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Abelian Confinement Mechanism in QCD*)

Tsuneo SUZUKI

Department of Physics, Kanazawa University, Kanazawa 920

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It is shown explicitly that quarks and gluons, dynamical as well as static, are confined in QCD if color magnetic monopoles in the abelian gauge condense. The mass generation essential for the confinement occurs from the spontaneous breaking of color magnetic $U_m(1)$ symmetry, whereas color electric $U_e(1)$ remains unbroken.

To understand the mechanism of quark confinement is very important in QCD. Monopoles exist in QCD since color $SU(3)$ is compact. Mandelstam[1] and 'tHooft[2] suggested that the confinement phase may be realized as a superposition of the monopoles. Especially, the 'tHooft idea[3] of abelian projection is interesting. After the abelian projection is done, QCD is reduced to an abelian theory with charges and monopoles.[4)~6)] If the monopoles make Bose condensation, the confinement is expected to occur.

In a previous paper,[7] a confinement model based on the 'tHooft idea[3] has been proposed. Static confining potentials of a linear form are derived explicitly. Comparison of the results with experiments leads to interesting predictions in the case of physical $SU(3)$ QCD.[8]

It is the purpose of this paper to study the abelian confinement mechanism in detail and to show clearly that the only assumption of the monopole condensation in the maximally abelian gauge is enough to prove confinement of dynamical as well as static quarks and gluons. The idea of the monopole condensation is very hopeful. Kronfeld et al.[9] recently showed numerically that the monopole condensation occurs in the confining phase of QCD.

We start with the usual $SU(2)$ QCD Lagrangian

$$\mathcal{L}=-\frac{1}{4}(\partial_\mu \boldsymbol{A}_\nu-\partial_\nu \boldsymbol{A}_\mu+e\boldsymbol{A}_\mu\times \boldsymbol{A}_\nu)^2 . \tag{1}$$

The abelian projection[3] is to fix the gauge keeping the maximal torus group. An explicit form of the gauge-fixing condition is not necessary in the following discussion. A renormalizable gauge such as $\alpha^{-1}(D_\mu A^{+\mu})(D_\nu^* A^{-\nu})$ with $D_\mu=\partial_\mu+ieA_\mu$ which preserves the torus group $U(1)$ is sufficient. Here $\sqrt{2}A^\pm=A^1\pm iA^2$ and $A=A^3$. Then the system is regarded as an abelian $U(1)$ theory with charges and magnetic monopoles. The magnetic monopoles appear in the abelian channel.

To see the monopole contribution explicitly, the Lagrangian introducing an abelian dual vector field B_μ and a constant vector n_μ following Zwanziger are rewritten:[5]

$$\mathcal{L}=\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3 , \tag{2}$$

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$$\mathcal{L}_1=-\frac{1}{2n^2}[n\cdot(\partial_\wedge A)]^2-\frac{1}{2n^2}[n\cdot(\partial_\wedge A)]\cdot[n\cdot(\partial_\wedge B)^d]+(A\to B,\ B\to -A)\,, \qquad (3)$$

$$\mathcal{L}_2=D_\mu A_\nu^+(D^*{}_\wedge A^-)^{\nu\mu}-ie(\partial_\wedge A)_{\mu\nu}A^{+\mu}A^{-\mu}+\frac{e^2}{4}(A^+{}_\wedge A^-)^2\,, \qquad (4)$$

$$\mathcal{L}_3=k_\mu B^\mu\,, \qquad (5)$$

where the Zwanziger's notation was used:[5] $(\partial_\wedge A)_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $2(\partial_\wedge B)^d_{\mu\nu}=\varepsilon_{\mu\nu\alpha\beta}(\partial_\wedge B)^{\alpha\beta}$. k_μ is the monopole current expressed as an integral over the monopole trajectories.[7),8)] To get the partition function, one has to sum up all possible monopole contributions. The summation can be done and the result is expressed as a functional integral with respect to one (or more) complex scalar field(s).[10)] For example, if monopoles with the lowest magnetic charge $g=4\pi e^{-1}$ alone contribute, $\mathcal{L}_3$ in (5) is replaced by $\mathcal{L}_3=|\partial_\mu\varphi+igB_\mu\varphi|^2$ with a complex scalar field φ. Then k_μ becomes $ig(\varphi\partial_\mu\varphi^*-\varphi^*\partial_\mu\varphi)+2g^2B_\mu|\varphi|^2$. The color electric current j_μ is derived from (4) as $j_\mu=\delta(\int d^4x\mathcal{L}_2)/\delta A^\mu$. Note that System (2) is invariant under the color electric $U_e(1)$ and the color magnetic $U_m(1)$ symmetries.

The equation of motion of A_μ is

$$\frac{1}{n^2}[(n\cdot\partial)^2A^\mu-(n\cdot\partial)n^\mu(\partial\cdot A)-(n\cdot\partial)\partial^\mu(n\cdot A)+n^\mu\partial^2(n\cdot A)$$
$$-\varepsilon^{\mu\alpha\beta\nu}(n\cdot\partial)n_\alpha\partial_\beta B_\nu]=j^\mu\,. \qquad (6)$$

A similar equation holds for B_μ.

We quantize System (2) canonically. Since the quantization of the charged gluon part is conventional, we discuss only the abelian part. All A_μ and B_μ fields are not independent and there are constraints. Introduce new constant vectors $\boldsymbol{\tau}_i$ $(i=1,2)$ such that $\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2=\boldsymbol{n}$ ($n^0=0$ is adopted for simplicity). Define $V_\mu^{(i)}=(A_\mu, B_\mu)$ and canonical momenta of $V_\mu^{(i)}$ as $\pi^\mu_{(i)}$. There are four second-class constraints $\boldsymbol{\tau}_i\cdot(\boldsymbol{\pi}_{(j)}-2^{-1}\varepsilon_{jk}\boldsymbol{\nabla}\times\boldsymbol{V}^{(k)})=0$ and four first-class constraints $\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_{(1)}-j^0=\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_{(2)}-k^0=\pi^0_{(j)}=0$.[6)] Corresponding to the latter constraints, the following (abelian) gauge conditions are chosen:

$$\boldsymbol{n}\cdot\boldsymbol{A}=\boldsymbol{n}\cdot\boldsymbol{B}=A^0=B^0=0\,. \qquad (7)$$

Usual quantization gives all equal-time commutators. Using the constraints, it is possible to eliminate either $\boldsymbol{A}$ or $\boldsymbol{B}$ completely.[6)]

Let us next derive the propagators of the fields A_μ and B_μ. Write the Fourier transforms of $i^{-1}\langle 0|T(V^{(i)\mu}(x)V^{(j)\nu}(y))|0\rangle$ as $D_F^{\mu\nu}(i=j=1)$, $E_F^{\mu\nu}(i=j=2)$ and $Y_F^{\mu\nu}(i=1, j=2)$, respectively. It is easy to derive the free propagators

$$D_F^{\mu\nu}(k)=E_F^{\mu\nu}(k)=-\frac{1}{k^2}\omega^{\mu\nu} \quad \text{and} \quad Y_F^{\mu\nu}(k)=-\frac{1}{k^2(n\cdot k)}\zeta^{\mu\nu}\,,$$

where $\omega_{\mu\nu}=g_{\mu\nu}-(n\cdot k)^{-1}(n_\mu k_\nu+n_\nu k_\mu)+n^2(n\cdot k)^{-2}k_\mu k_\nu$ and $\zeta_{\mu\nu}=\varepsilon_{\mu\nu\rho\sigma}n^\rho k^\sigma$. $(n\cdot k)^{-1}$ is interpreted as the principal part.

Brandt et al.[11)] proved that the full Green functions of gauge-invariant operators like the currents are n_μ-independent to all orders. When use is made of the generating functional expressed in terms of the Hori operator,[12),13)] it is possible to derive the following integral equation with respect to the full propagator:

$$\langle\!\langle 0|T(V_\mu^{H(i)}(x)V_\nu^{H(j)}(y))|0\rangle\!\rangle=\langle 0|T(V_\mu^{(i)}(x)V_\nu^{(j)}(y))|0\rangle+\int d^4x'd^4y'$$

$$\times\langle 0|T(V_\mu^{(i)}(x)V_\alpha^{(a)}(x'))|0\rangle\langle 0|T(V_\nu^{(j)}(y)V_\beta^{(b)}(y'))|0\rangle L_{ab}^{\alpha\beta}(x'-y'), \qquad (8)$$

$$L_{ab}^{\alpha\beta}(x'-y')\equiv\langle\!\langle 0|T\left\{\frac{\delta L_{2+3}}{\delta V_\alpha^{H(a)}(x')}\frac{\delta L_{2+3}}{\delta V_\beta^{H(b)}(y')}+\frac{1}{i}\frac{\delta^2 L_{2+3}}{\delta V_\alpha^{H(a)}(x')\delta V_\beta^{H(b)}(y')}\right\}|0\rangle\!\rangle, \quad (9)$$

where $L_{2+3}\equiv\int d^4z(\mathcal{L}_2+\mathcal{L}_3)$, $V_\mu^{H(i)}$ stands for the Heisenberg field of $V_\mu^{(i)}$ and the vacuum $|0\rangle\!\rangle$ is the true vacuum. The operators in the T-product of (9) are all gauge-invariant. Since $L_{ab}^{\alpha\beta}(x'-y')$ are the full Green functions, they are n_μ-independent. The Fourier transforms of $L_{11}^{\alpha\beta}$ and $L_{22}^{\alpha\beta}$ are expressed as $i(g^{\alpha\beta}k^2-k^\alpha k^\beta)C(k^2)$ and $i(g^{\alpha\beta}k^2-k^\alpha k^\beta)D(k^2)$, respectively. $L_{12}^{\alpha\beta}$ and $L_{21}^{\alpha\beta}$ vanish since it is impossible to write them in an n_μ-independent way. Solving Eq. (8), the full propagators in terms of $C(k^2)$ and $D(k^2)$ are[14)]

$$D_F'^{\mu\nu}(k)=-\frac{1}{k^2}(1+C+D)\omega^{\mu\nu}+\frac{n^2}{(n\cdot k)^2}D\lambda^{\mu\nu},$$

$$E_F'^{\mu\nu}(k)=-\frac{1}{k^2}(1+C+D)\omega^{\mu\nu}+\frac{n^2}{(n\cdot k)^2}C\lambda^{\mu\nu},$$

$$Y_F'^{\mu\nu}(k)=-\frac{1}{k^2(n\cdot k)}(1+C+D)\zeta^{\mu\nu},$$

where $\lambda_{\mu\nu}=g_{\mu\nu}-(n^2)^{-1}n_\mu n_\nu$. Here $(n\cdot k)^{-2}$ should be interpreted as $2^{-1}[(n\cdot k+i\varepsilon)^{-2}+(n\cdot k-i\varepsilon)^{-2}]$.[8)]

It is possible, on the other hand, to express the full propagators in terms of the proper functions which are in general n_μ-dependent. The proper functions of $(A_\mu A_\nu)$, $(B_\mu B_\nu)$ and $(B_\mu A_\nu)=-(A_\mu B_\nu)$ are expressed as $i\Xi_{\mu\nu}$, $i\Sigma_{\mu\nu}$ and $i\Omega_{\mu\nu}$. Then there are four equations:

$$D_F'^{\mu\nu}=D_F^{\mu\nu}-(D_F^{\mu\alpha}\Xi_{\alpha\beta}-Y_F^{\mu\alpha}\Omega_{\alpha\beta})D_F'^{\beta\nu}+(Y_F^{\mu\alpha}\Sigma_{\alpha\beta}+D_F^{\mu\alpha}\Omega_{\alpha\beta})Y_F'^{\beta\nu}, \qquad (10)$$

$$Y_F'^{\mu\nu}=Y_F^{\mu\nu}-(D_F^{\mu\alpha}\Xi_{\alpha\beta}-Y_F^{\mu\alpha}\Omega_{\alpha\beta})Y_F'^{\beta\nu}-(Y_F^{\mu\alpha}\Sigma_{\alpha\beta}+D_F^{\mu\alpha}\Omega_{\alpha\beta})E_F'^{\beta\nu}, \qquad (11)$$

$$Y_F'^{\mu\nu}=Y_F^{\mu\nu}-(E_F^{\mu\alpha}\Sigma_{\alpha\beta}-Y_F^{\mu\alpha}\Omega_{\alpha\beta})Y_F'^{\beta\nu}-(Y_F^{\mu\alpha}\Xi_{\alpha\beta}+E_F^{\mu\alpha}\Omega_{\alpha\beta})D_F'^{\beta\nu}, \qquad (12)$$

$$E_F'^{\mu\nu}=E_F^{\mu\nu}-(E_F^{\mu\alpha}\Sigma_{\alpha\beta}-Y_F^{\mu\alpha}\Omega_{\alpha\beta})E_F'^{\beta\nu}+(Y_F^{\mu\alpha}\Xi_{\alpha\beta}+E_F^{\mu\alpha}\Omega_{\alpha\beta})Y_F'^{\beta\nu}. \qquad (13)$$

$\Xi_{\mu\nu}$ and $\Sigma_{\mu\nu}$ are symmetric tensors. There are four symmetric tensors $g_{\mu\nu}$, $k_\mu k_\nu$, $(n_\mu k_\nu+n_\nu k_\mu)$ and $n_\mu n_\nu$. But all propagators are orthogonal to n_μ, so that the last two tensors do not contribute. $\Xi_{\mu\nu}$ and $\Sigma_{\mu\nu}$ are defined as $\Xi_{\mu\nu}=\Xi_1(g_{\mu\nu}k^2-k_\mu k_\nu)+\Xi_2k_\mu k_\nu$ and $\Sigma_{\mu\nu}=\Sigma_1(g_{\mu\nu}k^2-k_\mu k_\nu)+\Sigma_2k_\mu k_\nu$. $\Omega_{\mu\nu}$ is antisymmetric and then is written as $\Omega\varepsilon_{\mu\nu\lambda\sigma}n^\lambda k^\sigma$. Ξ_i, Σ_i and Ω are functions of k^2 and $n\cdot k$.

Equations (10)~(13) can be solved. Comparing the two expressions of the full propagators, all proper functions are written only in terms of $C(k^2)$ and $D(k^2)$. Important are the following relations:

$$\Xi_1 = \frac{C(1+D)(n\cdot k)^2}{(1+C+D)(n\cdot k)^2 + CDn^2k^2}, \tag{14}$$

$$\Sigma_1 = \frac{D(1+C)(n\cdot k)^2}{(1+C+D)(n\cdot k)^2 + CDn^2k^2}. \tag{15}$$

It should be emphasized that the n_μ-dependences of Ξ_1 and Σ_1 have been determined.

Up to this point there was no assumption but here dynamical assumption is given: The monopole condensation in the abelian channel occurs in the infrared region in QCD.[3] This is the only but basic assumption of this paper. The monopole condensation means a spontaneous breaking of the color magnetic $U_m(1)$ symmetry. Then the Higgs mechanism works. The proper function $\Sigma_1(k^2, n\cdot k)$ has a massless pole:

$$\Sigma_1(k^2, n\cdot k) = \frac{m^2}{k^2} + \cdots, \tag{16}$$

where m^2 is a mass parameter. Using the fact that $C(k^2)$ and $D(k^2)$ are functions of k^2 alone, the behavior (16) is realized when $D(k^2) = m^2(k^2 - m^2)^{-1} + O((k^2)^2)$ and $C(k^2) = O((k^2)^2)$ around $k^2 \sim 0$. The behaviors of the full propagators are determined to be

$$D_F'^{\mu\nu}(k) = -\frac{1}{k^2 - m^2}\omega^{\mu\nu} + \frac{n^2}{(n\cdot k)^2}\frac{m^2}{k^2 - m^2}\lambda^{\mu\nu} + O\left(k^2, \frac{(k^2)^2}{(n\cdot k)^2}\right), \tag{17}$$

$$E_F'^{\mu\nu}(k) = -\frac{1}{k^2 - m^2}\omega^{\mu\nu} + O\left(k^2, \frac{(k^2)^2}{(n\cdot k)^2}\right), \tag{18}$$

$$Y_F'^{\mu\nu}(k) = -\frac{1}{(n\cdot k)}\frac{1}{k^2 - m^2}\zeta^{\mu\nu} + O\left(\frac{k^3}{(n\cdot k)}\right). \tag{19}$$

One finds that the massless poles disappear. The $(n\cdot k)^{-1}$ and the $(n\cdot k)^{-2}$ terms defined above do not produce a massless pole singularity.

The above behaviors lead to interesting results. First we discuss the vacuum energy under the influence of static charged sources such as $j^\mu(x) = eg^{\mu 0}\{\delta(\boldsymbol{x} - \boldsymbol{a}) - \delta(\boldsymbol{x} - \boldsymbol{b})\}$. To calculate the vacuum energy exactly needs a numerical method. But when the correlation length ξ included in the monopole condensation[8] is much less than the penetration length $\lambda(=m^{-1})$, the energy is well approximated by the following integral of the tree diagram with the $A-A$ propagator (17):[15),16)]

$$V(r) = -e^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\left\{\frac{1}{\boldsymbol{k}^2 + m^2} + \frac{m^2}{\boldsymbol{k}^2 + m^2}\frac{1}{(\boldsymbol{n}\cdot\boldsymbol{k})^2}\right\}, \tag{20}$$

where $\boldsymbol{r} = \boldsymbol{a} - \boldsymbol{b}$. This was calculated approximately in the previous paper:[7),8)]

$$V(r) = -\frac{e^2}{4\pi}\left[\frac{e^{-mr}}{r} + m^2K_0\left(\frac{\sqrt{2}\xi}{\lambda}\right)r\right]. \tag{21}$$

The abelian monopole condensation leads to the confinement of static charges.

How about dynamical charged particles? Neither the Wilson-loop nor the potential is useful. We discuss the color electric charge operator Q_e directly. Using the equation of motion (6), it is expressed as

$$Q_e = \int d^3x j^0(x) = \int d^3x (\boldsymbol{n}\cdot\boldsymbol{\nabla})(\boldsymbol{n}\times\boldsymbol{\nabla})\cdot\boldsymbol{B}\,, \tag{22}$$

where the gauge condition (7) was used. The integral is a surface term. As seen from (18) and (19), the B_μ field has no massless pole, so that we get a conclusion:

$$Q_e = 0\,. \tag{23}$$

The dynamical color-charged particles are also confined. On the other hand, the magnetic charge $Q_m = \int k^0 d^3x$ is ill-defined because Q_m does not annihilate the vacuum. We have used the economical gauge (7). When a covariant gauge is adopted, (23) should hold in the positive-definite physical Hilbert space. The physical space is composed of color-electric neutral states alone. They are bound states of quarks and gluons like $(Q\bar{Q})$ and (QQQ). Also there appear neutral axial-vector and scalar fields as shown in Ref. 8). The existence of the latter states is the prediction of the abelian confinement mechanism. Note that the massless charged ghosts related to the non-abelian gauge-fixing are also confined.

A few comments are in order.

(1) The mass generation which is essential in the proof of (23) originates from the spontaneous breaking of the color magnetic $U_m(1)$ symmetry. It is important that the color electric $U_e(1)$ symmetry remains exact, although A_μ has a massive pole. When the B_μ fields are integrated out in the presence of the spontaneously generated mass, there appears a new term added to the free Lagrangian of A_μ:

$$\mathcal{L}_0 = \frac{1}{2}A_\mu(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)A_\nu + \frac{1}{2}A_\mu X^{\mu\nu}A_\nu\,,$$

where $X_{\mu\nu} = m^2[(n\cdot\partial)^2 + n^2m^2]^{-1}\varepsilon_{\alpha\beta\gamma\mu}\varepsilon^{\alpha}_{\delta\eta\nu}n^\beta n^\delta\partial^\gamma\partial^\eta$. This new term gives rise to the massive pole without breaking the $U_e(1)$ symmetry.

(2) The Zwanziger model[5] (3) $+j_\mu A^\mu + k_\mu B^\mu$ is itself very interesting. Since it has $U_e(1)\times U_m(1)$ symmetries, four different phases exist in principle. The Coulomb phase where neither $U(1)$ is broken was studied by Zwanziger.[5] When $U_e(1)$ alone is broken spontaneously, it is the Higgs phase. Then Q_m vanishes. This corresponds to the (relativistic) Meissner effect. The case dual to this has been studied in this paper. It is the confinement phase. What happens when both $U(1)$ are broken spontaneously? This case does not give a consistent theory, because $\Xi_1 \sim m^2k^{-2}$ and $\Sigma_1 \sim m^2k^{-2}$ are not compatible with (14) and (15). The non-existence of this phase is in agreement with the assertion of Ref. 17).

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