interest, then, to calculate the relevant "variances" and "covariances," as well as to estimate the size and standard errors of (13.1) and (13.2). The relevant second moments for the four seasonally unadjusted data sets are displayed in Table VII. For two of the four data sets (aggregate non-durables and rubber), it can be concluded without calculating any parameter estimates that (13.1) will be rejected. (This follows since columns (5) and (6) are negative for these two data sets in both Table VII and Table II.) For the other two data sets, parameters do have to be estimated to sign (13.1), and, for all four data sets, parameter estimates are needed to sign (13.2). The model, then, seems to be qualitatively consistent with (13.1) to a slightly greater degree than with (7.1), in that two data sets rather than one have second moments that are consistent with one relevant inequality.

In a more formal, quantitative sense, however, the model performs as poorly with respect to (13.1) and (13.2) as it did with respect to (7.1) and (7.2).[12/] Once again, almost all the inequalities are wrong signed, about half of them significantly so (see Tables VIII and IX). The only exception, once again, is chemicals (which does, however, satisfy (13.1) and (13.2) in a statistically significant fashion).

For these data, as for the automobile data studied by Blanchard (1983), then, the seasonals appear to contain little evidence to suggest that manufacturers are selecting their inventories in accord with (1).

(II) Inequalities (7.1) and (7.2) were also tried for quarterly, seasonally adjusted data. These were constructed from the monthly data sales by adding the figures for the relevant three months, inventories by selecting the last month of the quarter.

Since the estimates were very similar to those for monthly data, only a summary of the final results seems worth reporting. Inequality (7.1) was wrong signed for six of seven data sets (the exception was tobacco, and resulted from wrong-signed estimates of a_0 and a_2). Inequality (7.2) was wrong signed for all seven data sets. Four of the fourteen wrong signs were significant at the 5 percent level; the correct sign for tobacco was not.

These additional tests, then, support the results reported in the previous subsection.[13/]

IV. CONCLUSIONS

This summarizes the basic conclusions of this paper. It would seem that the linear quadratic model does a poor job of rationalizing these inventory data. In effect, a contradiction results when it is assumed that the actual inventory path chosen is the one that is optimal according to the model. The allegedly optimal path is dominated by a naive alternative path.

In the model without a target level for inventories, this follows simply because production is more variable than sales. Inventories therefore cannot be chosen simply to perform their putative function, smoothing production.[14/] For the model with a target level, the matter is slightly more complicated. Inventories do usually track their target level (except in the petroleum industry). But this makes production and inventories so variable that inventories cannot be chosen as hypothesized, to minimize quadratic inventory, production and target-level costs.

The basic implication of this is that inventories appear to serve some role other than production smoothing. The inventory literature suggests two possible explanations of the excess volatility of production. The first is backlog costs. Now, as we have seen, the typical formulation -- a simple cost of having inventories deviate from a target level -- is inadequate, at least for these data. But this does not rule out more sophisticated formulations. Some encouraging evidence from a model that includes such a formulation may be found in West (1983a).

The second possible explanation relates to stochastic cost variability. It is possible that inventories serve mainly not to smooth production in the face of random varying demand, but to smooth it in the face of randomly varying costs. In this case production may be more variable than sales (as noted by Topel (1982)). Stochastic cost variability has been crudely allowed for in some recent work by calling the unobservable disturbances "cost shocks" (Blanchard (1983), Eichenbaum (1982)). But if cost variability is an important determinant of optimal inventory stocks, it clearly is essential to model the cost variations explicitly. Some encouraging evidence from a model that does such modelling may be found in Blinder (1983).

It seems fair to say, however, that a convincing explanation of the excess volatility of production has yet to be made (see Blinder (1983)).

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FOOTNOTES

1. Throughout this paper, the word "inventories" used without qualification refers to manufacturers' inventories of finished goods.

2. Eichenbaum's (1982) model does not fit precisely into this framework, even in its simplified version (1982, pp 24-25). He includes the term "$a_4 w_{t+j} Q_{t+j}$" in the cost function, where w_{t+j} is the wage and a_4 another positive parameter. As will become apparent, the inequality to be derived here is approximately correct if $a_4(\text{cov}(w,Q)-\text{cov}(w,S))$ is small compared to the other terms in the inequality.

3. That is, inventories serve two functions apart from any they may serve on the revenue side. In the general formulation of the model used here, inventories may also serve to, say, allow the price speculation by producers that is emphasized in Eichenbaum (1982). The comment in this footnote also applies the model without a target level.

4. I thank both R. Shiller and L. Summers for (independently) suggesting to me the basic argument of this section.

5. Except if the firm has some market power and demand depends on actual or expected production or inventories. As far as I know, this assumption has never been made in this class of models.

6. See Bertsekas (1976, pp. 191-2) for a definition of an "open loop" policy. Strictly speaking, setting $H_t^A = H_t^*$ is the open loop policy only if inventories are the only control.

7. An alternative method for calculating unadjusted constant dollar inventories would be to deflate book value inventories by the appropriate wholesale price index. Given the massive switch from FIFO to LIFO accounting in the 1970's and cyclical differences in output price versus input cost (see Foss, et al. (n.d.)), this is likely to lead to estimates substantially inferior than those derived as described in the text.

8. It should be noted that in Reagan and Sheehan (1982) time series study of precisely the unadjusted aggregate data used here, it was found that seasonal dummies alone successfully accounted for the seasonal variation in inventories. There appeared to be no need to allow for indeterministic seasonal components.

9. This assumes $dp_{t+j}S_{t+j}/dH_t=0$. This is consistent with any linear quadratic inventory model that I am aware of, including not only those in which sales are exogenous (e.g. Belsley (1969)) but also those in which they are jointly endogenous with inventories (Eichenbaum (1982), Blinder (1982)).

10. u_{1t} is MA(1) if production and sales decisions are made simultaneously. But if production is decided before sales are known, as in Blinder (1982), u_{1t} is MA(2). It seemed desirable to adopt a procedure that was consistent under those circumstances, so the estimation procedure allowed for a MA(2) disturbance.

11. This is not to say that the model (1) implies that inventories and sales follow such an autoregression. In general, however, it does imply that they follow a bivariate ARMA process of some order (Hansen and Sargent (1981)). The order of the ARMA process cannot be tied down without making auxiliary assumptions that we have been at pains to avoid making. The AR process assumed in the text should be considered an approximation to this ARMA process.

12. Total expected costs when the determinished seasonal component is accounted for are:

$$(14) \qquad (1-d)^{-1}\{[a_0 \mathrm{var}(\Delta Q) + a_1 \mathrm{var}(Q) + a_2 \mathrm{var}(H)$$

$$-2a_2 a_3 \mathrm{var}(h_1 S_{+1}) = a_2 a_3 \mathrm{var}(S)]$$

$$+[a_0 \mathrm{var}(\Delta Q^s) + a_1 \mathrm{var}(Q^s) + a_2 \mathrm{var}(H^s)$$

$$-2a_2 a_s \mathrm{var}(H^s, S^s{}_{+1}) + a_2 a_3 \mathrm{var}(S^s)]\}$$

13. One further set of estimates was obtained, but results were so poor that they do not appear to warrant reporting in the text. An independent measure of production was obtained by using the Federal Reserve Board's index of industrial production. This is an available seasonally adjusted, and (7.1) and (7.2) were estimated for five of the seasonally adjusted data sets (aggregate non-durables, chemicals, food, petroleum and rubber). (Naturally, the inventory and sales figures were also scaled down to a base of 1967=100.) Parameter estimates, unfortunately were uniformly nonsensical, with about three-fourths of them wrong signed. The tests of (7.1) and (7.2) therefore do not seem worth reporting. But it is perhaps worth noting that $\mathrm{var}\ Q < \mathrm{var}\ S$ for all five data sets.

Apparently, the FRB index of industrial production does not jibe with the Department of Commerce figures on sales and inventories. Alan Blinder has suggested to me that this is because the FRB measure includes production that adds value to works in progress inventories.

14. This has been conjectured by Blinder (1981b) and Blanchard (1983)

TABLE I

BASIC VARIANCES AND COVARIANCES
(RAW DATA SEASONALLY ADJUSTED)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	var(ΔS)	var(ΔQ)	var(S)	var(Q)	(1)-(2)	(3)-(4)	var(H)	cov(H,S_{+1})
Aggregate non-durables	124 428	170 407	1 288 650	1 311 890	-45 979	-23 239	1 326 370	715 300
Food (SIC 20)	39 789	51 744	46 801	50 628	-11 955	-38 732	97 779	21 828
Tobacco (SIC 21)	657	2 145	647	1 401	-1 488	-754	1 542	233
Apparel (SIC 23)	5 032	10 728	15 369	18 866	-5 698	-3 497	17 320	4 689
Chemicals (SIC 28)	8 548	9 405	85 260	84 293	-857	967	71 598	16 130
Petroleum (SIC 29)	3 003	5 189	21 519	22 358	-2 186	-838	11 601	1 123
Rubber (SIC 30)	2 475	3 688	22 188	23 186	-1 213	-998	26 961	14 312

Notes: 1. Units are millions of 1972 dollars squared. Data are described in text.
2. Variances and covariances are calculated around a time trend, as described in the text.

TABLE II

BASIC VARIANCES AND COVARIANCES
(RAW DATA SEASONALLY UNADJUSTED)

	(1) $var(\Delta S)$	(2) $var(\Delta Q)$	(3) $var(S)$	(4) $Var(Q)$	(5) (1)-(2)	(6) (3)-(4)	(7) $var(H)$	(8) $cov(H,S_{+1})$
Aggregate non-durables	217 727	331 732	1 258 260	1 326 950	-114 004	-68.686	1 179 870	774 956
Chemicals	20 647	18 649	105 840	102 751	1 998	3 089	69 803	24 932
Petroleum	3 462	5 940	20 964	22 275	-2 447	-1 311	11 259	-6 225
Rubber	4 172	4 977	19 317	20 471	-805	-1 154	17 896	8 399

Notes: 1. Units are millions of 1972 dollars squared. Data are described in the text.
2. Variances and covariances are calculated around a time trend and seasonal dummies, as described in the text.

TABLE III

STRUCTURAL PARAMETERS, MODEL WITHOUT A TARGET LEVEL

	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	J
Raw data seasonally adjusted				
Aggregate non-durables	.2443 (.0453)	.5126 (.0903)	.0129 (.0188)	12.69
Food	.3377 (.0585)	.3261 (.1167)	-.0000 (.0175)	6.61
Tobacco	.0373 (.0828)	.9256 (.1652)	.0311 (.0510)	4.27
Apparel	.3844 (.0539)	.2332 (.1075)	.0169 (.0123)	8.10
Chemicals	.4074 (.0627)	.1872 (.1251)	.0160 (.0172)	12.91
Petroleum	.1399 (.0826)	.7209 (.1648)	.0418 (.0235)	7.42
Rubber	-.0501 (.1150)	1.0999 (.2294)	-.0083 (.0354)	7.91
Raw data seasonally unadjusted				
Aggregate non-durables	-.1093 (.0931)	1.2182 (.1857)	-.0038 (.0273)	19.92
Chemicals	.3530 (.0839)	.2958 (.1674)	.0224 (.0198)	14.48
Petroleum	.3300 (.0555)	.3417 (.1107)	.0395 (.0127)	4.85
Rubber	.4204 (.1058)	.1612 (.2111)	-.0117 (.0254)	4.17

Notes:
1. Variables defined in text.
2. J distributed as chi-squared with four degress of freedom, critical levels: 9.48 at .05, 13.28 at .01, 14.86 at .005.
3. Asymptotic standard errors in parentheses; standard error on $a_1=1-(1+d)a_0=1-1.995a_0$ calculated as 1.995 times the standard error on a_0.

TABLE IV

STRUCTURAL PARAMETERS, MODEL WITH A TARGET LEVEL

	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	J
Raw data seasonally adjusted					
Aggregate non-durables	.1759 (.1113)	.6489 (.2220)	.0228 (.0232)	1.1249 (1.2178)	11.44
Food	-.0786 (.2914)	1.1568 (.5813)	.0839 (.0868)	6.4669 (3.4099)	2.62
Tobacco	.0241 (.0854)	.9520 (.1704)	.0420 (.0540)	1.2325 (2.0185)	3.76
Apparel	.1117 (.1276)	.7271 (.2546)	.0257 (.0283)	4.8653 (5.3242)	1.43
Chemicals	.3990 (.0671)	.2041 (.1339)	.0171 (.0177)	.3256 (.9832)	12.83
Petroleum	.0775 (.0908)	.8453 (.1811)	.0367 (.0263)	1.1048 (1.0980)	4.01
Rubber	-.2456 (.1189)	1.4900 (.2372)	.0199 (.0494)	4.5217 (10.6588)	1.79
Raw data seasonally unadjusted					
Aggregate non-durables	-.2419 (.1014)	1.4827 (.2023)	.0617 (.0464)	2.0416 (1.1844)	11.46
Chemicals	.2092 (.1392)	.5827 (.2777)	.0375 (.0282)	.8601 (.8514)	13.50
Petroleum	.2232 (.1029)	.5546 (.2053)	.0253 (.0206)	.8504 (1.3155)	3.53
Rubber	.3100 (.1722)	.3816 (.3435)	-.0085 (.0320)	-3.0046 (13.3695)	4.30

Notes:
1. See Notes to Table III.
2. J distributed as chi-squared with three degrees of freedom, critical levels: 7.81 at .05, 11.34 at .01, 12.84 at .005.

TABLE V

TEST STATISTICS, MODEL WITHOUT A TARGET LEVEL

	(1) Eq'n (7.1)	(2) Eq'n (12)	(3) 100 x (1)/(2)
Raw data seasonally adjusted			
Aggregate non-durables	-8 074 590 (6 779 480)	146 256 000	-5.50
Food	-1 056 690 (1 881 690)	6 797 350	-15.54
Tobacco	-160 232 (38 669)	284 920	-56.23
Apparel	-659 426 (97 754)	1 762 890	-37.41
Chemicals	-262 668 (276 638)	4 151 380	-6.33
Petroleum	-279 082 (124 475)	3 465 590	-8.05
Rubber	-162 299 (161 445)	5 018 480	-3.23
Raw data seasonally unadjusted			
Aggregate non-durables	-13 324 700 (6 961 700)	315 102 000	-4.23
Chemicals	11 111 (335 832)	7 708 310	.14
Petroleum	-339 895 (81 276)	2 001 050	-16.98
Rubber	-63 054 (99 154)	1 036 880	-6.08

Notes: Units are billions of "normalized dollars, obtained after normalizing one 1972 dollar to one unit of production and $a_0 + a_1(1 + d)$ = one dollar.

TABLE VI

TEST STATISTICS, MODEL WITH A TARGET LEVEL

	(1) Eq'n (7.2̂)	(2) Eq'n (12̂)	(3) 100 x (1)/(2)
Raw data seasonally adjusted			
Aggregate non-durables	-3 339 800 (6 904 450)	182 428 000	-1.83
Food	2 398 440 (3 050 810)	40 638 700	5.90
Tobacco	-158 798 (39 817)	293 430	-54.11
Apparel	-525 333 (97 687)	4 896 480	-10.73
Chemicals	-238 359 (279 689)	4 431 220	-5.37
Petroleum	-242 594 (137 816)	4 120 130	-5.89
Rubber	169 716 (382 009)	8 124 210	2.09
Raw data seasonally unadjusted			
Aggregate non-durables	9 642 140 (18 963 400)	417 671 000	2.31
Chemicals	241 510 (386 956)	13 544 900	1.78
Petroleum	-366 608 (187 799)	2 923 310	-12.50
Rubber	-21 256 (149 736)	1 456 590	-1.45

Notes: Units are billions of "normalized" dollars, obtained after normalizing one 1972 dollar to one unit of production and $a_0 + a_1 (1-d)$ = one dollar.

TABLE VII

BASIC VARIANCES AND COVARIANCES-SEASONAL COMPONENTS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$var(\Delta S^s)$	$var(\Delta Q^s)$	$var(S^s)$	$var(\dot{Q}^s)$	(1)-(2)	(3)-(4)	$var(H^s)$	$cov(H^s, S^s_{+1})$
Aggregate non-durables	2 109 340	2 641 980	1 115 950	1 238 090	-532 640	-122 136	50 935	23 749
Chemicals	57 917	48 951	40 877	29 718	8 967	11 161	3 133	2 727
Petroleum	4 129	2 017	1 371	816	2 111	555	780	559
Rubber	16 812	19 141	7 269	8 348	-2 330	-1 079	479	-25

Notes: Units are millions of 1972 dollars squared. Data and calculation are described in the text.

TABLE VIII

TEST STATISTICS, SEASONAL MODEL WITHOUT A TARGET LEVEL

	(1)	(2)	(3)
	Eq'n $(\hat{13}.1)$	Eq'n $(\hat{14})$	100 x (1)/(2)
Aggregate non-durables	-31 394 100 (8 448 700)	558 919 000	-5.61
Chemicals	1 290 390 (327 677)	12 936 200	9.98
Petroleum	-292 635 (97 243)	2 914 500	-10.04
Rubber	-168 585 (54 951)	2 195 920	-7.67

Notes:
1. See notes to Table V.
2. Equation (14) defined in footnote 12.

TABLE IX

TEST STATISTICS, SEASONAL MODEL WITH A TARGET LEVEL

	(1)	(2)	(3)
	Eq'n $(1\hat{3}.2)$	Eq'n $(1\hat{4})$	100 x (1)/(2)
Aggregate non-durables	-234 130 (21 052 500)	713 810 000	-0.00
Chemicals	1 929 020 (636 967)	1 927 400	10.1
Petroleum	-209 849 (162 745)	3 108 010	-6.75
Rubber	-247 514 (151 183)	3 167 820	-7.81

Notes:
See notes to Table VIII

Appendix

The appendix briefly outlines the procedure used to derive the asymptotic covariance matrix of the parameters needed to calculate (7.1) and (7.2). Much more detail may be found in West (1983a).

Write the three equation system consisting of (10) and (11) as:

$$y_1 = Xb_1 + u_1$$

$$y_2 = Zb_2 + u_2$$

$$y_3 = Zb_3 + u_3$$

y_1 is the vector of observations of the left hand side of (10); y_2 and y_3 contain vectors of inventories and sales. X contains the right hand side variables in (10) (including deterministic terms), Z the right hand side variables in (11). The error u_1 in MA(2) (see footnote 10), u_2 and u_3 are iid.

b_1 was estimated by two step, two stage least squares, $\hat{b}_1=(\hat{A}Z'X)^{-1}\hat{A}Z'y_1$. $\hat{A}$ is Hansen's (1982) optimal weighting matrix (no heteroscedasticity correction), $\hat{A}=X'Z(Z'\hat{\Omega}Z)^{-1}$. $\hat{\Omega}$, the variance-covariance matrix of u_1, was calculated from 2SLS estimates of u_1.

The numerical simulations in West (1984) suggest that b_1 is likely to be estimated only slightly less efficiently than it would have been had it been estimated by a "full information" technique that specified the demand side of the market, solved for the equilibrium of the model, and imposed cross-equation contraints.

$\hat{b}_2$ and $\hat{b}_3$ were estimated by OLS.

Let θ denote the parameter vector, $\theta = (b_1, b_2, b_3, \sigma_{22}, \sigma_{23}, \sigma_{33})$. $\sigma_{ij} = Eu_{it}u_{jt}$, and are needed to calculate (7.1) and (7.2) since they figure into the variances and covariances in these inequalities. (These second moments, again, were calculated as functions of b_2, b_3 and the σ_{ij} as described briefly in the text and in detail in West (1983a).) Now, the $\hat{b}_i$ were calculated as just described, the $\hat{\sigma}_{ij}$ from the moments of the OLS residuals. Thus, the $\hat{b}_i$ and $\hat{\sigma}_{ij}$ satisfy the orthogonality conditions.

$$0 = T^{-1}\Sigma h_t(\hat{\theta}) = \begin{bmatrix} T^{-1}\Sigma \hat{A} Z_t(y_{1t} - X_t'\hat{b}_1) \\ T^{-1}\Sigma Z_t(y_{2t} - Z_t'\hat{b}_2) \\ T^{-1}\Sigma Z_t(y_{3t} - Z_t'\hat{b}_3) \\ \hat{\sigma}_{22} - T^{-1}\Sigma(y_{2t} - Z_t'\hat{b}_2)^2 \\ \hat{\sigma}_{23} - T^{-1}\Sigma(y_{2t} - Z_t'\hat{b}_2)(y_{3t} - Z\hat{b}_3) \\ \hat{\sigma}_{33} - T^{-1}\Sigma(y_{3t} - Z_t'\hat{b}_3)^2 \end{bmatrix}$$

$$= \begin{bmatrix} T^{-1}\hat{A}\ Z'(y_1 - X\hat{b}_1) \\ T^{-1}Z'(y_2 - Z\hat{b}_2) \\ T^{-1}Z'(y_3 - Z\hat{b}_3) \\ \hat{\sigma}_{22} - T^{-1}\Sigma(y_{2t} - Z_t'\hat{b}_2)^2 \\ \hat{\sigma}_{23} - T^{-1}\Sigma(y_{2t} - Z_t'\hat{b}_2)(y_{3t} - Z_t'\hat{b}_3) \\ \hat{\sigma}_{33} - T^{-1}\Sigma(y_{3t} - Z_t'\hat{b}_3)^2 \end{bmatrix}$$

As proved by Hansen, then, the asymptotic covariance matrix of $\sqrt{T}(\theta - \overset{*}{\theta})$ is $(\text{plim}\ T^{-1}\Sigma h_{t\theta})^{-1} S (\text{plim} T^{-1}\Sigma h_{t\theta}')^{-1}$, where $\overset{*}{\theta}$ is the true but unknown θ and $S = \sum_{j=-2}^{2} Eh_t h_{t-j}^1$. Details on how this covariance matrix were calculated may be found in West (1983a).