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A Very Low Complexity QRD-*M* Algorithm Based on Limited Tree Search for MIMO Systems

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*Abstract***— We present a very low complexity QRD-*M* algorithm for MIMO systems. The original QRD-*M* algorithm decomposes the MIMO channel matrix into upper triangular matrix and applies a limited tree search. To accomplish near-MLD(Maximum Likelihood Detection) performance for QRD-*M* algorithm, number of search points at each layer must be the modulation size. In the proposed scheme, each of survival branches are extended only to the corresponding QR decomposition (QRD)-based detection symbol in the next layer and its neighboring symbols in the constellation. Using this approach, we can significantly decrease the complexity of conventional QRD-*M* algorithm. Simulation results show that the proposed algorithm scheme achieves the detection performance near to that of the MLD with negligibly low complexity.**

Keywords- MIMO, QRD-M

I. INTRODUCTION

Recently, several detection algorithms for MIMO systems achieving near-MLD performance have been proposed [1-6]. The tree search based QRD-*M* algorithm and sphere decoding (SD) are the most promising algorithms. Both algorithms are attracting special attention as they achieve near-MLD performance, while requiring substantially low complexity in comparison with MLD. The QRD-*M* algorithm reduces the complexity by selecting *M* candidates with the smallest accumulated metrics at each level of the tree search [8]. To accomplish near-MLD performance for QRD-*M* algorithm, *M* should be large enough and it still requires high computational complexity.

To reduce the complexity of QRD-*M* algorithm, an adaptive QRD-*M* algorithm proposed in [11] calculates a threshold value at each stage and selects paths with accumulated metrics less than the threshold. The threshold is the sum of minimum accumulated metric and scaled noise power. Thus the complexity in [11] is less than that in the conventional non-adaptive algorithms by the effective reduction of search candidates at every stage. However, improperly estimated noise variance causes performance degradation or complexity increase, which is the main drawback of this method.

In this paper, we introduce the new approach which reduces the complexity of conventional QRD-*M* algorithm by using QRD based detection following with reduced search limited to the neighboring constellation points. The rest of this paper is organized as follows. After introducing MIMO system model in Section II, the basics of conventional QR based detection algorithms are introduced in Section III. In Section IV, we describe the proposed algorithm and its architecture. The improvement of the proposed algorithm and the complexity comparisons will be discussed in Section V. Finally, Section VI concludes this paper.

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II. MIMO SYTEM MODEL

Consider a MIMO system with n_T transmit and $n_R \geq n_T$ receive antennas. First, the data is demultiplexed into n_T data substreams. These sub streams are mapped onto *M*-QAM (in pour case) symbols and transmitted over the n_T antennas simultaneously. In the traditional MIMO system the received signal can be expressed as:

$$y = Hs + w \tag{1}$$

where H -is the channel matrix, $s \in A$ - transmitted signal vector (where $A \in Z$ denotes the lattice of the symbol alphabet), the entries of w are independent Gaussian noise component with variance σ^2. The channel matrix contains uncorrelated Gaussian channel coefficients with unit variance. The channel is assumed to be flat fading and quasi-static, i.e., the channel matrix is constant within the frame and changes independently from one frame to another. We also assume the perfect knowledge about the channel state information in the receiver.

III. CONVENTIONAL QRD-BASED DETECTION ALGORITHMS

A. QR Detection

In QR Detection algorithm, the QR-decomposition of channel matrix is obtained first as follows:

$$H = Q\tilde{R} \tag{2}$$

where Q is a $n_R \times n_R$ unitary matrix,

$$\tilde{R} = \begin{bmatrix} R \\ 0_{(n_R - n_T)\times n_T} \end{bmatrix} \tag{3}$$

R is $n_T \times n_T$ upper triangular matrix, and $0_{(n_R - n_T)\times n_T}$ is zero matrix of size $(n_R - n_T)\times n_T$.

Nothing that,

$$Q^H Q = I \tag{4}$$

After pre-multiplying received signal by Q^H, we can rewrite (1) as:

$$\tilde{y} = Rs + \tilde{w} \tag{5}$$

where $\tilde{y}$ is the first n_T rows of $Q^H y$ and $\tilde{w}$ is the first n_T rows of $Q^H w$.

In QR detection, estimation process is started from the last n_T layer, since it has no interference from other antennas. The estimates of n_T-th layer signal is obtained by canceling the fading and applying quantization operation.

$$\hat{s}_{n_T} = Q(\tilde{y}_{n_T} / q_{n_T, n_T}) \tag{6}$$

where $Q(.)$ denotes quantization operation, $\tilde{y}_{n_T}$ is the n_T element of the modified received signal vector $\tilde{y}$, and $\hat{s}_{n_T}$ is the estimated signal for n_T layer. Estimated signal of each next layer has interference from all previous layers. In order to obtain estimates of transmitted signal for each next layer, the interference from all previous layers should be subtracted before quantization operation. The general procedure of estimation process is given in Fig. 1:

$\hat{s} = 0$
for $i = n_T, \ldots, 1$
  if $i < n_T$
    for $j = i+1, \ldots, n_T$
      $\tilde{y}_i = \tilde{y}_i - r_{i,j} \cdot \hat{s}_j$
    end
  end
  $\hat{s}_i = Q(\tilde{y}_i / r_{i,i})$
end

Figure 1. QR detection

The complexity of QR detection is significantly low than MLD case. In terms of performance, the gap between QR detection performance and MLD is significant. The use of efficient sorted QR (SQR) Decomposition proposed in [13] can significantly improve the performance, however, remaining gap to optimal SER is still significant.

B. QRD Based Tree Search Algorithm (QRD-M)

Recently, several algorithms achieving near-MLD performance have been proposed. The QRD-*M* algorithm is especially of interest as it achieves near-MLD performance, while requiring comparatively low complexity [8]. The algorithm also has the advantage of variable complexity depending on the number of branches it chooses to retain. Unlike QR based detection, QRD-*M* algorithm is a tree search algorithm. At each detection layer, instead of deciding the signal, QRD-*M* algorithm keeps *M* reliable candidates. Decision is made after all detection layers processed.

The concept of QRD-*M* is to apply the tree search to detect the symbols in a sequential manner. Starting from the last layer (first detection layer), i.e., $i = n_T$, the algorithm calculates the metrics for all possible values of $\hat{s}_i$ from the constellation using Euclidean distance given as [8]:

$$|\, \tilde{y}_{i,cancelled} - r_{i,i}\hat{s}_i \,| \tag{8}$$

where

$$\tilde{y}_{i,cancelled} = \begin{cases} \tilde{y}_{n_T} & i = n_T \\ \tilde{y}_i - \sum_{j=i+1}^{n_T} r_{i,j}\hat{s}_j & i < n_T \end{cases} \tag{9}$$

and { $\hat{s}_{i+1}$, $\hat{s}_{i+2}$, …, $\hat{s}_{n_T}$ } are the estimated symbols of the considered survivor path in the previous layers. The metrics of these points or nodes are then ordered, and only *M* nodes with the smallest metrics are retained and the rest of the list is deleted. The same procedure is applied to the nodes of the next layer, and this process continues to the first layer (last detection layer, $i = 1$) as shown in fig. 2.

To accomplish near-MLD performance for QRD-*M* algorithm, *M* should be large enough for the selected paths to include the correct one. However, in the subsequent layers, we have to repeatedly extend to *MC* branches and calculate the corresponding *MC* Euclidean distances in order to select *M* survivor paths. Simulation results reveal that with the range of *M* for satisfactory performance, the computational complexity of QRD-*M* algorithm increases significantly as the number of antennas and the size of modulation set are large.

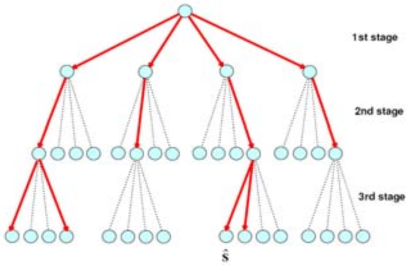

Figure 2. Tree structure of 3x3 MIMO system with QRD-*M* (*M* = *4*) and QPSK

C. Adaptive QRD-M Algorithm

To accomplish near-MLD performance for QRD-*M* algorithm, *M* must be large enough, which still requires high computational complexity. In order to reduce the complexity an adaptive control of surviving candidates has been proposed in [11]. Adaptive QRD-*M* algorithm calculates a threshold value at each stage and selects paths with accumulated metrics less than the threshold value. The threshold at the *m* th stage Δ_m is calculated as:

$$\Delta_m = E_{m,\min} + X\sigma^2 \tag{10}$$

where $E_{m,\min}$ is the minimum accumulated branch metric at the *m* th stage, *X* is a predetermined fixed value, and σ^2 is the noise power. Adaptive QRD-*M* algorithm adaptively controls the number of survival branches by selecting the branches whose metric is smaller than a threshold value calculated at each stage. Therefore, it brings about reducing the complexity of tree searching. At each stage, the sum of minimum accumulated metric and estimated noise power multiplied by constant scaling term *X* is acquired as the threshold value to prune off the unnecessary paths. As shown in Fig. 2.5, the flow of the selection of the surviving symbol replica candidates at the *m* th stage in the adaptive QRD-*M* method is as follows:

- Calculate accumulated branch metrics for all symbol replica candidates at the *m* th stage.
- Select the lowest accumulated branch metric
- Calculate Δ_m using (10).
- Select symbol replica candidates that have accumulated branch metrics lower than Δ_m. If the number of survived replica candidates becomes larger than predetermined maximum possible value $\widehat{M}$, $\widehat{M}$ symbol replica candidates with the lowest accumulated branch metrics are selected.

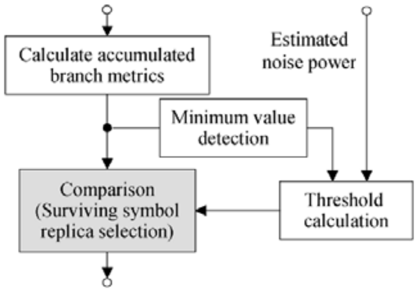

Figure 3. Block diagram of Adaptive QRD-*M* algorithm

Adaptive QRD-*M* method significantly reduces the complexity of the original QRD-*M* method while maintaining close to optimal performance, however, its complexity and performance strongly depends on reliability of estimated noise power.

IV. Proposed Detection Algorithm

Original QRD-*M* algorithm achieves near-MLD performance in case that the number of survivor candidates at each stage is sufficiently large so that the survivor paths include the correct one. So, the complexity is still too high to be applied for practical systems. If the number of survival candidates is small, it results in performance degradation. To alleviate the complexity of original QRD-*M* algorithm with near-MLD performance, we try to decrease the number of searching points at each layer without sacrificing the performance and thus, we decrease number of Euclidian path metric computations. Also as an additional difference to the conventional QRD-*M* and adaptive QRD-*M* algorithms, the proposed scheme uses sorted one (SQRD) instead of original QR decomposition, which gives us more reliable estimates with slight complexity overhead.

The general flowchart of the proposed algorithm can be illustrated as follows in Fig. 4. In the proposed scheme, we calculates the metrics for not all possible values of $\hat{s}_i$ in (12) but only neighboring points around the estimated centering point which is obtained by using QR detection symbol for $\hat{s}_i$ explained in previous section. So, the number of extending branches at each node is 5 not *C*, which results in the drastic reduction of metric computation complexity compared to the conventional QRD-*M* for high order modulation.

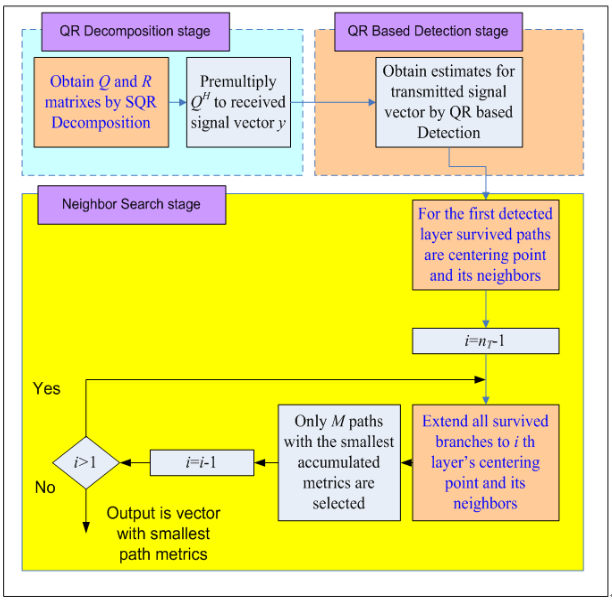

Figure 4. General Flowchart for Proposed Detection Algorithm

Use of the SQRD increases the reliability of centering point estimation. Even if the centering point gives an incorrect decision, there is a high probability that the actual point will be within the neighboring point. In order to show this, we tabulated the probability of centering point error correction with only four neighboring points consideration (Table 1). Let us denote the probability of error correction with neighboring points by P(A|B), where B is the wrong decision event of the centering point, and A is the event of correct decision existence among the four neighboring points. We simulated and tabulated the probability P(A|B) for different kinds of QAM-modulation and QR decomposition. According to the results, use of sorted QR decomposition gives us significantly higher probability of error correction for both 16 and 64 QAM modulations. Based on our simulation results we can say that use of the sorted QR Decomposition allows us consideration of only limited points since it gives a high probability of correct decision existence among them.

Table 1. Probability of Error Correction with Neighboring Points

Schemes	SNR	15 dB	20 dB	25 dB
		Probability of error correction with neighboring points		
16 QAM	**Proposed with QRD**	0.3821	0.7451	0.8547
	Proposed with SQRD	0.5113	0.9148	0.9901
	QRD All Possibilities Consideration	0.5717	0.9431	0.9912
64 QAM	**Proposed with QRD**	0.2431	0.6744	0.8063
	Proposed with SQRD	0.4688	0.8856	0.9719

As explained before, the detection process in proposed algorithm begins with setting the centering point which is the estimate of transmitted signal for current layer obtained by QR based detection. The centering point and four of its closest neighboring points are assumed as survived paths for last layer. The Fig. 5 illustrates the process of obtaining neighboring points based on centering point. The Euclidian metric computations are not applied for the n_T layer (first detection layer).

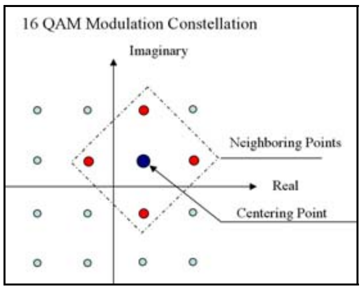

Figure 5. Centering point and its closest four neighbors

At the second stage, the survived paths are extended with centering point for n_T-1 layer and its neighboring points. The Euclidean distance calculations are obtained for all of these candidates and only *M* paths with the smallest accumulated metrics are selected. The same process is repeated until n_T-1 tree depth. Throughout paper we set the value of *M* equal to five. In the simulation results, the proposed algorithm still achieves near MLD performance even with *M* equal to five.

While original QRD-*M* algorithm extends survived paths to all kinds of candidates whose number is equal to constellation size, the proposed scheme extends with only candidates restricted to centering point and its neighbors, which results in significant low complexity. Besides this, the proposed algorithm doesn't require SNR knowledge as in the conventional adaptive QRD-*M* algorithm [11].

V. PERFORMANCE EVALUATION AND ANALYSIS

A. *Performance Evaluation*

In this section, we compare the average complexity and the error rate performance of the proposed scheme with MLD and other previously developed conventional methods for 2x2 and 4x4 MIMO systems. Original QRD-*M* [8] and adaptive QRD-*M* algorithms [11] have been also illustrated. Well known Sphere Decoding (SD) algorithm have not been included in performance comparisons, since detailed comparisons already have been described in [7]. We focused on the main drawbacks of the conventional QRD-*M* based detectors. The promising technique LR(Lattice reduction) aided detection is also included in simulations. We chose the simplest ZF/LR method for comparisons. To see the performance achieved by our proposed scheme, average symbol-error-rate (SER) curves plotted in Fig. 6 for 2x2 MIMO systems with 16-QAM modulation. It is observed that the proposed adaptive scheme achieves almost near MLD SER performance. For simulations, we set the value of *M*, which is the number of survival paths at each layer, is set to five. The original QRD-*M* algorithm uses *M* = 16. For adaptive QRD-*M* algorithm the value of *X* is set to 4.0.

The SER curves for 64-QAM modulation are also provided in Fig. 7. We can see that even for high level modulation the proposed scheme shows close to optimal SER performance. Fig. 6 and 7 show that for 2x2 MIMO systems for both 16 and 64 QAM modulations the proposed algorithms SER performance converges to optimal one. The SER curves for QRD-*M* (*M*=16), conventional adaptive QRD-*M* (X=4.0) algorithms are also almost same to optimal MLD algorithm.

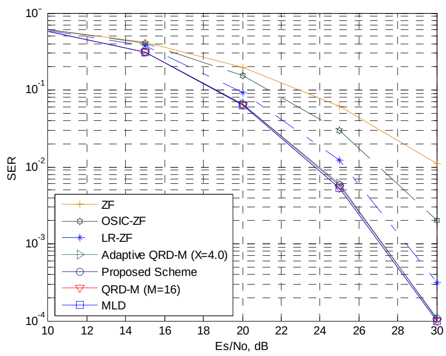

Figure 6. SER Performance comparisons, 2x2 MIMO System, 16 QAM Modulation

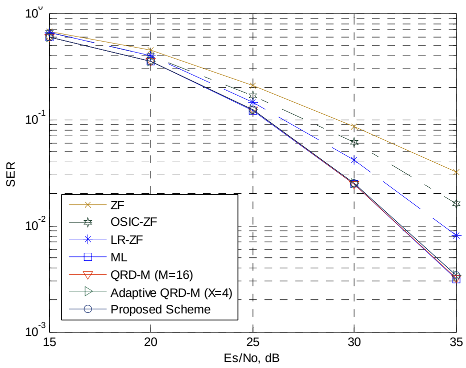

Figure 7. SER Performance comparisons, 2x2 MIMO System, 64 QAM Modulation

The simulation results for 4x4 MIMO system is given in Fig. 8. Which shows that even for higher number of transmit and receive antennas the proposed scheme maintains its high performance. Because the proposed algorithm uses reduced search among high reliable candidates, the proposed scheme gains the same near MLD performance as original QRD-*M*, and adaptive QRD-*M* algorithms.

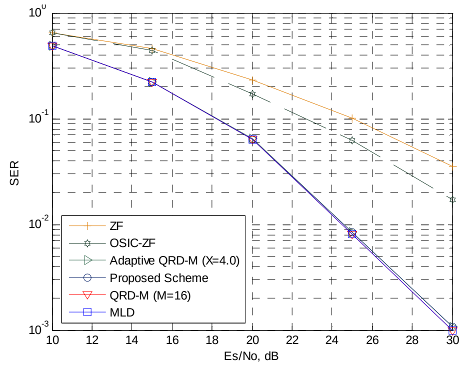

Figure 8. SER Performance comparisons, 4x4 MIMO System, 16 QAM Modulation

B. *Complexity Comparisons*

The simulation results show that the SER performance of proposed and two conventional schemes (Original and Adaptive QRD-*M*) are same irrespective of SNR. Therefore, the complexity should be considered in order to compare the efficiency of these schemes. Complexity is another criterion to evaluate the performance of developed algorithm. In this paper, the complexity is compared by using simulations and analytical expressions. The complexity comparison simulations are evaluated by the average number of metric computations, and the analytical comparisons are evaluated by the amount of real multiplicative operations, which are the main factors consider impacting overall complexity.

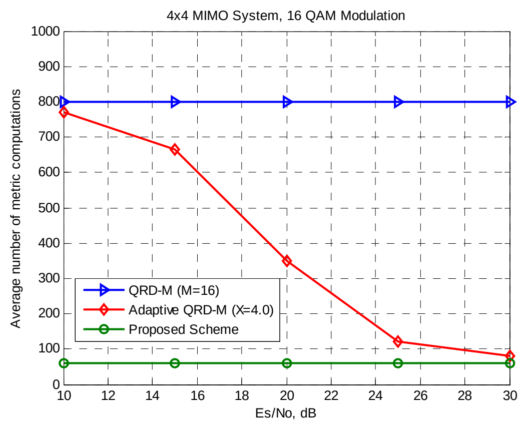

Figure 9. Average number of metric computations of conventional and proposed algorithms

Fig. 9 illustrates the number of branch metric computations needed by conventional QRD-*M* algorithms and the proposed scheme. Original QRD-*M* algorithm has constant complexity irrespective of *Es/No*. The complexity of an adaptive QRD-*M* algorithm depends on *Es/No*, since it chooses number of survival path according to noise variance. At *Es/No* =10 dB, for example, adaptive QRD-*M* algorithm is almost same complexity as original QRD-*M* algorithm, while proposed scheme has significantly low complexity. Also the proposed algorithm requires about 50% branch metric computations compared with adaptive QRD-*M* at *Es/No*=25 dB while both methods achieve the identical MLD performance. With larger *X*, adaptive QRD-*M* algorithm considers more branches of surviving candidates for near-MLD performance. Its branch metric computations are much larger as *X* increases. Furthermore, in order to achieve the near-optimal performance while maintaining low complexity, noise variance should be properly set for adaptive QRD-*M* algorithm.

VI. Conclusions

Throughout this paper, we focused on maintaining low complexity with optimal performance for MIMO detection. We introduced the new approach which significantly reduces the complexity of conventional QRD-*M* algorithm by using QRD based detection following with reduced search MLD detection. This kind of approach leads to reducing the number of branches to be searched at each stage. The performance and complexity analysis showed that the proposed scheme achieves near ML performance while maintaining significantly reduced complexity compared to the conventional QRD-*M* algorithms.

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