

A “Working” Solution to the Question of Nominal GDP Targeting*

Michael T. Belongia
Otho Smith Professor of Economics
University of Mississippi
Box 1848
University, MS 38677
mvpt@earthlink.net

and

Peter N. Ireland
Department of Economics
Boston College
140 Commonwealth Avenue
Chestnut Hill, MA 02467
peter.ireland@bc.edu

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Abstract: Although a number of economists have tried to revive the idea of nominal GDP targeting since the financial crisis of 2008, very little has been said about how this objective might be achieved in practice. This paper adopts and extends a strategy first outlined by Holbrook Working (1923) and later employed by Hallman, et al. (1991) in the P-Star model. It presents a series of theoretical and empirical results to argue that Divisia monetary aggregates can be controlled by the Federal Reserve and that the trend velocities of these aggregates exhibit the stability required to make long-run targeting feasible.

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A “Working” Solution to the Question of Nominal GDP Targeting

Although stabilizing nominal GDP has been suggested before as an objective for monetary policy actions, an increasing number of economists have tried to revive the idea since the financial crisis of 2008 and the apparent ineffectiveness of manipulating the federal funds rate when the zero bound constraint has been met. But while the merits of nominal GDP stabilization as a final objective for monetary policy have been emphasized in recent discussions, very little has been said about how this goal might be achieved in practice. Indeed, whereas earlier discussions offered explicit strategies and established linkages between, for example, nominal GDP and the monetary base (see, e.g., McCallum (1988) and Meltzer (1987)) or a broader monetary aggregate (Feldstein and Stock (1994)), the recent discussions have been relatively strong on the goal and relatively silent on how a path to the goal might be implemented.¹ Indeed, with the recent innovations of payment of interest on reserves and unusual behavior of the monetary base in the aftermath of the financial crisis, even thoughts of reviving some of the older, well-articulated strategies have been put on hold. Thus, for all of the attention that nominal GDP targeting has received as a potential goal for monetary policy, a practical means of achieving that end has yet to be offered.²

In this paper we propose a strategy for nominal GDP targeting based on a framework first outlined by Holbrook Working (1923) and used, with only minor modifications, by Hallman, et al. (1991) in the P-Star model. In these earlier applications, a policymaker is able to evaluate whether a value for the money stock is consistent with long-run price stability. Using essentially the same derivation found in Working’s original paper and making appropriate changes to the practical adaptations employed by Hallman et al., we find a path for money that is consistent with any desired long-run trajectory for nominal GDP. Unlike

¹ An exception is Sumner (1989, 1995) and the suggestion of implementing monetary policy through the use of a nominal GDP futures market.

² For a survey of issues regarding nominal GDP targeting, see Bean (1983). Clark (1994) offers some evidence on lagged adjustment v. forecast adjustment rules when NGDP targeting is implemented.

previous applications of this framework, we employ Divisia monetary aggregates in establishing a path for money that the central bank should try to maintain and use a one-sided filtering algorithm that can be implemented in real time to control for slow-moving trends in velocity.³

In what follows, we first explain the basic analytics of Working's framework and how we have adapted it to nominal GDP. Then, after reproducing Hallman et al.'s regression results to show that movements in the Divisia aggregates consistently anticipate movements in nominal income over a sample period that extends from 1967 through the present, we compare actual paths for the Divisia monetary aggregates to alternative trajectories that, according to our framework, would have been consistent with more stable nominal GDP growth since 1985. After using this comparison to discuss, in particular, the stance of current monetary policy, we examine how the Fed might control the behavior of these Divisia aggregates within an intermediate targeting strategy. Overall, we conclude that *if* nominal GDP is chosen as the central bank's objective (a question on which this paper takes no position), the strategy outlined in this paper has several virtues: It is transparent to outside observers, it is forward-looking, and yet it can be implemented in a fairly straight-forward manner.⁴

³ In fact, one might speculate that one reason for the demise of the P-star model was the post-sample instability of the velocity of M2, something which can be traced to the financial innovations era but can be attributed more specifically to the problems inherent in simple sum aggregation methods that fail to internalize pure substitution effects and would have been a consequence of such things as the payment of interest on deposits, the availability of a broader array of deposit accounts, and the greater substitution among these accounts by consumers in response to changes in user costs. In this context, it is interesting to note that Working, nearly ninety years ago, devoted an appendix of his paper to an attempt to create an "Index for a Medium of Exchange." Even though he was writing long before the era of financial innovations and the payment of interest on checkable deposits, he intuited that different components of a monetary aggregate should be weighted differently and in this appendix he made an early attempt to do just that.

⁴ One reason we take no position on the desirability of NGDP targeting is the results in West (1986). Using a model presented in Bean (1983), West demonstrated that the preference of NGDP targeting over, say, money supply targeting depends on values of certain parameters and, *a priori*, there is no clear reason to believe why those should take a value that would lead a policymaker to prefer one option over the other. We also take no position on whether targeting the level of nominal GDP is to be preferred to targeting the growth rate of NGDP. Throughout, our purpose is derive a practical approach to targeting the level of NGDP *if* that is to become the central bank's adopted goal.

Working's Framework

Working's (1923) objective was to find a value for the money supply that would be consistent with long-run price stability. At the time of his writing, many others had investigated Quantity Theory relationships empirically.⁵ From this research, strategies to stabilize the price level emerged but even Fisher's (1920) plan did not incorporate a method for dealing with lags in the process. Thus, Working's innovation was to recognize the role of lags and to establish a policy framework that embedded a long-run desired path for price stability. A central bank then could compare the current price level against the desired long-run path and evaluate whether the stance of policy was too accommodative or too restrictive.

Using Quantity Theory relationships, Working re-wrote the basic expression as $(V/T) = (P/M)$. Because (P/M) did not have a "definite conception," Working dealt with its reciprocal. To find a long-run path for it, he estimated a trend value for the price level using a regression of the log value of the price level on time, time squared, and time cubed; future values for the price level were extrapolations from this trend regression. With this information, Working then could plot, on a log scale, values for (M/P) to illustrate the value of circulating medium that would be consistent with his long-run trend path for the aggregate price level.

In adapting Working's framework for the P-Star model, Hallman, et al. (1991) expressed their basic relationship as:

$$(1) P^*_t = (M2_t V^*_t) / Q^*_t.$$

In this expression, P^*_t is the long-run target value for the price level at time t , V^*_t is the long-run equilibrium value for velocity, taken by Hallman, et al, to be the sample mean for $M2$ velocity, and Q^*_t is the value for potential real GDP at time t .⁶ Rearranging terms so as to apply

⁵ For more background, see the surveys in Humphrey (1973) and Laidler (2011).

⁶ Although Taylor (1993) published his famous paper on a rule for the implementation of monetary policy after the P-star paper was published, he did not cite it. Nonetheless, he had this to say about an alternative rule in that paper (pp. 209 – 210): "Since the mid-1970s monetary targets have been used in many countries to state targets for inflation. If money velocity were stable, then, given an estimate of potential output growth, money targets would imply a target for the price level; given velocity and a real output target, the target price level would obviously fall out algebraically from the money supply target. Even though the 1980s

the framework more directly to nominal income targeting and making some desirable changes in empirical choices, the framework to be employed in this paper is:

$$(2) PQ_t^* = M_t V_t^*,$$

where PQ_t^* is the long-run target value for nominal GDP, M_t is the value of a Divisia monetary aggregate, and V_t^* is trend velocity for that chosen monetary aggregate. Equation (2) highlights one key advantage of any nominal income targeting scheme, relative to the price-level or inflation targeting framework implied by the P-star model in (1): Nominal income targeting allows one to sidestep the challenge of estimating accurately potential output in real time. Meanwhile, the use of a Divisia monetary aggregate in (2) in place of simple-sum M2 in (1) is motivated by Barnett's (1980) classic work, which introduced monetary economists to the logic behind, and the practical benefits of, Divisia monetary aggregation; this empirical choice also distinguishes our approach from that of Feldstein and Stock (1994), which like the P-star model, uses simple sum M2 as an intermediate target within a nominal GDP targeting strategy.⁷

In (2), we also depart from the P-star framework in yet another way, by calculating trend velocity V_t^* using the one-sided version of the Hodrick-Prescott (1997) filter described by Stock and Watson (1999). Figure 1 uses quarterly data to compare the actual velocities of Divisia M1 and MZM to the trend values obtained with this one-sided H-P filter.⁸ The choice, both here and below, to focus on M1 and MZM together allows us to assess the robustness of our findings to the choice of narrow versus broad monetary aggregates. Our series on Divisia

have shown that money velocity is not stable in the short run, the long-run stability of the velocity of some monetary measures allows one to state targets for the price level. For example, with an estimated secular growth of real output of 2.5 percent and a steady velocity, a money growth range of 2.5 percent to 6.5 percent – the Fed's targets for 1992 – would imply that the price level target grows at 0 to 4 percent per year. Given biases such as index number problems in measuring prices, the 2-percent per year implicit target inflation rate is probably very close to price stability or 'zero' inflation."

⁷ For a more recent discussion and survey of the extensive literature on the Divisia monetary aggregates, see Barnett (2012).

⁸ The MZM aggregate – “money, zero maturity” – includes those assets in M2, less small time deposits, plus institution-only money market mutual funds. It first was discussed in detail by Motley (1988), who referred to it as “non-term M3.” It later picked up the label of MZM.

M1 and MZM are drawn from the Federal Reserve Bank of St. Louis' FRED database; Anderson and Jones (2011) describe their construction in detail. With quite similar results, not shown, we also replicated the analysis using Anderson and Jones' Divisia M2 series, as well as the much broader, Divisia M4 aggregate provided by the Center for Financial Stability and described in Barnett, et al. (2012).

The graphs in Figure 1 reveal quite clearly the shifting, but slow-moving, trends in velocities that, Reynard (2007) finds, must be accounted for in identifying the long-run linkages between money and prices, not just in the U.S. but in Switzerland and the Euro Area as well.⁹ Our one-sided version of the H-P filter imposes the same setting $\lambda = 1600$ for the smoothing parameter as is commonly used in the two-sided H-P filter for quarterly data. It produces a similar, but somewhat more volatile, measure of the trend, reflecting the fact that, unlike the standard H-P filter, the one-sided variant only uses data up through period t in constructing the value for the trend at period t . This feature, however, is precisely what allows our algorithm to be implemented in real time and also makes our measure suitable for use in the forecasting equations described below. An added advantage of this one-side filter is that once the parameter λ is fixed, no additional parameters need to be estimated or calibrated in constructing the series for trend velocity: As explained by Stock and Watson (1999, p. 301), values for the trend can be generated quickly and easily using the equations of the standard Kalman filter.

Otherwise, equation (2) parallels (1) for the P-star model by depicting the nominal GDP target PQ_t^* for time t as one that is implied by the level of the Divisia monetary aggregate M_t for that period, given the value of V_t^* , and by suggesting that the actual value for nominal income PQ_t should tend to gravitate, over time, towards the target PQ_t^* . To test this hypothesis, we estimate a set of regression equations that mirror Hallman et al.'s (1991, p. 847) in their

⁹ Along the same lines, it is interesting to note once again that Working (1923) himself found it necessary to control for slow-moving shifts in trends by including time squared and cubed in addition to time itself in his regression equations. Since Working's regression-based approach might well be considered an early version of the modern, though only slightly more elaborate, filtering procedures used here, we find it especially useful to trace the origins of our own approach back to his as well as to the more familiar P-star model.

specification. Specifically, Hallman et al. find that in quarterly data running from 1955.1 through 1988.4, inflation tends to rise when the long-run price target P_t^* implied by (1) is above the actual price level P_t ; likewise, inflation falls when P_t^* is below P_t . They confirm the statistical significance of this result by regressing the change in inflation on four of its own lags and the lagged value of the price gap, defined as the difference between p_t^* and p_t , the natural logarithms of P_t^* and P_t , and rejecting the null hypothesis that the coefficient on the lagged price gap equals zero. Here, similarly, we regress $\Delta^2 pq_t$, the change in nominal income growth (and hence the analog to Hallman et al.'s $\Delta \pi_t = \Delta^2 p_t$, the change in the inflation rate) on four of its own quarterly lags and on the lagged value of the nominal income gap, defined as the difference between pq_t^* , the natural log of the nominal income target in (2), and pq_t , the log of the actual value of nominal GDP during period t .

Although the availability of data on the Divisia monetary aggregates pushes the starting date for our own quarterly sample ahead to 1967.1, we can now extend that sample well beyond Hallman et al.'s, all the way through 2012.3. Our estimates, with the absolute value of the associated t statistic below each coefficient, are

$$\Delta^2 pq_t = \begin{array}{ccccc} -0.605\Delta^2 pq_{t-1} & -0.357\Delta^2 pq_{t-2} & -0.287\Delta^2 pq_{t-3} & -0.071\Delta^2 pq_{t-4} & +0.339(pq_{t-1}^* - pq_{t-1}) \\ (8.3) & (4.4) & (3.5) & (1.0) & (3.7) \end{array}$$

for Divisia M1 and

$$\Delta^2 pq_t = \begin{array}{ccccc} -0.612\Delta^2 pq_{t-1} & -0.358\Delta^2 pq_{t-2} & -0.286\Delta^2 pq_{t-3} & -0.071\Delta^2 pq_{t-4} & +0.245(pq_{t-1}^* - pq_{t-1}) \\ (8.3) & (4.4) & (3.5) & (1.0) & (3.4) \end{array}$$

for Divisia MZM.¹⁰ In both cases, the large and statistically significant coefficient on the lagged nominal GDP gap indicates that nominal income growth accelerates when the gap is positive and decelerates when the gap is negative, so that actual nominal GDP converges over time to the long-run target defined in (2). Table 1 shows, additionally, that the lagged nominal GDP

¹⁰ Again following Hallman et al. (1991), quarterly changes in nominal GDP growth are multiplied by 400, so that they are expressed in annualized percentage points, and the nominal GDP gap is multiplied by 100, so that it is measured in percentage points, in these regressions. A constant term, shown in table 1 but not in the equations as displayed here, is also included in each regression.

gap retains its significance across subsamples running from 1967.1 through 1979.4 and from 1980.1 through 2012.3. Thus, a nominal income target set with reference to either a narrow or a broad Divisia monetary aggregate proves useful in forecasting future nominal GDP growth, even in the most recent data. Most importantly from a practical perspective, no breakdowns of the forecasting equation are observed, and no special “shift-adjustments” beyond accounting for the slow-moving trends in V_t^* using the one-sided filter are needed to maintain the stability of these empirical relationships.

Equation (2) follows the approach in Hallman et al. (1991) by defining the long-run target for nominal GDP in terms of the observed value of the monetary aggregate and the trend value of velocity. It is equally useful, however, to turn the equation around, and use it to identify the path for a monetary aggregate that is consistent with a desired trajectory for nominal GDP. Towards this end, let

$$(3) M_t^* = PQ_t^*/V_t^*$$

define the target M_t^* for money that is consistent with a chosen target PQ_t^* for nominal income, given the long-run value for velocity V_t^* . In the United States between 1985 and 2007, in fact, nominal GDP grew at an average annual rate of almost exactly 5.5 percent. The top panel of Figure 2 plots the actual series for the logarithm of nominal GDP against a trend line with this slope, fitted via a least-squares regression over the 23-year period. The bottom panel, meanwhile, shows deviations of nominal GDP from this trend, highlighting the modest swings experienced during the “Great Moderation” as well as the much more pronounced gap that opened during the most recent recession and continues to widen today. As noted by Woodford (2012), nominal GDP now lies more than 15 percent below a trend line estimated with data from the period before the financial crisis.

Interpreting the trend line in Figure 2 as a target path for nominal GDP that extends through 2012:3, Figure 3 plots the gaps between the logs of actual Divisia M1 and MZM and the corresponding target values for money implied by equation (3). With the regression results from above in mind, one can view positive values for these money gaps as putting upward pressure on nominal GDP growth and negative values as putting downward pressure on

nominal GDP; the gaps thereby indicate whether monetary policy was too accommodative, too restrictive, or appropriately neutral during any given period. In fact, negative values for both the M1 and MZM gaps are observed just before the recession of 1990-91, and both series decline, while remaining slightly positive, before the recession of 2000-01. Larger positive gaps, meanwhile, appear during the economic recoveries of the middle 1980s and early 1990s. Most significantly, however, both panels of Figure 3 suggest that the stance of monetary policy shifted gradually from ease to tightness towards the middle of the last decade and, in fact, began to exert a considerable drag on nominal income growth in 2005 and 2006, thereby supporting Hetzel's (2012) claim that Federal Reserve policy was itself a key factor in triggering the initial slowdown and severe recession that followed. What's more, both figures suggest that despite the Federal Reserve's efforts to lower interest rates and increase dramatically the supply of bank reserves, insufficient growth in the monetary aggregates, particularly against the backdrop of heightened demand for safe and highly liquid assets reflected by the downward movements in trend velocity shown in Figure 1, continues to severely depress nominal GDP in the U.S. economy today.¹¹

Overall, the picture that emerges from Figure 3 is one of persistent volatility in the stance of monetary policy, switching from periods of ease to contraction and back again.¹² This volatility is not entirely unexpected, however, because, under a regime of interest-rate-targeting, a central bank will have to change the quantity of reserves (and money) to maintain its interest rate peg. Thus, in addition to offering a perspective on whether monetary policy has been relatively easy or restrictive at various points in time, Figure 3 also can be interpreted as offering evidence on one consequence implementing monetary policy through an interest rate

¹¹ Once again, the results shown in Figure 3 appear similar when the analysis is applied to Divisia M2 and M4, except that weakness in large time deposits, repurchase agreements, and commercial paper – highly liquid money market instruments included in the M4 aggregate but not in M1 or MZM – make monetary policy look even more restrictive throughout the period since 2008.

¹² Hetzel (2008, Chapter 23, and 2012, Chapter 8) characterizes these variations as “stop-go” monetary policy and offers a detailed explanation for why it may have evolved in this manner over the past five decades.

target: Judged in reference to a smooth path for nominal GDP, targeting the federal funds rate apparently has created an inherent instability in monetary policy.

In summary, the foregoing discussion has tried to establish that monetary policy has the potential to hit a long-run path for nominal GDP if it can control the behavior of a Divisia monetary aggregate that would keep nominal GDP on such a target path. It is to this question of monetary control we now turn.

Money Multipliers for Simple Sum and Divisia Aggregates

Spindt (1983) extends Barnett's (1980) work by deriving general expressions for the multipliers of Divisia monetary aggregates. Here, these expressions are reproduced for the special case of aggregates formed from currency and a single type of interest-bearing deposit. The results make clear how the appearance of user-cost terms in the budget-share weights of the Divisia index can – and seemingly do – help dampen volatility in the behavior of its companion multiplier. A series of numerical examples, based on these expressions together with a model of the demand for currency and deposits drawn from Belongia and Ireland (2012), reveals that for a wide range of plausible parameterizations, the multiplier for the Divisia monetary aggregate is likely to be more stable than the multiplier for the corresponding simple sum measure. We find that this same pattern appears in the U.S. data.

Let D_t , C_t , and R_t denote the dollar values of deposits, currency, and bank reserves.

The simple sum monetary aggregate M_t^s and the monetary base H_t are then defined by

$$(4) \quad M_t^s = D_t + C_t$$

and

$$(5) \quad H_t = R_t + C_t.$$

Following the usual route towards obtaining an expression for the money multiplier of the simple sum aggregate, let

$$(6) \quad k_t = C_t / D_t$$

denote the currency-deposit ratio and

$$(7) r_t = R_t / D_t$$

denote the reserve ratio. Using (4)-(7), the simple sum multiplier can be calculated as

$$(8) m_t^s = \frac{M_t^s}{H_t} = \frac{D_t + C_t}{R_t + C_t} = \frac{1 + k_t}{r_t + k_t}.$$

Equation (8) depicts the textbook result that the money multiplier depends inversely on both the currency-deposit ratio and the reserve ratio.

Because Divisia indexes are growth rate indexes, however, it is useful for the sake of comparison to express the multiplier for the simple sum aggregate in its less familiar growth rate form as well. Spindt (1983) accomplishes this task using the approximations

$$(9) \Delta \ln(M_t^s) = \left(\frac{w_t^D + w_{t-1}^D}{2} \right) \Delta \ln(D_t) + \left(\frac{w_t^C + w_{t-1}^C}{2} \right) \Delta \ln(C_t)$$

and

$$(10) \Delta \ln(H_t) = \left(\frac{v_t^R + v_{t-1}^R}{2} \right) \Delta \ln(R_t) + \left(\frac{v_t^C + v_{t-1}^C}{2} \right) \Delta \ln(C_t)$$

for the growth rates of the simple sum aggregate and the money base, where

$$(11) w_t^D = \frac{D_t}{M_t^s} = \frac{D_t}{D_t + C_t} = \frac{1}{1 + k_t}$$

and

$$(12) w_t^C = \frac{C_t}{M_t^s} = \frac{C_t}{D_t + C_t} = \frac{k_t}{1 + k_t}$$

represent the quantity shares of deposits and currency in the simple sum aggregate and, analogously,

$$(13) v_t^R = \frac{R_t}{H_t} = \frac{R_t}{R_t + C_t} = \frac{r_t}{r_t + k_t}$$

and

$$(14) \quad v_t^C = \frac{C_t}{H_t} = \frac{C_t}{R_t + C_t} = \frac{k_t}{r_t + k_t}$$

represent the quantity shares of reserves and currency in the monetary base. Equations (9)-(14) combine to yield

$$(15) \quad \begin{aligned} \Delta \ln(m_t^s) = & \frac{1}{2} \left(\frac{k_t}{1+k_t} + \frac{k_{t-1}}{1+k_{t-1}} \right) \Delta \ln(k_t) \\ & - \frac{1}{2} \left(\frac{r_t}{r_t+k_t} + \frac{r_{t-1}}{r_{t-1}+k_{t-1}} \right) \Delta \ln(r_t) - \frac{1}{2} \left(\frac{k_t}{r_t+k_t} + \frac{k_{t-1}}{r_{t-1}+k_{t-1}} \right) \Delta \ln(k_t), \end{aligned}$$

restating (8) in growth rate form.

Meanwhile, the growth rate of the Divisia quantity aggregate M_t^d of deposits and currency is defined in discrete time by

$$(16) \quad \Delta \ln(M_t^d) = \left(\frac{s_t^D + s_{t-1}^D}{2} \right) \Delta \ln(D_t) + \left(\frac{s_t^C + s_{t-1}^C}{2} \right) \Delta \ln(C_t),$$

where (16) replaces the quantity shares that appear in (9) with expenditure shares on the monetary services provided by deposits and currency. These shares are computed using Barnett's (1978) formulas for the user costs u_t^D and u_t^C of deposits and currency:

$$(17) \quad u_t^D = \frac{\rho_t^B - \rho_t^D}{1 + \rho_t^B}$$

and

$$(18) \quad u_t^C = \frac{\rho_t^B}{1 + \rho_t^B},$$

where ρ_t^B denotes the rate of return on a benchmark asset that provides no monetary services, ρ_t^D denotes the own-rate of return of deposits, and (18) reflects the fact that currency does not pay interest. Let

$$(19) \quad u_t = \frac{u_t^C}{u_t^D} = \frac{\rho_t^B}{\rho_t^B - \rho_t^D}$$

denote the ratio of the user cost of currency to the user cost of deposits. Using this expression together with the formula (6) defining the currency-deposit ratio, the expenditure shares appearing in (16) may be computed as

$$(20) \quad s_t^D = \frac{u_t^D D_t}{u_t^D D_t + u_t^C C_t} = \frac{1}{1 + u_t k_t}$$

and

$$(21) \quad s_t^C = \frac{u_t^C C_t}{u_t^D D_t + u_t^C C_t} = \frac{u_t k_t}{1 + u_t k_t}.$$

Equations (10), (13), (14), (16), (20), and (21) combine to yield an expression for the growth rate of the money multiplier m_t^d for the Divisia aggregate:

$$(22) \quad \begin{aligned} \Delta \ln(m_t^d) = & \frac{1}{2} \left(\frac{u_t k_t}{1 + u_t k_t} + \frac{u_{t-1} k_{t-1}}{1 + u_{t-1} k_{t-1}} \right) \Delta \ln(k_t) \\ & - \frac{1}{2} \left(\frac{r_t}{r_t + k_t} + \frac{r_{t-1}}{r_{t-1} + k_{t-1}} \right) \Delta \ln(r_t) - \frac{1}{2} \left(\frac{k_t}{r_t + k_t} + \frac{k_{t-1}}{r_{t-1} + k_{t-1}} \right) \Delta \ln(k_t). \end{aligned}$$

Spindt (1983) shows how (22) extends to the more general case, with multiple types of deposits and reserve assets.

Comparing (15) and (22) reveals that the relative user cost term u_t defined in (19) enters into the money multiplier formula for the Divisia aggregate but not for the corresponding simple sum measure. Intuitively, the two multipliers coincide when $u_t = 1$; in this case, deposits pay no interest, implying that an optimizing agent will be indifferent between the monetary services provided by an additional dollar in deposits and the monetary services provided by an additional dollar in currency and will, in that sense, view deposits and currency as perfect substitutes at the margin. Equations (15) and (22) indicate that movements in the reserve ratio r_t affect the multipliers for the Divisia and simple sum aggregates symmetrically. Since $u_t > 1$ whenever deposits do pay interest, however, the first term inside brackets on the right-hand side of (22) will typically be a larger positive number than the corresponding term in

(15), suggesting that, in particular, a decrease in the currency-deposit ratio k_t that increases the money multiplier for the simple sum aggregate will tend to produce a smaller-sized increase in the money multiplier for the Divisia aggregate.

Two observations, however, force us to stop short of using this comparison between (15) and (22) alone to claim that the money multiplier for the Divisia aggregate will surely be more stable than the money multiplier for the simple sum measure. First, while the growth rate formula (15), like the more familiar level formula (8), implies that a fall in the currency-deposit ratio will always cause the money multiplier for the simple sum aggregate to rise, sufficiently large values of the relative user cost variable u_t may cause the money multiplier to *fall* in response to the same change in k_t : Under such circumstances, m_t^d could exhibit movements that are larger, in absolute value, than the corresponding changes in m_t^s . Second, if most changes in the currency-deposit ratio reflect underlying changes in user costs brought about by exogenous shocks or monetary policy actions that change either the benchmark interest rate ρ_t^B or the spread between the benchmark rate and the own rate on deposits $\rho_t^B - \rho_t^D$, then u_t will vary together with k_t , producing movements in the money multiplier for the Divisia aggregate that are difficult to pin down from an inspection of (22) alone.

To resolve these ambiguities, we combine (15) and (22), which are, by themselves, simply accounting formulas that identify the more fundamental determinants of the money multipliers, with elements drawn from the more detailed, general equilibrium model of the demand for monetary assets presented in Belongia and Ireland (2012). In this model, a representative household economizes on shopping time using an aggregate M_t^a of monetary services obtained from currency C_t and deposits D_t , where the monetary aggregator takes the constant elasticity form

$$(23) M_t^a = [\nu^{1/\omega} C_t^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} D_t^{(1-\omega)/\omega}]^{\omega/(\omega-1)}$$

and the parameters satisfy $0 < \nu < 1$ and $\omega > 0$. With this specification, the household optimally chooses the currency-deposit ratio k_t as a function of the same opportunity cost variable u_t defined above, in (19). In particular,

$$(24) \quad k_t = \left(\frac{\nu}{1-\nu} \right) \left(\frac{1}{u_t} \right)^\omega,$$

a relation that associates an increase in the opportunity cost of currency relative to deposits with a decline in currency-deposit ratio. Equation (24) can be combined with either (15) or (22) to obtain a model of how the money multiplier for either the simple sum or the Divisia aggregate changes in response to movements in the currency-deposit ratio that are ultimately driven by changes in the user cost variable u_t .

Monthly data covering the period from 1980.01 through 2007.12 guide us in calibrating this model: The sample's starting date marks the beginning of the era in which consumers have had access to a wide range of interest-earning deposits, while the terminal date ensures that the figures are not influenced unduly by the extreme fluctuations in monetary variables witnessed (and shown, for instance, in our own Figure 3 from above) during and since the financial crisis. Over this period, the average ratio of Federal Reserve Bank of St. Louis adjusted reserves to deposits was 0.116 for M1 and 0.026 for MZM; hence, in evaluating (15) and (22), the reserve ratio is fixed at either $r = 0.12$ or $r = 0.025$. The average ratio of currency to deposits was 0.620 for M1 and 0.124 for MZM; hence, in (24), the parameter ν is chosen to match a value of $k = 0.60$ or $k = 0.125$.¹³

As noted in Belongia and Ireland (2012), the price aggregator

$$(25) \quad \rho_t^B - \rho_t^a = [\nu(\rho_t^B)^{1-\omega} + (1-\nu)(\rho_t^B - \rho_t^D)^{1-\omega}]^{1/(1-\omega)},$$

¹³ Since (24) implies that the average currency-deposit ratio also depends on the elasticity of substitution parameter, the setting for ν is adjusted as ω varies across the range of examples considered below to maintain these constant values of k .

is dual to the quantity aggregator (23), where ρ_t^a denotes the own rate of return on the true monetary aggregate M_t^a and ρ_t^B and ρ_t^D are, as in (17)-(19), the benchmark return and the own rate on deposits. Data provided through the Center for Financial Stability and also described by Barnett, et al. (2012) include readings on benchmark rates of return ρ_t^B as well as on interest rate aggregates for Divisia M1 and Divisia MZM that can serve as measures of ρ_t^a . Average values over the period 1980.01 – 2007.12 in these data are $\rho^B = 0.078$ for the benchmark rate, $\rho^B - \rho^a = 0.070$ for the M1 aggregate, and $\rho^B - \rho^a = 0.048$ for MZM. We use these figures, together with (25), to back out implied values for ρ^D , the average own rate on deposits in each monetary aggregate, then substitute the average benchmark and deposit rates into (19) to obtain an initial setting for u_t when evaluating (15), (22), and (24) numerically.

With the model thereby calibrated for both M1 and MZM cases, Table 2 shows values of the derivatives $\partial \ln(m_t^s) / \partial u_t$ and $\partial \ln(m_t^d) / \partial u_t$ computed numerically using (15), (22), and (24), for various values of the parameter ω measuring the elasticity of substitution between currency and deposits. Thus, each entry in the table quantifies the response of the money multiplier for either the simple sum or the Divisia aggregate to a shock or monetary policy action that increases the relative user cost of currency and thereby leads, through (24), to a decrease in the currency-deposit ratio. In every case, the results confirm the intuition suggested, earlier, by a direct comparison of (15) and (22). The positive values reported for $\partial \ln(m_t^s) / \partial u_t$ indicate that a shock that causes the currency-deposit ratio to fall causes the simple sum multiplier to rise; but the values reported for $\partial \ln(m_t^d) / \partial u_t$ – still positive, yet distinctly smaller in magnitude – show that the Divisia multiplier rises as well, but by a smaller amount. Thus, while it is possible to concoct examples in which the opposite is true, this realistically calibrated model consistently suggests that the money multiplier for a Divisia aggregate is likely to be more stable than the money multiplier for the corresponding simple sum measure.

Table 3 shows that the relationship predicted by the model also holds true in the U.S. data. For the same period used in the calibration exercise, and for three additional sample periods considered in the forecasting exercises below, the money multiplier for the Divisia M1 or MZM aggregate has a standard deviation that is smaller than that of the multiplier for the corresponding simple sum measure. Whether these smaller month-to-month movements in the Divisia money multiplier are also forecastable is the subject of the next section.

Forecasting Experiments

The multiplier relationships explored above suggest several hypotheses and related experiments that would update the results reported by Spindt (1984). Because one of the potential errors that could move GDP off the target path would be control errors that result from an inability to forecast movements in the Divisia money multiplier out-of-sample, our specific goal here is to evaluate, within the context of a nominal GDP targeting framework, which pair of instrument and monetary aggregate would be most likely to keep nominal GDP on a target path. With two Divisia aggregates – M1 and MZM – as the basis for calculating a future path for money, the central bank must decide which instrument is most closely linked to the behavior of these measures. In the forecasting exercise below, we will consider multipliers derived from four potential instruments of control: Adjusted reserves, total reserves, non-borrowed reserves, and the adjusted monetary base.

Across the interval 1967.01 – 2007.12, we first estimate univariate ARMA models for each multiplier series over three sub-samples. The results of those estimations then are used to calculate errors from static, out-of-sample forecasts over horizons of three years following the terminal data point of the estimation interval. The sub-samples were chosen to evaluate the effects of notable institutional changes, thereby confronting the models with their greatest challenge. The first forecast period covers the period of the Fed's experiment with monetary targeting (1979 – 1982). The second estimation period ends at the time of the Y2K injection of reserves such that the forecast period covers a sample period when the Fed was draining reserves from the system and then dealing with a recession that may have been caused by its

excessively restrictive actions post-Y2K.¹⁴ The third estimation period spans the Great Moderation and ends just prior to the onset of the most recent downturn; most notably, however, this is a period in which any emphasis on money and monetary control had disappeared from discussions of monetary policy.

Before proceeding with the forecasting experiment, it is instructive to present the data in broad overview. Also, because of the wholesale changes in financial markets that occurred in the early 1980s, these summary statistics are reported for three sample periods: 1967.01 – 1979.09, 1984.01 – 1999.12, and the entire 1967.01 – 2007.12 period under study. Although the data in Table 4 reveal very broad similarities across alternative money multipliers and over time, the multiplier derived from non-borrowed reserves exhibits a standard deviation that is substantially larger than that of the base or adjusted reserves; somewhat surprisingly, this result prevails even in the sample period prior to the advent of financial innovations. On its face this does not mean that non-borrowed reserves cannot be used as the central bank's instrument of control or that movements in this multiplier cannot be forecasted out-of-sample, but its consistently larger standard deviation is something to note as the forecasting exercises are undertaken.

The results of the static forecasts are reported in Table 5. Because the foregoing examples for nominal GDP examined only Divisia M1 and MZM we limit our analysis to those variables but these analytics could be applied to other Divisia aggregates as well. Again, we conduct the forecasting experiment over three different periods of time to minimize the chances that any particular result is due to happenstance.¹⁵ Variables chosen to represent the central bank's policy instrument (H) in each table include adjusted reserves (ADJ RES), nonborrowed

¹⁴ In the middle of this estimation period, the Fed reduced reserve requirements on demand deposits from twelve to ten percent in April 1992 and eliminated reserve requirements on nonpersonal time deposits in December 1990.

¹⁵ For example, the relatively low and stable rates of base/reserves/money growth over the last decade may introduce an "illusion" of more precise monetary control. Stability in inflation and interest rates coupled with generally stable real growth also could contribute to this illusion. Or, these results may suggest that the standard money multiplier model be re-examined in the context of modern institutional arrangements with special attention to changes that would tend to enhance monetary control.

reserves (NBR), total reserves (TOT RES), and the adjusted monetary base (BASE) as reported by the Federal Reserve Bank of St. Louis. The cell entries include two error statistics: Root mean squared error (RMSE) and mean absolute error (MAE). We first discuss results for each monetary aggregate in turn, then attempt to draw more general conclusions by reviewing the results as a group, and conclude with a final set of experiments that speak directly to the possibility of using a Divisia monetary aggregate as an intermediate target against the backdrop of the financial crisis of 2008 and the institutional disruptions and changes that followed.

The Divisia M1 Aggregate

The results for the Divisia measure of M1 and the four variables used to represent the Fed's policy instrument indicate that, in all cases and across all sample periods, the monetary base multiplier is associated with the smallest MAE and RMSE. Moreover, in many cases, the error statistics for the base multiplier are an order of magnitude smaller than those of the next closest competitor. Thus, if the Fed were to implement this particular approach to NGDP targeting with Divisia M1 as its guide, the monetary base would appear to be the policy instrument that would generate the smallest control error. With respect to the general results over sample periods, it is interesting to note that, for the most part, the forecast errors are not markedly different across time. This result is surprising because the introduction of new bank liabilities not subject to reserve requirements, the increasing use of "sweep" activities by banks, and the reduction in reserve requirements more generally should have made monetary control subject to *larger* errors.

The Divisia MZM Aggregate

Results for the MZM multipliers indicate that, as for Divisia M1, the multiplier derived from the monetary base produces the lowest forecast errors for each of the three sample periods and those errors are lower by a substantial margin compared to the three other alternatives. Also, as in the case of Divisia M1, the nonborrowed reserves instrument produces the highest MAE and RMSE values. Finally, it is interesting to note that the control errors for the much broader MZM liabilities grouping are similar to those for the narrow M1 aggregate.

Thus, while one reason to choose between a narrow and broad intermediate target often is how closely it is associated with the central bank's instrument of control, there is nothing in Table 5 that would lead one to prefer strongly one Divisia measure to the other; it seems as if the central bank could use the monetary base to influence the path of either with comparable success.

The Recent Financial Crisis and Monetary Control

The foregoing experiments all were conducted over sample periods prior to the recent financial crisis and responses to it by the Federal Reserve that have made, in the minds of many observers, reserves and the monetary base uninformative indicators of central bank actions. Moreover, the introduction of payment of interest on reserves would have weakened, if not severed, any link between traditional measures of central bank liabilities and money to a degree that discussions of monetary control would be all but a moot point, post-2008. Taken at face value, these points might seem correct.

For practical purposes, however, the question facing a central bank always becomes one of what it wishes to accomplish. For example, there is little doubt that sweep accounts represent an effort by banks to evade reserve requirements and this evasion complicates measurement of the money supply. As a bank regulator, however, the Federal Reserve has a number of options to control or eliminate this behavior if, in fact, greater control and more accurate measurement of the money supply were a policy objective.¹⁶

With regard to the post-2008 environment, a similar logic applies. While it is true that the Federal Reserve has added a large volume of assets to its portfolio and begun to pay interest on reserves, these actions have not necessarily distorted all linkages between the Fed's balance sheet and the aggregate quantity of money. Tatom (2011), for example, has derived both balance sheet and multiplier relationships in the aftermath of the financial crisis and found that a relatively straightforward adjustment – subtracting excess reserves from the

¹⁶ Feldstein and Stock (1994, pp. 51-54) make a similar point with respect to their NGDP targeting framework based on simple sum M2. They argue that the Federal Reserve could exercise tighter control over sum M2 by re-extending reserve requirements to the non-M1 components of the broader monetary aggregate and paying interest on reserves as well.

monetary base – would provide an accurate representation of monetary policy actions that would affect the money supply. Plots of this adjusted series, both in log-levels and in growth rates, are shown in Figure 4. The data indicate that, popular discussions to the contrary, this series is relatively stable even across the turbulent period of 2008-2012.

Because the other measures that had been used to represent the monetary policy instrument have been distorted by recent events, we turn finally to this adjusted measure to examine whether the Federal Reserve still could have controlled the behavior of money through the period of the financial crisis. The bottom portion (section D) of Table 5 reports these results using an estimation period of 1993.01 – 2008.12 and an out-of-sample forecasting interval that spans 2009.01 – 2011.12. The results for the Divisia M1 and MZM multipliers indicate that, in both cases, the mean absolute error and RMSE statistics are about three times as large as the comparable statistics as those for the monetary base over the period immediately preceding the financial crisis. The same error statistics, however, remain comparable to those for the other three monetary instruments (non-borrowed reserves, total reserves and adjusted reserves) computed for earlier sample periods. In this context, a disinterested observer could conclude that the financial crisis and the introduction of innovations such as the payment of interest on reserves has weakened the link between the monetary base and both Divisia aggregates, making forecasting errors in its multiplier more like those of other potential instruments of control. On the other hand, because the Fed has been directing its efforts to targeting the federal funds rate which, by construction, has allowed money to vary freely, the same observer could conclude that much could be done to tighten the link between the adjusted monetary base and the monetary aggregates *if* the Fed wished to control their behavior rather than the funds rate.

Conclusion

Because monetary policy, when implemented by manipulation of the federal funds rate, has been viewed to be impotent when the funds rate reaches its zero bound, some observers have suggested that the Fed attempt to meet its dual mandate by setting a target for nominal

GDP. While taking no position on the merits of nominal GDP targeting relative to alternatives that stabilize the price level or the money supply instead, this paper modifies a framework suggested by Working (1923) and similar to that of the P-Star model and illustrates how it might be used to target nominal income. It shows that the central bank can use the monetary base to control the path for either a narrow or broad Divisia monetary aggregate and, through this device, it can keep nominal GDP growing along any desired long-run path. The framework is built on traditional, Quantity Theoretic, foundations, and draws directly from Barnett's (1980) economic approach to monetary aggregation. Its procedures are transparent, convenient to implement and monitor in real time, and therefore easy to communicate to the public as well. If stabilizing nominal income is to be recognized as an objective for monetary policy in the United States, our results pave a clear the path towards achieving that goal.

Figure 1. Actual and Trend Values of the Velocities of Divisia M1 and MZM

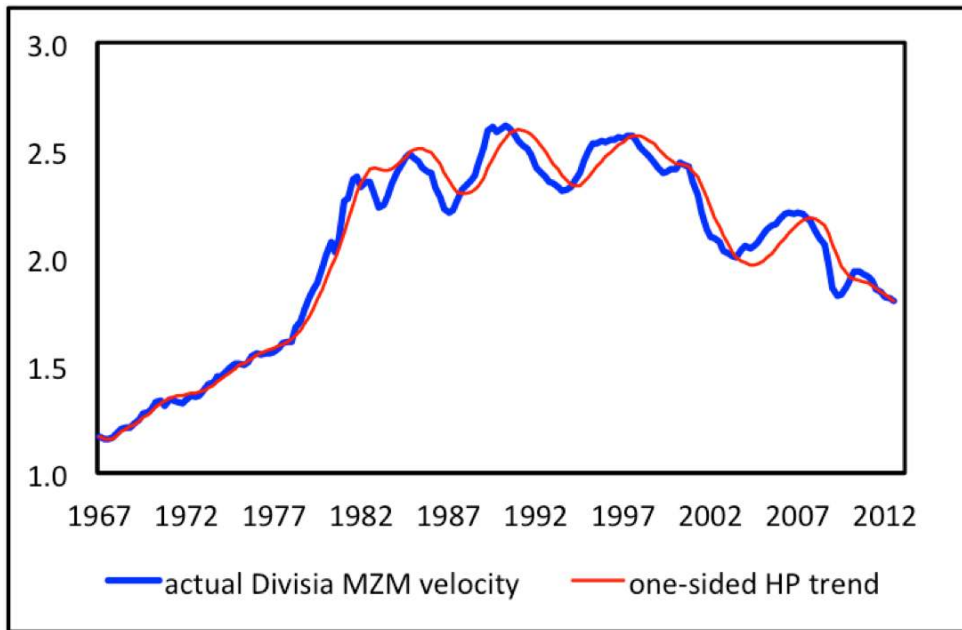
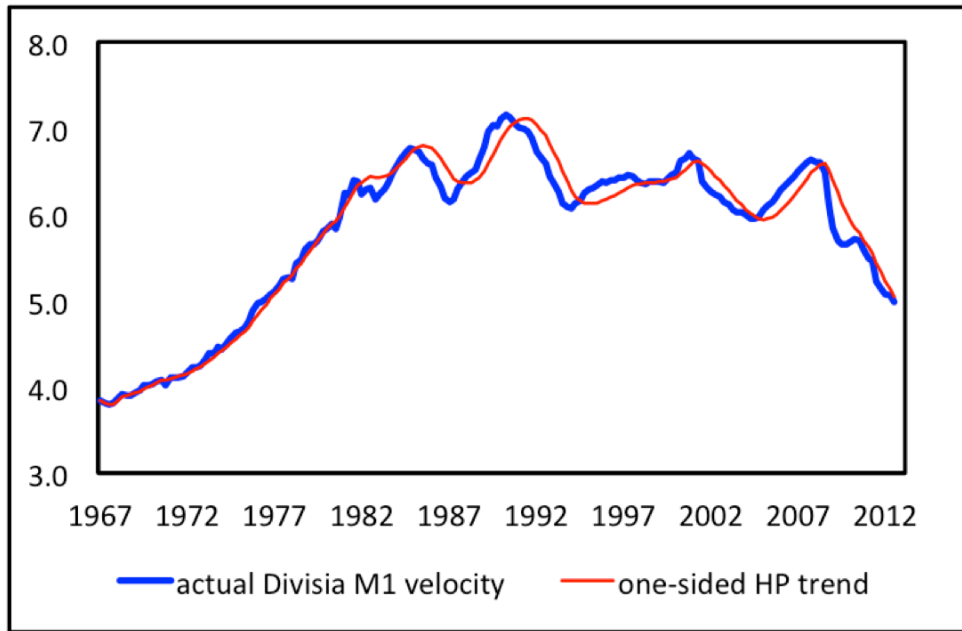
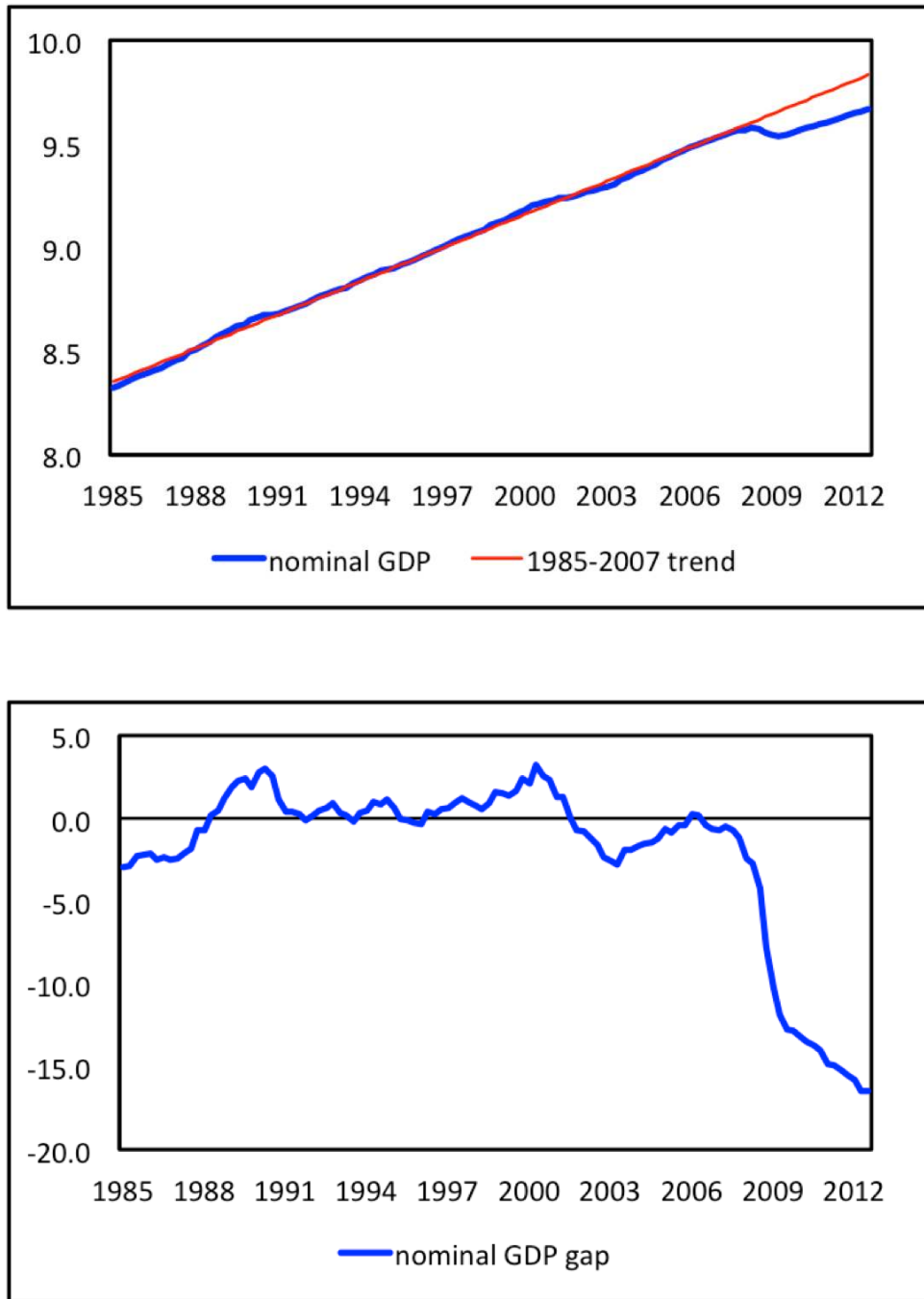
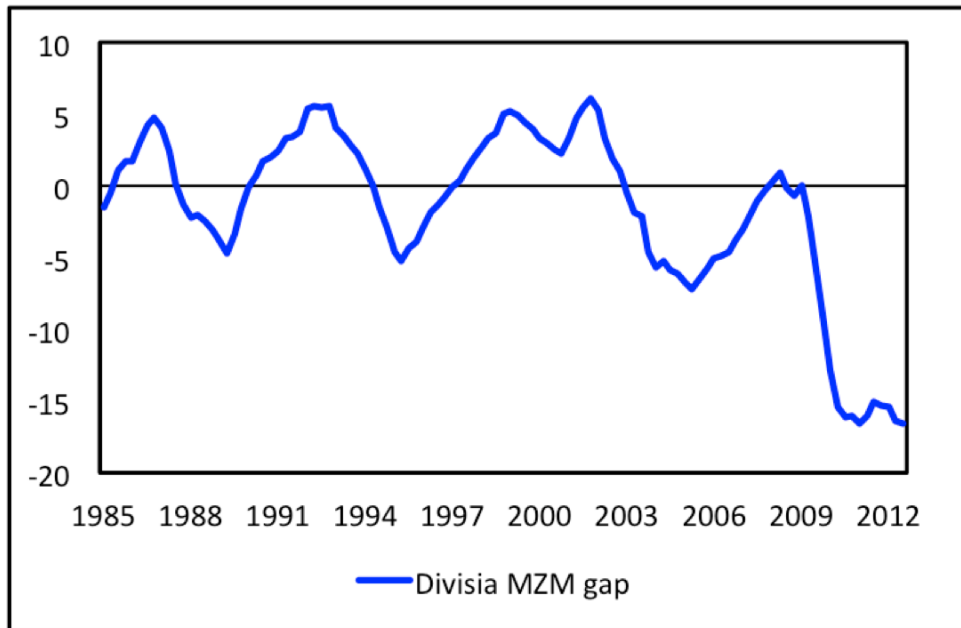
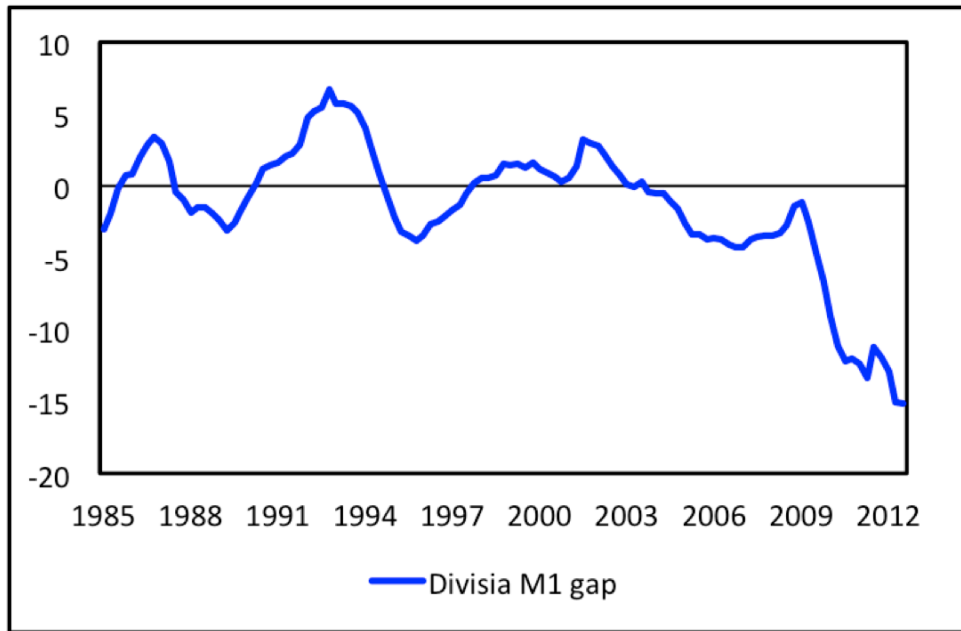


Figure 2. Nominal GDP Relative to Trend



The top panel compares the natural logarithm of nominal GDP to a trend line fitted by ordinary least squares to data from 1985 through 2007. The bottom panel plots percentage-point deviations of actual nominal GDP from the 1985-2007 trend, extrapolated out through 2012.

Figure 3. Gaps for Divisia M1 and Divisia MZM



The M1 and MZM gaps are measured as percentage-point differences between the actual value of each series and the desired value implied by equation (3), where the target value for nominal GDP is given by the trend line displayed in Figure 2.

Figure 4. The Recent Behavior of the Monetary Base Adjusted for Excess Reserves

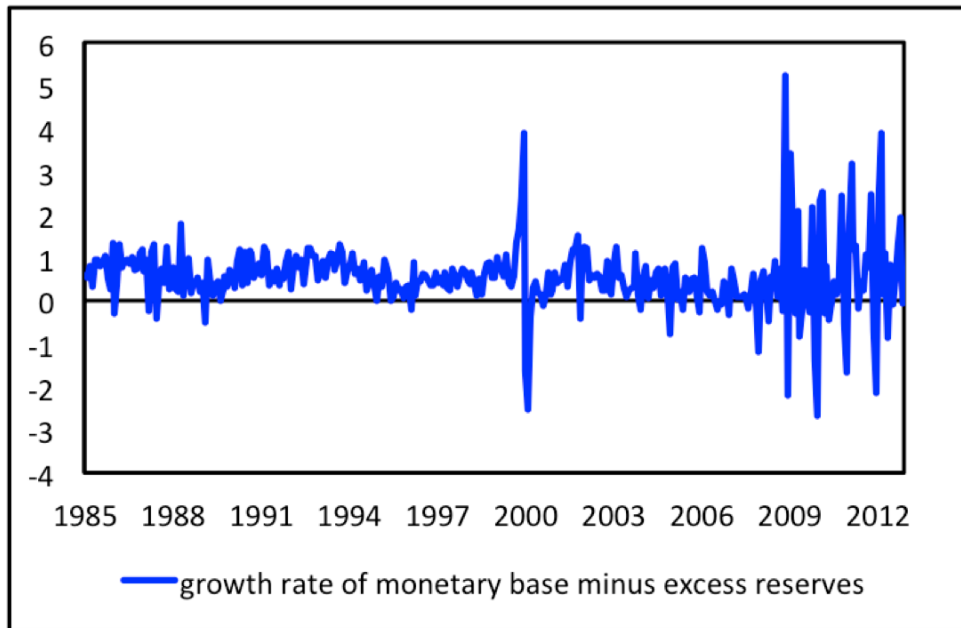
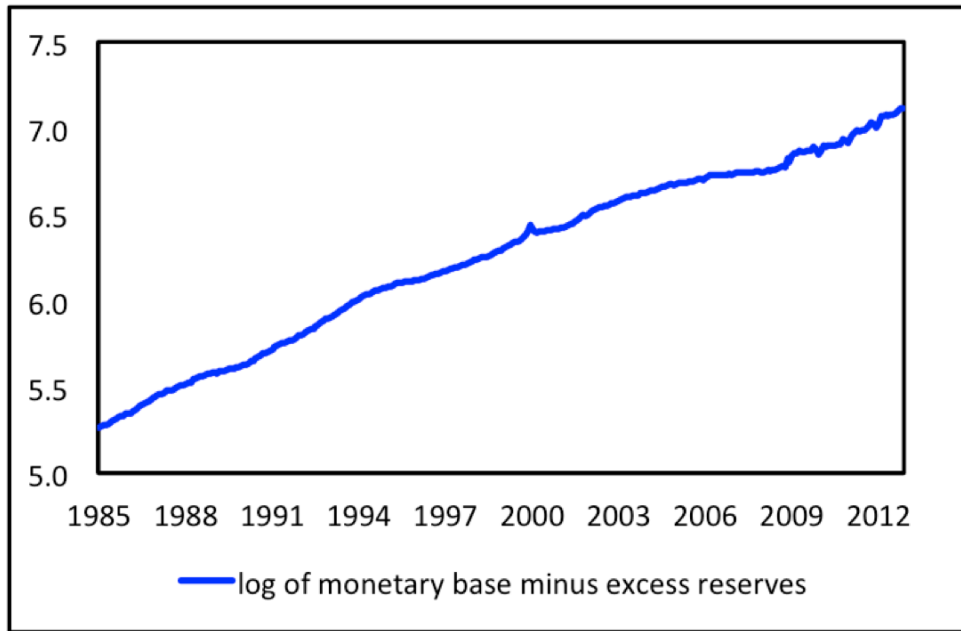


Table 1. Estimated Forecasting Equations for Changes in Nominal GDP Growth

Full Sample: 1967:1 – 2012:3

Dependent variable: $\Delta^2 pq_t$

	Divisia M1		Divisia MZM	
	<u>coefficient</u>	<u>t stat (p value)</u>	<u>coefficient</u>	<u>t stat (p value)</u>
constant	-0.114	-0.455 (0.65)	-0.062	-0.249 (0.80)
$\Delta^2 pq_{t-1}$	-0.605	-8.297 (0.00)	-0.612	-8.316 (0.00)
$\Delta^2 pq_{t-2}$	-0.357	-4.396 (0.00)	-0.358	-4.356 (0.00)
$\Delta^2 pq_{t-3}$	-0.287	-3.516 (0.00)	-0.286	-3.473 (0.00)
$\Delta^2 pq_{t-4}$	-0.071	-0.972 (0.33)	-0.071	-0.968 (0.33)
$pq^*_{t-1} - pq_{t-1}$	0.339	3.730 (0.00)	0.245	3.397 (0.00)
	$\bar{R}^2 = 0.31$	$DW = 2.00$	$\bar{R}^2 = 0.30$	$DW = 1.98$

Pre-1980 Subsample: 1967:1 – 1979:4

Dependent variable: $\Delta^2 pq_t$

	Divisia M1		Divisia MZM	
	<u>coefficient</u>	<u>t stat (p value)</u>	<u>coefficient</u>	<u>t stat (p value)</u>
constant	1.916	2.655 (0.01)	0.807	1.384 (0.17)
$\Delta^2 pq_{t-1}$	-0.714	-5.096 (0.00)	-0.871	-6.337 (0.00)
$\Delta^2 pq_{t-2}$	-0.453	-2.717 (0.01)	-0.662	-4.028 (0.00)
$\Delta^2 pq_{t-3}$	-0.377	-2.268 (0.03)	-0.596	-3.622 (0.00)
$\Delta^2 pq_{t-4}$	-0.093	-0.682 (0.50)	-0.225	-1.629 (0.11)
$pq^*_{t-1} - pq_{t-1}$	2.482	3.472 (0.00)	1.042	3.010 (0.00)
	$\bar{R}^2 = 0.53$	$DW = 1.83$	$\bar{R}^2 = 0.51$	$DW = 1.98$

Post-1980 Subsample: 1980:1 – 2012:3

Dependent variable: $\Delta^2 pq_t$

	Divisia M1		Divisia MZM	
	<u>coefficient</u>	<u>t stat (p value)</u>	<u>coefficient</u>	<u>t stat (p value)</u>
constant	-0.262	-0.989 (0.32)	-0.157	-0.596 (0.55)
$\Delta^2 pq_{t-1}$	-0.469	-5.485 (0.00)	-0.466	-5.371 (0.00)
$\Delta^2 pq_{t-2}$	-0.280	-3.052 (0.00)	-0.267	-2.878 (0.00)
$\Delta^2 pq_{t-3}$	-0.186	-2.038 (0.04)	-0.170	-1.825 (0.07)
$\Delta^2 pq_{t-4}$	-0.077	-0.916 (0.36)	-0.070	-0.817 (0.42)
$pq^*_{t-1} - pq_{t-1}$	0.293	3.486 (0.00)	0.196	2.885 (0.00)
	$\bar{R}^2 = 0.21$	$DW = 2.02$	$\bar{R}^2 = 0.19$	$DW = 1.99$

Table 2. Responsiveness of Simple Sum and Divisia Money Multipliers to Changes in User Costs and the Currency-Deposit Ratio

ω	M1 Calibration		MZM Calibration	
	$\partial \ln(m) / \partial u$		$\partial \ln(m) / \partial u$	
	<u>Simple Sum</u>	<u>Divisia</u>	<u>Simple Sum</u>	<u>Divisia</u>
0.1	0.0383	0.0347	0.0409	0.0369
0.5	0.1909	0.1727	0.2023	0.1823
0.9	0.3426	0.3094	0.3596	0.3231
1.1	0.4181	0.3772	0.4363	0.3914
1.5	0.5682	0.5116	0.5845	0.5222
2.0	0.7541	0.6772	0.7562	0.6708

Table 3. Standard Deviations of Money Multipliers for Simple Sum and Divisia Aggregates

<u>Sample Period</u>	M1		MZM	
	<u>Simple Sum</u>	<u>Divisia</u>	<u>Simple Sum</u>	<u>Divisia</u>
1980.01 – 2007.12	0.0059	0.0051	0.0094	0.0063
1967.01 – 1979.09	0.0039	0.0038	0.0048	0.0043
1984.01 – 1999.12	0.0054	0.0046	0.0062	0.0052
1967.01 – 2007.12	0.0054	0.0047	0.0083	0.0058

Table 4. Descriptive Statistics for Divisia Money Multipliers

A. Sample Period 1967.01 - 1979.09

	<u>Mean</u>	<u>Std. Dev.</u>	<u>Maximum</u>	<u>Minimum</u>
<u>Divisia M1</u>				
BASE	-0.000826	0.003829	0.011111	-0.012540
NBR	0.002128	0.017671	0.049436	-0.092050
TOTAL RES	0.001895	0.008794	0.024304	-0.026502
ADJ RES	0.000422	0.008848	0.032164	-0.025904
<u>Divisia MZM</u>				
BASE	-0.001378	0.004254	0.012706	-0.014226
NBR	0.001576	0.017510	0.049007	-0.092042
TOTAL RES	0.001342	0.008746	0.020768	-0.027951
ADJ RES	-0.000131	0.008956	0.035208	-0.026324

B. Sample Period 1984.01 - 1999.12

	<u>Mean</u>	<u>Std. Dev.</u>	<u>Maximum</u>	<u>Minimum</u>
<u>Divisia M1</u>				
BASE	-0.001473	0.004621	0.013624	-0.029317
NBR	0.002262	0.020722	0.128942	-0.074395
TOTAL RES	0.002371	0.008952	0.028951	-0.016123
ADJ RES	-0.001017	0.018064	0.043840	-0.094980
<u>Divisia MZM</u>				
BASE	-0.001586	0.005310	0.011216	-0.032295
NBR	0.002148	0.020750	0.126564	-0.071686
TOTAL RES	0.002257	0.009776	0.030670	-0.021619
ADJ RES	-0.001131	0.018449	0.043356	-0.096570

C. Sample Period 1967.01 - 2007.12

	<u>Mean</u>	<u>Std. Dev.</u>	<u>Maximum</u>	<u>Minimum</u>
<u>Divisia M1</u>				
BASE	-0.000941	0.004732	0.020715	-0.029317
NBR	0.003006	0.031682	0.438039	-0.279693
TOTAL RES	0.002174	0.022221	0.225323	-0.335268
ADJ RES	0.000652	0.020273	0.172632	-0.191587
<u>Divisia MZM</u>				
BASE	-0.001049	0.005961	0.027523	-0.032295
NBR	0.002898	0.032246	0.441042	-0.284587
TOTAL RES	0.002066	0.023205	0.252127	-0.340162
ADJ RES	0.000544	0.021200	0.176275	-0.196481

Table 5. Results for Forecasting Experiments with Money Multipliers

A. Estimation period: 1967.01 - 1979.09; out-of-sample period: 1979.10 - 1982.09

<u>Instrument</u>	Divisia M1			Divisia MZM		
	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>
BASE	ARMA(1,2)	0.004419	0.005239	ARMA(1,1)	0.006263	0.007945
NBR	ARMA(2,2)	0.024783	0.030728	ARMA(4,4)	0.020427	0.025344
TOT RES	ARMA(2,3)	0.006940	0.010241	ARMA(3,2)	0.009999	0.013780
ADJ RES	ARMA(1,2)	0.008876	0.011062	ARMA(1,2)	0.010545	0.014031

B. Estimation period: 1985.01 - 1999.12; out-of-sample period: 2000.01 - 2002.12

<u>Instrument</u>	Divisia M1			Divisia MZM		
	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>
BASE	ARMA(1,1)	0.004842	0.007828	ARMA(7,7)	0.009557	0.012701
NBR	ARMA(3,3)	0.026168	0.062926	ARMA(3,3)	0.028746	0.067466
TOT RES	ARMA(1,3)	0.029462	0.075954	ARMA(1,3)	0.030866	0.080885
ADJ RES	ARMA(5,2)	0.035735	0.062042	ARMA(4,2)	0.033044	0.061030

C. Estimation period: 1989.01 - 2004.12; out-of-sample period: 2005.01 - 2007.12

<u>Instrument</u>	Divisia M1			Divisia MZM		
	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>
BASE	ARMA(3,3)	0.003943	0.005058	MA(2)	0.003834	0.005130
NBR	ARMA(6,4)	0.027529	0.074589	ARMA(3,5)	0.029071	0.074203
TOT RES	AR(2)	0.015820	0.020533	AR(2)	0.015743	0.019947
ADJ RES	ARMA(2,1)	0.013571	0.017504	ARMA(2,2)	0.017504	0.017167

D. Estimation period: 1993.01 - 2008.12; out-of-sample period: 2009.01 - 2011.12

<u>Instrument</u>	Divisia M1			Divisia MZM		
	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>	<u>Model</u>	<u>MAE</u>	<u>RMSE</u>
ADJ BASE	ARMA(4,2)	0.014501	0.017471	ARMA(1,1)	0.011935	0.014888

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