Mon. Not. R. Astron. Soc. **278,** 251–264 (1996)

Absolute proper motions and Galactic orbits of M 5, M 12 and M 15 from Schmidt plates

R.-D. Scholz,[1] M. Odenkirchen,[2] S. Hirte,[1] M. J. Irwin,[3] F. Börngen[4] and R. Ziener[4]

[1] *WIP-Projekt Astrometrie der Universität Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany*
[2] *Sternwarte der Universität Bonn, Auf dem Hügel 71, D-53121 Bonn 1, Germany*
[3] *Royal Greenwich Observatory, Madingley Road, Cambridge CB3 0EZ*
[4] *Thüringer Landessternwarte Tautenburg, D-07778 Tautenburg, Germany*

Accepted 1995 August 4. Received 1995 July 25; in original form 1995 April 26

ABSTRACT

Within the Potsdam Globular Cluster Programme we have obtained absolute proper motions of M 5 (NGC 5904), M 12 (NGC 6218) and M 15 (NGC 7078) from measurements of Tautenburg, Palomar and UK Schmidt plates. The plates were scanned with the APM facility in Cambridge. The tangential motion of the globular clusters was obtained by a plate overlap solution using large numbers of background galaxies as reference objects. Before that an extensive error-removal technique was applied in order to exclude systematic positional plate-to-plate distortions of the different Schmidt plates and the known errors of the measuring machine. Combining our results with the known distances and radial velocities of the clusters, we obtained their space motions and Galactic orbits in a three-component Galactic potential. Whereas the presently observed Galactocentric distances of M 12 and M 15 are typical for their calculated orbits, we conclude from our results that M 5 is apparently an outer halo cluster only briefly visiting the nearer regions.

Key words: astrometry – globular clusters: individual: M 5 – globular clusters: individual: M 12 – globular clusters: individual: M 15 – Galaxy: kinematics and dynamics.

1 INTRODUCTION

The kinematics of the Galactic globular clusters and of Galactic dwarf spheroidal satellites (dSph) is of great interest in the discussion of the dark matter and of the dynamics of the Milky Way. Investigations of the Galactic globular cluster system including the cluster space motions and orbits are usually based on only one velocity component. Whereas radial velocities of ~70 per cent of the known Galactic globulars are available, tangential motions have been obtained for only ~20 per cent of them. Webbink (1981) and Monella (1985) list radial velocity data for 85 and 108 globular clusters, respectively, whereas in the recent list of Pryor & Meylan (1993) more detailed information concerning the radial velocities, their uncertainties and dispersions are given for 57 clusters. Dauphole et al. (1995) studied the orbits of 26 globular clusters for which not only radial velocities but also proper motions were found in the literature. However, for most of these globulars the proper motions were not directly determined with respect to a non-rotating, quasi-inertial reference system.

Mean proper motions of many Galactic globular clusters have been determined by Cudworth and coworkers (including M 15, Cudworth 1976 and M 5, Cudworth 1979). In their investigations they used refractor plates with a good astrometric scale (about 10 arcsec mm^{-1}), taken over a baseline of more than 70 yr, so that they obtained very high accuracies in the relative proper motions. Their reference frame was built up from small numbers of field stars, however, and therefore the derived proper motions had to be converted to absolute proper motions using various assumptions concerning the secular parallaxes of these field stars and the solar motion. Consequently, the uncertainty in the absolute proper motion of each cluster remained of the order of a few mas yr^{-1} (cf. Cudworth 1976, 1979). The recent re-reduction by Cudworth & Hanson (1993) using new assumptions concerning the motion of the field stars and of the sun derived from the Lick absolute proper-motion program, improved the absolute proper-motion accuracy $(\sigma_{\mu_x}, \sigma_{\mu_y})$ to (1.7, 1.3) mas yr^{-1} for M 5 and (1.0, 1.0) mas yr^{-1} for M 15.

In an alternative approach to connect globular cluster motions to an extragalactic reference frame, Brosche, Geffert & Ninkovic (1983) and Brosche et al. (1985, 1991) used Lick stars with known absolute proper motion as reference stars. The work of Brosche et al. (1991) includes the cluster

M 12 with an absolute proper-motion accuracy of (1.3, 1.3) mas yr^{-1}. Tucholke (1992a, b) used this technique to derive the proper motion of selected globular clusters relative to the background of the SMC.

Recently, Geffert et al. (1993a) used visual measurements of less than 50 selected galaxies on Schmidt plates (POSS I glass copies and CERGA Schmidt plates), in addition to long baseline refractor plates, in order to define the absolute reference frame for the proper-motion determination in globular cluster fields.

With the aim of extracting the information stored on deep Schmidt plates we started a programme to define absolute proper motions of Galactic globular clusters based on automated scans of Tautenburg Schmidt plates with the APM machine at Cambridge (Scholz, Odenkirchen & Irwin 1993, 1994; hereafter called papers I and II). At higher Galactic latitudes ($|b|>20°$) there are large numbers of both faint galaxies and cluster stars ($\sim 10^3$) on the Schmidt plates which can be used for the determination of an accurate mean absolute proper motion of the cluster.

In our first investigations of M 3 (paper I) and M 92 (paper II) we used, exclusively, plates of the Tautenburg Schmidt telescope (see Section 2), which had some advantages for astrometric studies in comparison with other large Schmidt telescopes. The number of reference galaxies used in the differential plate-to-plate solutions (which were done independently for each pair of plates) was always larger than 1500. Extending our work to the dSphs in Draco and Ursa Minor we combined APM measurements of POSS I glass copies, second-epoch Palomar and third-epoch Tautenburg Schmidt plates (Scholz & Irwin 1994).

The absolute proper motions of M 5, M 12 and M 15 presented here have been obtained from APM scans of Palomar, Tautenburg and UKST plates. In contrast to the reduction procedure used in papers I and II and in Scholz & Irwin (1994), we applied a plate overlap method for which we also had to change the process of plate matching and systematic error removal. Combining our absolute proper motions of the clusters with radial velocities and distances from the literature has enabled us to obtain their space motions. Our determination of the Galactic orbits of these clusters is based on the new three-component Galactic model of Allen & Santillan (1991). Our results for M 5 and M 12, on which we already reported at IAU Symposium 164 (Scholz et al. 1995), are also used in Dauphole et al. (1995), who applied a similar Galactic potential in their calculations. In papers I and II and in Odenkirchen, Scholz & Irwin (1994) our orbital calculations for the clusters M 3 and M 92 were based on the old Galactic model potential of Allen & Martos (1986). For comparison, we also present here the recalculated orbits of M 3 and M 92 in the new potential.

2 OBSERVATIONS AND MEASUREMENTS

Table 1 lists the plate material in the three globular cluster fields. The Tautenburg 2-m telescope with a 1.34-m Schmidt correction plate and a focal length of 4 m was not designed for a sky survey. With a useful plate size of 24 cm and a scale of 51.4 arcsec mm^{-1} each plate covers only about 3°.3 × 3°.3 of sky. In comparison to the Palomar, UK and ESO Schmidt telescopes, however, the larger focal length and the smaller plate size lead to less problems with the plate bending and a better scale for astrometric work. Owing to the relatively bright sky and moderate seeing conditions at Tautenburg (near Jena), the plates usually do not go as deep as the Palomar or UKST plates. The limiting magnitude of Tautenburg B plates varies between 20 and 21.5. In some cases, as in the M 5 field included in this work, the Tautenburg plates selected from a larger number of plates taken in that field go deeper than the POSS I plates. For M 15 there were also plates from different epochs available in the Tautenburg plate archive, however, for M 12 only new epoch plates were taken with the Tautenburg telescope.

All Schmidt plates used in this project were measured with the APM measuring machine in Cambridge/UK (Kibblewhite et al. 1984) with a pixel size of ~7.5 μm. For the details of the measuring process we refer the reader to papers I and II. The measured objects were classified into stars, non-stellar objects, noise images and merged objects, using the standard APM software. In order to prevent possible magnitude-dependent systematic errors, only unsaturated and faint stars and galaxies were used in the determination of proper motions. Objects appearing to have merged on at least one of the plates were not used in subsequent analysis. The galaxies providing the absolute reference frame in the field of a globular cluster were selected from the objects classified as non-stellar on the deepest plates in a field. Only galaxies outside a given cluster radius were selected because of possible overlapping images of cluster stars masquerading as galaxies in the classification procedure. The number density of the selected reference galaxies varied over the field mainly as a result of the different limiting magnitudes of the overlapping plates, but also owing to some galaxy clusters in the field (see e.g. Fig. 1b).

3 THE ABSOLUTE CLUSTER PROPER MOTIONS

Stellar proper motions can be obtained by positional astrometry using (hereafter so called) 'plate solutions', with an external reference catalogue at different epochs or by differential astrometry working with 'plate-to-plate solutions'. In papers I and II, but also in Scholz & Irwin (1994), we preferred to average the independent results from different plate-to-plate solutions for the determination of proper motions instead of combining all plates in an overlap solution. The iterative plate overlap method of Eichhorn (1960) consists of the stepwise improvement of a catalogue of positions α, δ and proper motions μ_α, μ_δ of stars. In the first step a plate solution is obtained for each plate by use of stars from a reference catalogue containing α, δ and μ_α, μ_δ. From the obtained positions of all measured stars at different epochs, a first approximation of their proper motions and of their positions at the mean epoch can be derived. The results are then used as a new reference catalogue in the next iteration step, and so on.

Our experience in Schmidt plate solutions with reference catalogues like the PPM (Röser & Bastian 1988) shows that because of the distortion of the plate during the exposure, the geometry of the Schmidt plates cannot be fully calibrated by the usually small numbers (~100) of reference stars. Therefore, the accuracy of the position α, δ of a target object measured with respect to the reference stars on a Schmidt plate depends not only on the accuracy of the reference

Table 1. Plate material.

Cluster	Telescope/ Plate No. (* ref. plate)	Passband	Distance of cluster from plate centre ° in α	° in δ	Epoch
M 5	POSSI/O1402	U+B	+0.17	−2.75	1955.30
	POSSI/E1402	R	+0.16	−2.73	1955.30
	Tautbg/3433	B	−0.07	−0.04	1972.35
	Tautbg/3436	V	−0.02	−0.04	1972.35
	UKST/J5193	B_J	+1.02	−2.38	1979.54
	Tautbg/5821*	V	+0.06	−0.03	1982.36
	Tautbg/8348	B	−0.04	−0.02	1993.37
	Tautbg/8353	B	−0.04	−0.02	1993.38
M 12	POSSI/O155	U+B	+2.05	+1.66	1950.46
	POSSI/E155	R	+2.05	+1.63	1950.46
	UKST/J6192*	B_J	−1.16	+1.77	1980.59
	Tautbg/8349	B	−0.09	−0.08	1993.37
	Tautbg/8361	B	−0.01	−0.04	1993.39
M 15	POSSI/O298	U+B	−2.74	+0.38	1951.58
	POSSI/E298	R	−2.76	+0.38	1951.58
	Tautbg/2618*	B	−0.08	−0.05	1967.82
	Tautbg/2657	V	−0.06	−0.08	1967.85
	Tautbg/7595	B	−0.07	−0.04	1991.53
	Tautbg/7615	B	−0.15	−0.06	1991.67
	Tautbg/7631	V	−0.15	−0.07	1991.69
	Tautbg/7633	V	−0.15	−0.05	1991.69

catalogue and the random measuring error on that plate, but also on the position of the target object in the field. In addition, there are systematic magnitude-dependent effects, so that a plate solution obtained with the bright stars from a reference catalogue is not representative of the majority of fainter objects measured on the plate. These magnitude-dependent effects are the result of the combination of asymmetric image profiles and photographic saturation on the Schmidt plates. Basically, in good seeing, at some 3 to 4 mag above the plate limit all the stellar images begin to saturate.

If we are interested only in the faint objects measured on the Schmidt plates and mainly in their proper motions, we can work without a reference catalogue. There are large numbers of faint stars (and galaxies) which can be used for highly accurate differential plate-to-plate solutions. With these solutions, not only the differential Schmidt plate geometry, but also any systematic field position distortions coming from the measuring machine, can be controlled and corrected for (see Section 3.1). Magnitude-dependent effects also appear in plate-to-plate solutions owing to the different observing conditions. Fortunately, the vast majority of the cluster stars and the background galaxies used in our investigation are not saturated, and therefore we do not expect magnitude-dependent systematic errors.

Considering the problems described above, we chose to combine the advantages of the differential method with those of an iterative improvement of the results in an overlap solution. Consequently, we first modelled the differential plate-to-plate geometry, transforming all measuring coordinates to one common system. In this process, described in Section 3.1, we also looked for and removed (a) residual large-scale systematic effects after a third-order polynomial plate-to-plate solution had been applied and (b) systematic errors of the measuring machine using the assumption that all faint stars outside the cluster have a constant motion. The following application of the iterative Eichhorn method (Section 3.2) is based on the use of the galaxies with only provisional α, δ but known zero proper motions as the reference catalogue. By this procedure we achieve an improvement of the μ_α, μ_δ of the stars without having to improve the absolute positions α, δ of the stars.

3.1 Plate matching and systematic error removal

With the intention of combining all plates covering a cluster field in an overlap solution, we selected one of these plates as the reference plate in the plate-matching process. Fig. 1(a) shows, as an example, matched objects in the M 12 field.

The reference plate of each field is marked in Table 1. There were several criteria for selecting a plate as a reference plate:

(i) the plate covers the maximum overlap field of all plates around the cluster;

(ii) the plate is one of the deepest plates;

(iii) there is a minimum of noise images and/or dirt on the plate.

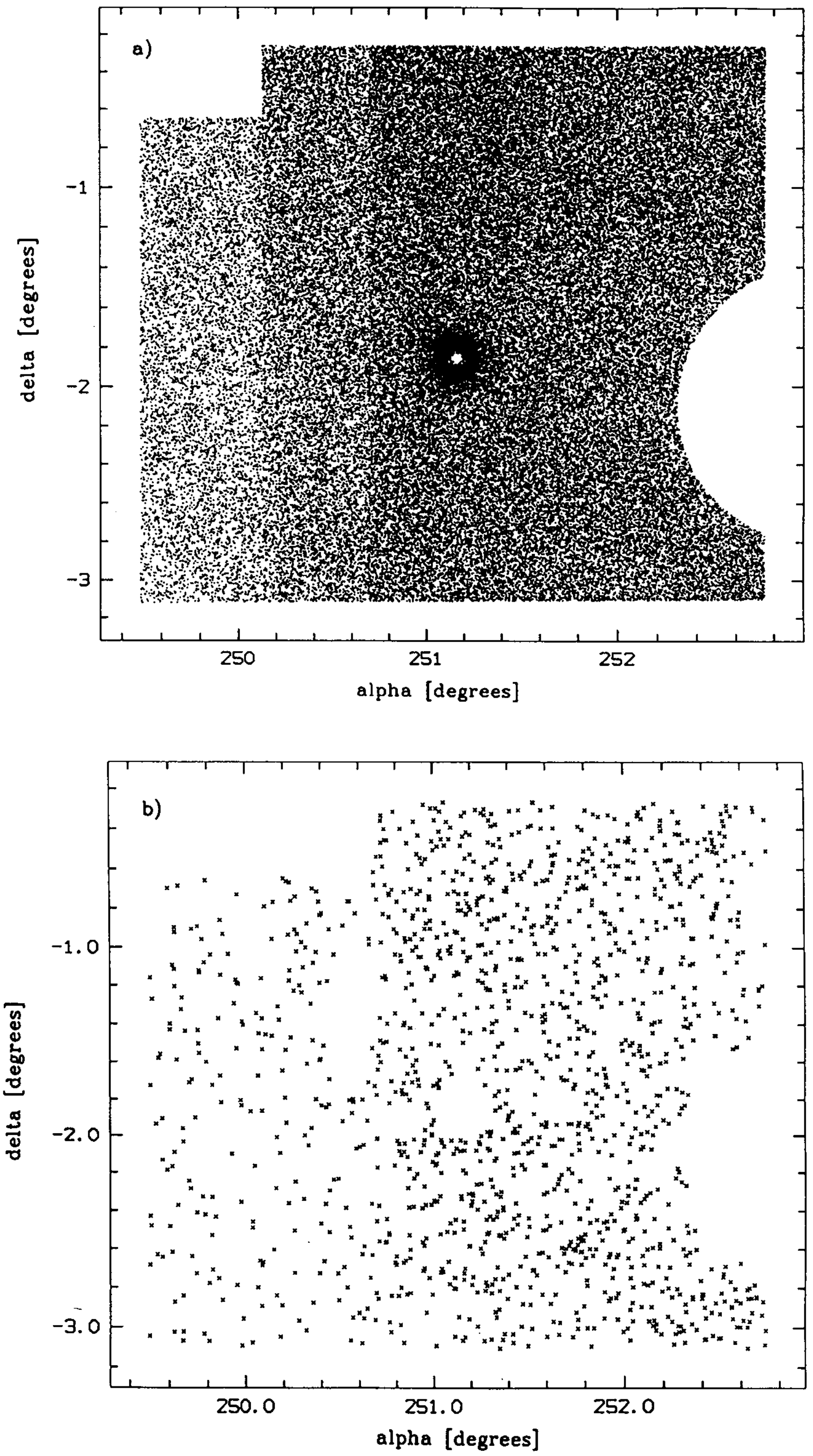


Figure 1. Overlapping plates in the M 12 field with objects measured on the reference plate and at least one other plate. (a) All objects (~100 000). (b) Reference galaxies (~1300). (Images in the empty regions of the rectangular overlap zone were affected by density wedges and not used in the reduction. In the cluster region no galaxies were selected owing to the crowding and classification problems.)

In the iterative plate-matching procedure we begin by finding the brighter stars on the reference plate and a comparison plate using a large search radius of 5 to 10 arcsec and a linear plate-to-plate model. The final iteration is based on a third-order polynomial plate-to-plate solution using all objects and a search radius of 1.5 arcsec.

After this transformation of the measuring coordinates of the comparison plate to reference plate system, we applied an error-reduction technique to the residual coordinate differences. This removal of position-dependent errors is based on the investigations of Evans (1988), and has already been described in papers I and II. In the first step a two-dimensional binning and smoothing of the coordinate differences was carried out using a bin size of 20×20 arcmin^2. This procedure removes residual large-scale systematic differences between the coordinates on the reference plate and those on the comparison plate that could not have been modelled by the third-order polynomial. The large-scale systematic effects are mainly the result of different observing conditions when the plates were taken. In the next step we investigated all systematic effects in the coordinate differences, binning and removing them iteratively along the coordinate axes with bin sizes of about 3 arcmin. This iterative procedure is necessary to exclude the periodic errors of the measuring machine.

Whereas in papers I and II the error removal was done separately for each independent pair of plates, we had to apply this technique here for each plate with respect to the same reference plate prior to the iterative overlap solution. An error reduction after the plate overlap solution would involve a consideration of the different number of plates each object was measured on. Note that all systematic error removals are done differentially, i.e. in comparison to the particular reference plate. Therefore, the coordinate system on the reference plate to which all other plates are transformed remains unchanged. That is, the error removal does not improve the absolute positional accuracy, but is important for estimating the stellar proper motions.

The cluster region itself was not used in the determination of either the two-dimensional or one-dimensional corrections. The removal of position-dependent systematic errors assumes that the field stars have a constant motion over the area processed. This is a valid assumption away from the Galactic plane, since all that we require is the average field star motion, over the magnitude range of the unsaturated images considered, to be a constant.

3.2 The plate-overlap method

In paper I and paper II we used only plates at two different epochs and determined the proper motions independently from different pairs of plates. In these plate-to-plate solutions with the galaxies as reference points we applied third-order polynomials and the method of stepwise regression described in Hirte et al. (1990) separately for each pair of plates.

In the present work we did not arrange the plate material from three different epochs in pairs, but applied the iterative plate overlap method of Eichhorn (1960). The basic modification of this method was the use of a reference catalogue of galaxies with zero proper motions. Therefore, the reference catalogue was given as the α, δ of the galaxies obtained from the standard APM plate solution for the reference plate (see plate matching) with the PPM catalogue. The standard APM plate solution with the PPM consists of a concentric projection of the measuring coordinates on the Schmidt plate in combination with a linear plate model.

In the plate matching and error removal process (see Section 3.1), the measuring coordinates from all plates in a field were transformed to the measuring coordinate system of the reference plate by applying third-order polynomials and the two-dimensional and one-dimensional binning and correction methods. Therefore, we preferred to use the simplest (i.e. linear six-plate constant) model in combination with the concentric Schmidt projection in the following determination of α, δ for each plate at its epoch during the iterative Eichhorn procedure. The proper motions were then determined from a linear fit of the obtained α, δ positions over the plate epochs. The reference catalogue of the galaxies with zero proper motions was used in the first approximation, and in all following iteration steps we used all objects with reliable proper motions (i.e. those obtained from at least three plates at different epochs) as the new reference catalogue. After two iteration steps, there were only small changes and improvements in the accuracy of the determined proper motions.

In the M 12 field, where we had chosen the UKST plate as the reference plate, we applied a complete quadratic polynomial (i.e. including terms x^2, xy, y^2) instead of the linear six-plate constant model. The reason for that was the large deviation between the tangential point of the reference plate and the centre of the overlap field. In the other fields, we also experimented with different models in the determination of α, δ during the iterative Eichhorn method, and found that the linear model provided the best results.

Owing to the non-uniform distribution of the reference galaxies over the field, and their large individual measuring errors, there remained some systematic errors in the field star proper motions, i.e. deviations from a constant motion. Therefore, we tried an alternative method using faint reference stars outside the cluster region, and in a small magnitude interval with smaller measuring errors instead of the galaxies in the first approximation of the iterative Eichhorn method. Large proper-motion stars were excluded by the plate matching with a final search radius of 1.5 arcsec (see Section 3.1) in combination with the condition that every reference object appeared on at least three plates at different epochs. The proper motions of the reference stars were simply set to zero in the first input catalogue. After a few iterations and the determination of relative proper motions for all objects, we subtracted the mean 'proper motion' of the galaxies. This procedure did not change the results for the mean absolute cluster proper motion, but improved the results in the field. Especially in the field of M 15, where we had only about 500 reference galaxies, we obtained more reliable results by incorporating a few thousand reference stars.

In the plots of the final field stars' proper motions against the coordinates, there was no evidence of significant systematic effects in the 100 per cent overlap zone of the plates. Therefore, the mean proper motion of the field stars, within the same small magnitude interval as used for the cluster stars, could be determined very accurately. Investigations of the field stars' proper motions with respect to magnitudes and colours will be the subject of further investigations concerning Galactic structure. We will also use the proper motions for a membership analysis of faint cluster stars (at least for the case of M 5) in our next paper. Here, we concentrate on the mean absolute cluster motions by averaging large numbers of cluster and field stars.

3.3 Proper-motion accuracy

If averaging the proper motion of all stars within a given cluster radius, the mean cluster proper motion has to be corrected for the contamination with field stars in the cluster region:

$$\mu_c = (1 + n_f/n_c)\mu_{c+f} - (n_f/n_c)\mu_f.$$

The cluster region was selected as $4 < r < 14$ arcmin for M 5, $3 < r < 13$ arcmin for M 12, and $3 < r < 12$ arcmin for M 15. In the core of the clusters no stellar images could be measured because of the image crowding on the Schmidt plates. Fig. 2 shows the radial change of the mean proper motion. The number of field stars n_f in the cluster region was estimated from the stellar density in the outer field. The ratio n_f/n_c in the selected cluster regions (see above) was quite different at the low Galactic latitude of M 12 (920/800) in comparison with M 5 (215/845) and M 15 (250/920).

The derived proper motions of the cluster stars were also investigated for a systematic effect dependent on their magnitudes, unless the magnitude interval had already been restricted in the reduction process. Only in one case, M 5, did we find a significant magnitude equation in μ_δ. As a zero point for the correction we took the mean magnitude of the reference galaxies. The correction for the magnitude equation changed the value of μ_δ for M 5 by -0.7 mas yr^{-1}. The

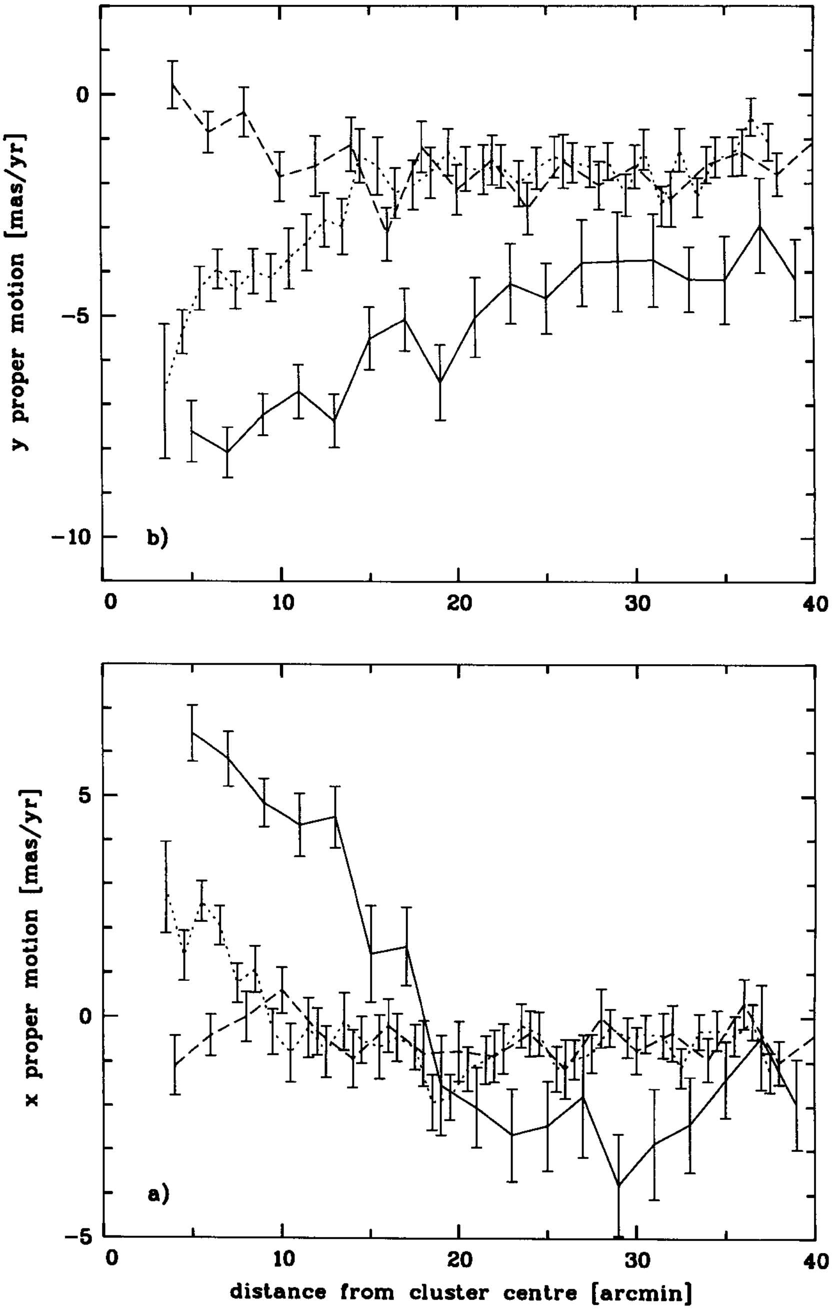
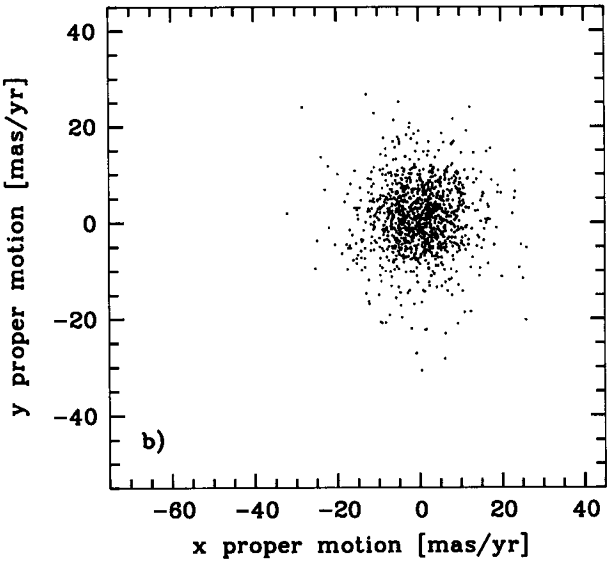


Figure 2. Contamination with field stars as seen in the radial change of the mean absolute proper motion (M 5: solid line, M 12: dotted line; M 15: dashed line).

reason for the magnitude effect in the M 5 field is not clear to us, but we note that the location of the cluster on the POSS I O and E plates is very close to the southern edge, where systematic errors are likely to be much larger than normal. The advantage of using centre-of-gravity position estimates for unsaturated images throughout our study is that we effectively sample the same part of the image profile for all images in the magnitude range. In general, we do not find a significant magnitude term for unsaturated images.

The formal error of the mean proper motion of all stars in the cluster region was only about 0.2 to 0.3 mas yr^{-1}. However, the main component in the error of the mean absolute cluster proper motion is dependent on the number of galaxies and on their proper-motion dispersion. The number of galaxies, n_g, used for the reference frame in the field around a globular cluster was 1160, 1385 and 510, respectively for M 5, M 12 and M 15. The formal error in their mean (zero) proper motion, determined as $d_g/\sqrt{n_g}$, where d_g is the proper-motion dispersion of the galaxies, varied between 0.2 and 0.5 mas yr^{-1}. For the most interesting case of M 5 we show the proper-motion diagram for all stars in the cluster region, and for the reference galaxies in the outer field, respectively, in Figs 3(a) and (b). Here we can distinguish (even in the Schmidt data with only ~4 mas yr^{-1} accuracy) for a single star the proper-motion distribution of the cluster from that of the field stars. Also, we see that the mean absolute proper motion of M 5 in μ_δ cannot be as large as given by Cudworth & Hanson (1993).

A more realistic error estimate (than the formal error described above) for the mean absolute proper motion of a cluster was found by comparing the results obtained by using different samples, both for the reference galaxies and for the cluster stars (cf. bootstrap resampling). We compared the results obtained from the full set of galaxies and stars with those derived using only the objects measured using more plates or having higher individual proper-motion accuracies. The absolute cluster proper motions given in Table 3 represent the mean results from about 10 alternative computations per cluster. The results of all of these computations with different object samples were within the errors given in Table 3.



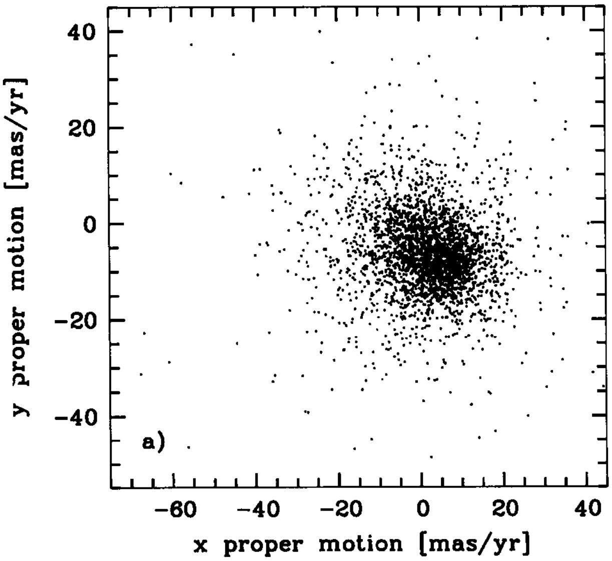


Figure 3. Proper-motion diagram for M 5. (a) All stars in the cluster region (here we selected $4 < r < 35$ arcmin in order to plot about equal numbers of cluster stars and field stars). Note that in this case, with a large difference between the cluster and the field star mean proper motion, we can see two distributions: one for the field stars with large dispersion and one for the cluster stars with lower dispersion. (b) The reference galaxies in the M 5 field.

3.4 Comparison with other results

A comparison of our results with former values of the absolute cluster proper motions is shown in Table 2.

The M 5 proper motion of Geffert et al. (1993b) and the first value of Geffert et al. (1993a) given for M 15 are mean relative proper motions with respect to the PPM system, whereas the second value of Geffert, Forner & Hiesgen (1993a) for M 15 is the result of a zero-point correction based on additional measurements of 28 star/galaxy pairs (i.e. nearby images) on POSS I and CERGA Schmidt plates. This, in our opinion, is nowhere near enough points to model the differential geometry of Schmidt plates taken with different telescopes and different plate centres. The results of Cudworth & Hanson (1993) are based on highly accurate relative proper motion measurements of Cudworth (1976) for M 15 and of Rees (1993) for M 5, which were converted to absolute proper motions using new assumptions concerning the motion of the field stars and of the sun, derived from the Lick absolute proper-motion program. Despite the high accuracy of the relative proper motion measurements, this method suffers from the small number of field stars available and their inherent proper-motion dispersion. Brosche et al. (1991) used Lick stars with known absolute proper motion as reference stars to connect the globular cluster motions to the extragalactic reference frame.

For M 5 we find an absolute proper motion in the same direction, but with only about half of the value in μ_δ as given by Cudworth & Hanson (1993), using the results of Rees (1993). Our results for M 12 are in good agreement with those of Brosche et al. (1991). The absolute proper motion of M 15 is in agreement with that of Cudworth & Hanson (1993) and Geffert et al. (1993a) in $\mu_\alpha \cos\delta$ only. For μ_δ

there is a wide spread of the results from 0 to -10 mas yr^{-1}. From Fig. 2(b) we can see that we would also get a negative value for the μ_δ of M 15, not taking into account the contamination by field stars.

4 SPACE MOTIONS AND GALACTIC ORBITS

Together with distances and radial velocities, which are well known for many of the Galactic globular clusters, absolute proper motions provide the key for the determination of the space motions of globular clusters in our Galaxy. We combined our proper-motion data for M 5, M 12 and M 15 with distances and radial velocities from the recent compilations of Peterson (1993) and Pryor & Meylan (1993) (see Table 3). Using standard values for solar motion, motion of the LSR and the distance between the Sun and the Galactic centre, i.e. (10, 15, 7.5) km s^{-1}, 220 km s^{-1} and 8.5 kpc, we derived from the data sets of Table 3 the present positions and velocities of the clusters in a Galactocentric inertial frame. These are reported in Table 4. The given uncertainties were obtained from error estimates for the observations (cf. Table 3) and for the solar parameters. They are of the order of 0.5 kpc for the position components and 25 km s^{-1} for the velocity components.

If the gravitational potential of the Galaxy is assumed to be known, the vectors from Table 4 uniquely determine the Galactic orbits of the clusters. In papers I and II our orbit calculations for the clusters M 3 and M 92 were based on the Galactic model potential of Allen & Martos (1986). Here, we adopt the more recent model by Allen & Santillan (1991). As before, this model consists of three components, which account for the Galactic bulge, disc and a massive halo. The mathematical formulation of the disc component has become simpler, and avoids certain shortcomings of the former model. For the Galactic bulge the new model is expected to give a more realistic representation than the former. In the halo the potential remains similar to the former model (for details see Allen & Santillan 1991). The new model incorporates somewhat different values for the velocity of the LSR and the Sun–Galactic centre separation, now following recent IAU recommendations. In order to be consistent we have used these values also for the calculation of the Galacto-

Table 2. Comparison with former results.

Cluster	absolute proper motion $\mu_\alpha \cos\delta$ [mas/a]	μ_δ [mas/a]	reference
M 5	$+6.1 \pm 2.9$	$+37.6 \pm 2.8$	Meurers & Hallermann (1966)
	$+5.2 \pm 1.7$	-14.2 ± 1.3	Rees (1993), Cudworth & Hanson (1993)
	-14.5	-1.0	Geffert *et al.* (1993)
	$+6.7 \pm 0.5$	-7.8 ± 0.4	this work
M 12	-5.8 ± 1.9	$+7.5 \pm 3.2$	Meurers & Hallermann (1966)
	$+1.6 \pm 1.3$	-8.0 ± 1.3	Brosche *et al.* (1991)
	$+3.1 \pm 0.6$	-7.5 ± 0.9	this work
M 15	-0.3 ± 1.0	-4.2 ± 1.0	Cudworth & Hanson (1993)
	$+1.3 \pm 4.4$	-6.8 ± 4.4	Geffert *et al.* (1993)
	-1.0 ± 1.4	-10.2 ± 1.4	Geffert *et al.* (1993)
	-0.1 ± 0.4	$+0.2 \pm 0.3$	this work

Table 3. Data from observations.

Cluster	α (1950) h	m	δ (1950) °	'	D [kpc]	v_r [km/s]	$\mu_\alpha \cos\delta$ [mas/a]	μ_δ [mas/a]
M 5	15	16.0	+2	16	7.6 ± 0.8	$+54.0 \pm 1.0$	$+6.7 \pm 0.5$	-7.8 ± 0.4
M 12	16	44.6	−1	52	5.6 ± 0.6	-42.6 ± 1.0	$+3.1 \pm 0.6$	-7.5 ± 0.9
M 15	21	27.6	+11	57	10.5 ± 1.1	-108.5 ± 1.5	-0.1 ± 0.4	$+0.2 \pm 0.3$

Table 4. Derived Galactocentric position and velocity.

Cluster	x [kpc]	y [kpc]	z [kpc]	U [km/s]	V [km/s]	W [km/s]
M 5	-3.3 ± 0.7	0.4 ± 0.1	5.5 ± 0.6	316 ± 31	190 ± 26	-203 ± 29
M 12	-3.8 ± 0.7	1.5 ± 0.2	2.6 ± 0.3	88 ± 17	126 ± 31	-166 ± 23
M 15	-4.6 ± 0.7	8.5 ± 0.9	-4.8 ± 0.5	33 ± 17	154 ± 22	66 ± 16

centric positions and velocities of the clusters. As in previous papers (e.g. Odenkirchen & Brosche 1992), the equations of motion were integrated backwards from the present time to −10 Gyr by means of a Bulirsch–Stoer numerical integrator. The resulting orbits for M 5, M 12 and M 15 are described by projections on to the meridional plane (Figs 4a, 5a and 6a) and on to the Galactic plane (Figs 4b, 5b and 6b), and by a collection of orbital parameters given in Table 5. In each of the three cases the motion is regular (i.e. non-chaotic).

For comparison, we also recalculated the orbits of the two clusters M 3 and M 92, in the new potential. Their new orbital parameters are also given in Table 5. In general, the new values are very similar to those found with the older model. As a result of the changes in the potential of the central bulge component, the new perigalactic distances are slightly shorter and the maximum velocities somewhat higher. Therefore, the mean period of revolution on the new orbits is found to be 5 per cent smaller in the case of M 3 and

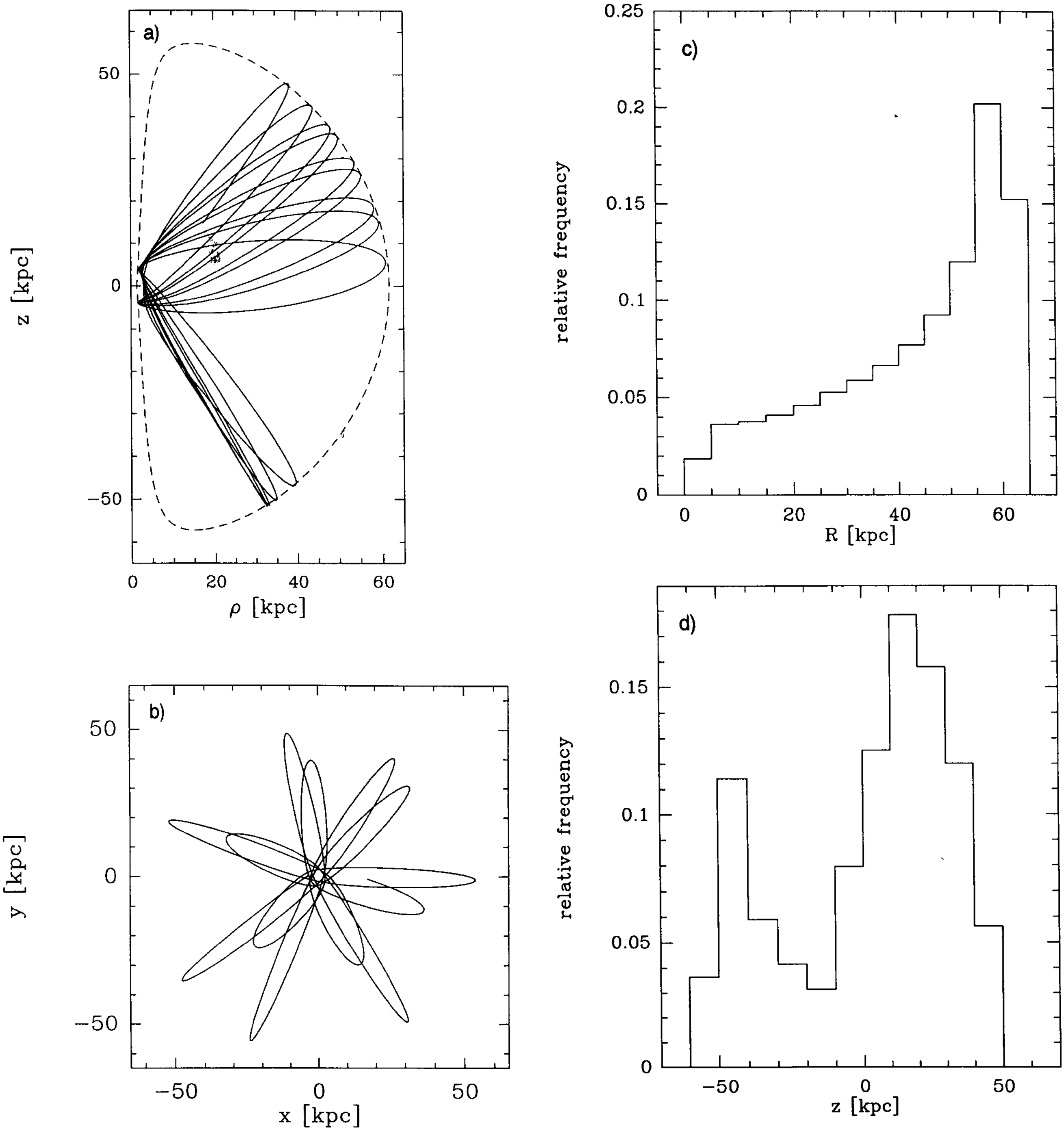


Figure 4. Orbit of M 5 in the time interval [−10, 0] Gyr. (a) Meridional section. (b) Projection on to Galactic plane. (c) Relative frequency of Galactocentric distance R along the orbit. (d) Relative frequency of distance from Galactic plane along the orbit.

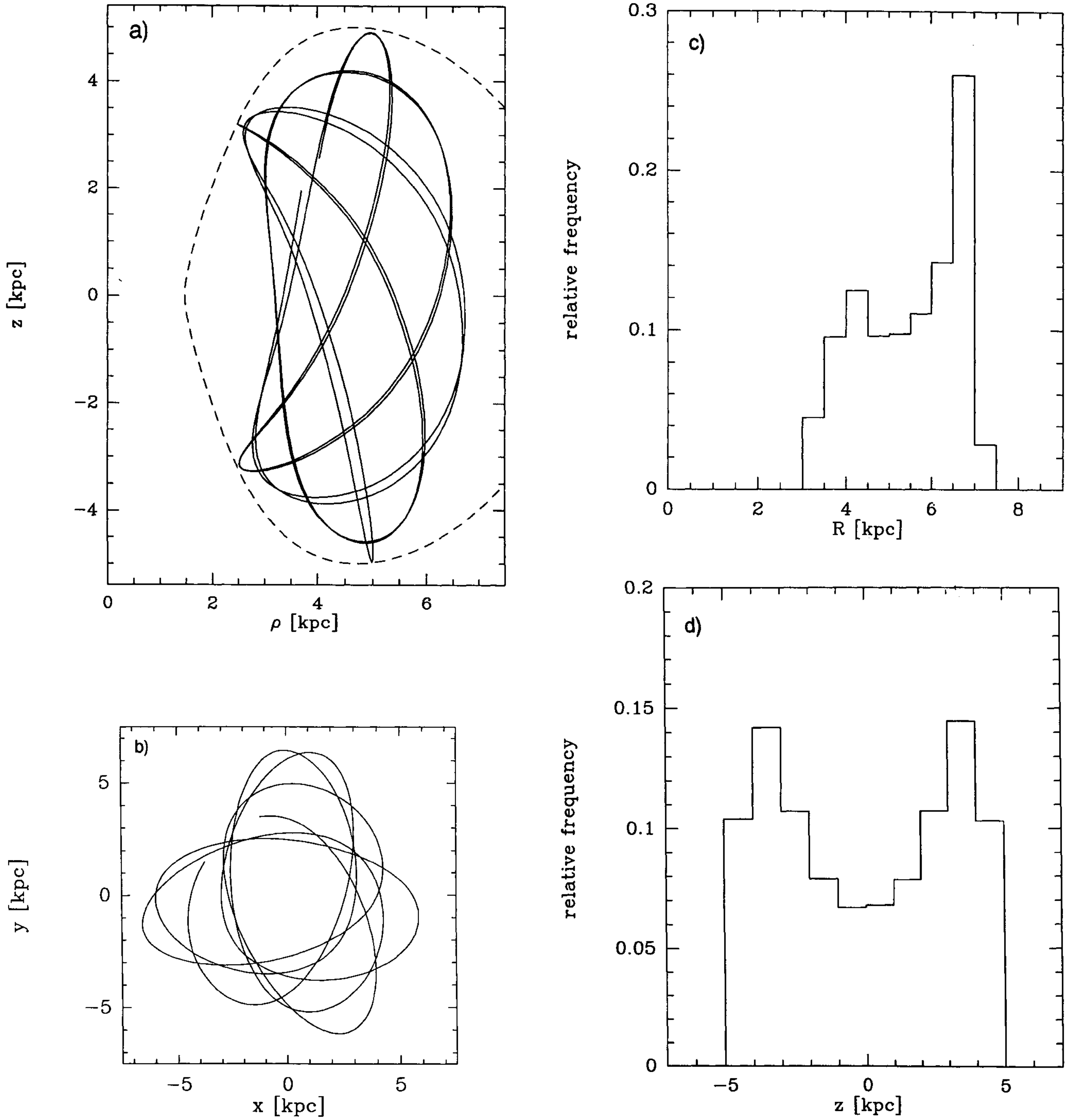


Figure 5. Orbit of M 12: (a) Meridional section, time interval [−1, 0] Gyr. (b) Projection on to Galactic plane, time interval [−1, 0] Gyr. (c) Relative frequency of Galactocentric distance R along the orbit. (d) Relative frequency of distance z from the Galactic plane along the orbit.

8 per cent smaller in the case of M 92. However, global features of the orbits, for example the box type for M 3 and 'tube' type for M 92, remain the same.

4.1 The orbit of M 5

Our astrometric measurements of M 5 yield a total proper motion of 10 mas yr^{-1}. Together with the heliocentric distance of the cluster, its radial velocity and its position on the sky, this transforms to a very high Galactocentric space velocity of 420 km s^{-1}. The resulting orbit of M 5 leads through the outskirts of the Galaxy. As can be seen from Figs 4(a) and (c), the apogalacticon lies at 61 kpc. The mean distance from the Galactic centre during 10 Gyr is 43 kpc. [Note that with the velocity values from Cudworth & Hanson (1993) the apogalactic distance of M 5 would rise as high as

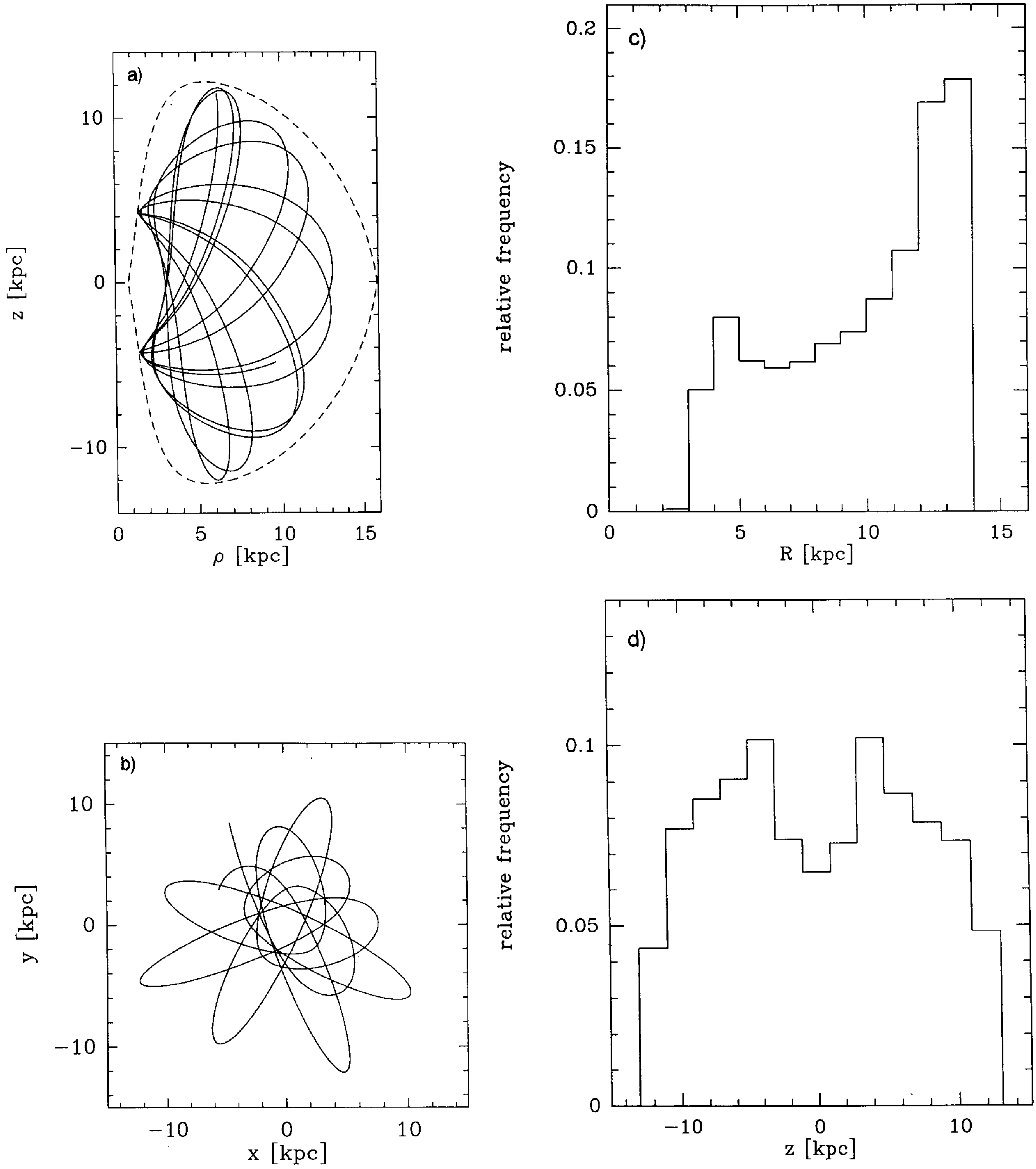


Figure 6. Oribt of M15. (a) Meridional section, time interval [−2, 0] Gyr. (b) Projection on to Galactic plane, time interval [−2, 0] Gyr. (c) Relative frequency of Galactocentric distance R along the orbit. (d) Relative frequency of distance z from the Galactic plane along the orbit.

140 kpc.] The time intervals at which the cluster is found to be within a distance of 10 kpc from the Galactic centre add up to only 6 per cent of the total time.

Fig. 4(b) illustrates that M 5 is in slow prograde rotation around the Galactic centre. It moves on highly eccentric loops, with a mean period of revolution of 1.3 Gyr, equivalent to only 7.5 revolutions in 10 Gyr. The orbit is highly inclined to the Galactic plane, reaching distances of up to 52 kpc from the Galactic plane. While the orbital plane slowly rotates, its inclination remains constant to within ±8°.

Table 5. Orbital parameters during [−10, 0] Gyr.

a) Geometry of the orbit

Cluster	R_{min}	R_{max} [kpc]	$\overline{R}$	$R_{t=0}$	e	z_{min} [kpc]	z_{max}
M 5	3.0	61.2	43.3	6.5	0.91	−51.7	47.9
M 12	3.2	7.1	5.5	4.8	0.38	−5.0	5.0
M 15	3.0	13.5	9.7	10.8	0.64	−12.2	12.2
M 3	4.6	12.4	9.3	11.8	0.46	−12.1	12.1
M 92	1.0	11.7	8.1	9.5	0.84	−1.3	5.7

b) Absolute value of velocity

Cluster	v_{min}	v_{max} [km/s]	$\overline{v}$	$v_{t=0}$
M 5	23	513	162	421
M 12	121	322	201	226
M 15	74	393	188	171
M 3	106	340	195	130
M 92	6	460	172	126

c) Rotation around z-axis

Cluster	Θ_{min}	Θ_{max} [km/s]	$\overline{\Theta}$	$\Theta_{t=0}$	$\overline{T}$ [Gyr]	n_U
M 5	12	451	36	222	1.332	7.5
M 12	89	243	142	149	0.171	58.6
M 15	32	306	82	44	0.286	35.0
M 3	24	260	66	39	0.277	36.1
M 92	−46	−6	−15	−7	0.267	−37.4

d) Integrals of motion and inclination of orbital plane

Cluster	J_z [kpc·km/s]	E [km^2/s^2]	$\overline{i}$	Δi [degree]
M 5	−739	−57290	116	+6 / − 8
M 12	−602	−136390	128	+7 / −10
M 15	−424	−113532	111	+4 / − 7
M 3	−282	−113724	101	±2
M 92	+61	−125407	71	+14 / −52

In the meridional projection shown in Fig. 4(a), the orbit appears to be asymmetric with respect to the Galactic plane. In the 10-Gyr time interval the probability of finding the cluster at northern Galactic latitudes is 64 per cent, but is only 36 per cent for southern latitudes. However, this effect is a result of the low number of revolutions within this time interval. On larger time-scales, the orbit would fill its 'cone'-like envelope more and more symmetrically. The degree of asymmetry also changes if the initial values for the orbit (present position and velocity) are varied within the given range of uncertainty.

According to Table 5 and Figs 4(c) and (d), the present values of the position of the cluster and its velocity are very unusual along the calculated orbit. One may conclude that either we are indeed observing M 5 in a very special state or that the proper motions or the Galactic model are incorrect. However, we have no reason to doubt the derived high space velocity of M 5, which together with its current Galactic location, leads us to the conclusion that M 5 is most likely an outer halo cluster briefly visiting the inner region. As such it provides an excellent example of an outer halo cluster for close-up study.

4.2 The orbit of M 12

For M 12 we find an orbit that belongs to the inner halo of the Galaxy. It remains within the solar circle at distances between 3.2 and 7.1 kpc from the Galactic centre, and not farther than 5 kpc from the Galactic plane (see Figs 5a to d). Hence the eccentricity of the orbit is only moderate compared with other clusters. M 12 rotates rapidly about the Galactic z-axis in a prograde sense, with a mean rotational velocity of 140 km s^{-1}. In 10 Gyr it completes nearly 59 revolutions around the Galactic centre. Thus, the mean period for one revolution is only 0.17 Gyr. In order to avoid overcrowding, Figs 5(a) and (b) only show 10 per cent of the calculated orbital path of M 12.

In projection on to the meridional plane the orbit keeps within a symmetric box-shaped envelope. This box lies within the so-called zero-velocity curve (dashed line in Fig. 5(a) given by the values of the two known constants of the motion), but takes a much smaller area than would be allowed by this curve. The meridional orbit also shows an approximate periodicity, which arises from 'reflections' in the upper-left and lower-right corners of the box. This feature is, however, not retained, when realistic variations of the initial values of the cluster are taken into account.

Several orbits were calculated, with the initial velocity of the cluster changed according to the probable errors in the observations. We found that under such variations the spatial extent of the orbit changes by about ±15 per cent, while the mean period of revolution ranges between 0.15 and 0.19 Gyr.

4.3 The orbit of M 15

The orbit of M 15 is a halo orbit of medium spatial extent. This can be seen in Figs 6(a) and (b), where projections of the orbital path for the interval $[-2, 0]$ Gyr are given. The characteristic parameters of the orbit are similar to those found for the cluster M 3 (see Table 5). Within the 10-Gyr interval the Galactocentric distance of the cluster varies from 3.0 to 13.5 kpc, but with a 55 per cent probability for values larger than 10 kpc. Its distance from the Galactic plane can rise to 12 kpc, yet the most probable value of $|z|$ is around 4 kpc (cf. Figs 6c and d). Therefore, one may say that the presently observed values of R, z (and also those of the velocity components) are quite typical for the calculated orbit. However, the orbit we present here is quite different from the one shown by Geffert et al. (1993a). This disparity is a result of considerably different results for the proper motions and hence for the space velocity of M 15.

Like the majority of globular clusters, M 15 is in prograde motion around the Galactic centre. The mean period of revolution is found to be about 0.29 Gyr, corresponding to 35 revolutions in 10 Gyr. The orbital plane is inclined against the Galactic plane at an angle of approximately 69° and has temporal variations of not more than ±7° along the orbit.

Again, we investigated how the orbit and its characteristic parameters change within the given range of uncertainty of the initial values. It turned out that the maxima of R and $|z|$ may differ by about 10 per cent from the values of the mean orbit, whereas the perigalactic distance could change up to 25 per cent. The mean period of the orbit of revolution ranges from 0.26 to 0.31 Gyr. The shape of the envelope of the meridional orbit does not change.

5 CONCLUSION

The determination of mean absolute proper motions of the majority of Galactic globular clusters remains as a task for the near future. We have shown that the use of fully scanned Schmidt plates allows the direct and accurate measurement of the cluster proper motions against the absolute reference frame represented by large numbers of galaxies. In our opinion, measuring the reference galaxies, the cluster and the field stars in one measuring process yields a better estimate and correction of systematic errors than other methods and should, therefore, provide the most reliable results. Our results partly agree with measurements made previously by other authors using different methods. For the very interesting case of M 5, we obtained a smaller absolute proper motion, which still led to a high space velocity, but not to the extremely large value found. For a comparison of Galactic models and of different parameters of the Galactic globular clusters with their space velocities, computed from both the radial and the tangential velocities, we need many more absolute proper-motion determinations. Among the next candidates to be studied in our programme, there are some more-distant clusters including NGC 2419, 5053, 5634, 6229 and 7492 for which absolute proper motions are not yet available, but also some clusters like M 2, M 13 and NGC 5466 for which direct or indirect measurements of absolute proper motions have already been published.

ACKNOWLEDGMENTS

This work is based on Schmidt plates taken at the Tautenburg, Palomar and UKST observatories. The APM is a national astronomy facility financed by the Particle Physics and Astronomy Research Council. MO acknowledges financial support from the Bundesministerium für Forschung und Technologie (FKZ 50 OO 9101 6). We thank the referee, Dr K. Cudworth, for his comments and suggestions, which improved the discussion of our results.

REFERENCES

Allen C., Martos M. A., 1986, Rev. Mex. Astron. Astrofis., 13, 137
Allen C., Santillan A., 1991, Rev. Mex. Astron. Astrofis., 22, 255
Brosche P., Geffert M., Ninkovic S., 1983, Publ. Astron. Inst. Czech. Acad. Sci., 56, 145
Brosche P., Geffert M., Klemola A. R., Ninkovic S., 1985, AJ, 90, 2033
Brosche P., Tucholke H.-J., Klemola A. R., Ninkovic S., Geffert M., Doerenkamp P., 1991, AJ, 102, 2022
Cudworth K. M., 1976, AJ, 81, 519
Cudworth K. M., 1979, AJ, 84, 1866
Cudworth K. M., Hanson R. B., 1993, AJ, 105, 168
Dauphole B., Geffert M., Colin J., Ducourant C., Odenkirchen M., Tucholke H.-J., 1995, A&A, in press
Djorgovski S., Meylan G., 1993, in Djorgovski S., Meylan G., eds, ASP Conf. Ser., Vol. 50, Structure and dynamics of globular clusters. Astron. Soc. Pac., San Francisco, p. 325
Eichhorn H., 1960, Astron. Nachr., 285, 233

Evans D. W., 1988, PhD Thesis, Cambridge University
Geffert M., Colin J., LeCampion J.-F., Odenkirchen M., 1993a. AJ, 106, 168
Geffert M., Forner C., Hiesgen M., 1993b, in Steinert K.-G., eds., Geodetical Astrometry, Proc. VIII Intern. Lohrmann Colloq. Wiss. Z. TU, Dresden, 42, 32
Hirte S., Dick W. R., Schilbach E., Scholz R.-D., 1990, in Jaschek C., Murtagh F., eds, Proc. of Conference on Errors, Uncertainties and Bias in Astronomy. Cambridge Univ. Press, p. 343
Kibblewhite E., Bridgeland M., Bunclark P., Cawson M., Irwin M., 1984, in Capaccioli M., ed., Proc. of Conference on Astronomy with Schmidt-Type Telescopes. Reidel, Dordrecht, p. 89
Meurers J., Hallermann L., 1966, Annal. der Univ. Sternw. Wien, 26, 127
Monella R., 1985, Coelum LIII, 287
Odenkirchen M., Brosche P., 1992, Astron. Nachr., 313, 69
Odenkirchen M., Scholz R.-D., Irwin M. I., 1994, in MacGillivray H. T. et al., eds, Proc. IAU Symp. 161, Astronomy from wide-field imaging, Kluwer, dordrecht, p. 453
Peterson C. J., 1993, in Djorgovski S., Meylan G., eds, ASP Conf. Ser., Vol. 50, Structure and dynamics of globular clusters. Astron. Soc. Pac., San Francisco, p. 337
Pryor C., Meylan G., 1993, in Djorgovski S., Meylan G., eds, ASP Conf. Ser., Vol. 50, Structure and dynamics of globular clusters. Astron. Soc. Pac., San Francisco, p. 357
Rees R. F., Jr, 1993, AJ, 106, 1524
Röser S., Bastian U., 1988, A&AS, 74, 449
Scholz R.-D., Irwin M. J., 1994, in MacGillivray H. T. et al., eds, Proc. IAU Symp. 161, Astronomy from wide-field imaging, Kluwer, Dordrecht, p. 535
Scholz R.-D., Odenkirchen M., Irwin M., 1993, MNRAS, 264, 579 (Paper I)
Scholz R.-D., Odenkirchen M., Irwin M., 1994, MNRAS, 266, 925 (Paper II)
Scholz R.-D., Hirte S., Irwin M. J., Odenkirchen M., 1995, in Gilmore G., Van der Kruit P. C., eds, Proc. IAU Symp. 164, Stellar Populations, p. 405
Tucholke H.-J., 1992a, A&AS, 93, 293
Tucholke H.-J., 1992b, A&AS, 93, 311
Webbink R. F., 1981, ApJS, 45, 259

APPENDIX A: DEFINITION OF SYMBOLS

D	distance from object to observer;
v_r	heliocentric radial velocity;
x, y, z	Galactocentric space coordinates in a right-handed Cartesian coordinate system with z pointing to North Galactic Pole, x away from the Sun and y in direction of Galactic rotation at the Sun;
U, V, W	corresponding velocity components;
R	distance from Galactic centre;
ρ	projected distance in Galactic plane;
min, max	as index means minimum and maximum during 10 Gyr;
–	above letter means time-average in 10 Gyr;
e	eccentricity defined as $(R_{\max}-R_{\min})/(R_{\max}+R_{\min})$;
v	absolute value of velocity;
Θ	component of velocity vector in the direction of Galactic rotation at the position of the cluster;
T	period of one revolution around z-axis;
n_U	number of revolutions in 10 Gyr;
J_z	z-component of specific angular momentum vector;
E	specific total orbital energy.