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Absolute Dissipative Drift-Wave Instabilities in Tokamaks*

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Contrary to previous theoretical predictions, we show that the dissipative drift-wave instabilities are absolute in tokamak plasmas. The existence of unstable eigenmodes is shown to be associated with a new eigenmode branch induced by the finite toroidal couplings.

The stability of drift-wave eigenmode in sheared magnetic fields has been intensively investigated due to its potential importance to the transport processes in magnetically confined plasmas such as tokamaks. Most of the theories are, however, limited to the slab model. It is now well established that in slab geometries, both the collisionless[1-5] and the collisional[6] (dissipative) electrostatic drift-wave eigenmodes are stable in the absence of ion temperature gradients.

The applicability of the slab approximation to tokamak plasmas is, however, not at all clear. In particular, Taylor[7] has suggested that the toroidal couplings may significantly affect the shear-damping mechanism and, thereby, the stability properties. On the other hand, stability studies in toroidal geometries[6,8] using Taylor's strong-coupling approximation[7] indicate that the eigenmodes are, again, stable for typical values of the shear in tokamaks, i.e. $rq'/q > 1/2$. Here, $q = rB_t/RB_\theta$ is the usual safety factor. In this work, we adopt the ballooning-mode formalism[9] and investigate the stability properties of dissipative drift-wave eigenmodes in toroidal plasmas without using the strong-coupling approximation. We find, both analytically and numerically, that, contrary to previous theoretical results, unstable eigenmodes do exist. We observe that these unstable eigenmodes are directly related to the appearance of a new toroidicity-induced branch[10] which experiences negligible shear damping.

Let us consider electrostatic drift waves in an axisymmetric tokamak with concentric, circular magnetic surfaces. Adopting here the usual (r,θ,ξ) coordinates corresponding respectively to the (minor) radial, poloidal and toroidal directions, we express the perturbed potential, ϕ, as

$$\phi(r,\theta,\xi,t) = \sum_j \hat{\phi}_j(s) \exp\left[i\left(m_o\theta + j\theta - n\xi - \omega t\right)\right], \quad (1)$$

where $|j| \ll |m_o|$, $s = (r-r_o)/\Delta r_s$, r_o is the reference mode-rational surface $m_o = nq(r_o)$, $\Delta r_s = 1/k_\theta \hat{s}$, $k_\theta = m_o/r_o$ and $\hat{s} = rq'/q$ at $r = r_o$. For simplicity, we ignore temperature gradients and consider only the resistive effects. The two-dimensional eigenmode equation can be straightforwardly derived using fluid descriptions for both the electrons and ions, and is given by[6,8]

$$\left[L(j,s) - f_t(j,s)\, T(j,s)\right] \hat{\phi}_j(s) = 0, \quad (2)$$

where

$$L = \left(1 - i\,\frac{\omega\,\nu_{ei}}{k_{||}^2 v_e^2}\right) b_\theta\left(\hat{s}^2 \frac{\partial^2}{\partial s^2} - 1\right) - 1 + \frac{\omega_{*e}}{\omega} + \frac{k_{||}^2 C_s^2}{\omega^2}, \quad (3)$$

$$f_t = \left(1 - i\omega\,\nu_{ei}/k_{||}^2 v_e^2\right)\left(\varepsilon_n\,\omega_{*e}/\omega\right), \quad (4)$$

$$T\,\hat{\phi}_j(s) = \hat{\phi}_{j+1}(s) + \hat{\phi}_{j-1}(s) + \hat{s}\,\frac{\partial}{\partial s}\left[\hat{\phi}_{j+1}(s) - \hat{\phi}_{j-1}(s)\right], \quad (5)$$

$k_{||} = (s-j)/qR$, $b_\theta = k_\theta^2 \rho_s^2$, $\rho_s = C_s/\omega_{ci}$, $C_s^2 = T_e/M_i$, $\varepsilon_n = r_n/R$,

$r_n^{-1} = |d \ln N(r)/dr|$, and the rest of notations is standard. In deriving Eq. (2), we also assume $\tau = T_e/T_i \gg 1$ and $|(\rho_s^2/\tau)(d^2/dr^2 - k_\theta^2)\phi| \ll |\phi|$. Note that T in Eq. (5) is the toroidal-coupling operator due to ion ∇B and curvature drifts.

Since, typically, $|m_o| \sim |n| \sim |r_n/\rho_s| \sim O(10^2\text{-}10^3)$, the large-n ordering, i.e., the ballooning-mode formalism[9] is appropriate here. In the zeroth order, we have, with $z = s-j$, $\hat{\phi}_j(s) = \Phi(z)$ and $\hat{\phi}_{j\pm 1}(s) = \Phi(z\mp 1)$; i.e., the eigenmodes are composed of identical structures centered at each mode-rational surface. Equation (2) then reduces to a one-dimensional differential-difference equation, i.e.,

$$\left[L(z) - f_t(z)\, T(z)\right] \Phi(z) = 0, \tag{6}$$

and $T(z)\,\Phi(z) = \Phi(z+1) + \Phi(z-1) + \hat{s}\,(d/dz)\,[\Phi(z-1) - \Phi(z+1)]$. Fourier transforming Eq.(6), we obtain the following eigenmode equation describing dissipative drift waves in toroidal plasmas

$$\left\{\left(d^2/d\hat{\theta}^2\right)\left[d^2/d\hat{\theta}^2 + Q_1(\hat{\theta})\right] + i\alpha\, Q_2(\hat{\theta})\right\}\hat{\phi}\,(\hat{\theta}) = 0; \tag{7}$$

where $\hat{\phi}$ is the fourier transform of Φ,

$$Q_1(\hat{\theta}) = \eta_s^2\, \Omega^2\, P(\hat{\theta}), \tag{8}$$

$$P(\hat{\theta}) = 1 - 1/\Omega + Q_2(\hat{\theta}), \tag{9}$$

$$Q_2(\hat{\theta}) = b_\theta\left(1 + \hat{s}^2\,\hat{\theta}^2\right) + \left(2\,\varepsilon_n/\Omega\right)\left(\cos\hat{\theta} + \hat{s}\,\hat{\theta}\,\sin\hat{\theta}\right), \tag{10}$$

$\Omega = \omega/\omega_{*e}$, $\eta_s^2 = b_\theta\, q^2/\varepsilon_n^2$, $\alpha = \bar{\nu}\,\Omega^3\left(q^2\, b_\theta/\varepsilon_n^2\right)^2$, and

$\bar{\nu} = \nu_{ei}\, m_e/\omega_{*e}\, m_i$. The boundary condition imposed on Eq.(7) is that the unstable ($\mathrm{Im}\,\Omega > 0$) eigenmodes decay asymptotically as $|\hat{\theta}| \to \infty$.

We first consider the $\alpha \propto \nu_{ei} = 0$ limit[10]. Here, the electron response is adiabatic and the relevant eigenmode equation is

$$\left[d^2/d\hat{\theta}^2 + Q_1\left(\hat{\theta}\right)\right]\hat{\phi}\left(\hat{\theta}\right) = 0 \ . \tag{11}$$

The boundary condition is then, $\hat{\phi}\left(\hat{\theta}\right) \rightarrow \exp\left(i\Omega\eta_s b_\theta^{1/2} \hat{s} \hat{\theta}^2/2\right)$ as $|\hat{\theta}| \rightarrow \infty$; i.e., the wave energy is outward propagating. Furthermore, an examination of $Q_1\left(\hat{\theta}\right)$, as defined by Eq.(8), indicates that the potential structure consists of a parabolic anti-well plus modulations due to toroidal couplings. Equation (11) has been analyzed using both the interactive WKB[11] and numerical shooting codes. In Fig.1, we plot the eigenmode frequencies $\Omega = \Omega_r + i\Omega_i$ versus the toroidicity ε_n for $b_\theta = 0.1$, $\hat{s} = 1$, and $q = 1$ for the lowest eigenstate. The results clearly show the existence of two damped eigenmode branches. One is a slab-like branch and the other is a new branch induced by the finite toroidicity. The slab-like eigenmodes, similar to the Pearlstein-Berk modes[12] found in the slab limit, correspond to unbounded eigenstates with anti-well potential structures and, hence, experience finite shear damping due to (free) outward energy convection. In fact, in this case, torodicity further enhances the shear damping rates. The toroidicity-induced (T-I) eigenmodes, however, experience negligible shear damping; typically, we find $-\Omega_i \sim O\left(10^{-3} - 10^{-4}\right)$. Typical potential structures corresponding to the weak (smaller ε_n) and strong (larger ε_n) T-I eigenmodes are shown in Fig.2. It is clear from Fig.2 that the T-I eigenmodes correspond to eigenstates quasibounded by local potential wells induced by finite toroidal couplings. The shear damping is negligible here because the convection of wave energies occurs only through the tunneling leakages. In this respect, the eigenmodes are quasimarginally stable. It is interesting to note that for a certain parameter regime both eigenmode branches can exist simultaneously. Furthermore, we note the slab-like eigenmodes, having turning points $\pm\theta_t$ close to $\hat{\theta}=0$ (i.e., $|\theta_t| << 1$), can be understood using Taylor's strong-coupling approximation, and the eigenmodes remain damped for $\nu_{ei} \neq 0$[6]. In this work, we therefore concentrate on the T-I eigenmodes.

We now consider the effects of finite electron resistive dissipation ($\nu_{ei} \neq 0$) on the T-I eigenmodes. For the purpose of this letter, we assume ν_{ei} is small and perform a perturbative analysis on the weak T-I eigenmodes. Since the tunneling effects are small, they may be ignored in the present perturbation theory. The corresponding potential structure in the $\nu_{ei} = 0$ limit, c.f. Fig.2(a), then suggests that the eigenmodes can be assumed to be localized at $\hat{\theta} = \theta_o$ where $\theta_o \neq 0$ and $Q_1'(\theta_o) = 0$; i.e.,

$$\theta_o b_\theta \hat{s}^2 + (\varepsilon_n/\Omega)\left[(\hat{s}-1)\sin\theta_o + \hat{s}\,\theta_o\cos\theta_o\right] = 0. \qquad (12)$$

Let $\eta = \hat{\theta} - \theta_o$ and expand Q_1 and Q_2 about $\hat{\theta} = \theta_o$ to $O(\eta^2)$; Eq.(7) becomes

$$\left[(d^2/d\eta^2)(d^2/d\eta^2 + Q_{10} + Q_{10}''\eta^2/2) + i\alpha(Q_{20} + Q_{20}''\eta^2/2)\right]\hat{\phi}(\eta) = 0, \qquad (13)$$

where $(Q_{1,2})_0 \equiv Q_{1,2}(\theta_o)$ and $\mathrm{Re}\, Q_{10}'' < 0$. Setting $\phi(t) = \int_{-\infty}^{\infty} d\eta\, \hat{\phi}(\eta) \exp(i\eta t)$ in Eq. (13), we obtain

$$\left[(Q_{10}''/2)(t^2 - it_k^2)\, d^2/dt^2 + t^4 - Q_{10}t^2 + i\alpha Q_{20}\right]\phi(t) = 0. \qquad (14)$$

Here, $t_k^2 = \alpha Q_{20}''/Q_{10}'' = \alpha/\eta_s^2\,\Omega^2$. Eq.(14) can be written as

$$\left[d^2/dy^2 + \lambda - y^2 - i\Lambda/\left(y^2 - iy_k^2\right)\right] \phi\left(y\right) = 0 . \tag{15}$$

where $y = t/\beta$, $\beta = \left(- Q_{10}''/2\right)^{1/4}$, Re $\beta > 0$, $y_k^2 = t_k^2/\beta^2$, $\lambda = \left(Q_{10} - it_k^2\right)/\beta^2$ and $\Lambda = \left[\alpha\left(1/\Omega - 1\right) + it_k^4\right]/\beta^4$. Noting that $|\Lambda| \propto |\alpha| \propto \nu_{ei}$, a perturbative treatment of Eq. (15) can be readily done and we find for the lowest eigenstate, $\lambda = 1 + \lambda_1$, where $\lambda_1 = - \sqrt{\pi}\ (\Lambda/y_k) \exp(-i\pi/4)$. The dispersion relation is then

$$P\left(\theta_o\right) \simeq \Gamma\left(1 + \lambda_1 - \delta\right); \tag{16}$$

here, $\Gamma = \left(\beta/\eta_s\Omega\right)^2$ and δ is included to represent the tunneling effects.

To further analyze Eq. (16), we need to solve θ_o from Eq. (12). For this purpose, we note that the T-I eigenmode branch generally exists for $|\varepsilon_n/\Omega| > |b_\theta \hat{s}|$ and $|\theta_o| > 1$ for $\hat{s} \sim 1$, so that $\theta_o \simeq \pi/2$. Thus,

$$P\left(\theta_o\right) \simeq 1 + b_\theta\left(1 + \hat{s}^2 \pi^2/4\right) - 1/\Omega + \varepsilon_n \pi \hat{s}/\Omega, \tag{17}$$

$$\Gamma \simeq \left(\varepsilon_n \hat{s}/q\Omega\right)\left(\varepsilon_n \pi/2\ \Omega b_\theta \hat{s} - 1\right)^{1/2}, \tag{18}$$

and $\lambda_1 = -\left(1/\Omega - 1\right)\left(\pi \hat{\nu}\ \Omega/\Gamma^3\right)^{1/2} \exp\left(-i\pi/4\right)$. (19)

For parameters of interest here, we note that $|\Omega\Gamma| < 1$. Thus, we have, with $\Omega = \Omega_r + i\Omega_i$ and $|\Omega_i/\Omega_r| < 1$,

$$\Omega_r \simeq \left(1 - \pi \varepsilon_n \hat{s}\right)/\left[1 + b_\theta\left(1 + \hat{s}^2\pi^2/4\right)\right], \tag{20}$$

and

$$\Omega_i = \gamma - \gamma_t, \tag{21}$$

where

$$\gamma = \left(1 - \Omega_r\right)\left(\pi\ \bar{\nu}/2\Omega_r\Gamma\right)^{1/2}/\left[1 + b_\theta\left(1 + \hat{s}^2\ \pi^2/4\right)\right], \qquad (22)$$

and γ_t is the small shear damping rate due to the tunneling, which can be estimated by examining the $\nu_{ei}=0$ limit[10].

Equation (21) shows that electron resistive dissipation can destabilize the T-I eigenmodes if $\bar{\nu} > \bar{\nu}_c$, where

$$\bar{\nu}_c = \left(\frac{\nu_{ei}\ m_e}{\omega_{*e}\ m_i}\right)_c = \frac{2\gamma_t^2}{\pi}\left[1 + b_\theta\left(1 + \frac{\hat{s}^2\pi^2}{4}\right)\right]^2 \left(\frac{\Gamma}{1-\Omega_r}\right). \qquad (24)$$

For $\bar{\nu} \gg \bar{\nu}_c$, *we have* $\gamma \propto \bar{\nu}^{1/2} \propto \nu_{ei}^{1/2}$; *i.e., the growth rates of* unstable eigenmodes scale as $\nu_{ei}^{1/2}$. We note that the above perturbative analysis is valid for $|\lambda_1| < 1$; i.e., $\bar{\nu} < \bar{\nu}_p = (\Omega_r\Gamma)^3(1-\Omega_r)^{-2}/\pi$.

Finally, we have also solved Eq. (7) numerically in order to verify as well as extend the above analytical results. Figure 3 plots Ω versus the resistivity parameter $\bar{\nu} = \nu_{ei}\ m_e/\omega_{*e}\ m_i$ for the case $b_\theta = 0.1$, $\varepsilon_n = 0.15$, $q = \hat{s} = 1$. The numerical results clearly demonstrate the properties predicted analytically; i.e., (i) the eigenmode is destablized for $\bar{\nu} > \bar{\nu}_c \simeq 2.5 \times 10^{-8}$, and (ii) $\gamma \propto \bar{\nu}^{1/2}$ for $10^{-2} \geq \bar{\nu} \gg \bar{\nu}_c$. For the present case, Eqs. (20) and (21) predict that $\Omega_r \simeq 0.4$ and $\gamma \simeq \bar{\nu}^{1/2} - \gamma_t$ with $\gamma_t \simeq 1.9 \times 10^{-4}$. To obtain a *quantitative comparison, we have also plotted the analytical results* in Fig. 3 up to the perturbation limit $\bar{\nu}_p \simeq 0.03$. We note the agreement is reasonably good. For $\bar{\nu} \gtrsim \bar{\nu}_p$. numerical results show that Ω_i starts decreasing with $\bar{\nu}$. Analytical theory for the large $\bar{\nu}$ limit as well as a more complete presentation of the numerical results will be published elsewhere.

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References

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[1]D. W. Ross and S. M. Mahajan, Phys. Rev. Lett. 40, 324(1978). Also, K. T. Tsang, P. J. Catto, J. C. Whitson, and J. Smith, Phys. Rev. Lett. 40, 327(1978).

[2]Liu Chen, P. N. Guzdar, R. B. White, P. K. Kaw and C. Oberman, Phys. Rev. Lett. 41, 649(1978).

[3]T. M. Antonsen, Jr., Phys. Rev. Lett. 41, 33(1978).

[4]Y. C. Lee and Liu Chen, Phys. Rev. Lett. 42,708(1979).

[5]Y. C. Lee, Liu Chen, and W. M. Nevins, Princeton Plasma Physics Laboratory Report PPPL-1544(1979).

[6]Liu Chen, P. N. Guzdar, J. Hsu, P. K. Kaw, C. Oberman, and R. White, Nucl. Fusion 19,373(1979). Also, P. N. Guzdar, Liu Chen, P. K. Kaw, and C. Oberman, Phys. Rev. Lett. 40, 1566(1978).

[7]J. B. Taylor, in Proc. of 6th Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Vol. 2, (IAEA, Vienna, 1977), P.323.

[8]W. M. Tang, Nucl. Fusion, 18, 1089(1978). Also, W. Horton, R. D. Estes, H. Kwak, and D. I. Choi, Phys. Fluids 21, 1366(1978).

[9]J. W. Connor, R. J. Hastie, and J. B. Taylor, Culham Laboratory Report CLM-P537(1978, to be published. A. H. Glasser in Proc. Finite Beta Theory Workshop, Varenna, 1977, ed - B. Coppi and W. Sadowski. Y. C. Lee and J. W. Van Dam, University of California at Los Angeles Report PPG-337(1978).

[10] Detailed discussions on the eigenmodes in this limit is given by Liu Chen and C.Z. Cheng, Princeton Plasma Physics Laboratory Report PPPL-1562(1979).

[11] R. B. White, J. Comput. Phys. 31, No.3 (1979), to be published.

[12] L. D. Pearlstein and H. L. Berk, Phys. Rev. Lett 23, 220(1969).

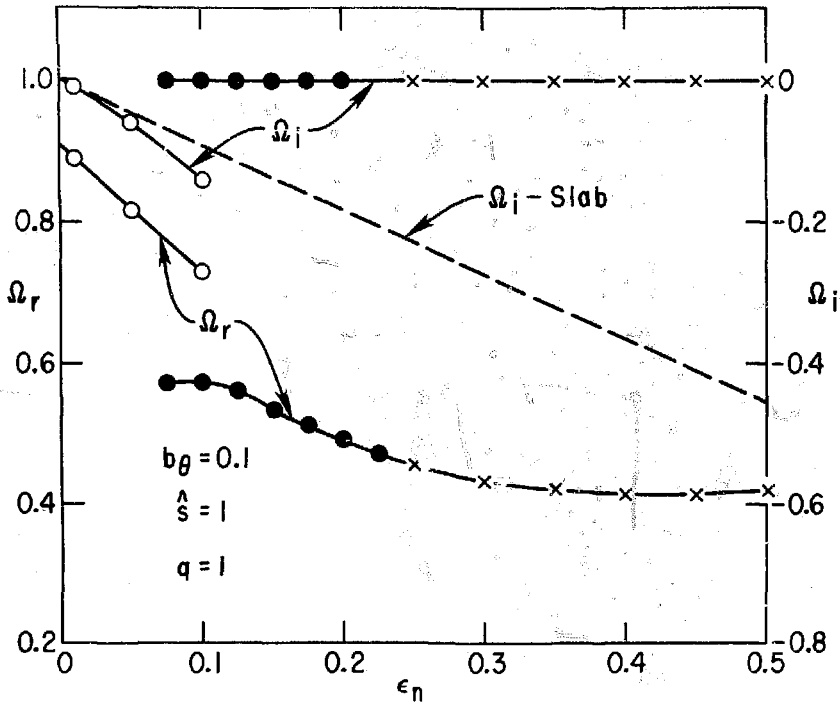

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Fig. 1. Eigenmode frequencies Ω versus ε_n in the $\nu_{ei} = 0$ limit. o, ●, and X correspond, respectivly, to the slab-like, and weak, and strong, toroidicity-induced eigenmodes. Ω_i in the slab limit is also shown.

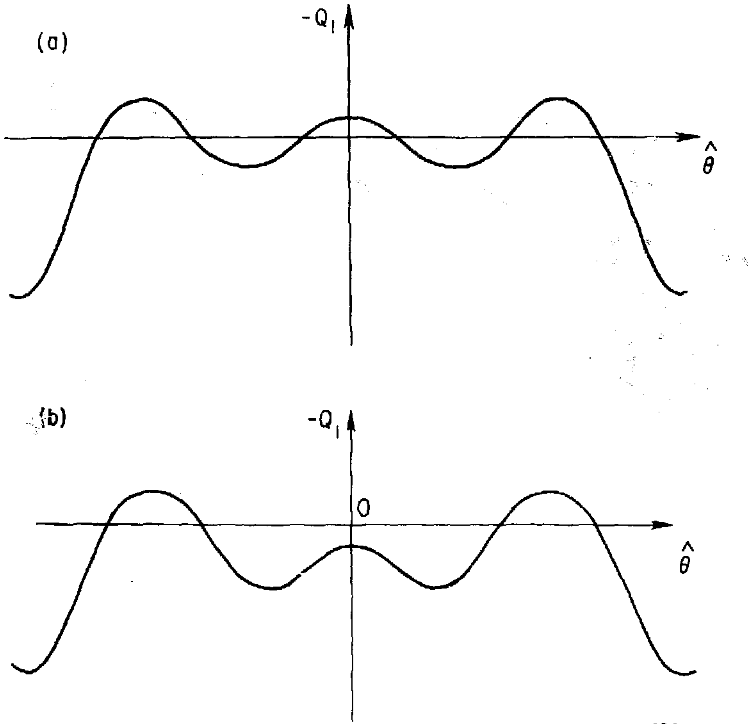

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Fig. 2. Typical potential structures, $-Q_1$, for the (a) weak and (b) strong toroidicity-induced eigenmodes in the $\nu_{ei} = 0$ limit.

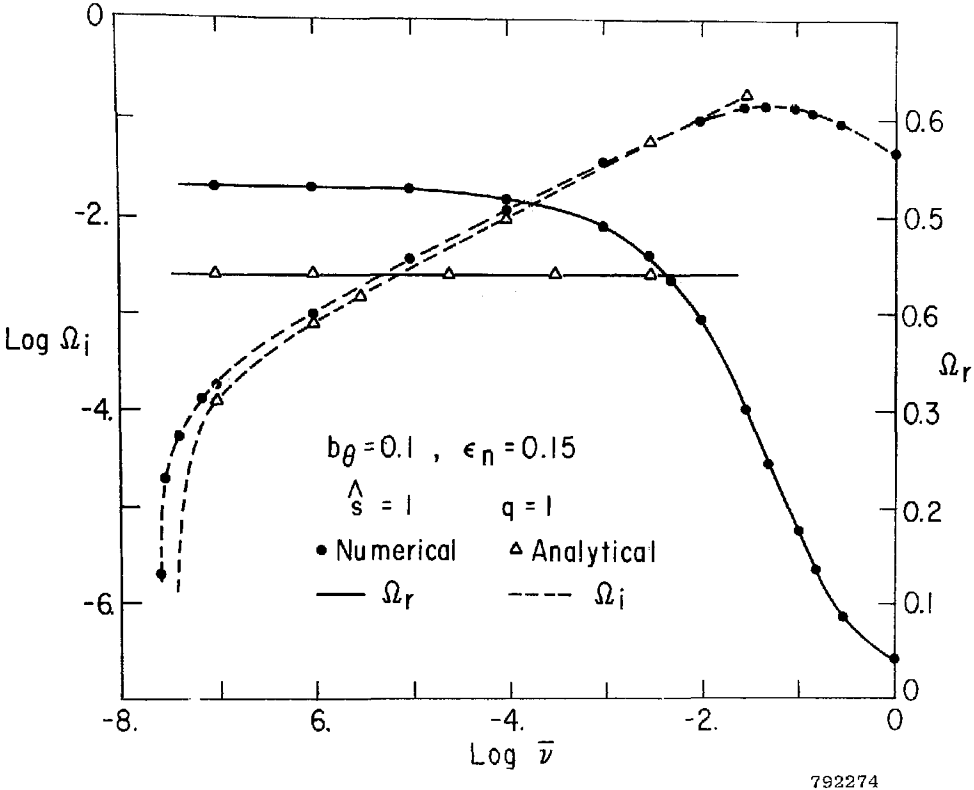

Fig. 3. A plot of $\Omega = \Omega_r + i\Omega_i$. $\Omega_i > 0$ versus $\bar{\nu} = \nu_{ei}\, m_e/\omega_{*e}\, m_i$.