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ABOUT THE SBRA METHOD APPLIED IN MECHANICS OF CONTINENTAL PLATES

O APLIKACI METODY SBRA V GEOMECHANICE KONTINENTÁLNÍCH DESEK

Abstract

This article shows how the probabilistic SBRA (Simulation-Based Reliability Assessment) Method (i.e. stochastic Monte Carlo approach) is applied to the model of the behaviour of the lithosphere of the Earth (geomechanical model). The main idea is based on the genesis of thermoelastic waves (i.e. influence of our Son) due to thermal expansion of the rock mass and the ratcheting mechanisms. SBRA method applied in this problem is a new and innovative trend for modelling in mechanics.

Abstrakt

Tento článek ukazuje jak je pravděpodobnostní SBRA (Simulation-Based Reliability Assessment) metoda (tj. stochastický Monte Carlo přístup) aplikovaný v modelu chování litosféry planet Země (geomechanický model). Základní myšlenka je postavena na genezi termoelastické vlny (tj. vliv našeho Slunce) vznikající vlivem teplotní roztažnosti masívu a západkovém mechanismu. Metoda SBRA použitá v tomto problému je novým a inovativním trendem modelování v mechanice.

Keywords

Lithosphere, continental plate, field of temperature, heat conductivity factor, ratcheting, stress tensor, thermoelastic wave, strain tensor, creep, SBRA method.

1 INTRODUCTION

Our work extends the works of M. Hvozdara et al. [3] and J. Berger [7], who detected and described the behavior of the thermoelastic wave, and the work of J.Croll [8], who described the ratcheting mechanism.

In the first step, we created the geomechanical model of the lithosphere, see Fig. 1.1, where we tested the directions of the relative expansion of the lithosphere plate in two places. This model assumes that the main part of the deformation depends on the solar irradiation, see Fig. 1.2. We used the simplified mathematical model which consists of linear differential equation focused to the Eurasian continental plate. The probabilistic Simulation-Based Reliability Assessment (SBRA) method showed the possibility to simulate some physical quantities (ratcheting, probability of a

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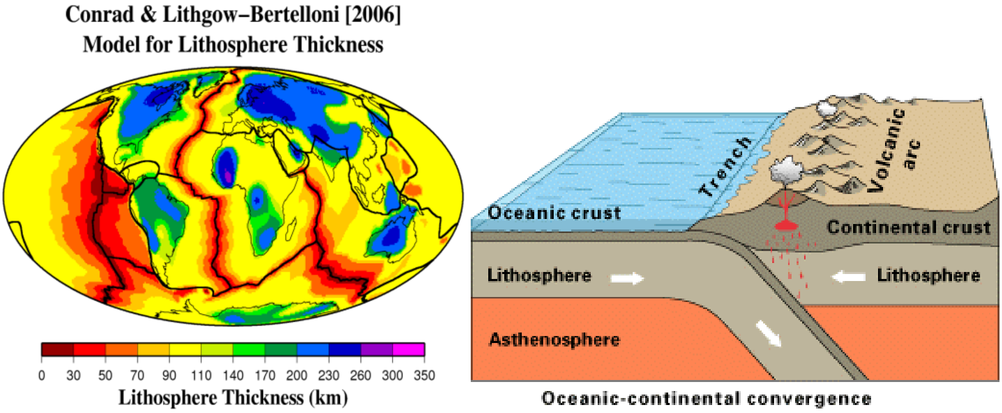
possible earthquake etc.) and the limits of the linear model where the non-linear behavior and the ratcheting begin.



Oceanic-continental convergence

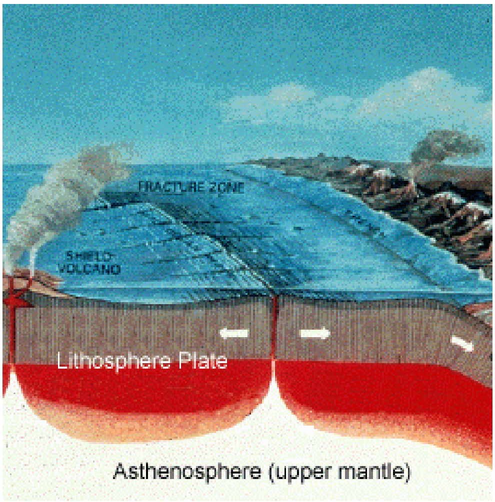


Fig. 1.1 Lithosphere thickness variations and its problems, see reference [9], [10] and internet

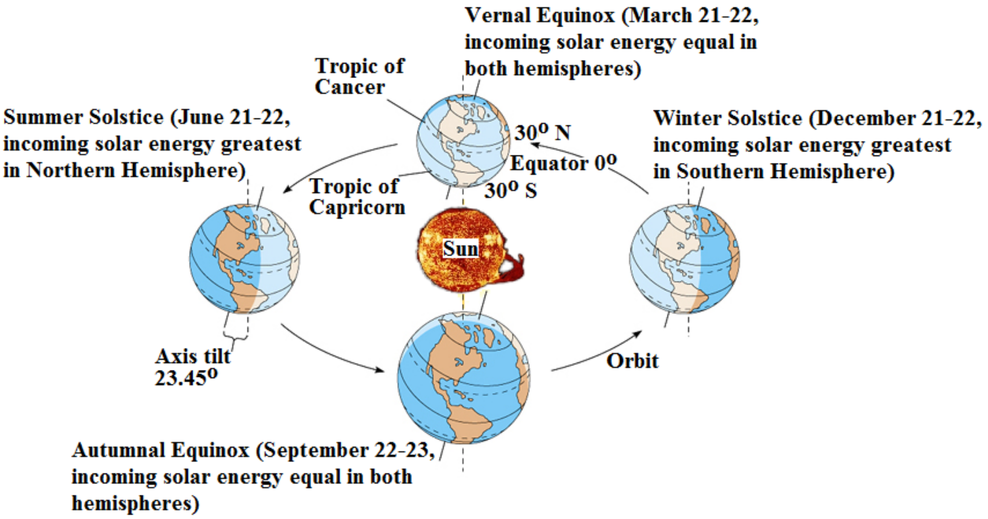


Fig. 1.2 Solar irradiation

The relative movement of the continent against oceanic crust can be explained in Fig.1.3. It was found that the diurnal cycle has larger impact to the movement of the plates than the annual cycle. Therefore, the occurrence time for the creep, slow slip events or earthquakes is not dominantly in the winter time, but at any time. The another impact factor is the Earth's rotation, which can accelerate or decelerate the proposed mechanism. For more information, see reference [11] and.

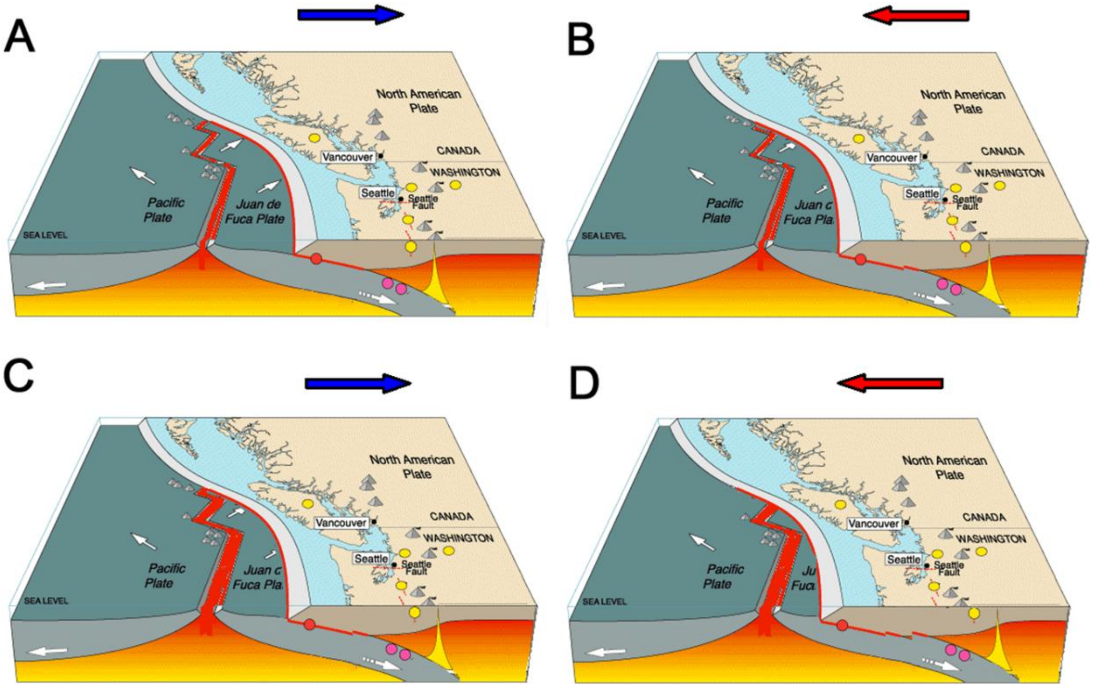


Fig. 1.3 Example of a relative movement of the continent against oceanic crust: A) – in summer is the continent contracted and pulls the ocean crust, the strength limit for the friction is higher than tensile strength limit . Rift is opened and lava can flow up to fill it. B) – in winter is the continent expanded and pushes the ocean crust against another continent. Rift is closed and the slip events, tremors, creep can occurred between continental and oceanic crusts because the strength limit in compression is higher than friction . C) - next summer is the situation similar with one exception – the rift is filled by lava, which creates the ratchets, which protect the movement of the ocean crust to the same position as before. This mechanism leads to the spreading of the sea floor. Only this ratcheting can explain the subduction of the whole rift under the continent (as it can be seen in the case of Juan de Fuca plate). It can be made due to another rift westerly of the Juan de Fuca plate. The convectional currents in the mantle cannot explain such behaviour.

2 NUMERICAL MODEL, PROCESS OF CALCULATION OF STRAIN ON BOUNDARIES OF THE CONTINENTAL PLATE

The temperature profile in continental rocks was calculated in one-day steps and in one-year cycles for each latitude. The same temperature profile was calculated in 30-minutes steps in one-day cycles. Both cycles were superimposed and the relative temperature development was calculated in 30-minutes steps during one year.

Than the relative strains ε_{xx} , ε_{yy} and ε_{zz} /1/ in the far field under a surface can be evaluated as an integral (or they are directly proportional to this integral) of temperature (depth) profile multiplied by linear thermal expansion coefficient α $/K^{-1}/$. Because the attenuation of thermal wave with the depth is high (due to low thermal expansion of rock), the far field is supposed to be in order of one kilometre outside the expanded block of lithosphere. We are able to evaluate the equivalent (normalised) relative strains of each block and to evaluate the principal component of stress tensor and its relative development in time. It depends mainly on the geometry and geographical position of the continents. We calculated the relative values of the principal component of the stress tensor.

The maximum and minimum strain in diurnal period was evaluated and the annual strain development was calculated for the points on the border of continents. Japan (140.625E, 50N) and Italy (16.47E, 40N) were chosen as examples. The principal component of relative strain changes its direction in time in both cases. At the end of March the direction is towards the continent. It is the result of the contraction of the Eurasian lithosphere plate after winter. In the case of Italy, it is in the direction of the NE and, in the case of Japan, it is in the direction of the NW. The opposite direction can be seen in September. This is the result of the expansion of the Eurasian lithosphere plate after summer. The results are in accordance with the field GPS measurement of continental deformations at both places.

Cyclic variations of temperature $\vartheta_{(h,t)}$ /K/ close to Earth‘s surface can be evaluated for homogeneous isotropic environment by equation:

$$\vartheta_{(h,t)} = \vartheta_0 e^{-h\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right), \tag{2.1}$$

where ϑ_0 /K/ is an amplitude of temperature variations on the Earth’s surface as a function of time t /s/, h /m/ is depth, $\omega = 2\pi/\tau$ /s^{-1}/ (τ /s/ is period), a $\in < 7.10^{-7};\ 22.10^{-7} >$ /m^{2}s^{-1}/ is temperature conductance coefficient.

The character of the stress tensor behavior of rock is entirely dependent on temperature variations in layers close to the surface due to the small velocity of the temperature penetration into depth and its large attenuation. The equivalent (normalised) relative strain at point is given:

$$\varepsilon \approx \int_0^H \alpha\ \Delta\vartheta_{(t,h)}\ dh\ , \tag{2.2}$$

H /m/ is depth, which the heat penetrates during five periods with $a = 13.10^{-7} m^2 s^{-1}$ (mean value). To estimate the ratio between the lowest and the highest strain and their direction at the border of the continent it will be sufficient to simplify the function $\alpha\,\Delta\vartheta_{(t,h)}$ inside the integral (2.2) in this way:

$$\Delta\bar{\vartheta}_{(t,h)} = \frac{T_{(t,h)} - T_s}{T_s}\ e^{-h\sqrt{\omega/2a}}, \tag{2.3}$$

T_s /K/ is the mean temperature on the surface, $T_{(t,h)}$ /K/ is the relative temperature curve on the surface:

$$T_{(t,h)} = T_s \cos(\omega t - h\sqrt{\frac{\omega}{2a}}), \tag{2.4}$$

the term $-h\sqrt{\omega/2a}$ represents the delay of the temperature variations at the depth h with the respect to the variations on the surface. When we substitute (2.4) into the relationship (2.3), we obtain:

$$\Delta\bar{\vartheta}_{(t,h)} = e^{-h.\sqrt{\omega/2a}} \cos(\omega t - h\sqrt{\frac{\omega}{2a}}). \tag{2.5}$$

When we substitute (2.5) into the integral (2.2), we obtain:

$$\varepsilon \approx \int_0^H e^{-h.\sqrt{\omega/2a}} \cos(\omega t - h\sqrt{\frac{\omega}{2a}})\ dh\ . \tag{2.6}$$

An primitive (antiderivative) function of the integral (2.6) is:

$$F(x) = \frac{e^{-h.\sqrt{\omega/2a}}}{2\sqrt{\omega/2a}} \left(\sin(h\sqrt{\omega/2a} - \omega t) - \cos(h\sqrt{\omega/2a} - \omega t)\right) + C\ . \tag{2.7}$$

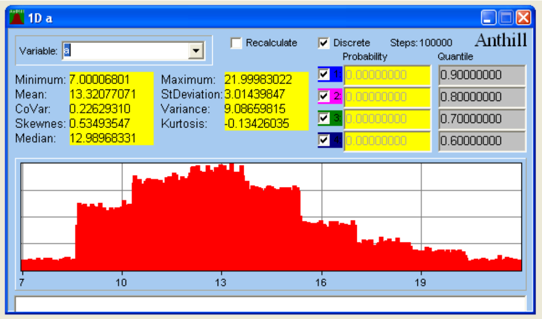
Due to a fast attenuation the enumeration was only done for five annual periods, which means $H = 113{,}48799\ m$.

3 THE PROBABILISTIC SBRA METHOD APPLICATION (ANTHILL SOFTWARE)

The first approximate calculation was modified in this way. We added a random variable, which simulated a temperature conductance coefficient a /m^2s^{-1}/ (see figure 3.1). The relationship (2.6) describes enumeration of the variable ε - the equivalent (normalised) relative strain**.** For this very reason, it is necessary to determine an initial condition – the actual time. The temperature field close to the surface is dependent on the temperature variations at the surface. It may be assumed that in the case of constant surface temperature, the strain and the evaluated value ε will tend towards zero. We made a simulation of the behaviour of this variable for the diurnal period. For the relationship (2.7), the random variable *t* was used with uniform distribution in the range <0; 86 400> /s/. Therefore, according to the relationship (2.7) we obtain:

$$\varepsilon_{\mathrm{vyp}} = \mathrm{F(H)} - \mathrm{F(0)}\,, \qquad (3.1)$$

where H /m/ is the depth, to which the heat penetrates during five periods. The value of the parameter H depends on the temperature conductance coefficient a; the example of generated values are in Fig. 3.2. and Fig. 3.3 shows the relationships of the calculation of the reliability (AntHill sw).

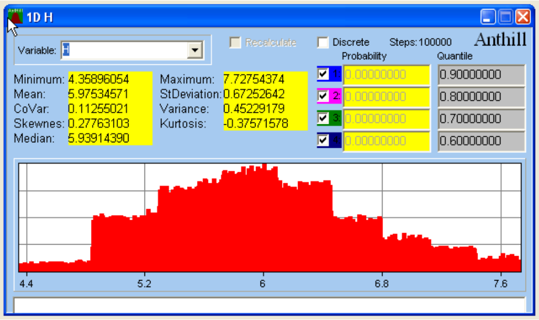


Variable: a
☐ Recalculate ☑ Discrete Steps: 100000

Minimum: 7.00006801
Mean: 13.32077071
CoVar: 0.22629310
Skewnes: 0.53493547
Median: 12.98968331
Maximum: 21.99983022
StDeviation: 3.01439847
Variance: 9.08659815
Kurtosis: -0.13426035

	Probability	Quantile
☑ 1:	0.00000000	0.90000000
☑ 2:	0.00000000	0.80000000
☑ 3:	0.00000000	0.70000000
☑ 4:	0.00000000	0.60000000

Fig. 3.1 The temperature conductance coefficient a



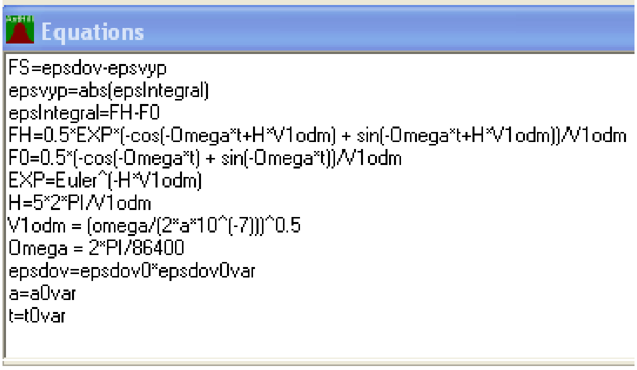
Variable: H
☐ Recalculate ☐ Discrete Steps: 100000

Minimum: 4.35896054
Mean: 5.97534571
CoVar: 0.11255021
Skewnes: 0.27763103
Median: 5.93914390
Maximum: 7.72754374
StDeviation: 0.67252642
Variance: 0.45229179
Kurtosis: -0.37571578

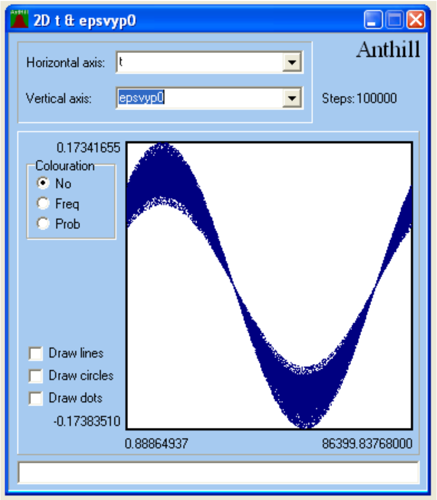
	Probability	Quantile
☑ 1:	0.00000000	0.90000000
☑ 2:	0.00000000	0.80000000
☑ 3:	0.00000000	0.70000000
☑ 4:	0.00000000	0.60000000

Fig. 3.2 Values of H - depth of temperature field

The behaviour of the variable ε_{vyp} is shown in figure 3.4, which shows that the strain inside the rocks varies during the diurnal period. Although the relative values are shown they respect the dynamics of the model well (maximum and minimum). We added the new variable "epsvyp0" to show the behaviour of strain, which corresponds to the relationship (3.1).



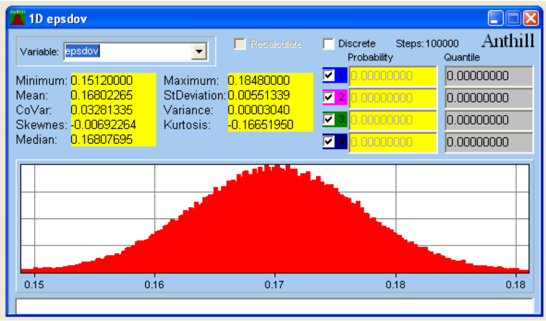
```
FS=epsdov-epsvyp
epsvyp=abs(epsIntegral)
epsIntegral=FH-F0
FH=0.5*EXP*(-cos(-Omega*t+H*V1odm) + sin(-Omega*t+H*V1odm))/V1odm
F0=0.5*(-cos(-Omega*t) + sin(-Omega*t))/V1odm
EXP=Euler^(-H*V1odm)
H=5*2*PI/V1odm
V1odm = (omega/(2*a*10^(-7)))^0.5
Omega = 2*PI/86400
epsdov=epsdov0*epsdov0var
a=a0var
t=t0var
```



Horizontal axis: t
Vertical axis: epsvyp0
Steps: 100000
Colouration: ● No ○ Freq ○ Prob
☐ Draw lines
☐ Draw circles
☐ Draw dots

Fig. 3.3 Calculation description (AntHill sw)

Fig. 3.4 Progression ε_{vyp} during diurnal period



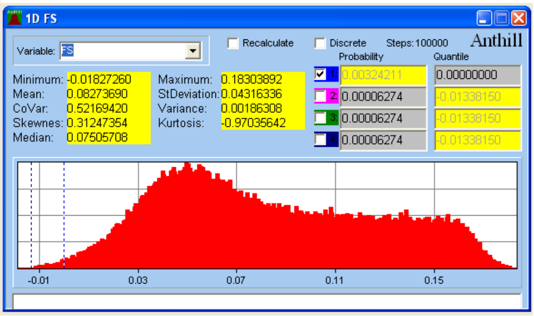
Variable: epsdov

☐ Discrete Steps: 100000

Minimum: 0.15120000
Mean: 0.16802265
CoVar: 0.03281335
Skewnes: -0.00692264
Median: 0.16807695
Maximum: 0.18480000
StDeviation: 0.00551339
Variance: 0.00003040
Kurtosis: -0.16651950

	Probability	Quantile
☑ 1:	0.00000000	0.00000000
☑ 2:	0.00000000	0.00000000
☑ 3:	0.00000000	0.00000000
☑ 4:	0.00000000	0.00000000

Fig. 3.5 Distribution of critical values ε_{dov}



Variable: FS

☐ Recalculate ☐ Discrete Steps: 100000

Minimum: -0.01827260
Mean: 0.08273690
CoVar: 0.52169420
Skewnes: 0.31247354
Median: 0.07505708
Maximum: 0.18303892
StDeviation: 0.04316336
Variance: 0.00186308
Kurtosis: -0.97035642

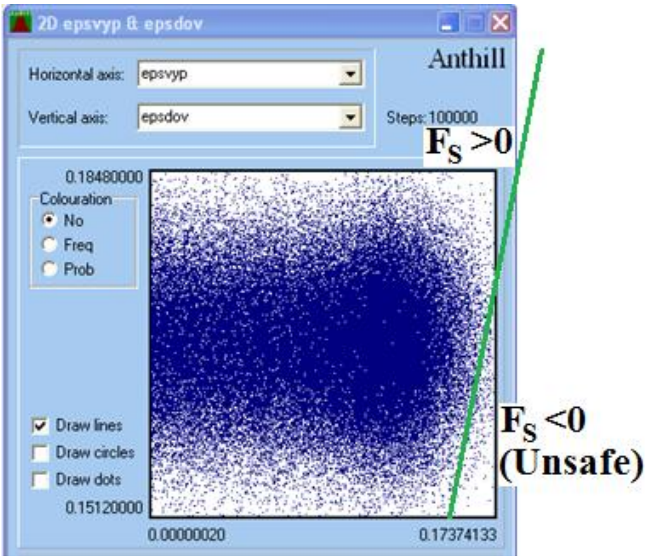
	Probability	Quantile
☑ 1:	0.00324211	0.00000000
☐ 2:	0.00006274	-0.01338150
☐ 3:	0.00006274	-0.01338150
☐ 4:	0.00006274	-0.01338150

Fig. 3.6 Reliability function F_S

There are two extremes in Fig.3.4. Both of them show the situation, when the model is approaching its limits when the linear law ceases to apply and the non-linear behaviour starts. One extreme describes the situation when the stress is approaching the ultimate compressive strength of the massif and the second describes the situation when the stress is approaching the ultimate tensile strength. The first of them, in a long perspective, leads to the creep or seismic events. The second extreme leads to the opening of the micro-cracks, cracks or faults, when ratcheting can occur.

To get a more accurate calculation, it will be necessary to take into account the dynamics of the strain growth and the relaxation of rocks for example a massif non-linearity, hysteresis or ratcheting. This is the subject of further work.

The distribution of frequencies corresponds to the chosen equation for computation of the temperature field close to the Earth's surface. To simulate a surface temperature, the model of the ideal periodic function was used. While, in this paper the ideal temperature function was used (defined by goniometric functions), the real temperature curves must be used in the real approach. These curves have asymmetric forms when the minimum of the diurnal temperature can be determined at the moment of the sunrise and the maximum can be determined approximately one hour after noon or we can use the real temperature development curves. This means that the heating of the Earth's surface is considerably faster then the cooling.



Horizontal axis: epsvyp

Vertical axis: epsdov

Steps: 100000

Colouration
● No
○ Freq
○ Prob

☑ Draw lines
☐ Draw circles
☐ Draw dots

Fig. 3.7 Distribution admissible values ε_{dov} and critical values ε_{vyp}

We used the numeric integration method for the evaluation of the temperature. If the surface temperature is constant then the integral will tend towards zero. Therefore, the computation was modified and the new variable $|\varepsilon_{vyp}|$ was set up.

Figure 3.5 shows the distribution of variable ε_{dov} minus critical value of the variable ε_{vyp} .

The value ε_{dov} was estimated by probabilistic approach (see figure 3.5). In this way the reliability function F_S could be estimated (see figure 3.6):

$$F_S = \varepsilon_{dov} - |\varepsilon_{vyp}|. \tag{3.2}$$

We also verified the methodology of the probabilistic SBRA method in the real geological situation and the number of simulations of 10^5 gave a fair result.

The probability of micro-movement P_f (i.e. the probability of the irreversible effect when $F_S < 0$) was evaluated as 0.00324 , i.e. 0.324%. The figures 3.6 and 3.7 show the principles of determining this value.

4 CONCLUSIONS

The aim of this paper was to test the possibility of using probabilistic calculations for modelling the behaviour of the lithosphere of the Earth (the primary methodology for setting the limit values, which leads to the irreversible movement of the lithospheric plates - creep). We can say that the method can be used for the modelling of the massif, when its parameters do not enter as mean (i.e. constant) values, but as a variable (i.e. real) values. This is done by using Monte Carlo method (i.e. probabilistic SBRA approach). The method could be used directly inside the geomechanical model.

Another applications of SBRA Method are presented for example in [2], [4], [5], [12] and [13].

ACKNOWLEDGEMENTS

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