Absolute Negative Mobility Induced by Thermal Equilibrium Fluctuations

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A novel transport phenomenon is identified that is induced by inertial Brownian particles which move in simple one-dimensional, symmetric periodic potentials under the influence of both a time periodic and a constant, biasing driving force. Within tailored parameter regimes, thermal equilibrium fluctuations induce the phenomenon of absolute negative mobility (ANM), which means that the particle noisily moves *backwards* against a small constant bias. When no thermal fluctuations act, the transport vanishes identically in these tailored regimes. ANM can also occur in the absence of fluctuations on grounds which are rooted solely in the complex, inertial deterministic dynamics. The experimental verification of this new transport scheme is elucidated for the archetype symmetric physical system: a convenient setup consisting of a resistively and capacitively shunted Josephson junction device.

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A central result of thermodynamics is due to Le Chatelier, which (loosely speaking) states that "a change in one of the variables that describe the system at equilibrium produces a shift in the position of the equilibrium that counteracts this change." In particular, if a system is at thermal equilibrium, its reaction to an applied bias is so that the response is in the same direction of this applied force, towards a new equilibrium.

Thus, the seemingly paradoxical situation that the system's response is opposite to a small external load is prohibited by the laws of thermodynamics; it would imply the phenomenon of an absolute negative mobility (ANM). A possibility to circumvent the stringent conditions imposed by thermodynamics is, however, to go far from equilibrium where these restrictions no longer possess validity. Known examples of such absolute negative mobilities, or likewise, absolute negative conductivity, have been experimentally observed before within a quantum mechanical setting in p-modulation-doped multiple quantum-well structures [1], in semiconductor superlattices [2], and have been studied as well theoretically for ac-dc-driven tunneling transport [3], in the dynamics of cooperative Brownian motors [4], for Brownian transport containing a complex topology [5,6], and in some stylized, multistate models with state-dependent noise [7], to name but a few.

This startling and counterintuitive transport phenomenon has spurred renewed interest, motivated by the quest to explore new transport scenarios in simple and easy to fabricate devices that intrinsically exploit the abundant source of thermal fluctuations to a constructive technological use. Physical systems that are most suitable for this purpose are periodic one-dimensional symmetric systems such as phase differences across Josephson junctions [8], rotating dipoles in external fields [9,10], superionic conductors [11], and charge density waves [12]. Another important area constitutes the noise-assisted transport of Brownian particles [13,14], as it occurs for Brownian motors possessing ample applications in physics and chemistry [15].

Our main objective is to detect noise-induced ANM in ordinary physical systems that are readily available and which can be put to immediate use without the need to go to extremely low temperatures and/or the use of advanced fabrication techniques of higher-dimensional stylized structures that generate the necessary trapping mechanism for the occurrence of ANM; cf. the nicely tailored two-dimensional trap geometries used in Refs. [5,6]. Our minimal prerequisites for detecting ANM therefore are the use of (i) simple, one-dimensional symmetric periodicity, (ii) symmetric external forcing, (iii) inertial dynamics, and (iv) thermal fluctuations. The following model setup fulfills all of these.

We formulate the problem in terms of a Brownian classical particle of mass M moving in a spatially periodic potential $V(x) = V(x + L)$ of period L and barrier height ΔV, subjected to an external, *unbiased* time-periodic force $A\cos(\Omega t)$ with angular frequency Ω and of amplitude strength A. Additionally, a constant external force F acts on the system.

The dynamics of the system is thus modeled by the inertial Langevin equation [16]

$$M\ddot{x} + \Gamma\dot{x} = -V'(x) + A\cos(\Omega t) + F + \sqrt{2\Gamma k_B T}\,\xi(t), \tag{1}$$

where a dot and prime denote differentiation with respect to time t and the Brownian particle's coordinate x, respectively. The parameter Γ denotes the friction coefficient, k_B the Boltzmann constant, and T the temperature. Thermal fluctuations due to the coupling of the particle with the environment are modeled by zero-mean, Gaussian white noise $\xi(t)$ with autocorrelation function $\langle\xi(t)\xi(s)\rangle = \delta(t - s)$. The spatially periodic potential $V(x)$ is assumed to be *symmetric* and chosen in its simplest form, namely,

$$V(x) = \Delta V \sin(2\pi x/L). \quad (2)$$

Upon introducing the period L and the parameter combination $\tau_0 = L\sqrt{M/\Delta V}$ as units of length and time [17], respectively, Eq. (1) can be rewritten in dimensionless form, reading

$$\ddot{\hat{x}} + \gamma\dot{\hat{x}} = -\hat{V}'(\hat{x}) + a\cos(\omega\hat{t}) + f + \sqrt{2\gamma D_0}\hat{\xi}(\hat{t}), \quad (3)$$

where $\hat{x} = x/L$ and $\hat{t} = t/\tau_0$. The remaining rescaled parameters are the friction coefficient $\gamma = (\Gamma/M)\tau_0$, the potential $\hat{V}(\hat{x}) = V(L\hat{x})/\Delta V = \hat{V}(\hat{x}+1) = \sin(2\pi\hat{x})$ with unit period and barrier height $\Delta\hat{V} = 2$, the amplitude $a = LA/\Delta V$, the frequency $\omega = \Omega\tau_0$, the load $f = LF/\Delta V$, the zero-mean Gaussian white noise $\hat{\xi}(\hat{t})$ with autocorrelation function $\langle\hat{\xi}(\hat{t})\hat{\xi}(\hat{s})\rangle = \delta(\hat{t} - \hat{s})$, and the noise intensity $D_0 = k_B T/\Delta V$. From now on, we will use only the dimensionless variables and shall omit the "hat" for all quantities occurring in Eq. (3).

This Langevin equation (3) provides a simple model of the diverse periodic systems specified in the introduction. Yet the deterministic inertial dynamics of the nonequilibrium system defined by Eq. (3) exhibits a very rich and complex behavior [18–20]. Depending on the parameter values, periodic, quasiperiodic, and chaotic motion can result in the asymptotic long time limit; see Fig. 1. Different initial conditions of position and velocity can also lead to different asymptotic behavior; i.e., various attractors may coexist.

A rough classification of the asymptotic behavior can be made into locked states in which the motion is confined to a finite number of spatial periods, and running states in which the motion is unbounded in space. For the deterministic transport properties the running states are crucial. A broad spectrum of various forms of running states exists which comprises—apart from periodic—also chaotic motions. By adding thermal fluctuations, one typically activates a diffusive dynamics leading to random transitions between possibly coexisting basins of attraction, which play an analogous role to potential wells in equilibrium systems. For example, stable locked states of the deterministic system are destabilized by noise: transitions between neighboring locked states will lead to diffusive or even directed transport.

The most important quantifier for characterizing directed transport is the asymptotic mean velocity $\langle\langle v\rangle\rangle$ [17], which is defined as the average of the velocity over the time and thermal fluctuations. The Fokker-Planck equation corresponding to Eq. (3) cannot be analytically solved; therefore, we carried out extensive numerical simulations of the Langevin equation. Details of the employed numerical scheme are described in Ref. [21]. Parts of our so obtained results are presented next.

The velocity of a stable running state mostly points into the direction of the force f. But there are also running stable states which on average move in the *opposite* direction of the constant driving force, hence displaying deterministic ANM [22]; see Fig. 1(b). In these deterministic cases the energy consumed for moving uphill is taken from the oscillating driving force.

Most remarkably, within particular parameter regimes ANM is solely induced by thermal noise. In the case depicted in Fig. 2 the deterministic average velocity vanishes for a whole range of forces around zero, whereas a

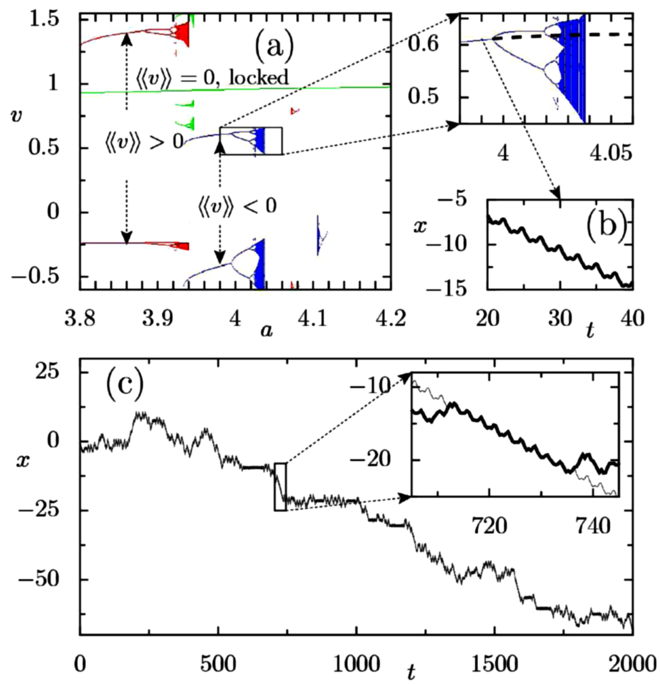

FIG. 1 (color online). In panel (a) we present a small part of the bifurcation diagram of the noiseless system as a function of the amplitude $a \in (3.8, 4.2)$. The ordinate indicates the stroboscopic velocity in the asymptotic long time limit. The system is biased by the positive force $f = 0.1$. The other parameter values are $\gamma = 0.9$ and $\omega = 4.9$. The straight line at $v \approx 0.9$ corresponds to a stable locked state. It persists for all shown values of the driving amplitude. For amplitudes up to $a \approx 4.04$ a running state with negative average velocity coexists. In panel (b) the corresponding orbit is depicted for $a = 3.99$. It is a period 2 state which stays within one potential well in the first period and moves to the left neighboring state in the second period. In panel (c) a realization of the stochastic system is displayed for $a = 4.2$ and $D_0 = 0.001$. Its average velocity is negative. In the inset we show a typical part of the stochastic trajectory (thick line) which contributes to the transport. For several periods of the driving force it closely follows a deterministic unstable periodic orbit (thin line). This orbit lies on the unstable branch emanating from a pitchfork bifurcation at $a \approx 3.99$ of the running state shown in panel (b). A part of the bifurcation diagram including the first bifurcation of the period two orbit to a stable period four orbit (solid line) and the mentioned unstable period two orbit (dashed line) is shown in the magnification in panel (a).

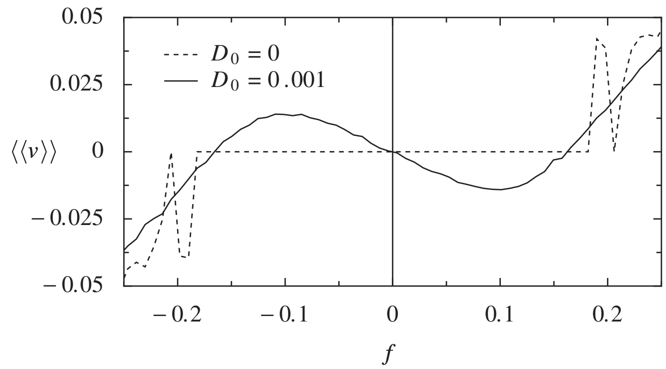

FIG. 2. The average velocity $\langle\langle v\rangle\rangle$ of an inertial Brownian particle described by Eq. (3) is depicted as a function of the external force f for the deterministic (dashed line) and noisy (solid line) dynamics. The system parameters are $a = 4.2$, $\omega = 4.9$, $\gamma = 0.9$, $D_0 = 0$ (dashed line), and $D_0 = 0.001$ (solid line). The absolute mobility defined as $\langle\langle v\rangle\rangle/f$ assumes a negative value for the noisy system in the range $|f| < 0.17$. The most pronounced ANM occurs for small absolute values of the bias f. For the bias force $f \in (-f_{stall}, f_{stall})$, with $f_{stall} \approx \pm 0.17$, the Brownian particle moves opposite to the applied bias f.

very small amount of noise yields a negative mobility at small forces. The region of ANM is bounded by the stall forces $\pm f_{stall}$ for which the average velocities vanish. With increasing temperature the stall force decreases and wears off at a finite temperature.

Although the above described ANM manifests itself only in the presence of thermal fluctuations, the underlying relevant mechanisms are strongly influenced by the deterministic dynamics of the system. At the driving strength $a = 4.2$ the deterministic system possesses only one stable orbit with period 1 which is a locked state and therefore does not contribute to the transport. Consequently, the current at $D_0 = 0$ is zero. There exists, however, a large number of unstable periodic orbits, transporting the particle in both positive and negative directions, which influence the relaxation dynamics from points lying far form the stable locked orbit. In presence of noise, the particle is permanently moved away from the stable orbit. In Fig. 1(c) we depict a single realization of the stochastic dynamics at $D_0 = 0.001$. One observes a systematic movement of the particle position into the negative direction. Moreover, between noisy bursts, periods of almost regular motion take place. All these regular parts are distinguished by approximately the same negative average velocity and therefore primarily contribute to the negative mobility. One of these regions has been enlarged in the inset of Fig. 1(c). The random trajectory strongly resembles a particular unstable orbit of the deterministic system. Upon reducing the driving amplitude a this orbit can be identified as the unstable branch emerging from a pitchfork bifurcation of a stable orbit; see the inset of Fig. 1(a). Before the bifurcation, the corresponding stable orbit also has negative average velocity. It is depicted in Fig. 1(b). Among many other unstable periodic orbits this unstable remnant of a stable running state apparently is most likely populated by the noise.

In Fig. 3, we depict the dependence of the mobility coefficient $\mu = (\partial\langle\langle v\rangle\rangle/\partial f)(f = 0)$ versus temperature for three cases: (i) the noise-induced ANM ($a = 4.2$), (ii) a regime with deterministic ANM ($a = 5.1$), and (iii) the "normal" or "positive" mobility regime ($a = 4.4$), when the velocity is positive for positive load, i.e., $\mu > 0$. For $a = 4.2$, there emerges an optimal temperature at which ANM is the most prominent. For $a = 5.1$, a continued increase of temperature eventually annihilates ANM. For the amplitude strength $a = 4.4$, an optimal temperature occurs at which the mobility is maximal. There also occur regimes of so termed *differential* negative mobility [21]; however, a more complete analysis is beyond the scope of this Letter but will be presented elsewhere. In particular, the effect of noise-induced ANM in (1) does not present an exception but can emerge as well in different regimes.

As an application of the above theoretical study we consider the resistively and capacitively shunted single Josephson junction for which the absolute negative conductance (ANC) can be measured, thus putting our predictions to a reality check. The phase difference ϕ across the junction obeys Eq. (1) with $x = \phi - \pi/2$, the mass is $M = (\hbar/2e)^2 C$, the friction coefficient $\Gamma = (\hbar/2e)^2(1/R)$, the barrier height $\Delta V = E_J = (\hbar/2e)I_0$, and the period $L = 2\pi$, where C denotes the capacitance, R the normal-state resistance of the junction, $E_J = (\hbar/2e)I_0$ the coupling energy of the junction, and I_0 the critical current. The load $F = (\hbar/2e)I_d$ corresponds to a dc-bias current, whereas the amplitude strength $A = (\hbar/2e)I_a$ and the frequency Ω define the external "ac-rocking" current. The average velocity $\langle\langle v\rangle\rangle$ translates into the voltage $\mathbb{V} = (\hbar/2e)\omega_0\langle\langle v\rangle\rangle$ across the junction, wherein $\omega_0 = (2/\hbar)\sqrt{E_J E_C}$ is the Josephson plasma frequency with the charging energy

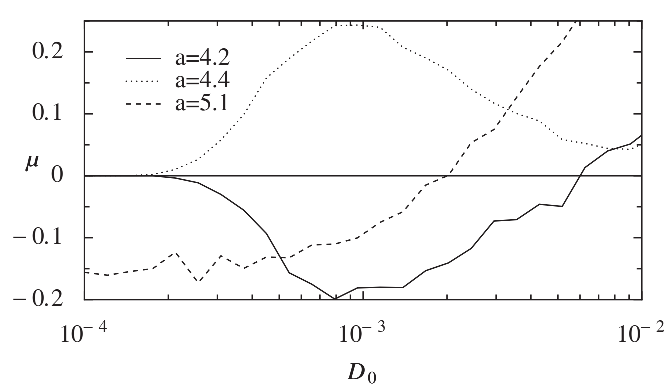

FIG. 3. The mobility coefficient $\mu = (\partial\langle\langle v\rangle\rangle/\partial f)(f = 0)$ depicted versus the dimensionless temperature strength, $D_0 \propto T$, for three values of the cosine-driving strength a; the strength $a = 4.2$ (solid line) corresponds to thermal noise-induced ANM. The driving strength $a = 5.1$ (dashed line) corresponds to a regime exhibiting deterministic ANM, and $a = 4.4$ (dotted line) is for a normal nonlinear response regime. The remaining parameters are $\omega = 4.9$ and $\gamma = 0.9$.

$E_C = e^2/C$. The dimensionless noise intensity $D_0 = k_B T/E_J$. Given the above relation, the dc-current-voltage characteristics can be obtained in the same form as presented in Fig. 2. The experimental setup to observe our novel noise-induced ANC necessitates the following parameter values: the amplitude of the ac-current $I_a = aI_0/2\pi \approx 0.67I_0$, an angular driving frequency $\Omega = \omega\omega_0/2\pi \approx 0.78\omega_0$, and a bias $I_d = fI_0/2\pi$ which becomes $I_d \approx 0.016I_0$ for the bias $f \approx 0.1$ which leads to an extremal velocity $\langle\langle v\rangle\rangle$; see Fig. 2. The frequency $\omega_r = 1/\tau_r = 1/RC$ which gives the relaxation time τ_r is $\omega_r = (\gamma/2\pi)\omega_0 \approx 0.14\omega_0$ and temperature is of the order $k_B T = E_J D_0 \approx 0.001(\hbar/2e)I_0$. For example, for a junction possessing a critical current of $I_0 = 0.1$ mA, a resistance $R = 2.9\ \Omega$, and a capacitance of $C = 20$ pF, the dc bias becomes $I_d = 1.6\ \mu$A, the amplitude of the ac current is $I_a = 67\ \mu$A, and the driving frequency is in the GHz regime, i.e., $\Omega = 96$ GHz. The optimal temperature occurs at $T = 2.4$ K and the maximal voltage is $\mathbb{V} = 0.6\ \mu$V. In turn, we predict a maximal voltage $\mathbb{V} = 2\ \mu$V for a junction at the temperature $T = 24$ K with $I_0 = 1$ mA, $R = 1\ \Omega$, and $C = 16$ pF, a dc bias $I_d = 16\ \mu$A, and an ac current with amplitude $I_a = 0.67$ mA and frequency $\Omega = 340$ GHz.

Our work has demonstrated that the surprising effect of a solely *noise-induced* absolute negative mobility can occur in generic, biased symmetric systems and devices that can be described by Eq. (1). In clear contrast, for noninertial, overdamped systems (i.e., with $M\ddot{x} = 0$) ANM cannot occur; this fact underpins the crucial role that inertial effects play for this anomalous transport feature. Moreover, the presence of a nonadiabatic, high-frequency external driving is an indispensable ingredient for ANM. Notably, the phenomenon of ANM with $F\langle\langle v\rangle\rangle < 0$ implies that beneficial power can be extracted upon the switch-on of a positive load force F. This phenomenon of either pure noise-induced ANM or deterministic ANM must clearly be distinguished from the phenomenon of a noise-assisted, directed transport occurring in Brownian motors [15] which use a symmetry breaking of either spatial or temporal origin; for (uncoupled) Brownian motors we have that for a bias $F = 0$ the current is finite (with a positive mobility) while for ANM it is zero (with negative mobility).

These surprising ANM findings can readily be experimentally tested with a single Josephson junction device when driven in the experimentally available GHz regimes. Other test systems described by the model (1) are superionic conductors or rotating dipoles in external fields.

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